Exercitiu PLS.

31.01.2022
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grupa 241.

$$X \sim \begin{pmatrix} -7 & -6 \\ 0.38 & 0.62 \end{pmatrix} \qquad Y \sim \begin{pmatrix} -1 & 6 \\ p_1 & p_2 \end{pmatrix}$$

a) $\mathbb{P}(X = -7,\ Y = 6) = 0.126667.$

$\mathbb{E}\{X \mid Y = 6\} = -6.5.$

$X \setminus Y$	-1	6	
-7	$0,2533333$	$0,126667.$	0.38
$-6.$	$p_1 - 0,2533333$	$p_2 - 0.126667.$	$0.62.$
	p_1	$p_2.$	

$$\mathbb{P}(X = -7,\ Y = 6) = \mathbb{P}(X = -7)\,\mathbb{P}(Y = 6 / X = -7) = 0,126667.$$

$$0.38 \cdot p_2 = 0,126667.$$

$$p_2 = 0,333333210.$$

$$p_1 = 1 - p_2$$

$$p_1 = 0,666666578$$

$$Y \sim \begin{pmatrix} -1 & 6 \\ 0,666666578 & 0,333333210 \\ \triangleq 0,6667 & \triangleq 0,3333 \end{pmatrix}$$

b)

$p_1 = 0,6667$.
$p_2 = 0,3333$

$$X \sim \begin{pmatrix} -7 & -6 \\ 0,38 & 0,62 \end{pmatrix} \quad Y \sim \begin{pmatrix} -1 & 6 \\ 0,6667 & 0,3333 \end{pmatrix}$$

$$X+Y \sim \begin{pmatrix} -8 & -1 & -7 & 0 \\ 0,253346 & 0,126654 & 0,413354 & 0,206646 \end{pmatrix}$$

$$X-Y \sim \begin{pmatrix} -6 & -13 & -5 & -12 \\ 0,253346 & 0,126654 & 0,413354 & 0,206646 \end{pmatrix}$$

$3X^2 + 5Y^2$

$$X^2 \sim \begin{pmatrix} 49 & 36 \\ 0,38 & 0,62 \end{pmatrix} \quad Y^2 \sim \begin{pmatrix} 1 & 36 \\ 0,6667 & 0,3333 \end{pmatrix}$$

$$3X^2 \sim \begin{pmatrix} 147 & 108 \\ 0,38 & 0,62 \end{pmatrix} \quad 5Y^2 \sim \begin{pmatrix} 5 & 180 \\ 0,6667 & 0,3333 \end{pmatrix}$$

$$3X^2 + 5Y^2 \sim \begin{pmatrix} 152 & 327 & 113 & 288 \\ 0,253346 & 0,126654 & 0,413354 & 0,206646 \end{pmatrix}$$

$\mathbb{E}[X] = (-7) * 0,38 + (-6) \times 0,62 = -6.38.$

$\mathbb{E}[Y] = (-1) \times 0,6667 + 6 \times 0,3333 = 1,3331$

$$\begin{aligned} \mathrm{Var}[X] &= \mathbb{E}[X^2] - (\mathbb{E}[X])^2 \\ &= 49 \cdot 0.38 + 36 \cdot 0,62 - (-6,38)^2 \\ &= 40,94 - (-6,38)^2 \\ &= 0,2356 \end{aligned}$$

$$Var[Y] = E[Y^2] - (E[Y])^2$$
$$= 12,6655 - (1,3331)^2$$
$$Var[Y] = 10,8883.$$

$$Var(7X - 7Y + 15)$$

$$7X \sim \begin{pmatrix} -49 & -42 \\ 0,38 & 0,62 \end{pmatrix} \qquad 7Y \sim \begin{pmatrix} -7 & 42 \\ 0,6667 & 0,3333 \end{pmatrix}$$

$$7X - 7Y = \begin{pmatrix} -42 & -91 & -35 & -84 \\ 0,253346 & 0,126654 & 0,413354 & 0,206646 \end{pmatrix}$$

$$7X - 7Y + 15 = \begin{pmatrix} -27. & -76. & -20. & -69. \\ 0,253346 & 0,126654 & 0,413354 & 0,206646 \end{pmatrix}$$
$$\parallel$$
$$B$$

