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141

exercițiul 3

$(f_n)_{n\geq 1}$ $f_n : [0,\infty) \to \mathbb{R}$ $[0,1]$; $[1,\infty)$

$$f_n(x) = \frac{nx^2+5}{nx+5}$$

- $\lim\limits_{n\to\infty} f_n(x) = \lim\limits_{n\to\infty} \frac{nx^2+5}{nx+5} = \frac{x^2}{x} = x$

$$f : [0,\infty) \to \mathbb{R}, \quad f_n \xrightarrow{s} f$$

- $a_n = \sup\limits_{x\in[0,1]} |f_n(x) - f(x)| = \sup\limits_{x\in[0,1]} \left| \frac{nx^2+5}{nx+5} - x \right| =$

$$= \sup_{x\in[0,1]} \left| \frac{\cancel{nx^2}+5-\cancel{nx^2}-5x}{nx+5} \right| = \sup_{x\in[0,1]} \left| \frac{5-5x}{nx+5} \right|$$

Fie $g_n : [0,\infty) \to \mathbb{R}$, $g_n(x) = \frac{5-5x}{nx+5}$

$$g_n'(x) = \frac{-5(nx+5)-(5-5x)n}{(nx+5)^2} =$$

$$= \frac{-\cancel{5nx}-25-5n+\cancel{5nx}}{(nx+5)^2} =$$

$$= \frac{-25-5n}{(nx+5)^2}$$

$n\geq 1 \Rightarrow g_n'(x) < 0 \Rightarrow$ funcția g_n este descrescătoare

$\Rightarrow$ pe $[0,1]$, $\sup\limits_{x\in[0,1]} g_n(x) = g_n(0) = \frac{5-5\cdot 0}{n\cdot 0+5} = 1$,

deci f_n nu converge uniform la f

pe $[4;\infty)$ $\Rightarrow$ $\sup\limits_{x\in[4;\infty)} g_n = g_n(4) = \frac{5-5\cdot 4}{n\cdot 4+5} = \frac{-15}{4n+5} \xrightarrow{\lim_{n\to\infty}} 0$

$\Rightarrow f_n \xrightarrow{u} f$