DPC for IHM

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1. NMPC formulation

1.1. IHM1

$$\begin{aligned}
\min \quad & \sum_{k=0}^{N_f-1} q_{\mathrm{XY}}\left(X_k - X_k^{\mathrm{ref}}\right)^2 + q_{\mathrm{XY}}\left(Y_k - Y_k^{\mathrm{ref}}\right)^2 + q_\varphi\left(\varphi_k - \varphi_k^{\mathrm{ref}}\right)^2 + q_v\left(v_k - v_k^{\mathrm{ref}}\right)^2 + q_\delta \delta_k^2 + q_T T_k^2 \\
\text{s.t.} \quad & x_{k+1} = f(x_k, u_k),\ k = 0, ..., N_f - 1 \\
& -T_{\max} \leq T_k \leq T_{\max},\ k = 0, ..., N_f - 1 \\
& -\delta_{\max} \leq \delta_k \leq \delta_{\max},\ k = 0, ..., N_f - 1
\end{aligned}$$

where $x = (X, Y, \varphi, v)^T$ denotes the state of the system, $u = (T, \delta)^T$ its control input, and f the discretized dynamics coming from the following ODE:

$$\begin{aligned}
\dot{X} &= v \cos(\varphi + \beta) \\
\dot{Y} &= v \sin(\varphi + \beta) \\
\dot{\varphi} &= v \frac{\sin(\beta)}{L} \\
\dot{v} &= \frac{F_x}{m}
\end{aligned}$$

where $\beta = \frac{1}{2}\delta$ denotes the kinematic slip angle, L the wheelbase, m the mass of the car, and $F_x = C_m T - C_{r0} - C_{r1} v - C_{r2} v^2$ the longitudinal force applied to the car.

1.2. IHM1.5

We only replace the costs on the XY by rotating the error accordingly to obtain longitudinal and lateral errors:

$$\begin{aligned}
e_{\mathrm{lon},k} &= \cos\left(\varphi_k^{\mathrm{ref}}\right)\left(X_k - X_k^{\mathrm{ref}}\right) + \sin\left(\varphi_k^{\mathrm{ref}}\right)\left(Y_k - Y_k^{\mathrm{ref}}\right) \\
e_{\mathrm{lat},k} &= -\sin\left(\varphi_k^{\mathrm{ref}}\right)\left(X_k - X_k^{\mathrm{ref}}\right) + \cos\left(\varphi_k^{\mathrm{ref}}\right)\left(Y_k - Y_k^{\mathrm{ref}}\right)
\end{aligned}$$

and then adding the cost $q_{\mathrm{lon}} e_{\mathrm{lon},k}^2 + q_{\mathrm{lat}} e_{\mathrm{lat},k}^2$.

Then only the weight matrix changes, but the cost function remains a linear least-squere loss. In particular, the weight matrix is different at each stage (because it depends on φ^{ref}).

We can also use these newly defined lateral error to enforce (approximate) track constraints as

$$e_{\mathrm{lat,min,k}} \leq e_{\mathrm{lat},k} \leq e_{\mathrm{lat,max,k}}$$

so in the end the OCP reads

$$\begin{aligned}
\min \quad & \sum_{k=0}^{N_f-1} q_{\mathrm{lon}} e_{\mathrm{lon},k}^2 + q_{\mathrm{lat}} e_{\mathrm{lat},k}^2 + q_\varphi\left(\varphi_k - \varphi_k^{\mathrm{ref}}\right)^2 + q_v\left(v_k - v_k^{\mathrm{ref}}\right)^2 + q_\delta \delta_k^2 + q_T T_k^2 \\
\text{s.t.} \quad & x_{k+1} = f(x_k, u_k),\ k = 0, ..., N_f - 1, \\
& e_{\mathrm{lat,min,k}} \leq e_{\mathrm{lat},k} \leq e_{\mathrm{lat,max,k}},\ k = 0, ..., N_f, \\
& -T_{\max} \leq T_k \leq T_{\max},\ k = 0, ..., N_f - 1, \\
& -\delta_{\max} \leq \delta_k \leq \delta_{\max},\ k = 0, ..., N_f - 1\ .
\end{aligned}$$

1.3. DPC

inputs: current state and state reference