17.02.2022

Seminar 1

1) $O(g(n)) = \{ f(n) \mid \exists c > 0,\ n_0 > 0$ a.î. $\boxed{f(n) \le c \cdot g(n)}, \forall n \ge n_0 \}$

$n^2 \in O(n^3)$? Aleg $c = 1,\ n_0 = 3$

$n^2 \le n^3,\ \forall n \ge 3$

SAU

$$\boxed{\lim_{n \to \infty} \frac{f(n)}{g(n)} < \infty}$$

2) $\Omega(g(n)) = \{ f(n) \mid \exists c > 0,\ n_0 > 0$ a.î. $\boxed{f(n) \ge c \cdot g(n)}, \forall n \ge n_0 \}$

$n^2 \in \Omega(n)$ DA

$n \log n \in \Omega(n\sqrt{n})$ NU

$$\boxed{\lim_{n \to \infty} \frac{f(n)}{g(n)} > 0}$$

3) $\Theta(g(n)) = \{ f(n) \mid \exists c_1 > 0,\ c_2 > 0,\ n_0 > 0$ a.î. $\boxed{c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n)}, \forall n \ge n_0 \}$

$n^2 \in \Theta(n^2)$ DA

$$\boxed{\lim_{n \to \infty} \frac{f(n)}{g(n)} \in \mathbb{R}_+}$$

$$\boxed{\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0}$$

4) $o(g(n)) = \{ f(n) \mid \forall c > 0,\ n_0 > 0$ a.î. $\boxed{f(n) < c \cdot g(n)}, \forall n \ge n_0 \}$

5) $\omega(g(n)) = \{ f(n) \mid \forall c > 0,\ n_0 > 0$ a.î. $\boxed{f(n) > c \cdot g(n)}, \forall n \ge n_0 \}$

$$\boxed{\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty}$$

Exerciții:

3.1-1 / 52 $\quad \max(f(n), g(n)) = \Theta(f(n)+g(n))$ (?)

$\Rightarrow \exists$ un pct. de la care f, g sunt poz (asimptotic pozitive)

f, g asimptotic pozitive $\Rightarrow \exists n_1 > 0$ a.î. $f(n) \geq 0, \forall n \geq n_1$
$\exists n_2 > 0$ a.î. $g(n) \geq 0, \forall n \geq n_2$.

~~a+b = a+b~~

$n_0 = \max(n_1, n_2)$

$$\boxed{\max(a,b) = \frac{a+b+|a-b|}{2}} \Rightarrow \max(f(n), g(n)) = \frac{f(n)+g(n)+\overbrace{|f(n)-g(n)|}^{\geq 0}}{2}$$

$$\geq \frac{f(n)+g(n)}{2} \Rightarrow \max(f(n), g(n)) \in \Omega(f(n)+g(n)) \quad (1)$$

(fct. asimp. nenegative)

$$\max(f(n), g(n)) \leq f(n)+g(n) \Rightarrow \max(f(n), g(n)) \in O(f(n)+g(n)) \quad (2)$$

$\Rightarrow \max(f(n), g(n)) \in O(f(n)+g(n))$

$\xrightarrow{(1),(2)}$ QED.
$\max(f(n), g(n)) \in \Theta(f(n)+g(n))$

$$f(n) \in \Theta(g(n)) \Leftrightarrow \begin{cases} f(n) \in \Omega(g(n)) \\ f(n) \in O(g(n)) \end{cases}$$

3.1-2 / 52 $\quad \forall a, b \in \mathbb{R}, b > 0 : (n+a)^b = \Theta(n^b)$

1) $\lim_{n \to \infty} \frac{(n+a)^b}{n^b} = \lim_{n \to \infty} \left(\frac{n+a}{n}\right)^b = 1 \in \mathbb{R}_+ \Rightarrow$ QED

2) $C_b^0 \cdot n^b \cdot a^0 + C_b^1 \cdot \underbrace{n^{b-1} \cdot a}_{\leq n^b} + C_b^2 \cdot n^{b-2} \cdot a^2 + \ldots + C_b^b \cdot n^0 \cdot a^b$.

$= (n+a)^b$

~~$\leq n^b(C_b^0 + C_b^1 + \ldots + C_b^b) = n^b \cdot a^b$~~

$\Rightarrow C_b^0 \cdot n^b \leq (n+a)^b$

$C_b^0 = 1$

$(n+a)^b = C_b^0 \cdot \underbrace{n^b \cdot a^0}_{\leq n^b \cdot a^b} + C_b^1 \cdot \underbrace{n \cdot a^1}_{\leq n^b \cdot a^b} + \ldots + C_b^b \cdot \underbrace{n^b \cdot a^b}_{\leq n^b \cdot a^b} \leq$

$\leq n^b \cdot a^b (\underbrace{C_b^0 + \ldots + C_b^b}_{2^b}) \leq n^b \cdot (2a)^b$

3.1-4 / 53

$2\cdot 2^n \ldots$
$2^{n+1} \in O(2^n)$? DA

$2^{2n} \in O(2^n)$? NU
$\Updownarrow$
$4^n \ldots$

3.1-5 / 53

$f(n) \in \Theta(g(n)) \iff \begin{cases} f(n) \in \Omega(g(n)) \\ f(n) \in O(g(n)) \end{cases}$!

"$\Rightarrow$" $f(n) \in \Theta(g(n)) \Rightarrow \exists c_1, c_2 > 0, n_0 > 0$ a.î. $\underbrace{c_1 \cdot g(n) \le f(n)}_{\Downarrow} \le c_2 \cdot g(n)$, $f(n) \in O(g(n))$

$f(n) \in \Omega(g(n))$

"$\Leftarrow$" $f(n) \in \Omega(g(n)) \Rightarrow \exists c_1 > 0, n_1 > 0$ a.î. $f(n) \ge c_1 \cdot g(n)$

$f(n) \in O(g(n)) \Rightarrow \exists c_2 > 0, n_2 > 0$ a.î. $f(n) \le c_2 \cdot g(n)$

Aleg $n_0 = \max(n_1, n_2) \Rightarrow \exists c_1, c_2 > 0, n_0 > 0$ a.î. $c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n)$

! $n! \in O(n^n)$

$n! = \underset{\le n}{1} \cdot \underset{\le n}{2} \cdot 3 \cdot \ldots \cdot \underset{\le n}{n} \le n \cdot n \cdot n \cdot \ldots \cdot n \le n^n$

SAU

$\log_2(n!) \in \Theta(n \log_2 n)$

$\log_a b = \frac{\log_c b}{\log_c a}$

$$\log_2 n! = \log_2(1\cdot 2\cdot 3\cdot \ldots \cdot n) = \log_2 1 + \log_2 2 + \ldots + \log_2 n \le$$

$$\le \log_2 n + \log_2 n + \ldots + \log_2 n = n\cdot \log_2 n \Rightarrow$$

$$\Rightarrow \log_2 n! \in O(n \log_n) \quad (1)$$

$$\log_2 n! = \log_2 1 + \log_2 2 + \ldots + \log_2 n \ge \log_2 \frac{n}{2} + \log_2\left(\frac{n}{2}+1\right) + \ldots + \log_2 n$$

$$\ge \log_2 \frac{n}{2} + \log_2 \frac{n}{2} + \ldots + \log_2 \frac{n}{2} = \frac{n}{2}\cdot \log_2 \frac{n}{2}$$

$$\frac{n}{2}\left(\log_2 n + \log_2 \frac{1}{2}\right)$$

$$\cancel{n \log n = \log n^n}$$

~~$P(n) = n! \ge$~~

~~$P(n) =$~~

$$\log_2 n! \le \frac{n}{2}\log_2 \frac{n}{2} = \frac{1}{2}n\left(\log_2 n - \log_2 2\right) = \frac{1}{2}n\log_n - \frac{n}{2} \Rightarrow$$

$$\Rightarrow \log_2 n! \in \Omega(n \log_n) \quad (2)$$

Din (1), (2) $\Rightarrow$ QED.

