

# Statistical Methods for Machine Learning

## Assignment 2

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### II.1 Classification

#### II.1.1 Linear discriminant analysis

We have implemented the LDA algorithm according to the slides from the Linear Classification lecture. We also checked with the MATLAB *predict* function from the *ClassificationDiscriminant* class and we have similar results with only a  $10^{-3}$  error. As expected the train error is lower than the test error

$$train_{ERR} = 0.1500$$

$$test_{ERR} = 0.2105$$

#### II.1.2 Linear discriminant analysis

We normalized both data sets and applied the LDA algorithm. We noticed that we get the same results as the non-normalized data. This would imply that the normalization has no effect on the accuracy of the LDA classifier. This happens because in

$$\delta_k(x) = x^T \Sigma^{-1} \mu_k - 1/2 \mu_k^T \Sigma^{-1} \mu_k + \ln Pr(Y = C_k)$$

the data points are multiplied by the covariance inverse and the mean is subtracted, thus doing a normalization inside the classifier. If we would normalize the data before applying the LDA classifier, the mean is 0 and the covariance is  $I$  which would imply that we obtain the same results as we did with the non normalized data set.

### II.1.3 Bayes optimal classification and probabilistic classification

For the given problem, we have the hypothesis class

$$H = \{h_0(x) = 0, h_1(x) = 1\}$$

because there is only one element in the input space and possible values are 0 or 1. The Bayes optimal classifier is the hypothesis for which we have the minimal risk:  $\min(1 * 1/4, 1 * 3/4) = 1/4$  which corresponds to the risk of hypothesis  $h_1(x)$ . The risk of the classifier is the sum of the risks for each classifier.

$$p(y = 0, h(x) = 1) = p(y = 0 | h(x) = 1)p(h(x) = 1) = 0.25 * 0.75 = 0.1875$$

$$p(y = 1, h(x) = 0) = p(y = 1 | h(x) = 0)p(h(x) = 0) = 0.75 * 0.25 = 0.1875$$

$$Rp(h) = 0.1875 + 0.1875 = 0.375$$

As a conclusion, the risk of this classifier is worse than using the Bayes optimal classifier.

## II.2 Regression: Sunspot Prediction

### II.2.1 Maximum likelihood solution

We used 3.15 and 3.16 to obtain the construct the design matrixes and train each of these models on the training set by finding the maximum likelihood estimate for the 3 selections. We have plotted for each selection the measured values from the test set and the predicted ones from our algorithm. As we can see from the 3 graphs the best results are obtained from considering all parameters.

The RMSs for each selection are:

$$RMS_1 = 35.4651$$

$$RMS_2 = 28.8398$$

$$RMS_3 = 18.7700$$

For the 2<sup>nd</sup> selection we have plotted the predicted values shown in Figure 4. Because we have only one set of parameters we have only a 1<sup>st</sup> degree equation which is shown as a straight line.

### II.2.2 Maximum a posteriori solution

We have implemented the Maximum a posteriori algorithm using (3.52-3.54) equations. We used  $\beta = 1$  and we chose  $\alpha$  with values between  $10^{-10}$  and  $10^{10}$ . As we can see from Figure 5 with the values of RMS spanned across our alpha interval the best outcome is set by using Selection 3. We will analyze only the  $\alpha$  values for selection 3 further. Between  $10^{-10}$  and  $10^{-2}$  we have the same RMS as the Maximum likelihood solution and for  $10^{-1}$  the RMS is lower and the best estimation we get from  $\alpha$  set to 10. Almost the same happens with the other selections, only on different  $\alpha$  values.