$$Var(B) = E[B^2] - (E[B])^2$$
$$= 2065,43 - (-38,9917)^2$$
$$= 545,073.$$

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

$$Cov(X,Y) = E[X \cdot Y] - E[X] \cdot E[Y]$$

$$Cov(X,Y) =$$

$$=$$

$$X \cdot Y \sim \begin{pmatrix} 7 & -42 & 6 & -36 \\ 0{,}253346 & 0{,}126654 & 0{,}413354 & 0{,}206646 \end{pmatrix}$$

$$\mathbb{E}[X \cdot Y] = -8{,}50518.$$

$$\mathbb{E}[X] = -6{,}35$$

$$\mathbb{E}[Y] = 1{,}3331$$

$$\mathrm{Var}[X] = 0{,}2356$$

$$\mathrm{Var}[Y] = 10{,}8883.$$

$$\mathrm{Cov}(X,Y) = \mathbb{E}[X \cdot Y] - \mathbb{E}[X] \cdot \mathbb{E}[Y]$$

$$= -8{,}50518 - (-6{,}35 \cdot 1{,}3331)$$

$$= -8{,}50518 - (-8{,}505178)$$

$$= 0{,}000622.$$

$$\rho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)} \cdot \sqrt{\mathrm{Var}(Y)}}$$

$$= \frac{0{,}000622}{0{,}4853 \times 3{,}2997.}$$

$$= 0{,}00038842\ldots$$

Exercitiul 2

1) $\mathbb{E}[\log(X)] \geq \log(\mathbb{E}[X])$

2) $\mathbb{E}[X] \geq \sqrt{\mathbb{E}[X]}$

E este pozitiv $\Rightarrow$ ~~Modul~~ se noteaza $A = \mathbb{E}[X] \Rightarrow$

$\Rightarrow A \geq \sqrt{A}$, $A > 0$

3) $\mathbb{E}[\sin^2(X)] + \mathbb{E}[\cos^2(X)] \geq 1$

4) $\mathbb{P}(X > c) \leq \frac{\mathbb{E}[X^3]}{c^3}$ Markov.

5) $\mathbb{P}(X \leq Y) \geq \mathbb{P}(X \geq Y)$

$X > 5$.

6) $\mathbb{P}(X+Y > 10) \leq \mathbb{P}(X > 5 \text{ sau } Y > 5)$

7) $\mathbb{E}[\min(X,Y)] \; ? \; \min \mathbb{E}[X]\,[Y]$

nu amestem
$\mathbb{P}(X)$ si alt $\mathbb{P}(Y)$

8) $\mathbb{E}\left[\frac{X}{Y}\right] \geq \frac{\mathbb{E}[X]}{\mathbb{E}[Y]}$

$$\left.\begin{aligned} \mathbb{E}\left[\frac{X}{Y}\right] &= \frac{X}{Y} \cdot \mathbb{P}(X) \cdot \mathbb{P}(Y) \\ \mathbb{E}\,\frac{[X]}{\mathbb{E}[Y]} &= \frac{\mathbb{P}(X) \cdot X}{\mathbb{P}(Y) \cdot Y} \end{aligned}\right\} \Rightarrow \mathbb{E}\left(\frac{[X]}{[Y]}\right) \leq \mathbb{E}\left[\frac{X}{Y}\right]$$

X - cel poate avea o valoare mult mai mică.

g) $\mathbb{E}[X^2(X^2+1)] \overset{?}{=} \mathbb{E}[X^2(Y^2+1)]$

nu avem informații despre Y

h) $\mathbb{E}\left[\frac{1}{X}\right] \overset{?}{=} \frac{1}{\mathbb{E}[X]}$

~~nu avem informații despre X~~

Exercițiul 3. 5-telefoane / 2-defecte.

~~X — # teste telefon unu.~~
~~Y — # teste telefon doi~~

$2 \leq X+Y \leq 4$.

$\mathbb{P}(X=1, Y=1) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$.

$\mathbb{P}(X=1, Y=2) = \frac{2}{5} \cdot \frac{3}{4_2} \cdot \frac{1}{3} = \frac{1}{10}$.

$\mathbb{P}(X=1, Y=3) = \frac{2}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{10}$.