3.1-7/53

$$\overbrace{o(g(n))}^{A_1} \cap \overbrace{\omega(g(n))}^{A_2} = \emptyset$$

„$\subseteq$" Fie $f \in A_1 \cap A_2$

$f \in A_1 \Rightarrow \left.\begin{matrix} \forall c_1 > 0 \\ \exists n_1 > 0 \end{matrix}\right\}$ a.î. $f(n) < c_1 \cdot g(n)$, $\forall n \geq n_1$

$f \in A_2 \Rightarrow \left.\begin{matrix} \forall c_2 > 0 \\ \exists n_2 > 0 \end{matrix}\right\}$ a.î. $f(n) > c_2 \cdot g(n)$, $\forall n \geq n_2$

! $\left.\begin{matrix} n_0 = \max(n_1, n_2) \\ c = c_1 = c_2 \end{matrix}\right\} \Rightarrow c \cdot g(n) < f(n) < c \cdot g(n)$ ①

„$\supseteq$" $\emptyset \in A_1 \cap A_2$

3.1-8/53

$O(g(n,m)) = \{ f(n,m) \mid \exists c > 0, n_0 > 0, m_0 > 0$ a.î. $f(n,m) \leq c \cdot g(n,m)$ $\forall n \geq n_0, m \geq m_0 \}$

generalizare $(f(n,m))$.

$a, b \in \mathbb{R}$; ct; $b > 0$

$(n+a)^b = \Theta(n^b) \Leftrightarrow (n+a)^b \in \Omega(n^b)$ și $(n+a)^b \in O(n^b)$ | $n^b \leq 0 \leq n^b$

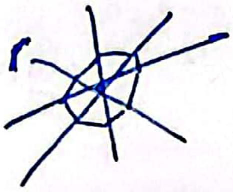
$(n+a)^b = C_b^0 \cdot n^b \cdot a^0 + C_b^1 \cdot n^{b-1} \cdot a + C_b^2 \cdot n^{b-2} \cdot a^2 + \ldots + C_b^b \cdot n^0 \cdot a^b$

$\underbrace{C_b^0 \cdot n^b \cdot a^0}_{\geq n^b} + \underbrace{C_b^2 \cdot n^{b-2} \cdot a^2 + \ldots}_{\geq 0} + a(C_b^1 \cdot n^{b-1} \cdot a^0 + C_b^3 \cdot n^{b-3} \cdot a^2 + \ldots)$

$(1-1)^n = C_b^0 \cdot 1 + C_b^1 (-1) + C_b^2 - C_b^3 + \ldots$

$(1+1)^n = C_b^0 + C_b^1 + C_b^2 + C_b^3 + \ldots$

$2^n = 2(C_b^0 + C_b^2 + \ldots + C_b^4 + \ldots) \Rightarrow \underbrace{C_b^0 + C_b^2 + \ldots}_{C_b^1} = \underline{2^{n-1}}$



03.02.2022.

Seminar 3

Inducție

1) $T(n) = T(n-1) + n$

$T(n) \le c \cdot n^2 + n(1-2c) + c \le c \cdot n^2$

$T(n) \le c \cdot n^2 \; ; c > 0$

Știm că $T(k) \le c \cdot k^2, \; \forall k < n$

$T(n) = T(n-1) + n \le c(n-1)^2 + n = c(n^2 - 2n + 1) + n$

$= cn^2 - 2cn + c + n = c \cdot n^2 + n(1 - 2c) + c \le c \cdot n^2$

$\Leftrightarrow c > \frac{1}{2}$

2) **Arbore**

$T(n) \quad n$

$\downarrow$

$T(n-1) \quad n-1$

$\downarrow$

$T(n-2) \quad n-2$

$\vdots$

$T(1) \quad 1$

$\frac{n(n+1)}{2} \qquad \cancel{\Theta(n^2)}$

$\qquad\qquad O(n^2)$

3) $T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$ → f - funcții nenegativă

Master

$\boxed{\text{caz 1:}}$ Dacă $\exists \, \varepsilon > 0$ și $f(n) \in O\left(n^{\log_b a - \varepsilon}\right) \Rightarrow$

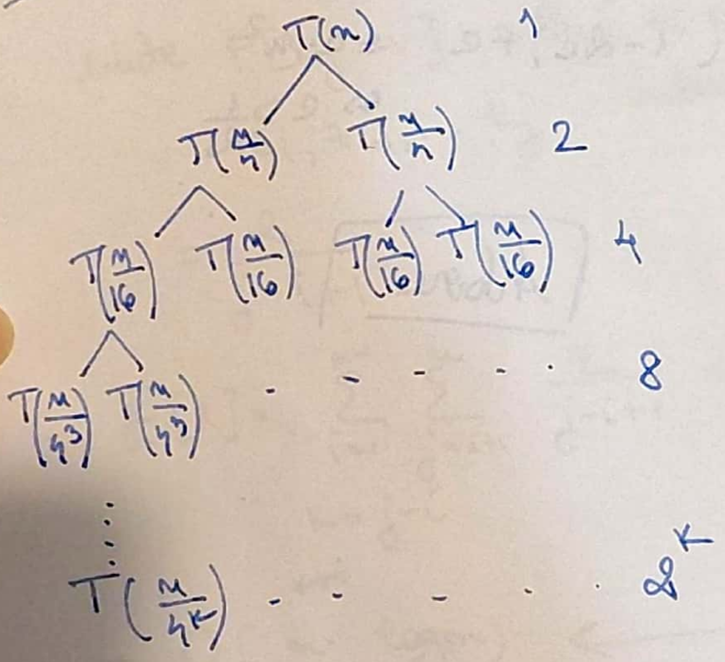
$\Rightarrow T(n) \in \Theta\left(n^{\log_b a}\right)$

$\boxed{\text{Caz 2:}}$ Dacă $f(n) \in \Theta\left(n^{\log_b a}\right) \Rightarrow T(n) \in \Theta\left(n^{\log_b a} \cdot \lg n\right)$

$\boxed{\text{Caz 3:}}$ Dacă $\exists\, \varepsilon > 0$ și $\exists\, c > 1$ a.î. $a \cdot f\left(\frac{n}{b}\right) \le c \cdot f(n)$, $\exists\, n_0 \in \mathbb{N}^*$,

