```
> eq:=x(n+1)=-1/2*x(n);
```

$$eq := x(n+1) = -\frac{1}{2}\,x(n)$$

```
> ans:=rsolve(eq,x(n));
```

$$ans := x(0)\left(-\frac{1}{2}\right)^{n}$$

```
> limit(ans,n=infinity);
```

$$0$$

```
> rsolve({eq,x(0)=0},x(n))
```

$$0$$

```
> f:=x->x^2-4;
```

$$f := x \rightarrow x^{2} - 4$$

```
> eq:=x(n+1)=f(x(n));
```

$$eq := x(n+1) = x(n)^{2} - 4$$

```
> rsolve(eq,x(n));
```

$$rsolve\left(x(n+1) = x(n)^{2} - 4, x(n)\right)$$

```
> eqp:=solve(x=f(x),x);
```

$$eqp := \frac{1}{2} - \frac{1}{2}\sqrt{17}, \frac{1}{2} + \frac{1}{2}\sqrt{17}$$

```
> D(f)(x);
```

$$2\,x$$

```
> eqp[1];evalf(%);
```

$$\frac{1}{2} - \frac{1}{2}\sqrt{17}$$

$$-1.561552813$$

```
> eqp[2];evalf(%);
```

$$\frac{1}{2} + \frac{1}{2}\sqrt{17}$$

$$2.561552813$$

```
> D(f)(eqp[1]);
```

$$1 - \sqrt{17}$$

```
> evalf(D(f)(eqp[1]));
```

$$-3.123105626$$

```
> abs(%);
```

$$3.123105626$$

```
> D(f)(eqp[2]);
```

$$1 + \sqrt{17}$$

```
> evalf(D(f)(eqp[2]));
```

$$5.123105626$$

```
> x[0]:=0;N:=5;
```

$$x_0 := 0$$

$$N := 5$$

```
> for i from 0 to N-1 do
    x[i+1]:=f(x[i])
  end do;
```

$$x_1 := -4$$

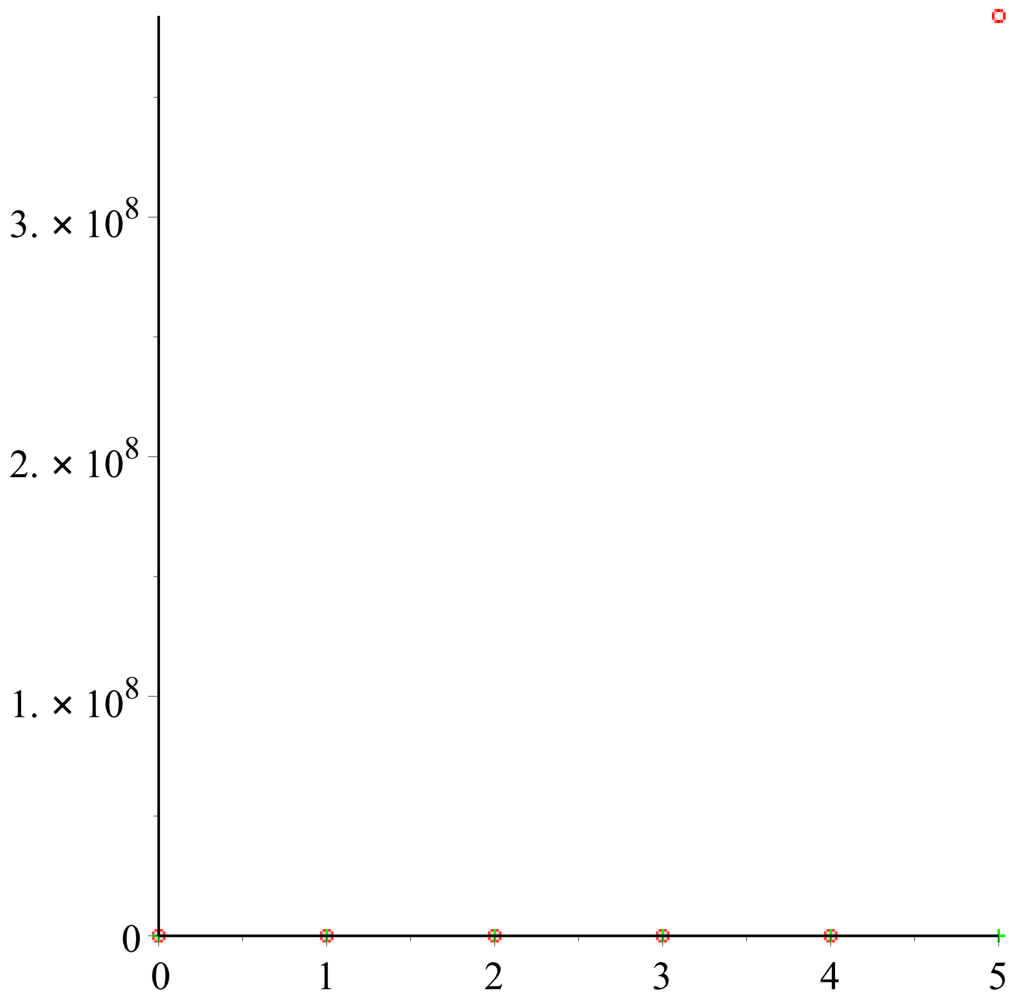
$$x_2 := 12$$

$$x_3 := 140$$

$$x_4 := 19596$$

$$x_5 := 384003212$$

```
> plot([[[n,x[n]]$n=0..N],[[n,eqp[1]]$n=0..N]],style=[point,point],
  symbol=[circle,cross],color=[red,green]);
```




```
> f:=x->2*x/(1+x);
```

