cursor imp. → select.
cursor par. → ~~ext~~ turnee cu id
cursor imbr. → premii.
tipul

Curs 14

Inegalități

1) $E[|xy|] \leq \sqrt{E[x^2]E[y^2]}$: Cauchy - Schwartz

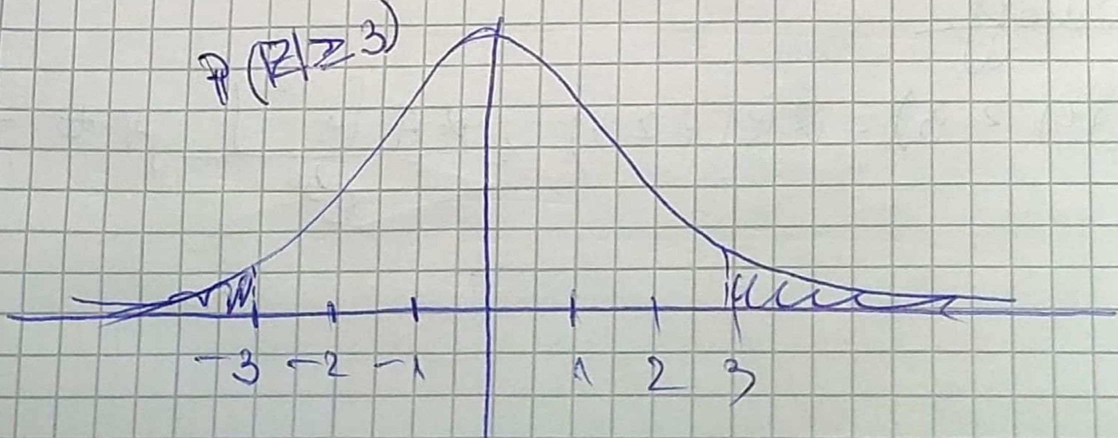
2) f convexă $E[f(x)] \geq f(E[x])$ Jensen
f concavă $E[f(x)] \leq f(E[x])$

3) $x > 0,\ a > 0$ $P(X > a) \leq \frac{E[x]}{a}$, Markov

4) X v.a. $E[x] = \mu$, $Var(X) = \sigma^2$
$P(|x-\mu| \geq a) \leq \frac{Var(x)}{a^2}$, $\forall a > 0$ Chebyshow

5) X v.a., $a > 0$, $t > 0$ atunci
$P(X \geq a) \leq \frac{E[e^{tx}]}{e^{ta}}$, $\forall t$ Chernoff

Exp: $Z \sim N(0,1)$. Vrem să mărginim superior
$P(|Z| \geq 3)$ folosind Markov, Chebyshow, Chernoff



Avem prop. 68-95-99,7

$\mathbb{P}(|Z| \leq 1) \simeq 0,68$

$\mathbb{P}(|Z| \leq 2) \simeq 0,95$

$\mathbb{P}(|Z| \leq 3) \simeq 0,997$

Astfel, avem $\mathbb{P}(|Z| \geq 3) \simeq 0,003$

a) Markov:

$$\mathbb{P}(|Z| \geq 3) \leq \frac{\mathbb{E}[|Z|]}{3} \leq \sqrt{\frac{2}{9\pi}} = 0,26$$

$$\mathbb{E}[Z] = \int |z| \varphi(z)\, dz$$

$$= \int |z| \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{z^2}{2}}\, dz$$

$$= 2\int_0^\infty \frac{z}{\sqrt{2\pi}} \cdot e^{-\frac{z^2}{2}}\, dz = \sqrt{\frac{2}{\pi}} \int_0^\infty z e^{-\frac{z^2}{2}}\, dz$$

$$= \sqrt{\frac{2}{\pi}} \left(-e^{-\frac{z^2}{2}}\right)\Big|_0^\infty = \sqrt{\frac{2}{\pi}}$$

b) Chebyshew

$$\mathbb{P}(|Z| \geq 3) = \mathbb{P}(|Z - \underset{\substack{\uparrow \\ \mu}}{0}| \geq 3) \leq \frac{Var(Z)}{9} = \frac{1}{9} = 0,11$$

c) Chernoff

$$\mathbb{P}(|Z| \geq 3) = 2\mathbb{P}(Z \geq 3) \leq \frac{2\mathbb{E}[e^{tZ}]}{e^{3t}}, \forall t$$

unde $\mathbb{E}[e^{tZ}] = \int_{-\infty} e^{tx} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} \, dx$