$\forall\, n \ge n_0$ și $f(n) \in \Omega\left(n^{\log_b a + \varepsilon}\right) \Rightarrow T(n) \in \Theta(f(n))$

Ex. 1 $\quad T(n) = 2 \cdot T\left(\frac{n}{4}\right) + 1$



$= 1 + 2 + 4 + \dots + 2^k = 2^k - 1$

$a^{\log_b c} = c^{\log_b a}$

$k = \log_4 n \Rightarrow S = 2^{\log_4 n} - 1 = 2\sqrt{n} - 1$

$n^{\log_4 2}$

$2n^{\log_4 2} - 1 = 2\sqrt{n} - 1$

$\Theta(\sqrt{n})$

Master:

$a = 2$

$b = 4$

$n^{\log_4 2} = \sqrt{n}$

pt. $\varepsilon = \frac{1}{2}$ $\quad 1 \in O\left(n^{\log_b a - \varepsilon}\right)$

$\Leftrightarrow O\left(n^{\log_4 2 - \frac{1}{2}}\right)$

Ex. 2

$T(n) = 2 \cdot T\left(\frac{n}{4}\right) + \underbrace{\sqrt{n}}_{= f(n)}$

$a = 2$

$b = 4$

$n^{\log_b a} = n^{\log_4 2} = \sqrt{n}$

$\| \quad \Theta \Rightarrow$ Caz 2 $\Rightarrow T(n) \in \Theta(\sqrt{n} \lg n)$

Ex. 3 : $T(n) = 2 \cdot T\left(\frac{n}{2}\right) + n$ (Merge sort)

$a = 2$

$b = 2$

$n^{\log_2 2} = n$

$\Theta \Rightarrow T(n) \in \Theta(n \log n)$

caz 2

Arbore :

$$T(n) \qquad n$$

$$T\left(\frac{n}{2}\right) \quad T\left(\frac{n}{2}\right) \qquad n$$

$$T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right) \qquad n$$

$$\vdots$$

$$T\left(\frac{n}{2^k}\right) \ \dots$$

$\left.\right\} n \cdot \log n$

Ex. 4 : $T(n) = 2T\left(\frac{n}{2}\right) + 1$

$a = 2$

$b = 2$

$n^{\log_2 2} = n$

$O(n - \varepsilon)$

$\Downarrow$

$T(n) \in \Theta(n)$

$$T(n) \qquad 1$$

$$T\left(\frac{n}{2}\right) \quad T\left(\frac{n}{2}\right) \qquad 2$$

$$T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right) \qquad 4$$

$$\vdots$$

$$T\left(\frac{n}{2^k}\right) \qquad 2^k$$

$1 + 2 + \dots + 2^k = 2^k - 1$

$k = \log_2 n$

$2^{\log_2 n} - 1 \qquad n^{\log_2 2} - 1$

$n - 1$

$\Downarrow$

$O(n)$

$2^0 \, 2^1 \, 2^2 \, 2^3 \dots 2^k$

Ex. 5: $T(n) = T\left(\frac{n}{2}\right) + 1$

$$T(n) \qquad 1$$
$$T\left(\frac{n}{2}\right) \quad T\left(\frac{n}{2}\right) \qquad 2$$
$$T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right) \qquad 4$$

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + 1$$

$$\left.\begin{array}{ll} T(n) & 1 \\ T\left(\frac{n}{2}\right) & 1 \\ T\left(\frac{n}{4}\right) & 1 \\ \vdots & \\ T\left(\frac{n}{2^k}\right) & 1 \end{array}\right\} \log n$$

$$O(\log n)$$

fast fourier transform FFT algorithm

Probleme

$x, y \to n$ cifre

baza 10

$O(n^2)$

$$\left.\begin{array}{l} x = x_H \cdot 10^{\frac{n}{2}} + x_L \\ y = y_H \cdot 10^{\frac{n}{2}} + y_L \end{array}\right\} \Rightarrow x \cdot y = \left(x_H \cdot 10^{\frac{n}{2}} + x_L\right)\left(y_H \cdot 10^{\frac{n}{2}} + y_L\right)$$

$$= x_H \cdot y_H \cdot 10^n + x_H \cdot 10^{\frac{n}{2}} \cdot y_L + x_L \cdot y_H \cdot 10^{\frac{n}{2}} + x_L \cdot y_L$$

$$= \underbrace{x_H \cdot y_H}_{a} \cdot 10^n + 10^{\frac{n}{2}} \underbrace{\left(x_H \cdot y_L + x_L \cdot y_H\right)}_{b} + \underbrace{x_L \cdot y_L}_{c} \Rightarrow 4 \text{ înmulțiri}$$

$$T(n) = 4 \cdot T\left(\frac{n}{2}\right) + n$$

$a = 4$

$b = 2$

$n^{\log_2 4} = n^2$

$O\left(n^{\log_2 4 - 1}\right) \xrightarrow{\text{caz 1}} T(n) \in \Theta(n^2)$

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$$b = x_H y_L + x_L \cdot y_H = \underbrace{(x_H + y_L)(y_H + x_L) - \underbrace{x_H y_H}_{a} - \underbrace{y_L x_L}_{c}}_{3 \text{ înmulțiri}}$$

alg. lui Karatsuba

$$T(n) = 3 \cdot T\left(\frac{n}{2}\right) + n$$

$a = 3$

$b = 2$

$n^{\log_b 3}$

$$\xrightarrow[n \in \Theta(n^{\log_2 3 - \varepsilon})]{\text{caz } 1} \xrightarrow{T.M} T(n) \in \Theta\ n^{\log_2 3}$$

$$T(n) = 4 \cdot T\left(\frac{n}{3}\right) + n$$

$a = 4$

$b = 3$

$n^{\log_b a} = n^{\log_3 4}$

$$\Rightarrow n < n^{\log_3 4}$$

$\Downarrow$ caz 1

$$T(n) \in \Theta(n^{\log_3 4})$$

$$T(n) \le c \cdot n^{\log_3 4}$$

$$T(n) = 4 \cdot T\left(\frac{n}{3}\right) + n \le 4 \cdot c \left(\frac{n}{3}\right)^{\log_3 4} + n$$

$$= \cancel{4} c \cdot \frac{n^{\log_3 4}}{\cancel{4}} + n = c \cdot n^{\log_3 4} + n \quad \text{nu putem compara}$$

$$\boxed{T(n) \le c \cdot n^{\log_3 4} - d \cdot n}$$

$$T(n) \le 4\left(c \cdot \left(\frac{n}{3}\right)^{\log_3 4} - \frac{d}{3} \cdot n\right) + n$$

$$= \cancel{4} \cdot c \frac{n^{\log_3 4}}{\cancel{4}} - \frac{4d}{3} n + n$$

$$= c n^{\log_3 4} - \underbrace{n\left(-1 + \frac{4d}{3}\right)}_{\ge 0} \Rightarrow \frac{4d}{3} \ge 1 \Rightarrow d \ge \frac{3}{4}$$

$$\sum_{k=0}^{n} x^k = \frac{x^{k+1} - 1}{x - 1}$$

$$\sum_{k=0}^{n} x^k \begin{cases} \infty, & x \ge 1 \\ \text{divergent}, & x < -1 \\ \frac{1}{1-x}, & x \in (-1, 1) \end{cases}$$

