

Laborator 1

- 1) A n -size vector a is circular-sorted if, for some index i between 0 and $n-1$, the vector $\{a[i], a[i+1], \dots, a[n-1], a[0], a[1], \dots, a[i-1]\}$ is sorted.

Example: $a[] = \{4, 5, 6, 0, 1, 2, 3\}$ is circular sorted (take $i=3$).

- a. Write a function `int minCirc(vector<int>& a)` which takes as input a circular-sorted vector and outputs the minimum element of a .

Complexity: $O(\log(n))$, where $n = a.size()$

- b. Write a function `int firstCirc(vector<int>& a, int e)` which takes as input a circular-sorted vector and an integer e . It outputs either -1 if e is not present in the vector, or the least index i such that $a[i] = e$.

Complexity: $O(\log(n))$, where $n = a.size()$

- 2) A vector is k -almost sorted if it could be transformed into a sorted vector by moving every elements from at most k positions.

Example: a 0-almost sorted vector is a sorted vector. The vector $a[] = \{1, 0, 3, 2, 5, 4\}$ is 1-almost sorted. The vector $b[] = \{3, 2, 1, 4, 5, 6, 9, 8, 7\}$ is 2-almost sorted.

Write a function `int lastAlmost(vector<int>& a, int k, int e)` which takes as input a vector a , an integer k such that a is k -almost sorted, and an integer value e . It outputs either -1 if e is not present in the vector, or the largest index i such that $a[i] = e$.

Complexity: $O(k \cdot \log(n))$, where $n = a.size()$

- 3) A vector is alternate-sorted if the sub-vector of all elements at even positions (i.e., $\{a[0], a[2], a[4], \dots\}$) is sorted by increasing values and the sub-vector of all elements at odd positions (i.e., $\{a[1], a[3], a[5], \dots\}$) is sorted by decreasing values.

Write a function `int existsAlternate(vector<int>& a, int e)` which takes as input an alternate-sorted vector a and an integer value e . It outputs -1 if e is not present in the vector, or any index i such that $a[i] = e$.

Complexity: $O(\log(n))$, where $n = a.size()$

- 4) A vector is bitonic if it can be partitioned into a vector sorted by increasing values and a vector sorted by decreasing values. The peak of the vector is its largest element.

Example: $a[] = \{1, 2, 3, 4, 5, 4, 3, 2, 1\}$ is bitonic and its peak equals 5.

- a. Write a function `int peak(vector<int>& a)` that takes as input a bitonic vector. Its output is any position in the vector that is equal to its peak.

Example: if the input is $a[] = \{1, 2, 3, 4, 5, 4, 3, 1\}$ then the output must be 4.

Complexity: $O(\log(n))$ where $n = a.size()$.

- b. Write a function `int existsBitonic(vector<int>& a, int e)` that takes as value a bitonic vector a and an integer value e . It outputs -1 if e is not present in the vector, or any index i such that $a[i] = e$.

Complexity: $O(\log(n))$, where $n = a.size()$

- 5) Given a vector a , a threshold-sum query on a requires as input a triple (i, j, e) . Its output consists of the sum of all elements greater than e between positions i and j .

Example: if $a[] = \{0, 3, 44, 5, 11, 2, 1, 6, 78, 9\}$ and the query input is $(3, 7, 4)$, then the query only considers the sub-vector $a[3, 7] = \{5, 11, 2, 1, 6\}$. It outputs $55 + 11 + 6 = 72$.

We define a query as follows:

```
struct query { int i, j, e; };
```

Write a function

```
vector<int> threshold-sum(vector<int>& a, vector<query>& q);
```

Its output is a vector of size `q.size()` whose entries are the outputs for each query in vector q .

Complexity: $O(N\sqrt{N} \cdot \log(N))$, where $N = a.size() + q.size()$

- 6) Implement MergeSort, HeapSort and Quicksort.

Link: <https://www.infoarena.ro/problema/algsort>

- 7) Implement RadixSort.

Link: <https://www.infoarena.ro/problema/radixsort>