$= \frac{1}{\sqrt{2\pi}} \int e^{-\frac{x^2}{2} + tx} \, dx$

$= \frac{1}{\sqrt{2\pi}} \int e^{-\frac{1}{2}(x^2 - 2tx + t^2 - t^2)} \, dx$

$= \frac{1}{\sqrt{2\pi}} \int e^{-\frac{(x-t)^2}{2} + \frac{t^2}{2}} \, dx$

$= e^{\frac{t^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} \int e^{-\frac{(x-t)^2}{2}} \, dt = 1, \quad X \sim N(t, 1)$

$= e^{\frac{t^2}{2}}$

$\rightarrow \mathbb{P}(|Z| \geq 3) \leq \frac{2 e^{\frac{t^2}{2}}}{e^{3t}} \; ; \forall t$

Pt. $t \geq 3$ avem $\mathbb{P}(|Z| \geq 3) \leq 2 e^{-\frac{9}{2}} = 0,02$

Exp. X v.a. ; $X \in [a, b]$; $\mathbb{E}[X] = \mu$; $Var(X) = \sigma^2$

$\mathbb{P}(|X - \mu| \geq t) \leq \frac{\sigma^2}{t^2} \; ; \forall t > 0$

necunoscut

dar aprox. $\leq \frac{(b-a)^2}{4t^2}$

↓

var. e mărg. între a și b

Vrem să arătăm:

$\cancel{\sigma^2 \geq \frac{(b-a)^2}{4}}$ $\qquad \sigma^2 \leq \frac{(b-a)^2}{4}$

$$g(\gamma) = E[(x-\gamma)^2]$$

$$\gamma = E[x]$$

$$E[(x-\gamma)^2] \geq E[(x-E[x])^2], \forall \gamma \in \mathbb{R}$$

$$E[(x-\gamma)^2] = E[(\underbrace{x-E[x]}+\underbrace{E[x]-\gamma})^2]$$

$$= E[(x-E[x])^2] + 2\underbrace{E[(x-E[x])(E[x]-\gamma)]}_{=0} +$$
$$+ (E[x]-\gamma)^2$$

$$\geq E[(x-E[x])^2], \forall \gamma \in \mathbb{R}$$

!

$$\sigma^2 = Var(x) = E[(x-E[x])^2] \leq E[(x-\gamma)^2], \forall \gamma$$

pt. $\gamma = \frac{a+b}{2}$

$$E[(x-\tfrac{a+b}{2})^2] = E[(x-a)(x-b) + \tfrac{(b-a)^2}{4}]$$

$$= \underbrace{E[(x-a)(x-b)]}_{\leq 0} + \frac{(b-a)^2}{4} \leq \frac{(b-a)^2}{4}$$

!

QED

Cum a făcut?

$$(x-\gamma)^2 = \underset{\leq 0}{A} + \frac{(b-a)^2}{4}$$

↓ îl scriem

↓ găsim ceva ≤ 0

pe γ ca să fie îndeplinită cond.

Scanned with CamScanner

Teoreme limită. Legea nr. mari

Def.: Fie $(X_n)_{n\geq 1}$ un șir de v.a. (funcții) și X o v.a. peste $(\Omega, \mathcal{F}, \mathbb{P})$.

Spunem că șirul $(X_n)_{n\geq 1}$ conv. la X <u>aproape sigur</u>, și notăm $X_n \xrightarrow{a.s.} X$ dacă

$$\boxed{\mathbb{P}(\underbrace{\lim_n X_n = X}) = 1}$$

$\downarrow$

eveniment $A = \{ w \in \Omega \mid \lim_n X_n(w) = X(w) \}$

$\rightarrow$ ex. pt. $\mathbb{P} \setminus 0$ $\rightarrow$ conv. apr. sigur pt că prob. lui 0 e 0 (?)

Fie $(X_n)_{\geq 1}$ un șir de v.a. și X o v.a. def. $(\Omega, \mathcal{F}, \mathbb{P})$

Spunem că șirul X_n conv. în plus la X, și notăm $X_n \xrightarrow{\mathbb{P}} X$ dacă $\forall \varepsilon > 0$

$$\lim_n \mathbb{P}(|X_n - X| \geq \varepsilon) \geq 0$$

Obs. $\forall \varepsilon > 0$, $\forall \delta > 0$, $\exists n_0 \in \mathbb{N}$ a.î. $n \geq n_0$

$$\mathbb{P}(|X_n - X| \geq \varepsilon) \leq \delta$$

nivel de încredere

acuratețea aproximării

Exp.: $X_n \sim \mathcal{U}([0,1])$ indep.