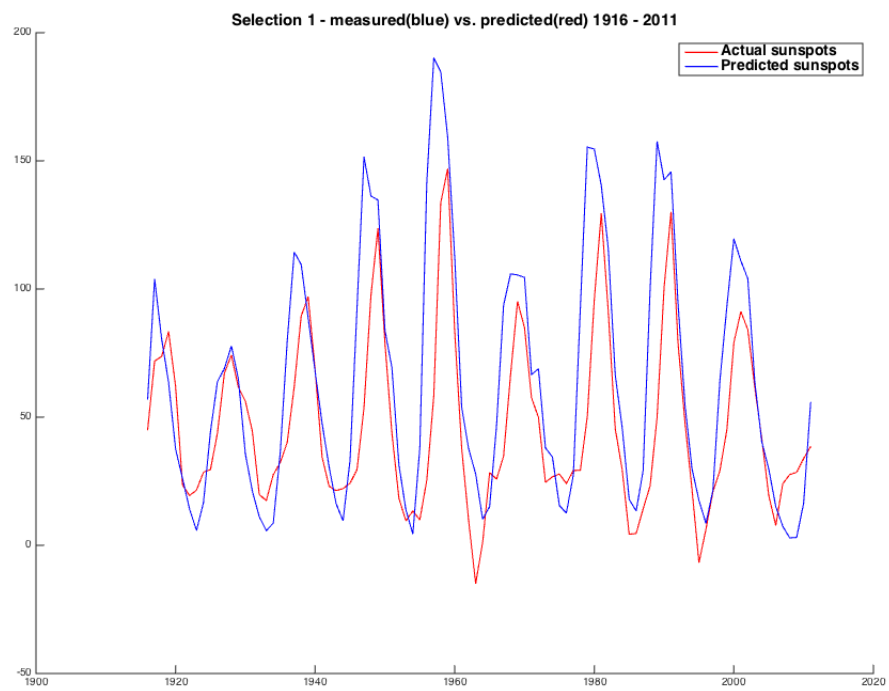


Figure 1: Selection 1 - measured(blue) vs. predicted(red) 1916 - 2011

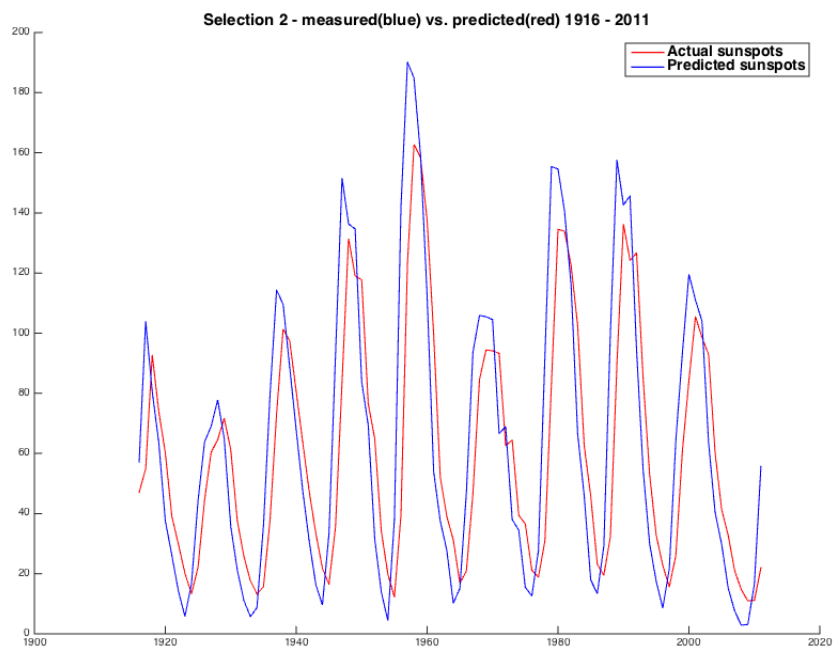


Figure 2: Selection 3 - measured(blue) vs. predicted(red) 1916 - 2011

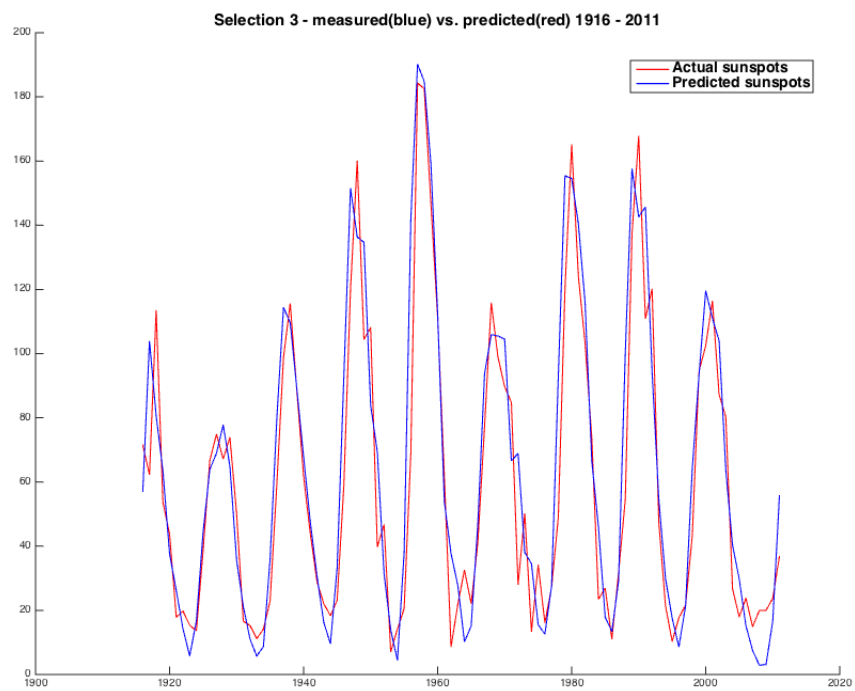