$$f := x \rightarrow \frac{2\,x}{x+1}$$

```
> eqp:=solve(x=f(x),x);
```

$$eqp := 0,\, 1$$

```
> D(f)(0);
```

$$2$$

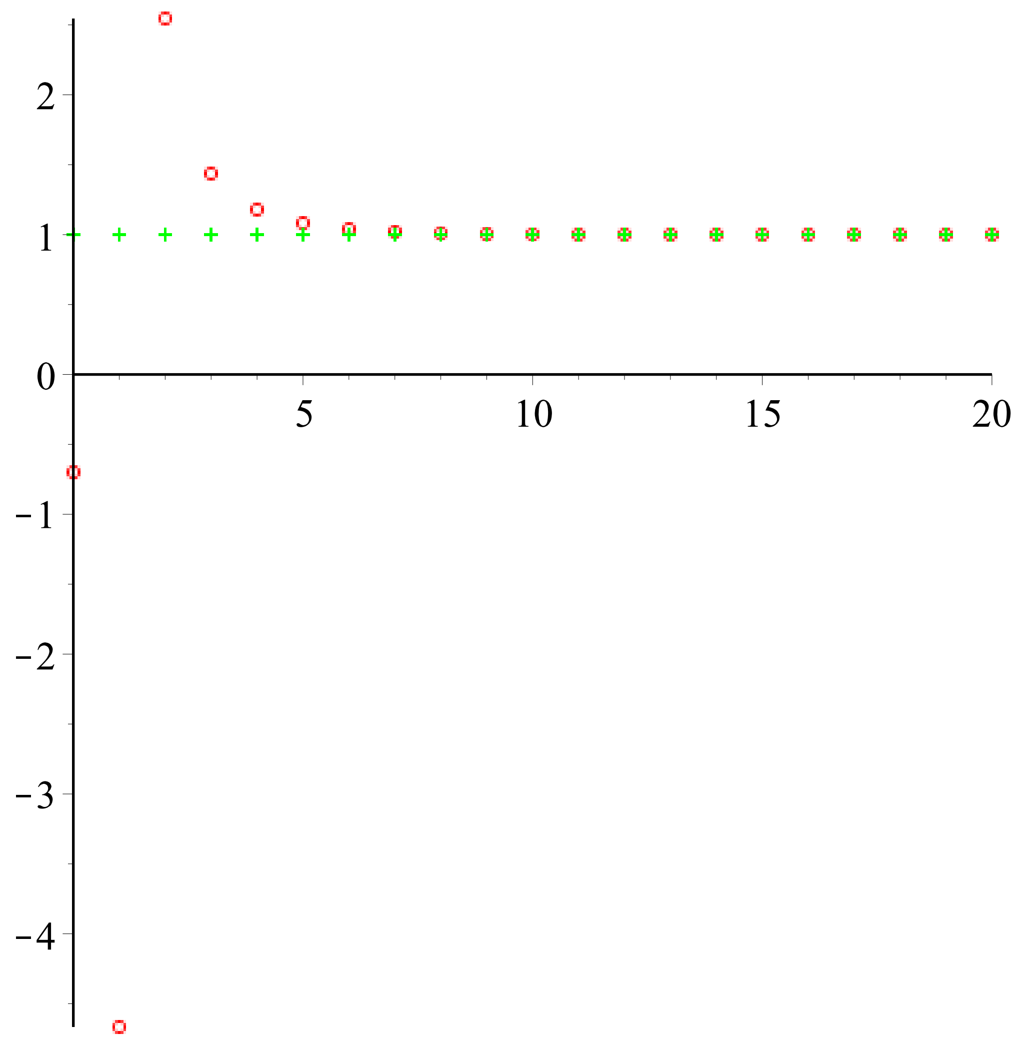
```
> D(f)(1);
```

$$\frac{1}{2}$$

```
> x[0]:=-0.7;N:=20;
```

$$x_0 := -0.7$$
$$N := 20$$

```
> for i from 0 to N-1 do
    x[i+1]:=f(x[i])
  end do:
> plot([[[n,x[n]]$n=0..N],[[n,1]$n=0..N]],style=[point,point],
  symbol=[circle,cross],color=[red,green]);
```



The Schur-Cohn Criterion

For the equation the equation

$$q^2 = p_1 q + p_2$$

we have:

$|q_{1,2}| < 1$ if and only if $|p_1| < 1 - p_2 < 2$

```
> restart;
```

```
> f:=(u,v)->(a*u+b*v*exp(-v))*exp(-u);
```

$$f := (u, v) \to \left(a\, u + b\, v\, \mathrm{e}^{-v}\right) \mathrm{e}^{-u}$$

```
> deq:=x(n+1)=f(x(n),x(n-1));
```

$$deq := x(n+1) = \left(a\, x(n) + b\, x(n-1)\, \mathrm{e}^{-x(n-1)}\right) \mathrm{e}^{-x(n)}$$

```
> eq:=x=f(x,x);
```

$$eq := x = \left(a\, x + b\, x\, \mathrm{e}^{-x}\right) \mathrm{e}^{-x}$$

```
> eqp:=solve(eq,x);
```

$$eqp := 0,\ -\ln\left(\frac{1}{2}\ \frac{-a+\sqrt{a^2+4\, b}}{b}\right),\ -\ln\left(-\frac{1}{2}\ \frac{a+\sqrt{a^2+4\, b}}{b}\right)$$

```