$Y_n = \min\{X_1, X_2, \dots X_n\}$

At. $Y_n \xrightarrow{\mathbb{P}} 0$

„Y_n converge în probabilitate la 0”

pt. $\varepsilon > 0$

$$\mathbb{P}(|Y_n - 0| \geq \varepsilon) \xrightarrow[n\to\infty]{} 0$$

$$\mathbb{P}(|Y_n| \geq \varepsilon) = \mathbb{P}(Y_n \geq \varepsilon) =$$

e pozitiv => sir form. din var. poz.
$Y_n \geq 0$

$$= \mathbb{P}(X_1 \geq \varepsilon, X_2 \geq \varepsilon, \dots, X_n \geq \varepsilon)$$

$$\overset{\text{indep.}}{=} \mathbb{P}(X_1 \geq \varepsilon)\mathbb{P}(X_2 \geq \varepsilon) \dots \mathbb{P}(X_n \geq \varepsilon)$$

$$= (1 - \underbrace{\mathbb{P}(X_1 < \varepsilon)}_{\varepsilon})(1 - \underbrace{\mathbb{P}(X_2 < \varepsilon)}_{\varepsilon}) \dots (1 - \mathbb{P}(X_n < \varepsilon))$$

$$\overset{\text{v. uniforme}}{=} (1-\varepsilon)(1-\varepsilon)\dots(1-\varepsilon) = (1-\varepsilon)^n$$

$$\mathbb{P}(|Y_n| \geq \varepsilon) = (1-\varepsilon)^n \text{ pt. } \varepsilon \in (0,1) \text{ pt. } \varepsilon \in [0,1]$$

$$(1-\varepsilon)^n \xrightarrow{n\to\infty} 0$$

Def. Numim eşantion de volum n din populaţia Q, v.a. $X_1, X_2, \dots, X_n$ indep. şi identic repartizate not. (i.i.d) cu $\mathbb{P} \circ X_i^{-1} = Q$

! Media eşantionului => $\overline{X_n} = \dfrac{X_1 + X_2 + \dots + X_n}{n}$

Pp. $X_1, X_2, \dots X_n$ eşantion de medie μ şi varianţă σ^2
$(\mathbb{E}[X_1] = \mu, \mathrm{Var}(X) = \sigma^2)$

$$\mathbb{E}[\overline{X_n}] = \mathbb{E}\left[\frac{X_1 + X_2 + \dots + X_n}{n}\right] = \frac{1}{n}(\underbrace{\mathbb{E}[X_1] + \mathbb{E}[X_2] + \dots + \mathbb{E}[X_n]}_{n\cdot\mu}) = \mu$$

Scanned with CamScanner

$$Vr(\overline{X_n}) = Vr\left(\frac{X_1+X_2+\dots+X_n}{n}\right) = \frac{1}{n^2}\Big(\underbrace{Vr(X_1)+Vr(X_2)+\dots+Vr(X_n)}_{n\cdot\sigma^2}\Big) = \frac{\sigma^2}{n}$$

Legea Nr. mari:

(LNM) (slabă)

Fie $(X_n)_n$ un șir de v.a. i.i.d. cu $\mathbb{E}[X_1] = \mu < \infty$, $Vr(X_1] =$ [redacted] $\sigma^2 < \infty$ e finit