~~$\mathbb{P}(X=2, Y=2) = \frac{2}{5} \cdot \frac{3}{4_2} \cdot \frac{1}{3} = \frac{1}{10}$~~

~~$\mathbb{P}(X=2, Y=3) = \frac{2}{5} \cdot \frac{2}{4_2} \cdot \frac{1}{3} = \frac{1}{10}$~~

$X \backslash Y$	0	1	2	3
1	0	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
2	0	$\frac{1}{10}$	$\frac{1}{10}$	0
3	0	$\frac{1}{10}$	0	0

6/10

Rep (conjunct)

X \ Y	0	1	2	3
1	0	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
2	0	$\frac{1}{10}$	$\frac{1}{10}$	0
3	0	$\frac{1}{10}$	0	0

Rep (marginale)

$$X \sim \begin{pmatrix} 1 & 2 & 3 \\ \frac{3}{10} & \frac{2}{10} & \frac{1}{10} \end{pmatrix} \qquad Y \sim \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & \frac{3}{10} & \frac{2}{10} & \frac{1}{10} \end{pmatrix}$$

b) $\mathbb{E}[X] = \frac{3+4+3}{10} = \frac{10}{10} = 1$

$\mathbb{E}[Y] = 1.$

$$X^2 \sim \begin{pmatrix} 1 & 4 & 9 \\ \frac{3}{10} & \frac{2}{10} & \frac{1}{10} \end{pmatrix}$$

$$Y^2 \sim \begin{pmatrix} 0 & 1 & 4 & 9 \\ 0 & \frac{3}{10} & \frac{2}{10} & \frac{1}{10} \end{pmatrix}$$

$\mathbb{E}[X^2] = \frac{3+8+9}{10} = 2$

$\mathbb{E}[Y^2] = 2.$

$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = 1$

$Var(Y) = 1$

7/10

$\mathbb{E}(X,Y)$

$\mathrm{Cov}(X,Y)$

$\rho(X,Y)$

c) $\mathbb{E}(X \mid Y=2) =$

$X \mid Y=2 \sim$

Exercițiul 5.

c) $\mathbb{E}[N] = 820$ $\quad (\mathrm{Pois})$

$\mathrm{Var}[N] = 820$

Exercițiul 4.

$$f(x) = \frac{x}{4} e^{-\frac{x^2}{8}} \mathbb{1}_{\{x>0\}}$$

$$F(x) = \int_{x\geq 0}^{x} f(t)\,dt \Bigg\} \Rightarrow F(x) = \int_0^x f(t)\,dt =$$

$$= \int_0^x \frac{t}{4} e^{-\frac{t^2}{8}} dt = -e^{-\frac{t^2}{8}} \Big|_0^x = -e^{-\frac{x^2}{8}} + 1 =$$

$$F^{-1}(x) = y \mid F \Rightarrow x = F(y) = 1 - e^{-\frac{y^2}{8}}$$

$$e^{-\frac{y^2}{8}} = 1 - x.$$

$$e^{-\frac{y^2}{8}} = 1 - x$$

$$\therefore \frac{y^2}{8} = \ln(1-x)$$

$$y = \sqrt{8 \ln\left(\frac{1}{1-x}\right)}$$

$$y = 2\sqrt{2 \ln\left(\frac{1}{1-x}\right)} \Rightarrow F^{-1}(x) = 2\sqrt{2 \ln\left(\frac{1}{1-x}\right)}$$

$$\frac{F^{-1}(0.75) - F^{-1}(0.25)}{\sqrt{\text{Var}(x)}} = \frac{2\sqrt{2 \ln \frac{1}{0.25}} - 2\sqrt{2 \ln \frac{1}{0.75}}}{}$$

Exercitiul 5.

2) Cst. $\mathbb{P}(820) = 0,4$

$\Rightarrow e^{-\lambda_1} \cdot \frac{\lambda_1^{820}}{820!} = 0,4 \Rightarrow e^{-\lambda_1} \lambda_1^{640} = 0,4$.

Olim. $\mathbb{P}(820) = 0,6$.

$\Rightarrow e^{-\lambda_2} \cdot \frac{\lambda_2^{820}}{820!} = 0,6$