Maximization of a Unimodal Function using Hill Climbing Algorithm with Best and First Improvements

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1 Abstract

In this paper, we conducted a case study aimed at maximizing a given function within a specific interval using both the first improvement and best improvement variants of the Hill Climbing algorithm. The function under examination is $f(x) = x^3 - 60x^2 + 900x + 100$, with x constrained to the integer interval [0, 31]. This function is unimodal, featuring a single maximum point.

Our investigation delves into the intricacies of these two algorithmic variants, with a keen focus on diverse metrics, including the function landscape, the attraction basin of all local maximum points, the average number of neighboring solutions evaluated at each point in the function's domain, and the average number of neighbors traversed to find the maximum point, all across multiple runs.

In conclusion, we observed that the best improvement variant explores a larger solution space compared to the first improvement variant, as it evaluates each neighbor. However, it requires a slightly smaller number of neighbors to reach the local maximum. Furthermore, we provide insights into the attraction basins for each case and offer various observations.

2 Introduction

This study aims to investigate the Hill Climbing method, specifically examining its first improvement and best improvement variants. Our focus is on the maximization of the function $f(x) = x^3 - 60x^2 + 900x + 100$, within the integer interval [0, 31], with a single maximum point at x = 10.

Given the small function domain, solutions are represented using 5 bits, encompassing integers from 0 to 31. We employ a mapping technique to understand attraction basins, revealing the relationship between initial starting points and convergence towards local maxima. Since we run the algorithm for every integer value, the best improvement variant of the Hill Climbing algorithm becomes a deterministic approach, consistently yielding the same results.

Our investigation delves into the function landscape for each algorithmic variant, examining key metrics and analyzing the attraction basins associated with each approach.

3 Methods

For both variants of the Hill Climbing algorithm, we adopted a binary representation with a fixed length of 5 bits, aligning with the nature of the function domain, which comprises integer values. We meticulously examined all 32 values ranging from 0 to 31 and encoded them into their binary counterparts. Subsequently, we applied the Hill Climbing algorithm with a minor modification.

In our approach, we utilized an evaluation function to calculate the value of the objective function. We generated neighboring solutions by flipping individual bits, always selecting the one resulting in the most significant improvement according to the chosen improvement method. We diligently updated the best solution encountered throughout the process. Unlike conventional Hill Climbing, we omitted the random initialization of the bitstring, starting from the encoded binary representations.

To maintain thorough records of our exploration, we employed a counter to track the number of evaluations required for convergence. Additionally, in each variant, we maintained a count of the total number of neighbors explored for each candidate point, further providing insights into the search process.

Moreover, we employed a mapping technique to analyze and visualize attraction basins, shedding light on how different starting points influence the convergence towards local maxima.

4 Function

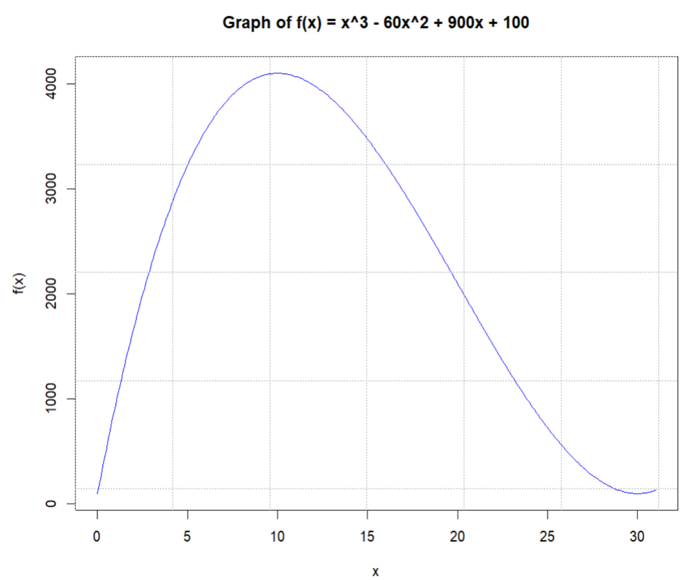
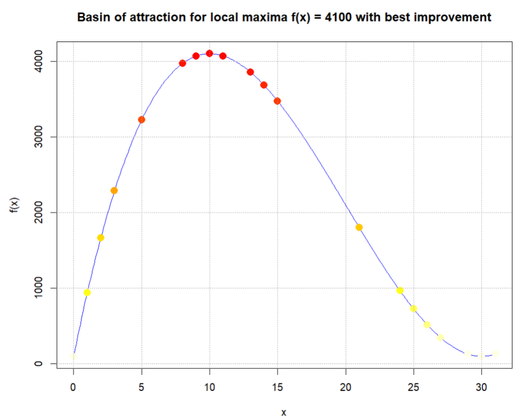
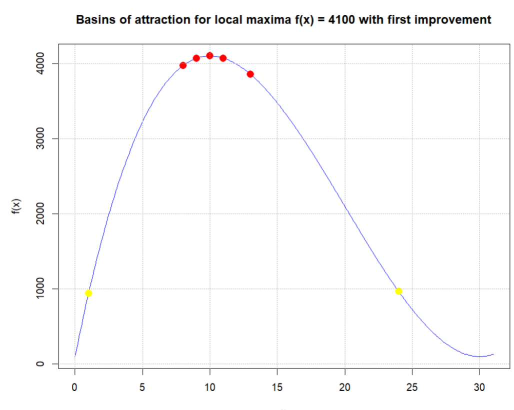