Atunci $\Rightarrow$ $\overline{X_n} \xrightarrow{P} \mu$

Versiunea tare

X_n v.a. i.i.d. $\mathbb{E}[|X_1|] < \infty$; $\mathbb{E}[X_1] = \mu$

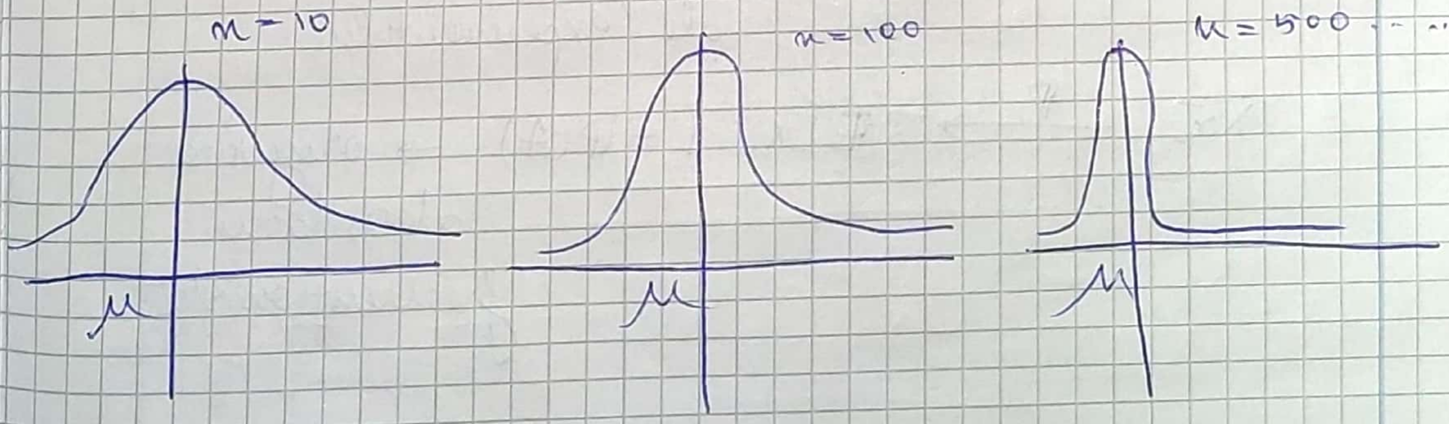
$$\overline{X_n} \xrightarrow{a.s.} \mu$$

Dem. Pt. $\varepsilon > 0$

$$\mathbb{P}(|\overline{X_n} - \mu| \geq \varepsilon) \xrightarrow{n\to\infty} 0$$

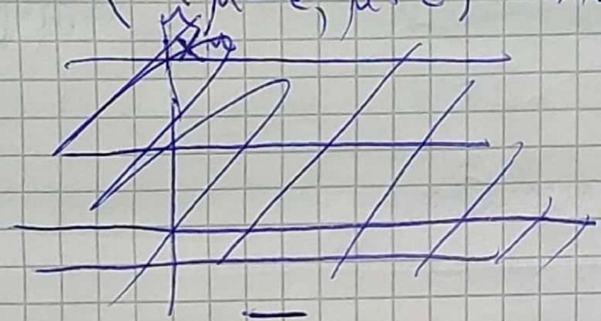
Ineg. Chebyshev:

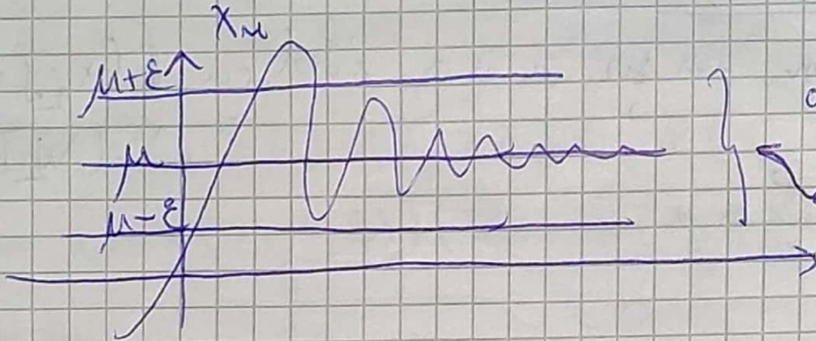
$$\mathbb{P}(|\overline{X_n} - \mu| \geq \varepsilon) \leq \frac{Vr(\overline{X_n})}{\varepsilon^2} = \frac{\sigma^2}{n\varepsilon^2} \xrightarrow{n\to\infty} 0$$



Pentru un nivel de acuratete dat

$$\mathbb{P}\left((\mu-\varepsilon, \mu+\varepsilon) \to \overline{X_n} \right) \xrightarrow[n\to\infty]{} 1$$





de la un rang prob. e cuprinsă în porțiune

~~var. de tip~~

~~$B(p$~~

Exp. $(\Omega, \mathcal{F}, \mathbb{P})$ c.p.; $A \in \mathcal{F}$

Fie $X_i = \begin{cases} 1, & \omega \in A \\ 0, & \text{altfel} \end{cases}$

$X_i \sim B(p)$

$p = \mathbb{P}(X_i = 1) = \mathbb{P}(A)$

$\overline{X_n} = \dfrac{X_1 + X_2 + \dots X_n}{n}$ frecvența relativă de apariție a ev. A în n repetiții ale experimentului

$\overline{X_n} \xrightarrow{\mathbb{P}} \mathbb{E}[X_i] = \mathbb{P}(A)$ → originea abordării frecvenționiste

Exp. Fie p procentul din populație care votează cu A.

