Heuristic vs. Deterministic Function Minimization

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1 Abstract

In this paper, we explore and compare two established problem-solving methods, namely heuristic and deterministic approaches, with the objective of identifying their respective strengths and weaknesses. Our study focuses on solving the global minimum optimization problem for functions with an arbitrary number of variables. We investigate and analyse the practical performance of these methods, presenting valuable insights into their execution times and the quality of solutions they yield. The paper's findings shed light on the suitability of these methods in real-world applications, addressing the need for informed decision-making when selecting a problem-solving approach.

2 Introduction

In recent years, the debate between heuristic and deterministic algorithms has gained prominence. Heuristic algorithms have witnessed substantial improvements, thanks to technological advancements. In numerous real-world scenarios, determining the global minimum of a multi-variable function is essential, yet often quite challenging.

This study aims to explore two distinct methods for locating global minima or maxima of functions with any number of variables. We will compare the performance of heuristic and deterministic approaches across four benchmark functions, including Rastrigin's and Michalewicz's functions. Our analysis will encompass both two-dimensional and ten-dimensional versions of these functions, with results computed in Python.

Through rigorous experimentation, we will report the minimum, average, and maximum execution times, as well as the best and worst solutions obtained by each method. We will also calculate the average solution quality across multiple runs. By contrasting these outcomes, we intend to highlight notable differences and understand the contributing factors.

This research not only seeks to determine which method is more effective for finding global minima but also discusses the practical implications of these findings. In conclusion, we will summarize the key insights and their relevance in the realm of optimization problems.

3 Methods

In this study, we implemented two methods, one heuristic and the other deterministic, to optimize functions with an arbitrary number of variables. We chose to employ the simplest and most intuitive implementations of both approaches to highlight the fundamental differences and their implications.

For the heuristic approach, we generated parameter sets within the function's domain for both two-dimensional and ten-dimensional cases. Subsequently, we evaluated the function at these coordinates and repeated this process a predefined number of times, serving as the stopping criterion. We recorded the minimum function value obtained during these iterations. Additionally, we measured the execution time (minimum, maximum, and average) and kept track of the overall solutions (best, worst, and average) to facilitate comparisons with the deterministic method.

On the other hand, for the deterministic method, we initialized a vector with a length equal to the number of function parameters, all set to the minimum value within the domain. We then conducted a linear iteration for each parameter, traversing the specified range with a defined step size. Similar to the heuristic approach, we recorded relevant metrics, such as execution times and solution statistics.

These comprehensive experiments allow us to draw meaningful comparisons between the two methods, elucidating the differences and outcomes as requested in the research objectives.

4 Functions

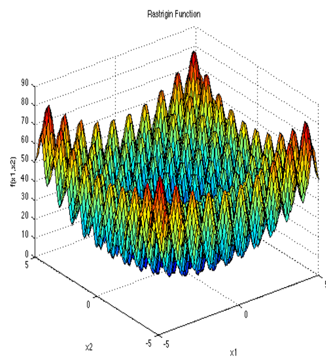

Figure 1: Rastrigin's function: $f_n(x) = 10n + \sum_{i=1}^{n}(x_i^2 - 10cos(2\pi x_i))$
$-5.12 \leq x_i \leq -5.12$
global minimum: $f(x) = 0$
https://www.sfu.ca/ ssurjano/rastr.png [2]

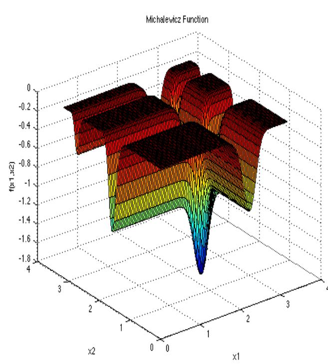

Figure 2: Michalewicz's function: $f_n(x) = -\sum_{i=1}^{n} sin(x_i)(sin\frac{ix_i^2}{\pi})^{20}$
$0 \leq x_i \leq \pi$
global minimum: $f_2(x) = -1.8013, f_{10}(x) = -9.66$
https://www.sfu.ca/ ssurjano/michal.png [3]

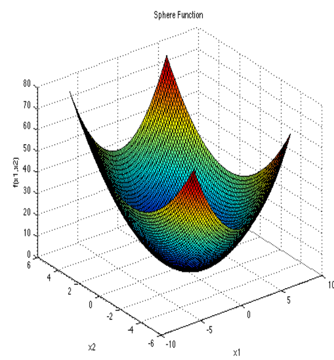

Figure 3: Sphere function: $f(x) = \sum_{i=1}^{D} x_i^2$
$-5.12 \leq x_i \leq 5.12$
global minimum: $f(x) = 0$
https://www.sfu.ca/ ssurjano/spheref.png [4]