Figure 1: $f(x) = x^3 - 60x^2 + 900x + 100$
$0 \leq x \leq 31$
global maximum: $f(x) = 4100$ when $x = 10$

5 Experimental Results

	Avg. of Scanned Neighbours	Avg. of Choosed Neighbours	Runs
HCBI	348.0	2.2	1
HCFI	283.347	2.2781	5000





Basins of Attraction - Best Improvement

Local Optimum	1 0 0 0 0 (3236)	0 0 1 1 1 (3803)	0 1 1 0 0 (3988)	0 1 0 1 0 (4100)
Basins	1 0 0 0 0 (16)	0 0 1 1 0 (6)	0 0 1 0 0 (4)	0 0 0 0 0 (0)
	1 0 0 0 1 (17)	0 0 1 1 1 (7)	0 1 1 0 0 (12)	0 0 0 0 1 (1)
	1 0 0 1 0 (18)	1 0 1 1 0 (22)	1 1 1 0 0 (28)	0 0 0 1 0 (2)
	1 0 0 1 1 (19)	1 0 1 1 1 (23)		0 0 0 1 1 (3)
	1 0 1 0 0 (20)			0 0 1 0 1 (5)
				0 1 0 0 0 (8)
				0 1 0 0 1 (9)
				0 1 0 1 0 (10)
				0 1 0 1 1 (11)
				0 1 1 0 1 (13)
				0 1 1 1 0 (14)
				0 1 1 1 1 (15)
				1 0 1 0 1 (21)
				1 1 0 0 0 (24)
				1 1 0 0 1 (25)
				1 1 0 1 0 (26)
				1 1 0 1 1 (27)
				1 1 1 0 1 (29)
				1 1 1 1 0 (30)
				1 1 1 1 1 (31)

Basins of Attraction - First Improvement

Local Optimum	1 0 0 0 0 (3236)	0 0 1 1 1 (3803)	0 1 1 0 0 (3988)	0 1 0 1 0 (4100)
Basins	0 0 0 1 0 (2)	0 0 0 0 1 (1)	0 0 0 0 0 (0)	0 1 0 0 1 (9)
	0 0 0 1 1 (3)	0 0 1 0 0 (4)	0 0 1 0 1 (5)	0 1 0 1 0 (10)
	1 0 0 0 0 (16)	0 0 1 1 1 (7)	0 0 1 1 0 (6)	0 1 0 1 1 (11)
	1 0 0 0 1 (17)	0 1 1 1 1 (15)	0 1 1 0 0 (8)	1 0 1 1 0 (22)
	1 0 0 1 0 (18)		0 1 1 0 0 (12)	1 1 0 1 0 (26)
	1 0 0 1 1 (19)		0 1 1 0 1 (13)	1 1 0 1 1 (27)
	1 0 1 0 1 (21)		0 1 1 1 0 (14)	1 1 1 1 1 (31)
	1 1 0 0 0 (24)		1 0 1 0 0 (20)	
	1 1 0 0 1 (25)		1 0 1 1 1 (23)	
	1 1 1 0 0 (28)		1 1 1 1 0 (30)	
	1 1 1 0 1 (29)			

6 Experimental Description

In our experimental setup, we meticulously investigated both the first improvement and best improvement variants of the Hill Climbing algorithm. We conducted an extensive analysis comprising 5000 iterations on average to thoroughly explore the solution space.

To ensure comprehensive coverage of the function's domain, we employed a step size of 1, given the integral nature of the problem. This choice allowed us to navigate through integer values effectively, enabling a detailed examination of neighboring points and enhancing the accuracy of our results.

Throughout the experiments, we consistently recorded essential metrics, providing insights into the performance and convergence characteristics of both the first improvement and best improvement Hill Climbing variants. This rigorous approach ensures robust and reliable findings.

7 Comparison and Interpretation

In our comparative analysis of the first improvement and best improvement variants of the Hill Climbing algorithm, we conducted 5000 iterations to explore their behaviors. The key observation lies in the fact that the algorithm, both in its first and best improvement forms, identifies values such as 3236, 3803.0, 3988.0, and 4100.0 as potential local maxima. However, in practical terms, only 4100 is the true local maximum, as the variable selection ensures that it prevails across the entire domain [0, 31].

A noteworthy aspect of our findings is the consistency in the attraction basins for Hill Climbing Best Improvement. The algorithm invariably leads to the single local maximum at 4100.0. However, for Hill Climbing First Improvement, while the attraction basins exhibit variation, the identified local maxima remain the same. These local maxima, specifically x = 16, x = 7, x = 12, and x = 10, will always be part of their respective attraction basins. This permanence arises from the absence of neighbors with superior function values within these basins.

Furthermore, we observed that Hill Climbing Best Improvement consistently explores more neighbors, averaging 348 neighbors per run, and typically requires 2.2 neighbors visited to reach the local maximum at 4100. In contrast, Hill Climbing First Improvement explores an average of 283.347 neighbors and usually requires approximately 2.278 neighbors visited to reach the same local maximum.

In summary, our observations reveal that Hill Climbing Best Improvement covers a broader exploration space, consistently visiting more neighbors, yet requiring fewer neighbors to reach the practical local maximum at 4100. On the other hand, Hill Climbing First Improvement traverses slightly more neighbors but, in practice, scans less neighbors to achieve the same result.

8 Conclusion

Our study of the Hill Climbing algorithm's first improvement and best improvement variants revealed that both methods successfully identified the same local maximum in the given function landscape. Hill Climbing First Improvement traverses slightly more neighbors but, in practice, scans less neighbors to achieve the same result.

References

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