~~dx eșantionare stratificată (ad. când alegem pers. pe care le întrebăm pt. sondaj după diverse criterii)~~

aici $\rightarrow$ alegere aleatorii

$X_1, X_2, \dots X_n \sim B(p)$ indep.

$$\overline{X_n} = \frac{X_1 + X_2 + \dots + X_n}{n}$$ = frecvența relativă a pers. intervievați care votează cu A din totalul de n.

$\Rightarrow$ Ineg. Chebyshev :

Bernoulli

$$\mathbb{P}(|\overline{X_n} - p| \geq \varepsilon) \leq \frac{Vr(\overline{X_n})}{\varepsilon^2} = \frac{p(1-p)}{n\varepsilon^2}$$

Dar p - necunoscut $\xrightarrow{\text{dar putem spune că}}$ $p \in (0,1) \Rightarrow p(1-p) \leq \frac{1}{4}$

$$\Leftrightarrow \mathbb{P}(|\overline{X_n} - p| \geq \varepsilon) \leq \frac{1}{4n\varepsilon^2} \leq \frac{1}{4n\varepsilon^2}$$

$\varepsilon = 0{,}01$ (acuratețea)

$\frac{1}{4n\varepsilon^2} = 0{,}05$ (nivel de încredere)

$\Rightarrow n \simeq 50\,000$

$\downarrow$

trebuie să întreb multă lume

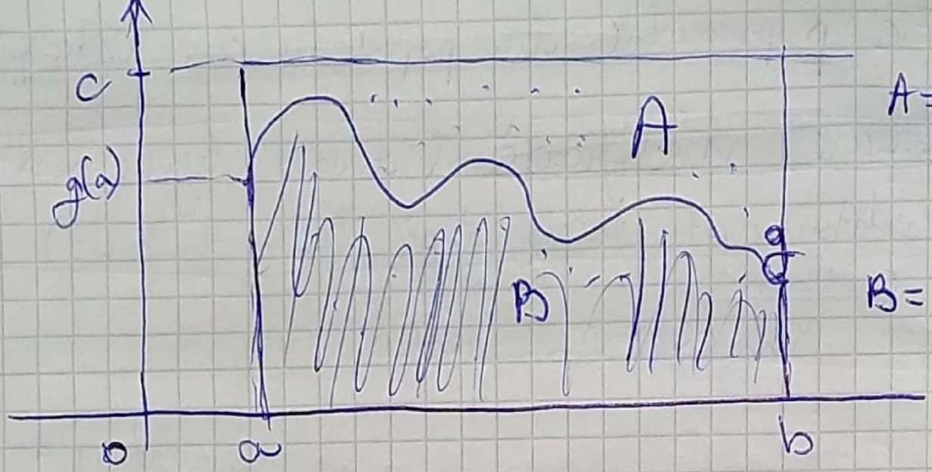
Integrarea Monte - Carlo

$\rightarrow$ metode de aprox. a unor cantit. integrale fol. v. a. și legea lui Markov (?)

Pp. că avem o fct. g și vrem să calculăm

$$\int_a^b g(x)\,dx$$

Pp. că pe $[a, b]$ avem $0 \leq g(x) \leq c$



$A = [a,b] \times [0,c]$ dreptunghi

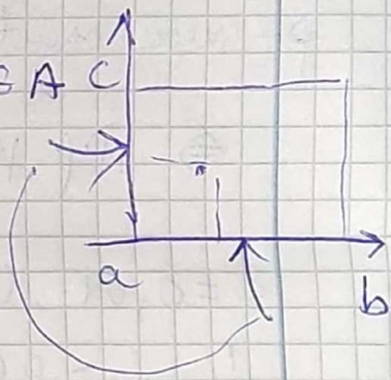
$B = \{(x,y) \mid a \le x \le b,\ 0 \le y \le g(x)\}$

