

# DPC for IHM

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## 1. NMPC formulation

### 1.1. IHM1

$$\begin{aligned} \min \quad & \sum_{k=0}^{N_f-1} q_{XY} (X_k - X_k^{\text{ref}})^2 + q_{XY} (Y_k - Y_k^{\text{ref}})^2 + q_{\varphi} (\varphi_k - \varphi_k^{\text{ref}})^2 + q_v (v_k - v_k^{\text{ref}})^2 + q_{\delta} \delta_k^2 + q_T T_k^2 \\ \text{s.t.} \quad & x_{k+1} = f(x_k, u_k), \quad k = 0, \dots, N_f - 1 \\ & -T_{\max} \leq T_k \leq T_{\max}, \quad k = 0, \dots, N_f - 1 \\ & -\delta_{\max} \leq \delta_k \leq \delta_{\max}, \quad k = 0, \dots, N_f - 1 \end{aligned}$$

where  $x = (X, Y, \varphi, v)^T$  denotes the state of the system,  $u = (T, \delta)^T$  its control input, and  $f$  the discretized dynamics coming from the following ODE:

$$\begin{aligned} \dot{X} &= v \cos(\varphi + \beta) \\ \dot{Y} &= v \sin(\varphi + \beta) \\ \dot{\varphi} &= v \frac{\sin(\beta)}{L} \\ \dot{v} &= \frac{F_x}{m} \end{aligned}$$

where  $\beta = \frac{1}{2}\delta$  denotes the kinematic slip angle,  $L$  the wheelbase,  $m$  the mass of the car, and  $F_x = C_m T - C_{r0} - C_{r1}v - C_{r2}v^2$  the longitudinal force applied to the car.

### 1.2. IHM1.5

We only replace the costs on the XY by rotating the error accordingly to obtain longitudinal and lateral errors:

$$\begin{aligned} e_{\text{lon},k} &= \cos(\varphi_k^{\text{ref}}) (X_k - X_k^{\text{ref}}) + \sin(\varphi_k^{\text{ref}}) (Y_k - Y_k^{\text{ref}}) \\ e_{\text{lat},k} &= -\sin(\varphi_k^{\text{ref}}) (X_k - X_k^{\text{ref}}) + \cos(\varphi_k^{\text{ref}}) (Y_k - Y_k^{\text{ref}}) \end{aligned}$$

and then adding the cost  $q_{\text{lon}} e_{\text{lon},k}^2 + q_{\text{lat}} e_{\text{lat},k}^2$ .

Then only the weight matrix changes, but the cost function remains a linear least-square loss. In particular, the weight matrix is different at each stage (because it depends on  $\varphi^{\text{ref}}$ ).

We can also use these newly defined lateral error to enforce (approximate) track constraints as

$$e_{\text{lat},\min,k} \leq e_{\text{lat},k} \leq e_{\text{lat},\max,k}$$

so in the end the OCP reads

$$\begin{aligned} \min \quad & \sum_{k=0}^{N_f-1} q_{\text{lon}} e_{\text{lon},k}^2 + q_{\text{lat}} e_{\text{lat},k}^2 + q_{\varphi} (\varphi_k - \varphi_k^{\text{ref}})^2 + q_v (v_k - v_k^{\text{ref}})^2 + q_{\delta} \delta_k^2 + q_T T_k^2 \\ \text{s.t.} \quad & x_{k+1} = f(x_k, u_k), \quad k = 0, \dots, N_f - 1, \\ & e_{\text{lat},\min,k} \leq e_{\text{lat},k} \leq e_{\text{lat},\max,k}, \quad k = 0, \dots, N_f, \\ & -T_{\max} \leq T_k \leq T_{\max}, \quad k = 0, \dots, N_f - 1, \\ & -\delta_{\max} \leq \delta_k \leq \delta_{\max}, \quad k = 0, \dots, N_f - 1. \end{aligned}$$

### 1.3. DPC

inputs: current state and state reference