> (-a+sqrt(a^2+4*b))<2*b
```

$$-a + \sqrt{a^2+4\, b} < 2\, b$$

```
> sqrt(a^2+4*b)<a+2*b;
```

$$\sqrt{a^2+4\, b} < 2\, b + a$$

```
> (sqrt(a^2+4*b))^2<(a+2*b)^2;
```

$$a^2 + 4\, b < (2\, b + a)^2$$

```
> (sqrt(a^2+4*b))^2<expand((a+2*b)^2);
```

$$a^2 + 4\, b < a^2 + 4\, a\, b + 4\, b^2$$

```
> 4*a*b+4*b^2-4*b>0;
```

$$0 < 4\, a\, b + 4\, b^2 - 4\, b$$

```
> factor(4*a*b+4*b^2-4*b)>0;
```

$$0 < 4\, b\, (a + b - 1)$$

the second equilibrium point is positive if a+b>1
the third equilibrium point is a complex number

```
> eqp[1];eqp[2];
```

$$0$$

$$-\ln\left(\frac{1}{2}\ \frac{-a+\sqrt{a^2+4\, b}}{b}\right)$$

```
> p1:=D[1](f)(0,0);
```

$$p1 := a$$

```
> p2:=D[2](f)(0,0);
```

$$p2 := b$$

```
> lineq:=y(n+1)=p1*y(n)+p2*y(n-1);
```

$$lineq := y(n+1) = a\, y(n) + b\, y(n-1)$$

```
> chareq:=q^2=p1*q+p2;
```

$$chareq := q^2 = a\, q + b$$

```
> rr:=solve(chareq,q);
```

$$rr := \frac{1}{2}\, a + \frac{1}{2}\sqrt{a^2+4\, b},\ \frac{1}{2}\, a - \frac{1}{2}\sqrt{a^2+4\, b}$$

```
> p1<1-p2;
```

$$a < -b+1$$

```
> 1-p2<2;
```

$$-b < 1$$

if a+b<1 then x=0 is locally asymptotically stable

```
> p1:=D[1](f)(eqp[2],eqp[2]);
```

