Temă :

$M_{B''B'} = ?$

Metoda I

$B' = \{e_1'(1,1,0), e_2'(1,0,1), e_3'(1,0,-1)\}$

$B'' = \{e_1''(1,0,0), e_2''(1,1,0), e_3''(1,1,1)\}$

$e_1' = \alpha e_1'' + \beta e_2'' + \gamma e_3''$

$$\begin{pmatrix}1\\1\\0\end{pmatrix} = \alpha \cdot \begin{pmatrix}1\\0\\0\end{pmatrix} + \beta \cdot \begin{pmatrix}1\\1\\0\end{pmatrix} + \gamma \cdot \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$\Leftrightarrow \begin{cases}\alpha + \beta + \gamma = 1 \Leftrightarrow \alpha = 0\\ \beta + \gamma = 1 \Leftrightarrow \beta = 1\\ \gamma = 0\end{cases}$$

$$\Rightarrow e_1' = \begin{pmatrix}0\\1\\0\end{pmatrix}$$

$e_2' = \alpha e_1'' + \beta e_2'' + \gamma e_3''$

$$\begin{pmatrix}1\\0\\1\end{pmatrix} = \alpha \cdot \begin{pmatrix}1\\0\\0\end{pmatrix} + \beta \cdot \begin{pmatrix}1\\1\\0\end{pmatrix} + \gamma \cdot \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$\begin{cases}\alpha + \beta + \gamma = 1 \Leftrightarrow \alpha = 1\\ \beta + \gamma = 0 \Leftrightarrow \beta = -1\\ \gamma = 1\end{cases}$$

$$\Rightarrow e_2' = \begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$e_3' = \alpha e_1'' + \beta e_2'' + \gamma e_3''$

$$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \alpha \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \gamma \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} \alpha + \beta + \gamma = 1 & \Leftrightarrow \alpha = 1 \\ \beta + \gamma = 0 & \Leftrightarrow \beta = 1 \\ \gamma = -1 \end{cases}$$

$$e_3' = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$M_{B'B''} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

Metoda II

$M_{B''B'} = ?$

$M_B \to M_{B''}$

$M_{B''} \xrightarrow[B''^{-1}]{} M_B \; , \; M_B \xrightarrow[B']{} M_{B'} \Rightarrow M_{B''B'} = B''^{-1} \cdot B'$

$B''^{-1} = ?$

$$B'' = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\det B = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1 + 0 + 0 - 0 - 0 - 0 = 1 \neq 0$$

$\Rightarrow \exists B''^{-1}$

$$B^{u\,T} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$B^{u*} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B^{u-1} = 1 \cdot \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B^{u-1} \cdot B' = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$M_{B^u B^*} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

B'