Generăm pct. Unif (A)
uniform repartizate pe A

$(X_1, Y_1) \ldots (X_n, Y_n) \sim \mathcal{U}(A)$

$(X, Y) \sim \mathcal{U}(A)$

$$f_{(X,Y)}(x,y) = \begin{cases} \dfrac{1}{\underset{\text{aria}}{\mathcal{A}}(A)}, & (x,y) \in A \\ 0, & \text{altfel} \end{cases}$$



unif
⇓
perechea e unif. pe supr. descrisă

$\mathcal{A}(A) = c(b-a)$

$X \sim \mathcal{U}([a,b])$
indep.
$Y \sim \mathcal{U}([0,c])$

$f_X(x) = \dfrac{1}{b-a} \cdot \mathbb{1}_{[a,b]}(x)$

$f_Y(y) = \dfrac{1}{c} \cdot \mathbb{1}_{[0,c]}(y)$

$$f_{(X,Y)}(x,y) \overset{\text{indep.}}{=} f_X(x)\, f_Y(y) = \frac{1}{c(b-a)} \mathbb{1}_A(x,y)$$

~~$Z_i \sim \beta(p)$~~

~~$p = \mathbb{P}(Z$~~

Fie $Z_i = \begin{cases} 1, & (x_i, y_i) \in B \\ 0, & \text{altfel} \end{cases}$, $Z_i \sim \beta(p)$

$$p = \mathbb{P}(Z_i = 1) \overset{(?)}{=} \mathbb{P}((x_i, y_i) \in B)$$

$$= \iint_B f_{(x,y)}(x,y)\, dx\, dy = \frac{A(B)}{A(A)}$$

γ în LNM: $\overline{Z_1} = \frac{Z_1 + \ldots + Z_n}{n} \xrightarrow{P} p = \frac{A(B)}{c(b-a)}$

$$= \frac{\int_a^b g(x)\, dx}{c(b-a)}$$

$$\boxed{\int_a^b g(x)\, dx = c(b-a) \cdot \overline{Z_n}}$$

<u>în 2:</u>

Fie $U_1, U_2, \ldots U_n \sim \mathcal{U}[a,b]$ i.i.d.

$X_1 = g(U_1)$; $X_2 = g(U_2) \ldots X_n = g(U_n)$ i.i.d.

<u>LNM</u>: $\overline{X_n} \xrightarrow{P} \mathbb{E}[X_1] = \mathbb{E}[g(U_1)]$

$$= \int g(x) f_{U_1}(x)\, dx$$

$$= \int g(x) \frac{1}{b-a} \mathbb{1}_{[a,b]}(x)\, dx$$

$$= \frac{1}{b-a} \int_a^b g(x)\, dx$$

Scanned with CamScanner

$$\cancel{\int_a^b g(x)\,dx = (b-a)\,x}$$ $\cancel{\forall x \in [a,b]}$

$$\frac{x_1+x_2+\dots+x_n}{n}$$

$$\int_a^b g(x)\,dx = (b-a)\cdot\frac{x_1+x_2+\dots+x_n}{n}$$

$$\int_a^b g(x)\,dx = (b-a)\cdot\frac{g(U_1)+\dots+g(U_n)}{n}$$

$$U_i \sim \mathcal{U}[a,b]$$

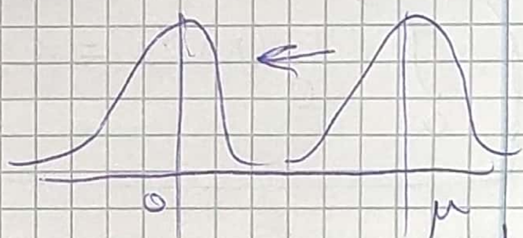
$$\boxed{\text{Th. limită Centrală}}$$

(TLC)

Din LNM: $\overline{X_n} \xrightarrow{P} \mathbb{E}[X_i]$

Fie $X_1, X_2, \dots X_n$ iid $\mathbb{E}[X_1]=\mu$, $Vr(X)=\sigma^2$

$$Vr\left(\frac{X-\mathbb{E}[X]}{\sqrt{Var(X)}}\right)=1$$



translatie

$$z=\frac{X-\mathbb{E}[X]}{\sqrt{Vr(X)}}$$ s.n. z-scor sau variabilă normalizată

$$z_n=\frac{X_1+X_2+\dots+X_n-\mathbb{E}[X_1+\dots+X_n]}{\sqrt{Var(X_1+\dots+X_n)}}$$

$$=\frac{X_1+\dots+X_n-n\mu}{\sqrt{n\sigma^2}}$$ variabilă de scor / variabilă normalizată

$$\boxed{Z_n = \sqrt{n}\left(\frac{\overline{X_n} - \mu}{\sigma}\right)}$$

(T) (Teorema limită centrală)