$$p1 := \frac{1}{2}\frac{a\left(-a+\sqrt{a^2+4b}\right)}{b} - \frac{1}{2}\frac{1}{b}\left(\left(-a\ln\left(\frac{1}{2}\frac{-a+\sqrt{a^2+4b}}{b}\right) - \frac{1}{2}\ln\left(\frac{1}{2}\frac{-a+\sqrt{a^2+4b}}{b}\right)\left(-a+\sqrt{a^2+4b}\right)\right)\left(-a+\sqrt{a^2+4b}\right)\right)$$

```
> simplify(%);
```

$$-\frac{1}{4}\frac{1}{b}\left(\left(-a+\sqrt{a^2+4b}\right)\left(\ln(2)\sqrt{a^2+4b}+\ln(2)\,a-\ln\left(\frac{-a+\sqrt{a^2+4b}}{b}\right)\sqrt{a^2+4b} - \ln\left(\frac{-a+\sqrt{a^2+4b}}{b}\right)a-2a\right)\right)$$

```
> p2:=D[2](f)(eqp[2],eqp[2]);
```

$$p2 := \frac{1}{2}\frac{1}{b}\left(\left(-\frac{1}{2}a+\frac{1}{2}\sqrt{a^2+4b}+\frac{1}{2}\ln\left(\frac{1}{2}\frac{-a+\sqrt{a^2+4b}}{b}\right)\left(-a+\sqrt{a^2+4b}\right)\right)\left(-a+\sqrt{a^2+4b}\right)\right)$$

```
> simplify(%);
```

$$-\frac{1}{4}\frac{1}{b}\left(\left(\ln(2)\sqrt{a^2+4b}-\ln(2)\,a-\ln\left(\frac{-a+\sqrt{a^2+4b}}{b}\right)\sqrt{a^2+4b} + \ln\left(\frac{-a+\sqrt{a^2+4b}}{b}\right)a-\sqrt{a^2+4b}+a\right)\left(-a+\sqrt{a^2+4b}\right)\right)$$

```
> chareq:=q^2=p1*q+p2;
```

$$chareq := q^2 = \left(\frac{1}{2}\frac{a\left(-a+\sqrt{a^2+4b}\right)}{b} - \frac{1}{2}\frac{1}{b}\left(\left(-a\ln\left(\frac{1}{2}\frac{-a+\sqrt{a^2+4b}}{b}\right) - \frac{1}{2}\ln\left(\frac{1}{2}\frac{-a+\sqrt{a^2+4b}}{b}\right)\left(-a+\sqrt{a^2+4b}\right)\right)\left(-a+\sqrt{a^2+4b}\right)\right)\right)q + \frac{1}{2}\frac{1}{b}\left(\left(-\frac{1}{2}a+\frac{1}{2}\sqrt{a^2+4b}+\frac{1}{2}\ln\left(\frac{1}{2}\frac{-a+\sqrt{a^2+4b}}{b}\right)\left(-a+\sqrt{a^2+4b}\right)\right)\left(-a+\sqrt{a^2+4b}\right)\right)$$

```
> rr:=solve(chareq,q);
```

$$rr := \ln\left(\frac{1}{2}\frac{-a+\sqrt{a^2+4b}}{b}\right)+1,\ \frac{1}{2}\frac{\sqrt{a^2+4b}\,a-a^2-2b}{b}$$

```
> (1/2)*(sqrt(a^2+4*b)*a-a^2-2*b)/b
```

```
> eqp[2];
```

$$-\ln\left(\frac{1}{2}\,\frac{-a+\sqrt{a^2+4\,b}}{b}\right)$$

```
> p1<1-p2;
```

$$\frac{1}{2}\,\frac{a\left(-a+\sqrt{a^2+4\,b}\right)}{b}-\frac{1}{2}\,\frac{1}{b}\left(\left(-a\ln\left(\frac{1}{2}\,\frac{-a+\sqrt{a^2+4\,b}}{b}\right)-\frac{1}{2}\ln\left(\frac{1}{2}\,\frac{-a+\sqrt{a^2+4\,b}}{b}\right)\left(-a+\sqrt{a^2+4\,b}\right)\right)\left(-a+\sqrt{a^2+4\,b}\right)\right)<1-\frac{1}{2}\,\frac{1}{b}\left(\left(-\frac{1}{2}\,a+\frac{1}{2}\,\sqrt{a^2+4\,b}+\frac{1}{2}\ln\left(\frac{1}{2}\,\frac{-a+\sqrt{a^2+4\,b}}{b}\right)\left(-a+\sqrt{a^2+4\,b}\right)\right)\left(-a+\sqrt{a^2+4\,b}\right)\right)$$

```
> simplify(%);
```

