Laboratory 4: Solving Differential Equations with MAPLE

1. Find the general solution of the differential equations:

   (a) $2x^2y' = x^2 + y^2$

   (b) $y' = -\frac{x+y}{y}$

   (c) $y' + y\tan x = \frac{1}{\cos x}$

   (d) $y' + \frac{2}{x}y = x^3$

   (e) $y'' + y = \sin x + \cos x$

   (f) $y'' - y = e^{2x}$

   (g) $y'' - y' = \frac{1}{1+e^x}$

2. Solve the following IVPs and plot the solution graph:

   (a) $y' = 1 + y^2$, $y\,(0) = 1$

   (b) $y' = \frac{1}{1-x^2}y + 1 + x$, $y\,(0) = 0$

   (c) $y' - 2y = -x^2$, $y\,(0) = \frac{1}{4}$

   (d) $y'' - 5y' + 4y = 0$, $y(0) = 5$, $y'(0) = 8$;

   (e) $y'' - 4y' + 5y = 2x^2e^x$, $y(0) = 2$, $y'(0) = 3$;

   (f) $y'' + 4y = 4\,(\sin 2x + \cos 2x)$, $y(\pi) = y'(\pi) = 2\pi$;

3. Consider the differential equation

$$y'\,(x) + \frac{k}{x}y\,(x) = x^3,$$

   where $k \in \mathbb{R}$.

   (a) Find the general solution

   (b) For $k = 1$ draw the solution curve.

   (c) For $k = 1$ solve the IVP $\begin{cases} y'\,(x) + \frac{k}{x}y\,(x) = x^3 \\ y(1) = 0 \end{cases}$ and draw the graph of solution

   (d) Use `animate` command to see the dependence of the solution for the IVP $\begin{cases} y'\,(x) + \frac{k}{x}y\,(x) = x^3 \\ y(1) = 0 \end{cases}$ with respect to the parameter k.

4. Find the solution of the following IVP

$$\begin{cases} y'' - y' - 2y = 0 \\ y(0) = a \\ y'(0) = 2 \end{cases}$$

   and the value of the parameter a such that $y(x) \to 0$ as $x \to +\infty$.