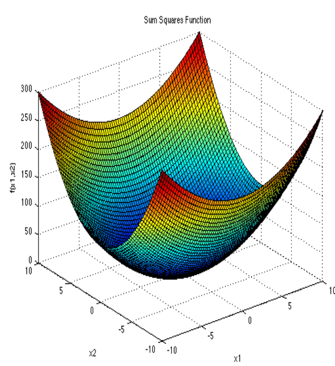

Figure 4: Sum Squares function: $f(x) = \sum_{i=1}^{n} x_i^2$
$-10 \leq x_i \leq 10$
global minimum: $f(x) = 0$
https://www.sfu.ca/ ssurjano/sumsqu.png [5]

5 Experiment Description

In our experimental setup, we conducted a comprehensive analysis using two distinct optimization methods: a heuristic approach and a deterministic linear traversal technique. We aimed to identify the optimal points (maxima or minima) of functions with an arbitrary number of variables, ensuring precision to at least 5 decimal places.

For each method and function, we performed 100 trials, capturing key metrics such as minimum, average, and maximum execution times, as well as the best and worst solutions achieved. In the heuristic approach, each trial comprised 10^5 iterations of random point generation, enabling us to converge towards potential global minima.

For the deterministic approach, we employed a step size of 10^{-3} to efficiently traverse the 2-dimensional function space. However, for the 10-dimensional function, the deterministic method became infeasible due to the astronomical number of operations involved, estimated at 10^{40}.

6 Experimental results

6.1 Heuristic

Rastrigin						
Dimensions	Best Solution	Worst Solution	Average Solution	Minimum Time	Average Time	Maximum Time
2	0.00057	0.18209	0.04798	528.404 ms	541.635 ms	579.575 ms
10	36.565	66.74	57.741	1294.909 ms	1322.204 ms	1458.423 ms

Michalewicz						
Dimensions	Best Solution	Worst Solution	Average Solution	Minimum Time	Average Time	Maximum Time
2	-1.8013	-1.798	-1.8004	566.155 ms	580.774 ms	636 ms
10	-6.222	-4.722	-5.339	1809.52 ms	1849.5332 ms	1956.56 ms

Sphere						
Dimensions	Best Solution	Worst Solution	Average Solution	Minimum Time	Average Time	Maximum Time
2	0.00001	0.00148	0.0003	753.044 ms	892.778 ms	1024.904 ms
10	4.416	11.572	8.167	914.14 ms	961.08 ms	1082.847 ms

Sum Squares						
Dimensions	Best Solution	Worst Solution	Average Solution	Minimum Time	Average Time	Maximum Time
2	0.0	0.0103	0.0018	1387.909 ms	1427.541 ms	1555.575 ms
10	47.307	193.345	137.916	1716.836 ms	1780.626 ms	1932.7 ms

6.2 Deterministic

Rastrigin						
Dimensions	Best Solution	Worst Solution	Average Solution	Minimum Time	Average Time	Maximum Time
2	0.0	0.0	0.0	959.879 s	962.316 s	966.251 s

Michalewicz						
Dimensions	Best Solution	Worst Solution	Average Solution	Minimum Time	Average Time	Maximum Time
2	-1.8013	-1.8013	-1.8013	100.579 s	103.381 s	105.683 s

Sphere						
Dimensions	Best Solution	Worst Solution	Average Solution	Minimum Time	Average Time	Maximum Time
2	0.0	0.0	0.0	1640.775 s	1651.428 s	1662.806 s

Sum Squares						
Dimensions	Best Solution	Worst Solution	Average Solution	Minimum Time	Average Time	Maximum Time
2	0.0	0.0	0.0	7119.834 s	7142.892 s	7170.261 s

7 Comparison and Interpretation

In analyzing the results of our study, we observe notable disparities between the deterministic and heuristic approaches. Specifically, the most striking difference lies in the execution times, with the deterministic method operating in the order of seconds and the heuristic method in milliseconds.

For the 2-dimensional case, the deterministic approach yields precise results, albeit at a considerable time cost. Conversely, the heuristic method produces results with a negligible error ($\varepsilon < 0.05$) close to the true values, making it a practical choice due to its faster computation.

Turning to the 10-dimensional scenario, the heuristic method's error increases due to the vast solution space. Nevertheless, it still maintains reasonable execution times. In contrast, the deterministic approach fails to provide results, rendering the heuristic approach the preferred choice.

8 Conclusion

This study highlights the overarching significance of adopting heuristic approaches over deterministic ones in general. It underscores that, in practical scenarios where time is a critical resource, heuristics tend to yield superior outcomes in terms of both result quality and time efficiency. Consequently, prioritizing heuristic methodologies can lead to more pragmatic and effective problem-solving strategies, offering a valuable alternative to exhaustive deterministic computations.

References

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