$$-\frac{1}{4}\,\frac{1}{b}\left(\left(-a+\sqrt{a^2+4\,b}\right)\left(\ln(2)\sqrt{a^2+4\,b}+\ln(2)\,a-\ln\left(\frac{-a+\sqrt{a^2+4\,b}}{b}\right)\sqrt{a^2+4\,b}-\ln\left(\frac{-a+\sqrt{a^2+4\,b}}{b}\right)a-2\,a\right)\right)<-\frac{1}{2}\,\frac{1}{b}\left(\ln(2)\sqrt{a^2+4\,b}\,a-\ln(2)\,a^2-\ln\left(\frac{-a+\sqrt{a^2+4\,b}}{b}\right)\sqrt{a^2+4\,b}\,a+\ln\left(\frac{-a+\sqrt{a^2+4\,b}}{b}\right)a^2-2\ln(2)\,b+2\ln\left(\frac{-a+\sqrt{a^2+4\,b}}{b}\right)b-\sqrt{a^2+4\,b}\,a+a^2\right)$$

```
> 1-p2<2;
```

$$-\frac{1}{2}\,\frac{1}{b}\left(\left(-\frac{1}{2}\,a+\frac{1}{2}\,\sqrt{a^2+4\,b}+\frac{1}{2}\ln\left(\frac{1}{2}\,\frac{-a+\sqrt{a^2+4\,b}}{b}\right)\left(-a+\sqrt{a^2+4\,b}\right)\right)\left(-a+\sqrt{a^2+4\,b}\right)\right)<1$$

```
> simplify(%);
```

$$\frac{1}{4}\,\frac{1}{b}\left(\left(\ln(2)\sqrt{a^2+4\,b}-\ln(2)\,a-\ln\left(\frac{-a+\sqrt{a^2+4\,b}}{b}\right)\sqrt{a^2+4\,b}+\ln\left(\frac{-a+\sqrt{a^2+4\,b}}{b}\right)a-\sqrt{a^2+4\,b}+a\right)\left(-a+\sqrt{a^2+4\,b}\right)\right)<1$$

```
> a:=0.2;b:=0.6;
```

$$a:=0.2$$
$$b:=0.6$$

```
> solve(x=f(x,x),x);
```

$$0.,\ -0.1266693134,\ -0.3841563104-3.141592654\,I$$

```
> p1:=D[1](f)(0,0);
```

$$p1 := 0.2$$

```
> p2:=D[2](f)(0,0);
```

$$p2 := 0.6$$

```
> lineq:=y(n+1)=p1*y(n)+p2*y(n-1);
```

$$lineq := y(n+1) = 0.2\,y(n) + 0.6\,y(n-1)$$

```
> chareq:=q^2=p1*q+p2;
```

$$chareq := q^2 = 0.2\,q + 0.6$$

```
> rr:=solve(chareq,q);
```

$$rr := 0.8810249676,\ -0.6810249676$$

```
> a:=0.4;b:=0.8;
```

$$a := 0.4$$
$$b := 0.8$$

```
> f(u,v);
```

$$\left(0.4\,u + 0.8\,v\,\mathrm{e}^{-v}\right)\mathrm{e}^{-u}$$

```
> eqp:=solve(x=f(x,x),x);
```

$$eqp := 0.,\ 0.1102123515,\ -0.3333559028 - 3.141592654\,\mathrm{I}$$

```
> p1:=D[1](f)(0,0);
```

$$p1 := 0.4$$

```
> p2:=D[2](f)(0,0);
```

$$p2 := 0.8$$

```
> chareq:=q^2=p1*q+p2;
```

$$chareq := q^2 = 0.4\,q + 0.8$$

```
> rr:=solve(chareq,q);
```

$$rr := 1.116515139,\ -0.7165151390$$

```
> eqp[2];
```

$$0.1102123515$$

```
> p1:=D[1](f)(eqp[2],eqp[2]);
```

$$p1 := 0.2480452180$$

```
> p2:=D[2](f)(eqp[2],eqp[2]);
```

$$p2 := 0.5710144882$$

```
> chareq:=q^2=p1*q+p2;
```

$$chareq := q^2 = 0.2480452180\,q + 0.5710144882$$

```
> rr:=solve(chareq,q);
```