Fie $(X_n)_n$ un şir de v.a. iid de $E[X_1] = \mu < \infty$ (finită)

$Vr(X_1) = \sigma^2 < \infty$. Atunci

$$\lim_n P(Z_n \leq x) = \Phi(x), \forall x$$

unde $Z_n = \sqrt{n} \cdot \frac{\overline{X_n} - \mu}{\sigma}$, $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$

(f. de rep. a $N(0,1)$) $\rightarrow$ ajung la o normală

Obs $X_1, X_2, \dots X_n$ iid., $E[X_1] = \mu$

$Vr[X_1] = \sigma^2$

$S_n = X_1 + \dots + X_n$

$$P(S_n \leq c) = P\left(\frac{S_n - E[S_n]}{\sqrt{Vr(S_n)}} \leq \frac{c - E[S_n]}{\sqrt{Vr(S_n)}}\right)$$

$$= P\left(Z_n \leq \frac{c - n\mu}{\sqrt{n\sigma^2}}\right) \overset{TLC}{\simeq} \Phi\left(\frac{c - n\mu}{\sigma\sqrt{n}}\right)$$

phi

Pt. n suficient de mare

$$\begin{cases} S_n \simeq N(n\mu, n\sigma^2) \\ \overline{X_n} \simeq N\left(\mu, \frac{\sigma^2}{n}\right) \end{cases}$$

$n\sigma^2$ → dispersie

$\overline{X_n} \simeq N\left(\mu, \frac{\sigma^2}{n}\right)$ ⟷ „e aproximativ normal"

Exp. : 100 pachete

$\mathcal{U}([5,50])$

Care e probabilitatea ca greutatea totală să fie ≥ 3000 kg ?

Astfel,

Fie $X_1, \dots X_{100}$ pachete $\sim \mathcal{U}([5,50])$

$S_{100} = X_1 + \dots + X_{100}$ greutatea totală

$\mathbb{P}(S_{100} \geq 3000) = ?$

Rez.

$$\mathbb{P}(S_{100} \geq 3000) = \mathbb{P}\left(\frac{S_{100} - \mathbb{E}[S_{100}]}{\sqrt{Var(S_{100})}} \geq \frac{3000 - \mathbb{E}[S_{100}]}{\sqrt{Var(S_{100})}}\right)$$

$Z_{100} \sim N(0,1)$

$$\overset{TLC}{=} 1 - \Phi\left(\frac{3000 - \mathbb{E}[S_{100}]}{\sqrt{Var(S_{100})}}\right)$$

$$\mathbb{E}[S_{100}] = 100\, \mathbb{E}[X_i] = 100 \cdot \frac{55}{2} = 27{,}5 \cdot 100 = 2750$$

$$Var(S_{100}) = 100 \cdot Var(S_1) = 100 \cdot \frac{(50-5)^2}{12} = 16875$$

$X \sim \mathcal{U}[a,b]$

$Var(X) = \frac{(b-a)^2}{12}$ $\qquad X = a + (b-a)Y$

$Y \sim \mathcal{U}[0,1)$ $\qquad Var(Y) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

$\mathbb{E}[Y] = \frac{1}{2}$

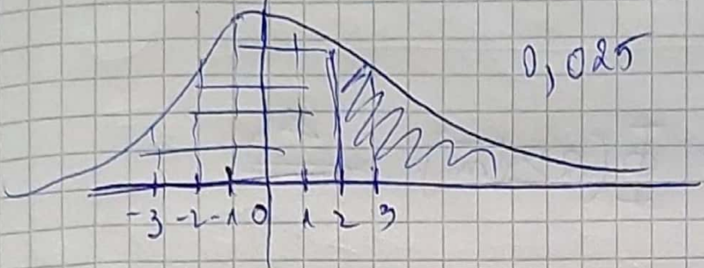
$$\mathbb{E}[Y^2] = \int_0^1 y^2\, dy = \frac{1}{3}$$