1) $T(n) = T\left(\frac{n}{2}\right) + n^2$

$$\begin{array}{ll} T(n) & n^2 \\ | & \\ T\left(\frac{n}{2}\right) & \left(\frac{n}{2}\right)^2 \\ | & \\ T\left(\frac{n}{4}\right) & \left(\frac{n}{2^2}\right)^2 \\ \vdots & \\ T\left(\frac{n}{2^k}\right) & \left(\frac{n}{2^k}\right)^2 \end{array}$$

$\Rightarrow \quad n^2 + \frac{n^2}{2^2} + \frac{n^2}{2^4} + \frac{n^2}{2^6} + \dots + \frac{n^2}{2^{2k}}$

$= n^2\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \dots + \frac{1}{2^{2k}}\right)$

$= n^2\left(1 \cdot \frac{\left(\frac{1}{2}\right)^{\frac{2k+1}{2}} - 1}{\frac{1}{2} - 1}\right)$

$= -2n^2\left(\frac{1}{2}\right)^{\frac{2k+1}{2}} - 2n^2$

$= -2n^2\left[\left(\frac{1}{2}\right)^{\frac{2k+1}{2}} - 1\right]$

1

$T(n) = n^2 \sum_{i=0}^{\log_2 n} \frac{1}{2^{2i}} = n^2 \sum_{i=0}^{\log_2 n} \left(\frac{1}{4}\right)^i < n^2 \sum_{i=0}^{\infty} \left(\frac{1}{4}\right)^i$

$n^2 \cdot \frac{4}{3} \leftarrow \frac{1}{1-\frac{1}{4}}$

$x \in (-1,1)$

curs 3

2) $T(n) = 2T(n-1)+1$

$$T(n) \quad 1$$
$$T(n-1) \quad T(n-1) \quad 2$$
$$\vdots \quad 4$$
$$2^n$$

$1+2+\dots+2^n$
$= 2^n - 1$

$2^n - 1$

$\boxed{\leq c \cdot 2^n + n}$

$T(n) \leq c \cdot 2^n$

$\leq 2c \cdot 2^n + 1 \leq nu$

$T(n) = 2T(n-1)+1 \leq 2\left(c \cdot 2^{n-1} + n_{-1+1}\right)+1$

$= 2c \cdot 2^{n-1} + 2n + 1 - \cancel{2} = 2 \cdot 2^n + 2n$

Laborator 2

Merge sort:

5 2 3 4 | 1 2 8 7

5 2 | 3 4   1 2 | 8 7

5 | 2   3 | 4   1 | 2   8 | 7

2 5   3 4   1 2   7 8

2 3 4 5   1 2 7 8

1 2 2 3 4 5 7 8

Master

$T(n) = aT\left(\frac{n}{b}\right) + O(n^d)$ $d=1$

$a = 2, b = 2$ $n^{\log_2 2} = n$

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17. 02. 2022.

Seminar 5

Probabilități

$c_1, c_2, \ldots, c_n$

$c_i \longrightarrow f_i$, $f_i \neq f_j \ \forall 1 \leq i,j \leq n$

best = 0

for i = 1 ... n

  if $f_i > f_{best}$

    best = i

$\frac{(n-1)!}{n!} = \frac{1}{n}$ o angajare $\downarrow$ cel mai bun pe prima poziție

exact n angajări $\frac{1}{n!}$ $\swarrow$ șirul e în ord. cresc.

$$X = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$E(X) = 1\cdot\frac{1}{6} + 2\cdot\frac{1}{6} + 3\cdot\frac{1}{6} + 4\cdot\frac{1}{6} + 5\cdot\frac{1}{6} + 6\cdot\frac{1}{6} = 3{,}5$$

$$E(X+Y) = E(X) + E(Y)$$

$P = (P_1, P_2 \ldots P_n)$

(i,j) $i<j$ și $P_i > P_j$ inversiune

$X_{ij} = \frac{1}{2}$

X – numărul de inversiuni

$$E(X) = E\left(\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} X_{ij}\right) = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} E(X_{ij})$$

$$= \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \frac{1}{2} = \frac{1}{2}\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \qquad \frac{n(n+1)}{2}$$

$$= \frac{n(n-1)}{4}$$

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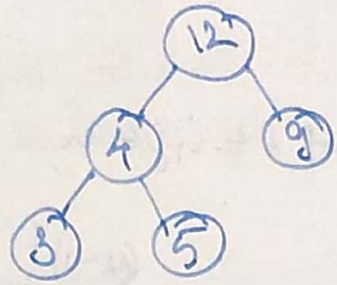
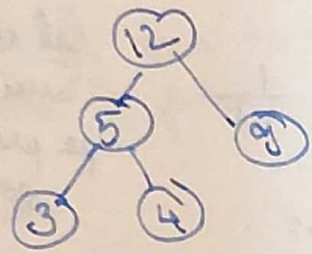
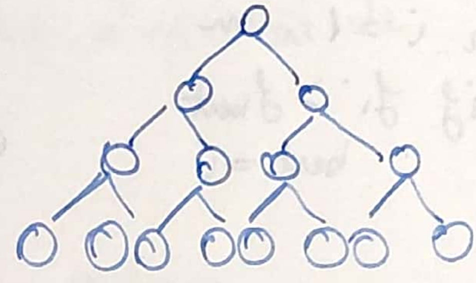
~~8~~ 12 4 9 3 5





8 ← 

$H = 1 \dots \cancel{15}$

$$0\cdot\frac{n}{2}+\frac{n}{4}\cdot 1+\frac{n}{8}\cdot 2+\frac{n}{16}\cdot 3+\dots+\frac{n}{2^k}(k-1)$$

$$\le \sum_{i=0}^{\log_2 n} \frac{n\cdot i}{2^i}$$

$$= n\cdot \sum_{i=0}^{\log_2 n} i\cdot\left(\frac{1}{2}\right)^i \le n\cdot\sum_{i=0}^{\infty} i\cdot\left(\frac{1}{2}\right)^i = \frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2}\cdot n$$

$$= 2n$$

$$\boxed{\sum_{k=0}^{\infty} k\cdot x^k = \frac{x}{(1-x)^2}}$$

for($i = \frac{n}{2} \dots 1$)
MaxH(i)

mai eficient de la jos => $O(n)$

în loc de $O(n\log n)$

$x_{11} \le x_{12}, \le \dots \le x_{1n}$

$x_{21} \le x_{22}, \dots x_{2n}$

$\vdots$

$x_{k1} \le x_{k2}, \dots x_{kn}$

$\downarrow$

scoatem minimul

$2n + 3n + 4n + \dots kn$

$\frac{k(k-1)}{2} n$

$k^2 \cdot n$

$kn \cdot \log(kn)$

$nk\log(k)$ cu Heap-uri

V . . .

al k-lea el. dacă ar fi sortat

$n\left(\underbrace{1 + \frac{1}{2} + \dots + \frac{1}{2^k}}_{\le 2}\right)$

I $n \log n$ sortare

II Minheap $\rightarrow$ $k \log n$.