$$rr := 0.8897876485,\ -0.6417424305$$

```
> a:=0.9;b:=50;
```

$$a := 0.9$$
$$b := 50$$

```
> f(u,v);
```

$$\left(0.9\,u + 50\,v\,\mathrm{e}^{-v}\right)\mathrm{e}^{-u}$$

```
> eqp:=solve(x=f(x,x),x);
```

$$eqp := 0.,\ 2.019608234,\ 1.892414771 - 3.141592654\,\mathrm{I}$$

```
> p1:=D[1](f)(eqp[2],eqp[2]);
```

$$p1 := -1.900171535$$

```
> p2:=D[2](f)(eqp[2],eqp[2]);
```

$$p2 := -0.8978295916$$

```
> chareq:=q^2=p1*q+p2;
```

$$chareq := q^2 = -1.900171535\ q - 0.8978295916$$

```
> rr:=solve(chareq,q);
```

$$rr := -0.8805632963,\ -1.019608239$$

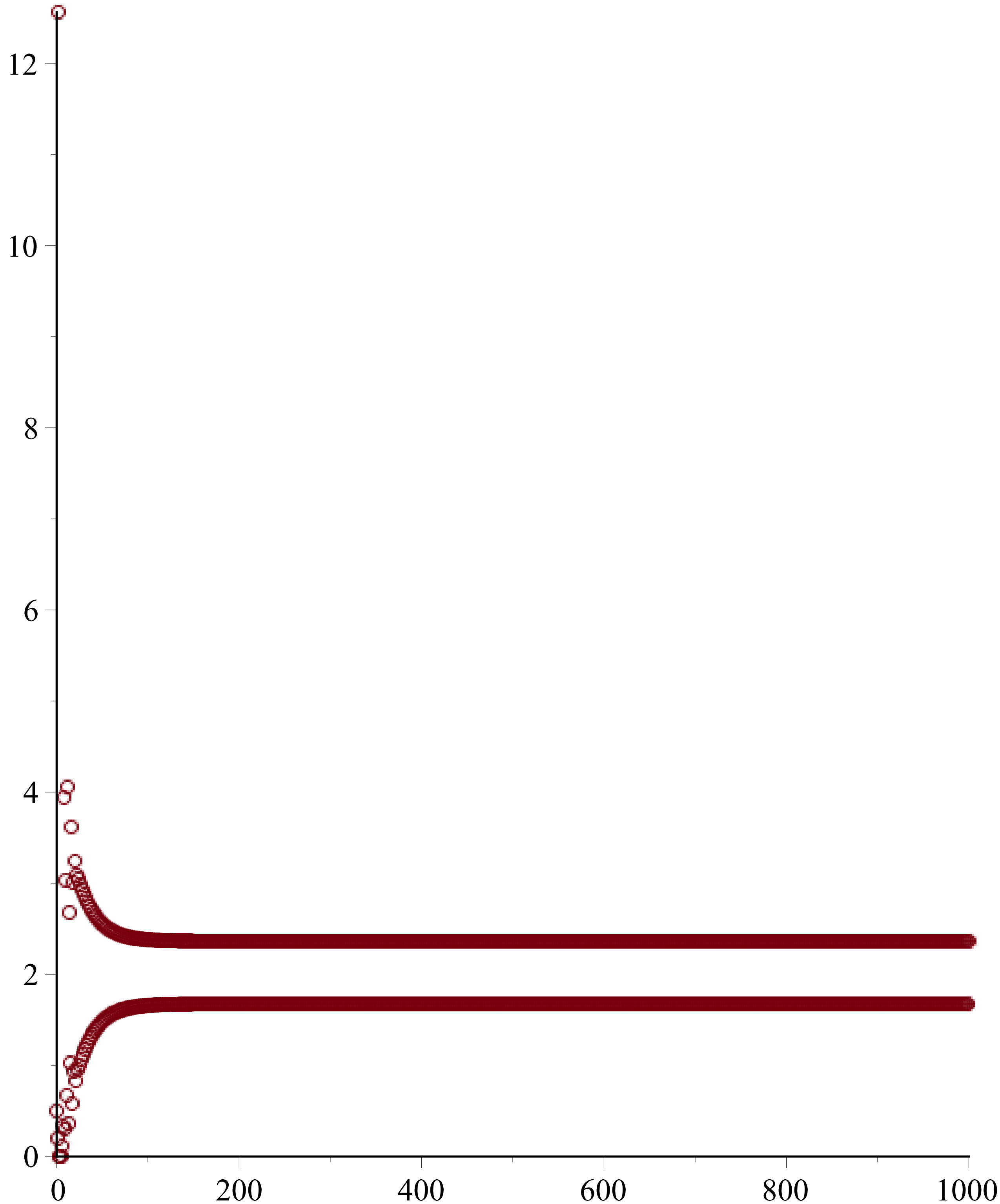
```
> x[0]:=0.5;x[1]:=0.2;N:=1000;
```

$$x_0 := 0.5$$

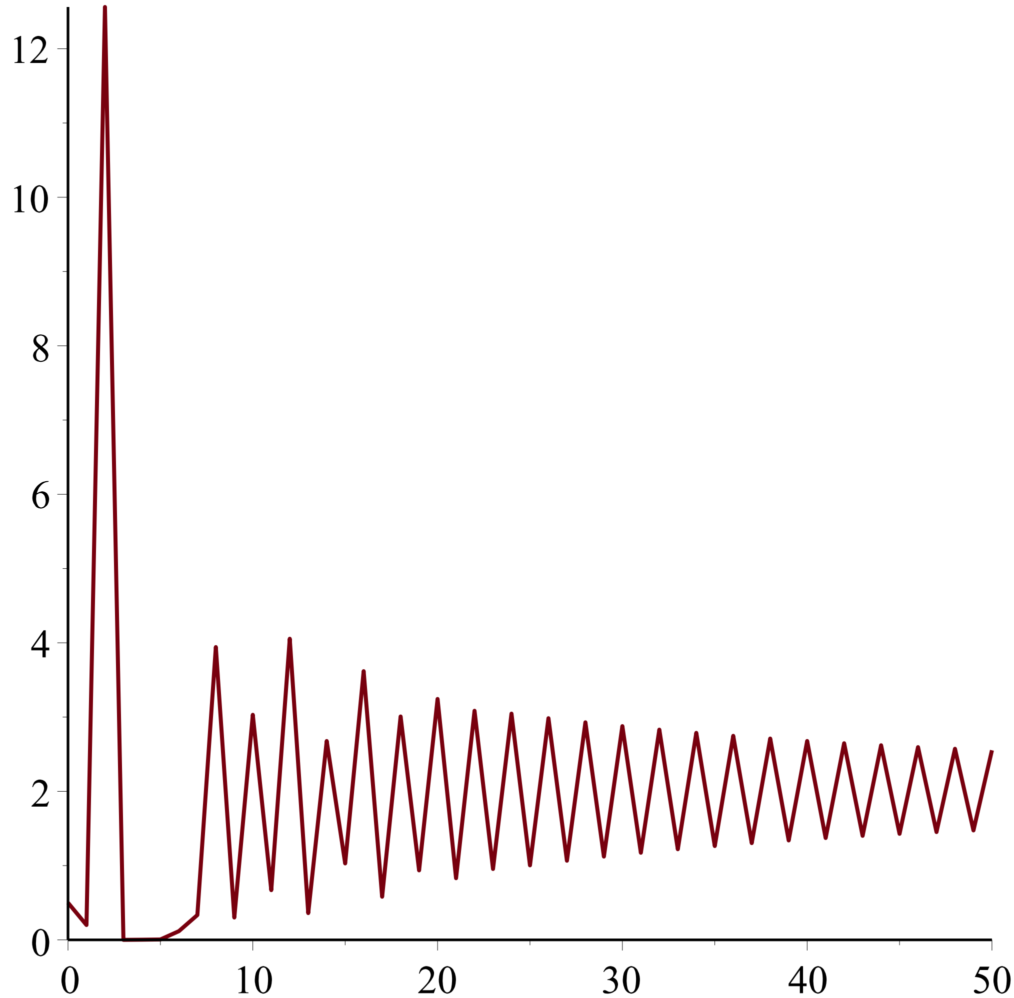
$$x_1 := 0.2$$

$$N := 1000$$

```
> for i from 0 to N-2 do
    x[i+2]:=f(x[i+1],x[i])
  end do:
> plot([[n,x[n]]$n=0..N],style=point,symbol=circle);
```




```
> plot([[n,x[n]]$n=0..N]);
```



12
10
8
6
4
2
0
0
10
20
30
40
50