Scanned with CamScanner

$$\mathbb{P}(S_{100} \geq 3000) \simeq 1 - \Phi\left(\frac{3000 - 2750}{\sqrt{16875}}\right) = 0,0274$$

$$\cong 1 - \Phi(1,92)$$

(?) $1 -$ pnorm$(1,92)$

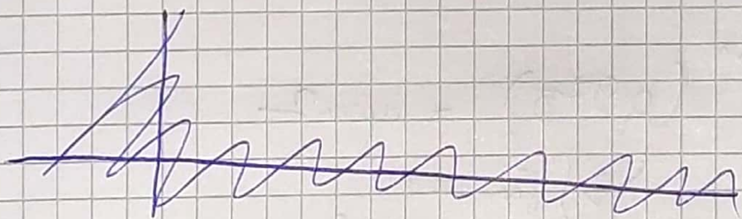


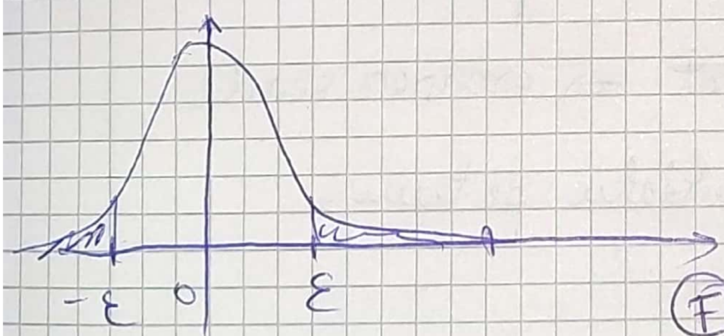
Exp. p pe A în ~~proporție~~ populație

$$X_1 \dots X_n \sim \mathcal{B}(p)$$

(TLC) $\overline{X_n} \simeq N\left(\mu, \frac{\sigma^2}{n}\right) - N\left(p, \frac{p(1-p)}{n}\right)$

(F) $\overline{X_n} - p \simeq N\left(0, \frac{p(1-p)}{n}\right)$





$$\mathbb{P}(|\overline{X_n} - p| \geq \varepsilon) = 2\mathbb{P}(\overline{X_n} - p \geq \varepsilon)$$

din simetrie

(F) $\overline{X_n} - p \simeq N\left(0, \frac{p(1-p)}{n}\right)$

$$\mathbb{P}(|\overline{X_n} - p| \geq \varepsilon) = 2\mathbb{P}(\overline{X_n} - p \geq \varepsilon)$$

$$= 2\mathbb{P}\left(\frac{\overline{X_n} - p}{\sqrt{\frac{(1-p)p}{n}}} \geq \frac{\varepsilon}{\sqrt{\frac{p(1-p)}{n}}}\right)$$

$$\overset{(TLC)}{=} 2\left(1 - \Phi\left(\frac{\varepsilon}{\sqrt{\frac{p(1-p)}{n}}}\right)\right)$$

$$\xrightarrow{(\Phi)} 2\left(1 - \Phi\left(\frac{\varepsilon}{\sqrt{\frac{p(1-p)}{n}}}\right)\right) \leq 2\left(1 - \Phi\left(2\varepsilon\sqrt{n}\right)\right)$$

$$p(1-p) \leq \frac{1}{4} \Rightarrow \frac{p(1-p)}{n} \leq \frac{1}{4n} \Rightarrow \frac{1}{\sqrt{\frac{p(1-p)}{n}}} \geq \frac{1}{\sqrt{\frac{1}{4n}}}$$

$$\Rightarrow \frac{\varepsilon}{\sqrt{\frac{p(1-p)}{n}}} \geq 2\varepsilon\sqrt{n}$$

$$\Rightarrow \Phi\left(\frac{\varepsilon}{\sqrt{\frac{p(1-p)}{n}}}\right) \geq \Phi(2\varepsilon\sqrt{n}) \quad \underline{\underline{*}}$$

$$P(|\overline{X_n} - p| \geq \varepsilon) \leq 2\underbrace{\left(1 - \Phi(2\varepsilon\sqrt{n})\right)}_{0,05}$$

$\varepsilon = 0,01$

$$\begin{cases} 2(1 - \Phi(2\varepsilon\sqrt{n})) = 0,05 \Rightarrow \Phi(2 \times 0,01\sqrt{n}) = 0,975 \\ \varepsilon = 0,01 \end{cases}$$

$$\Rightarrow 2 \times 0,01\sqrt{n} = \Phi^{-1}(0,975) = \text{aprox } 2$$

$$\rightarrow \begin{cases} \sqrt{n} \simeq 100 \\ n \simeq 10000 \end{cases} \text{ persoane}$$

eșantion stratificat $\Rightarrow$ eroarea scade

sâmbăta <u>10</u> consultații pe teams