Quicksort.

$p = k$

$p < k$ ( )

$p > k$ ( )

QuickSelect

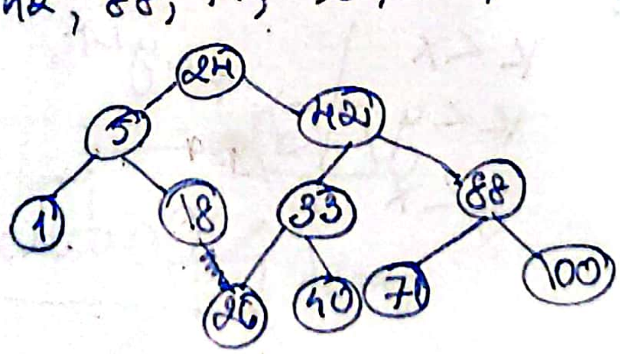
$\downarrow$

n

Arbori binari de căutare

24, 5, 18, 42, 88, 71, 33, 1, 40, 26, 100

RSD



Preordine (RSD)

24, 5, 1, 18, 19, 42, 33, 26, 40, 88, 71, 100

Inordine (SRD)

1, 5, 18, 19, 24, 26, 33, 40, 42, 71, 88, 100

Postordine (SDR)

1, 19, 18, 5, 26, 40, 33, 71, 100, 88, 42, 24

Fie T un ABC și X un nod cu 2 copii.

Arătați că succ(X) nu are fiu stâng și pred(X) nu are fiu drept.

- Pp. că succ(x) are fiu stâng. (y).

$$x < succ(x) = K$$
$$y < succ(x) = K$$
$$x < y < K$$

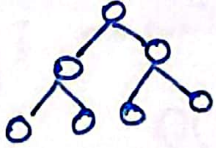
Contradicție

- Pp. că Pred(x) are fiu drept (y)

$$x > pred(x) = K$$

$$\left.\begin{array}{l} K < x \\ K < y \\ y < x \end{array}\right| \Rightarrow K < y < x$$

Contradicție

h  $h = \log_2(n) \Rightarrow n = 2^h$

$\boxed{2^{h+1} - 1}$ (max)

rămâne doar cu 1 pe ult. nivel (min)

$\boxed{2^h}$

---

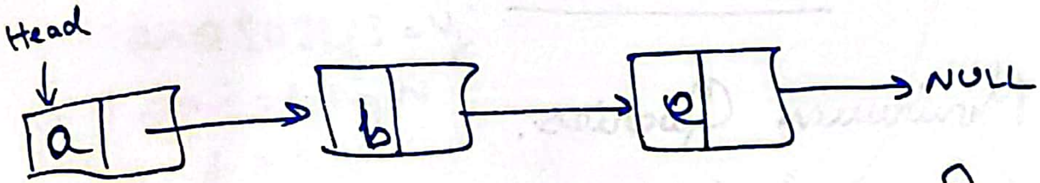
(27), 17, 14, 6, 13, 10, 15, 7, 12

$a_i \geq \begin{cases} a_{2i} \\ a_{2i+1} \end{cases}$

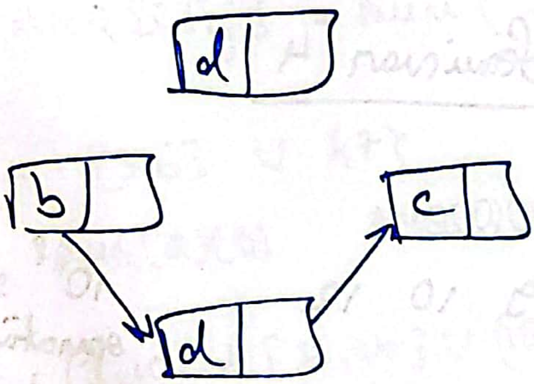
---

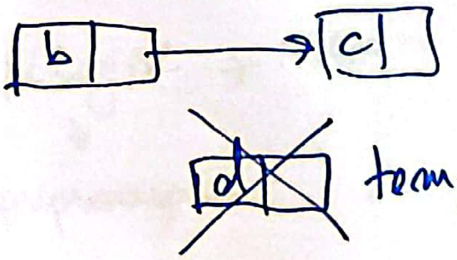
21. 03. 2022.

Liste:



1) adăugare nod





ștergerea unui nod

Alegerea medianei

```
randomize-select(A, p, r, i)
    if p == r
        return A[p]
    q = randomized-partition(A, p, r)
    k = q - p + 1
    if i == k
        return A[q]
    else if i < k
        return rand-select(A, p, q-1, i)
    else return rand-select(A, q+1, r, i-k)
```

$$T(n) = T(\max(n-k, k-1)) + O(n)$$

$$E(X) = \sum_{i=1}^{n} p_i v_i$$

$$E(T(n)) \leq E\left(\sum_{k=1}^{n} p_k \cdot T(\max(k-1, n-k)) + O(n) \right)$$

$$= E\left(\frac{1}{n} \sum_{k=1}^{n} T(\underset{\max}{n-k, k-1}) \right) + O(n)$$

se repetă max

$$= \frac{1}{n} \cdot 2 \sum_{k=\frac{n}{2}}^{n-1} E(T(k)) + O(n)$$

$$E(T(n)) = O(n) \leq c \cdot n$$

Vrem $E(T(n)) \leq c \cdot n$

$$E(T(n)) \le \frac{2}{n}\sum_{k=\lfloor \frac{n}{2}\rfloor}^{n-1} c\cdot k + a\cdot n$$

$$= \frac{2}{n}\left(\sum_{k=1}^{n-1} ck - \sum_{k=1}^{\lfloor \frac{n}{2}\rfloor -1} ck\right) + a\cdot n$$

$$= \frac{2}{n}\cdot c\left(\frac{n(n-1)}{2} - \frac{\lfloor \frac{n}{2}\rfloor\left(\lfloor \frac{n}{2}\rfloor -1\right)}{2}\right) + a\cdot n$$

$$= \frac{2}{n} c$$

$$\boxed{\le cn - \left(\frac{cn}{4} - \frac{c}{2} - an\right)}$$

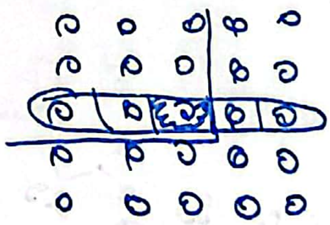
Adv. pt. $\frac{cn}{4} - \frac{c}{2} - an > 0$