Figure 3: Selection 3 - measured(blue) vs. predicted(red) 1916 - 2011

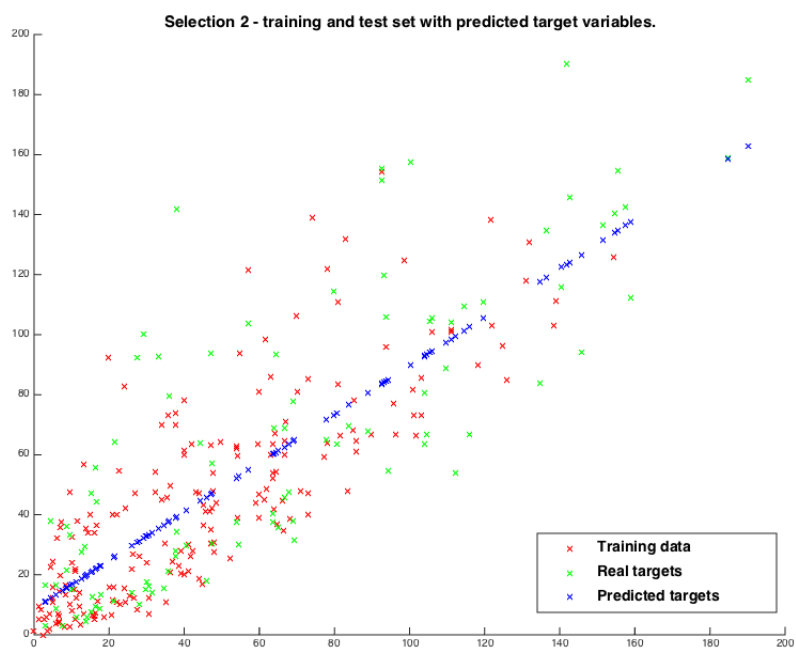


Figure 4: Predicted values for Selection 2

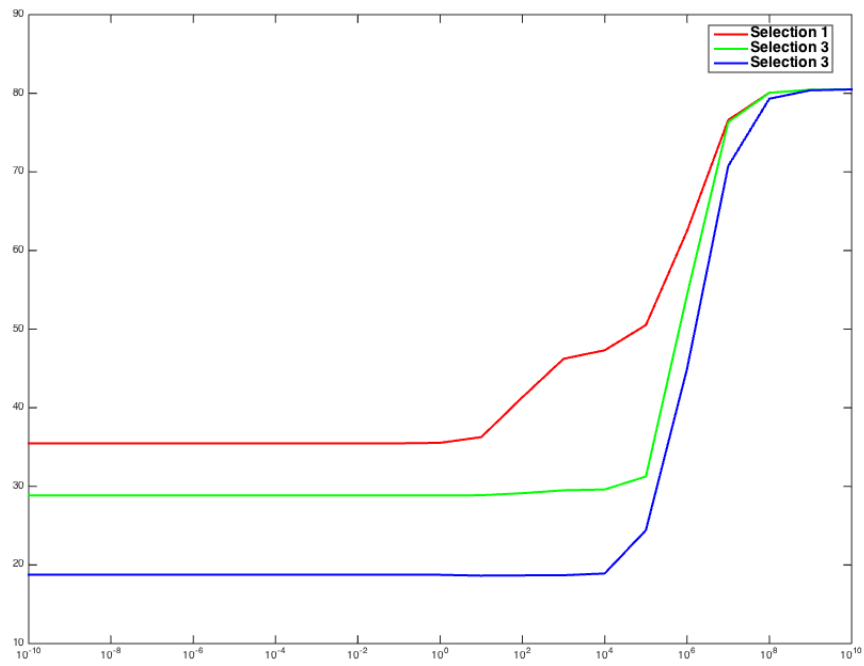


Figure 5: RMS error plotted over alpha values between  $10^{-10}$  and  $10^{10}$