Mediana $O(n)$

$V:\ 2\ \ 1\ \ 7\ \ 5\ \ 10\ \ 13 \rightarrow [1\ \ 2\ \ 5\ \ 7\ \ 10\ \ 13]$

$|V| = n$

$k = \lfloor \frac{n}{2} \rfloor$



$\frac{n}{5}\cdot\frac{1}{2}\cdot 3$ el. mai mici $\le$ pivot

$\frac{7n}{10}$ el. mai mari pivot

$$T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + O(n)$$

$$T(n) \le c\cdot n$$

:) Știm că $T(k) \le ck, \forall k < n$

$$T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + n$$

$$\le c \cdot \frac{n}{5} + c \cdot \frac{7n}{10} + n$$

$$\le n\left(\frac{c}{5} + \frac{7c}{10} + 1\right) \le n\left(1 + \frac{9c}{10}\right) \le c_n$$

$\cancel{1 + \frac{9c}{10}}$

$1 + \frac{9c}{10} - c \le 0$

$-\frac{c}{10} + 1 \le 0$

$\forall c \ge 10$

nth-element

$$n\left(\frac{1c}{3} + \frac{2c}{3} + 1\right) \le c$$

$c + 1 \le c$ $\quad T(n) \le c \cdot n \log n$

Fals

$\frac{5}{7} + 1$

$\frac{1}{7} \cdot \frac{1}{2} \cdot n \quad \frac{1}{7} \quad + \frac{5}{7} \quad c$

$\frac{9c}{7} + 1$

$\frac{2c}{7} + 1 \le 0$

$c \le -\frac{7}{2}$

5	4	93	6	23	8
2	1	100	7	30	15

$k = 2$

1) Aplicăm alg. de găsire al celui mai mic
$k\left(\frac{n}{2}\right)$ el în timp liniar.
mediana

2) $d_i = (|a_i - Med|)$ șir

3) găsim al k-lea cel mai mic el. din d.

4) iterăm prin d și alegem și elementele mai mici sau = cu x.

1

X : 1 4 6 8 10 12

Y : 1 2 3 4 5 6

$X_{mid} = X\left[\frac{1+n}{2}\right]$ între ele ~~căut~~ căutăm

$Y_{mid} = Y\left[\frac{1+n}{2}\right]$

$\underbrace{el_2 \quad x_m}_{sau} \underbrace{Y_m \quad el_1}$

sau

a) $X_{mid} = Y_{mid} \Rightarrow \frac{X_{mid}+Y_{mid}}{2}$

b) $X_{mid} < Y_{mid}$

$X_{mid} \ldots a,b \quad Y_{mid}$

$\Rightarrow \begin{cases} X[mid \ldots n] \\ Y[1 \ldots mid] \end{cases}$

c) $X_{mid} > Y_{mid}$ (Analog invers)

$T(n) = T\left(\frac{n}{2}\right) + 1 = \log n$

mediana a 2 vectori sortați

TRIE

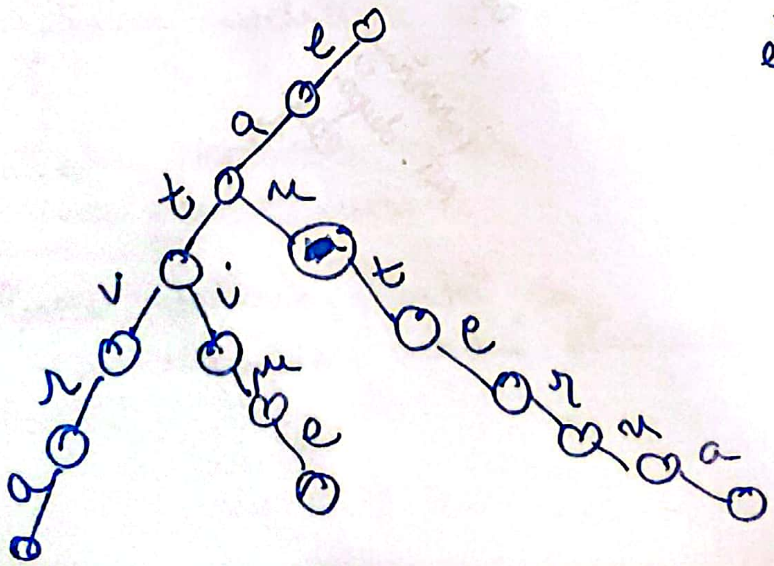
~~latitudine~~ lanterna
lanternă
latime
latitudine

$\rightarrow$ prefix com $\begin{cases} 2 \\ 3 \\ 4 \end{cases}$

$O(n \cdot |S|)$

$\downarrow$

lung. cuv.



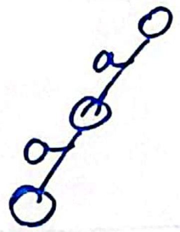
aaba

a
aa
aab
aaba

a
ab
aba

b
ba

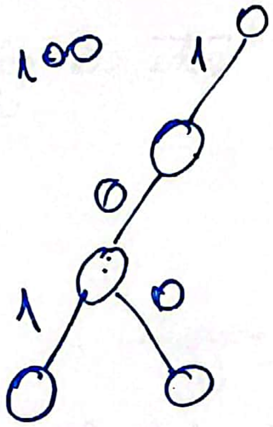
a



n^2

101 10

100



$n \le 10^5$

1 0 5 4 2

subsecv. xor maximă

$S_i = a_1 \wedge a_2 \dots \wedge a_i$

XOR max

xoruri parțiale

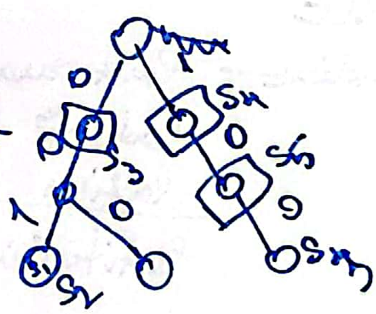
$S_1 = 001$

$S_2 = 1 \wedge 0 = 001$

$S_3 = 1 \wedge 0 \wedge 5 = 100$

$S_4 = 000$

$S_5 = 010$



mergeam pe complement

\+ inserare nr. după calc

Algoritmul de găsire a medianei în $O(n)$ determinist

3 1 7 4 10 2 5

1 2 3 2 5 7 10

La alg. aleator, în cazul favorabil:

$T(n) = T(n/2) + O(n)$
$= O(n)$

În cazul cel mai defavorabil:

$T(n) = T(n-1) + O(n) = O(n^2)$

$\boxed{\text{Algoritmul determinist}}$

$k = 5$
pivot = 2

2 1 5 | 10 4 7 5

$k = 2$
pivot = 4

4 7 5 10

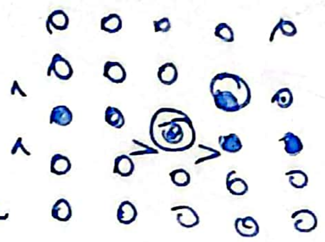
$k = 1$

1) Împărțim șirul în grupe de câte 5 (de ce nu 3, 7?)

2) Găsim mediana din fiecare grupă.

3) Găsim mediana celor $n/5$ mediane (recursiv).

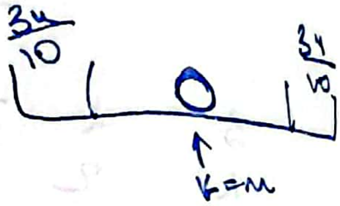
4) Alegem mediana găsită ca pivot și urmăm aceeași strategie ca la alg. recursiv.



$\frac{n}{5}$ grupe cu câte 5 elemente

$\frac{n}{5} \cdot \frac{1}{2}$ grupe care au fiecare câte 3 el. mai mici decât mediana medianelor

$\hookrightarrow \frac{3n}{10}$ elemente



$$T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + O(n)$$

Vrem $T(n) \le c \cdot n$

Pp. că $T\left(\frac{n}{5}\right) \le c \cdot \frac{n}{5}$

$T\left(\frac{7n}{10}\right) \le c \cdot \frac{7n}{10}$

$$\Rightarrow T(n) \le c \cdot \frac{n}{5} + c \cdot \frac{7n}{10} + n$$
$$= n\left(\frac{c}{5} + \frac{7c}{10} + 1\right)$$
$$= n \cdot \left(\frac{9c}{10} + 1\right) \le c \cdot n$$

$\frac{9c}{10} - c + 1 \le 0$

$\frac{c}{10} \ge 1 \Rightarrow c \ge 10$

$\frac{\frac{n}{3}}{2} \cdot 2 \quad \frac{n}{3} \to \frac{2n}{3}$

grupe 3:

$$T(n) = T(n/3) + T\left(\frac{2n}{3}\right) + O(n) = O(n \log n)$$

grupe 7:

$$T(n) = T(n/7) + T\left(\frac{5n}{7}\right) + O(n)$$

$\frac{\frac{n}{7}}{2} \cdot 4 \quad \frac{2n}{7} \quad \frac{5n}{7}$

Temă $\rightarrow$

Seminar (săpt. 13)

Exerciții ușor: 3

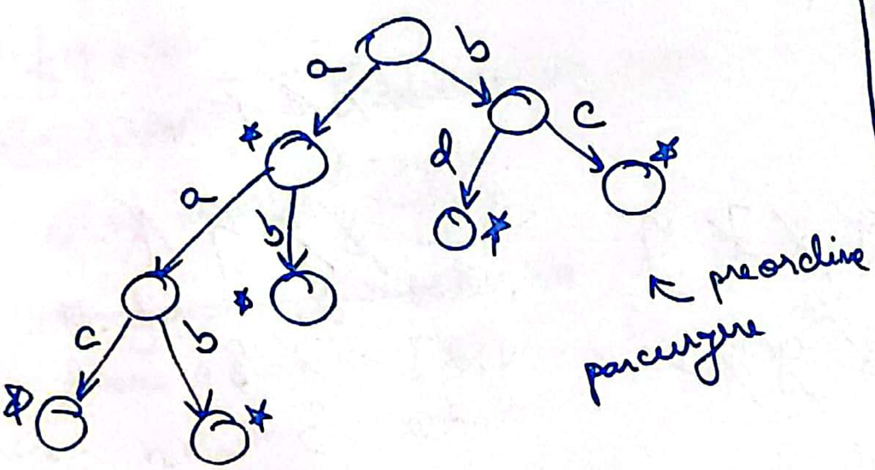
- N șiruri
  a, ab, bc, aab, aac, bd →
- M nr.
  1, 2, 5

→ a, aab, aac, ab, bc, bd

! ⇒ dc. sortăm: $n \log n$ (lungime cuv max.) + M

```
struct trie
{ char c;
  Trie * next[26];
  bool isword;
};
```

isword → var. care spune dc. se term sau nu un cuv



Const. trie: $\sum |s_i| + M$ → suma lungimilor cuv.

mai eficient: ~~folosire~~ utilizare trie: liniară

→ e suficient să explicăm algoritmul și cum funcționează tria.

Alg. AHO CORASICK

T
P_1
P_2
P_3
⋮

în $\Sigma P + T$ complex: să găs. câte p sunt în T

$\Downarrow$

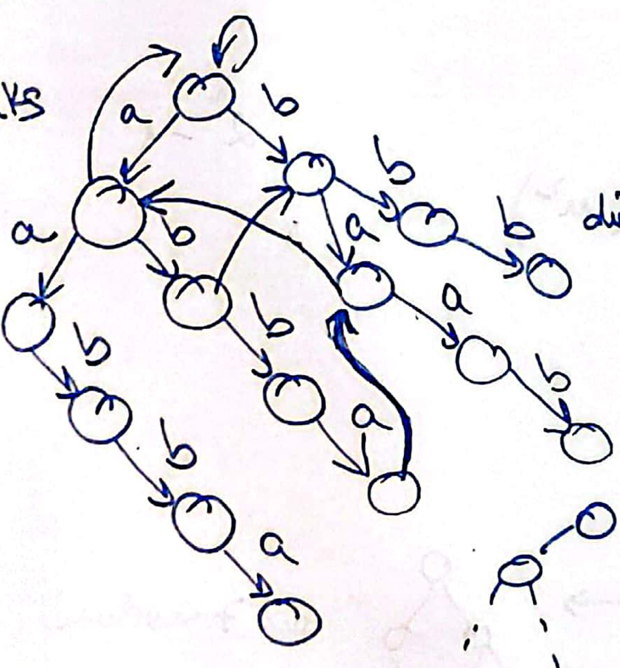
caută mai multe texte dintr-o cavitură

T abbaabbabab

P_1 aaba
P_2 abba
baab
P_3 bbb

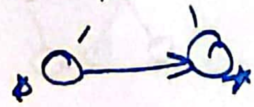
cel mai lung sufix care e prefix
↓
acolo ne-a trimis sufix link

1) Sufixlinks



abba

- mă uit la părinte din op. sau cu sufix linkul până găsesc cont. sau ajung la rad.
- văd dc. am cont. cu lit. resp.
da ↙
leg ~~litera~~ nodul cu sufix link de cel. echiv.



2) Output links
↳

abba ba
abba
verif sit. asta ↑

cuv. în cuv.
↳ afișăm 2

dc. se term. cuv. mergem cu sufix până găsim un alt șir term.

și leg.

→ când parc. sărim și cu suffix și cu output links.

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Subiect - examen $\quad 0^{0}\ 1.$

1) a) $\lg(n!) \in \Theta(n\log n)$

$\lg(n!) = \lg(1\cdot 2\cdot \ldots \cdot n) = \lg 1 + \lg 2 + \ldots + \lg n \le n\log n$

$\Rightarrow \lg(n!) \in O(n\log n)$

sau $\lg(n!) \le \lg(n^n) \in n\log n$

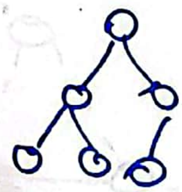
$n! \ge n^{\frac{n}{2}} \quad \forall n \ge 5$ (inductie)

$\lg(n!) \ge \log n^{\frac{n}{2}} \ge \frac{n}{2}\lg n \Rightarrow \lg(n!) \in \Omega(n\log n)$ $\Rightarrow$

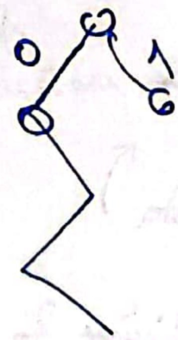
b) $(n+2^5)^3 \in \Theta(n^3)$ $\qquad 2^n - 1$

c) $\lg(n^{10}) + n^2 \in \Theta(n^2)$

d) $\lg n^5 \in \Theta(\lg n)$

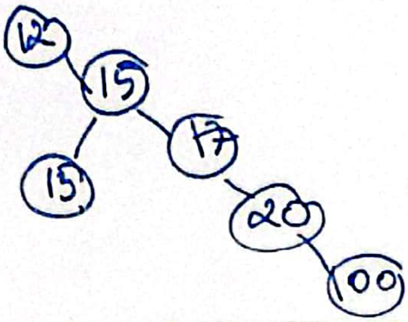
2) arbore plin. complet $\longrightarrow$  cu 7 noduri

aaab
↓



3) 12 15 13 17 20 100

→



4) $n^2 \log n \in O(n^3)$ mai mic strict.

putem să facem cu lim. $\lim_{n \to \infty} \frac{n^3}{n^2 \log n} = \infty$

sau → cu arbore de decizie → n! frunze

$h \geq \lg(n!)$

sau recurente : $T(n) = 2T(n-1) + 2$ (Masters.)

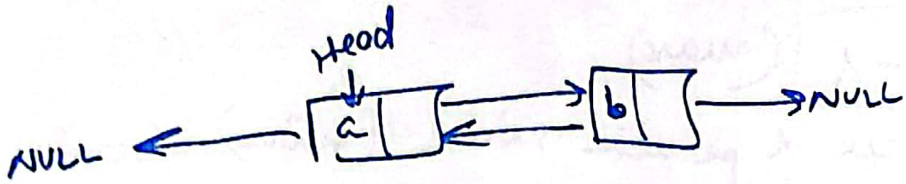
sau inducție

sau arb. recurență

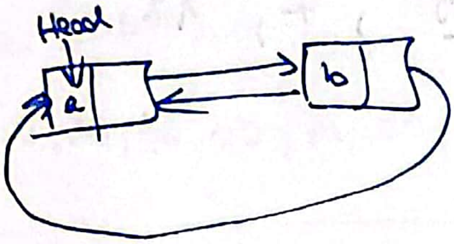
$T(n) \leq \ldots$

O

Liste dublu înlănțuite :



Liste circulare



Curs 6

Range Minimum Queries.

Seminar 4

RMQ

3 1 2 4 6 9 10 12

$10^8 \simeq 1s$ operații

$N \leq 10^5$

$M \leq 10^5$

$i, j \rightarrow min(v_i, \dots v_j)$

$O(m \cdot n)$ abordare brută

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$D[i][j] \to \min(v_i, \ldots v_j)$

$O(N^2 + M)$
↓
pt. Q

SPARSE TABLE
↓
fol. la orice funcții asociative
↕
el. disjuncte
și idempotente

$RMQ[i][j] = \min(v_j, v_{j+1}, \ldots v_{j+2^i-1})$

$RMQ[0][j] = v_j$

$RMQ[1][j] = \min(v_j, v_{j+1})$

$RMQ[i][j] = [j, j+2^i-1]$

$= [j, j+2^{i-1}-1] \cup [j+2^{i-1}, j+2^i-1]$

$\min(RMQ[i-1][j], RMQ[i-1][j+2^i-1])$

$RMQ[0][j] = v_j$
for $i = 1, \log N$
  $j = 1, n$
    $RMQ[i][j] = \min(\ldots)$

$[3,7] = \underbrace{[3,6]}_{RMQ(2)(3)} \cup \underbrace{\{7\}}_{RMQ(0)(7)}$

$[3,15] = \underbrace{[3,10]}_{8} \cup \underbrace{[11,14]}_{4} \cup \underbrace{[15,15]}_{1}$

$O(N\log N + M\log N)$
↓ preprocesare
↓ Q

funcții idempotente : $F(F(\ldots F(x))) = y$ mereu

$[a,b] = [a, a+2^p-1] \cup [b-2^p+1, b]$

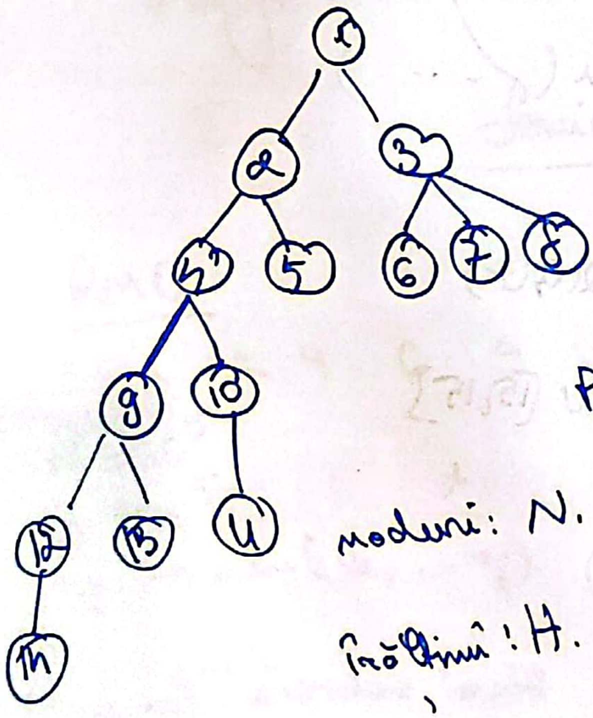
p maxim a.î. $2^p \le b-a+1$

$p = \lfloor \log_2(b-a+1) \rfloor$

$RMQ[p][a] \cup RMQ[p][b-2^p+1]$

$\log_2(i) = \log_2\left(\frac{i}{2}\right)+1$ → complexitate de preprocesare de 1

$\Rightarrow O(N\log N + M)$



LCA(a,b)
lowest common ancestor

* binary lifting

Parcurgere Euler:

noduri: N. 1 2 4 ... 14 12 9 (curs)

înălțimi: H. 1 2 3 4 5 ... 6 5 4 ....

min H (dintre primele apariții ale nodurilor în N)

lambda closure

1) Se dă un arbore bin. de căutare.

$O(n) \rightarrow$ parc. inordine SRD

↓

dacă e ord. cresc. → → e AB

DFS sau parcurgere în adâncime a arborelui.

$MIN[i]$ = nodul minim al subarborelui cu răd. în i.

$MAX[i]$ = — " —

↓ verif. la întoarc. din recursivitate.

---

1) Fie v un șir de nr. întregi și s un nr. întreg.

Să se găsească (i, j) a.î. $v_i + v_j = s$.

MAP

[redacted] MAP

UNORDERED

$O(n^2)$

$O(n \log n)$: $v_1 < v_2 < \ldots < v_n$

$HASH(i) \begin{cases} 1, & \text{dacă se află în mulțime} \\ 0, & \text{altfel} \end{cases}$

↓

$O(1)$

↓ $O(n)$

10^9+7, 666103

Metode de Hashing:

1) met. cu modulo ; → $p = 7)$

L_0 (nr. care dau 0 rest. la împ. 7)
p

⋮

L_6

p lungu, nr. prim

și după le extragem.