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> eqd:=diff(y(x),x)=- (x+y(x))/y(x);
eqd :=  $\frac{d}{dx} y(x) = - \frac{x + y(x)}{y(x)}$ 

> dsolve(eqd,y(x));
 $y(x) = \frac{1}{2} \sqrt{3} x \tan\left(\text{RootOf}\left(\sqrt{3} \ln\left(\frac{3}{4} x^2 + \frac{3}{4} x^2 \tan(_Z)^2\right) + 2 \sqrt{3} _C1 - 2 _Z\right)\right) - \frac{1}{2} x$ 

> ans:=dsolve(eqd,y(x),implicit);
ans := - $\frac{1}{2} \ln\left(\frac{x^2 + x y(x) + y(x)^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2 y(x) + x) \sqrt{3}}{x}\right) - \ln(x) - _C1 = 0$ 

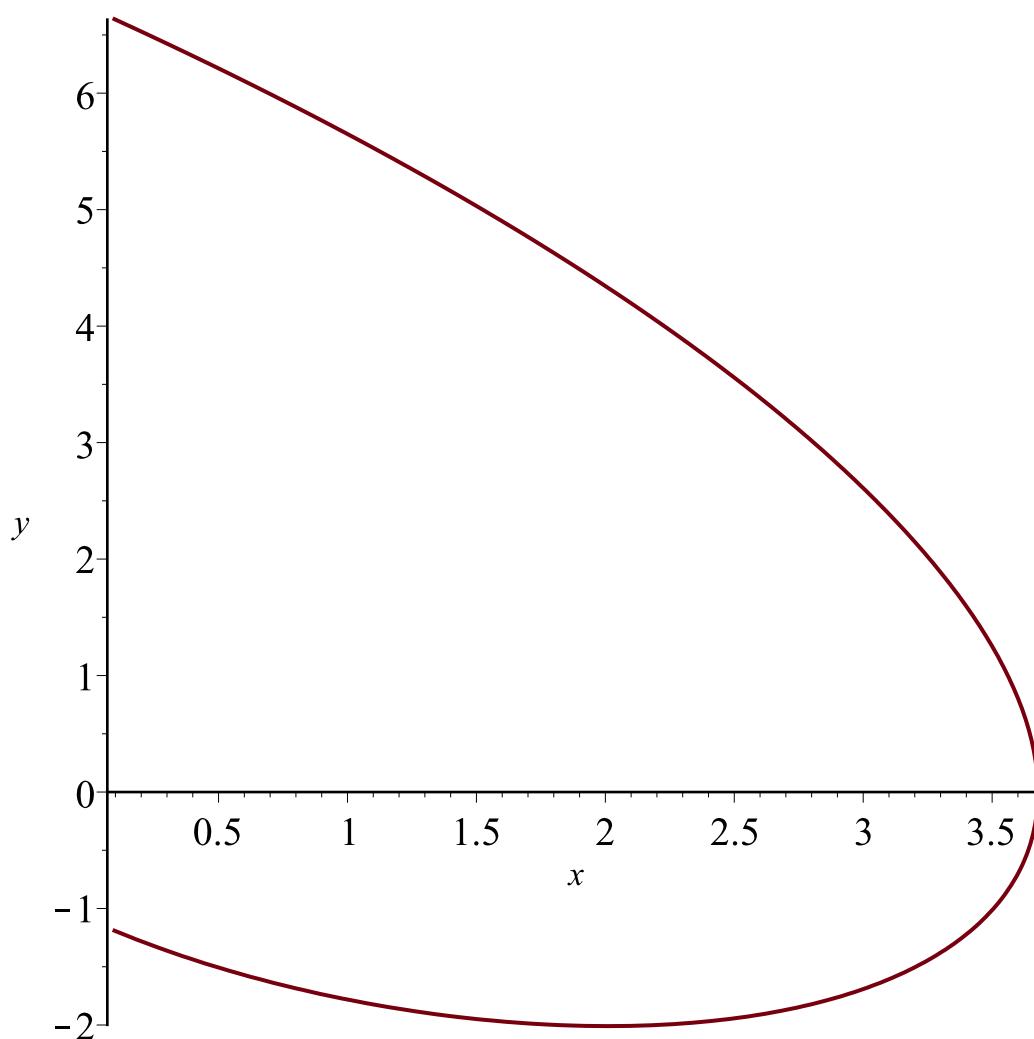
> lhs(ans);
 $-\frac{1}{2} \ln\left(\frac{x^2 + x y(x) + y(x)^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2 y(x) + x) \sqrt{3}}{x}\right) - \ln(x) - _C1$ 

> ans2:=subs(y(x)=y(lhs(ans)));
ans2 := - $\frac{1}{2} \ln\left(\frac{x^2 + x y + y^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2 y + x) \sqrt{3}}{x}\right) - \ln(x) - _C1$ 

> fsol:=unapply(ans2,x,y,_C1);
fsol := (x,y,_C1)  $\rightarrow$  - $\frac{1}{2} \ln\left(\frac{x^2 + x y + y^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2 y + x) \sqrt{3}}{x}\right) - \ln(x) - _C1$ 

> with(plots):
> implicitplot(fsol(x,y,-1)=0,x=-10..10,y=-10..10,numpoints=50000);

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> fsol(x,y,-1);

$$-\frac{1}{2} \ln\left(\frac{x^2 + xy(x) + y(x)^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2y(x) + x)\sqrt{3}}{x}\right) - \ln(x) + 1 = 0$$

> fsol(1,0,-1);

$$1 + \frac{1}{18} \sqrt{3} \pi = 0$$

> ans3:=dsolve({eqd,y(1)=3},y(x),implicit);

$$\begin{aligned} ans3 := & -\frac{1}{2} \ln\left(\frac{x^2 + xy(x) + y(x)^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2y(x) + x)\sqrt{3}}{x}\right) - \ln(x) \\ & + \frac{1}{2} \ln(13) - \frac{1}{3} \sqrt{3} \arctan\left(\frac{7}{3} \sqrt{3}\right) = 0 \end{aligned}$$

> ans3:=subs(y(x)=y,ans3);

$$\begin{aligned} ans3 := & -\frac{1}{2} \ln\left(\frac{x^2 + xy + y^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2y + x)\sqrt{3}}{x}\right) - \ln(x) + \frac{1}{2} \ln(13) \\ & - \frac{1}{3} \sqrt{3} \arctan\left(\frac{7}{3} \sqrt{3}\right) = 0 \end{aligned}$$

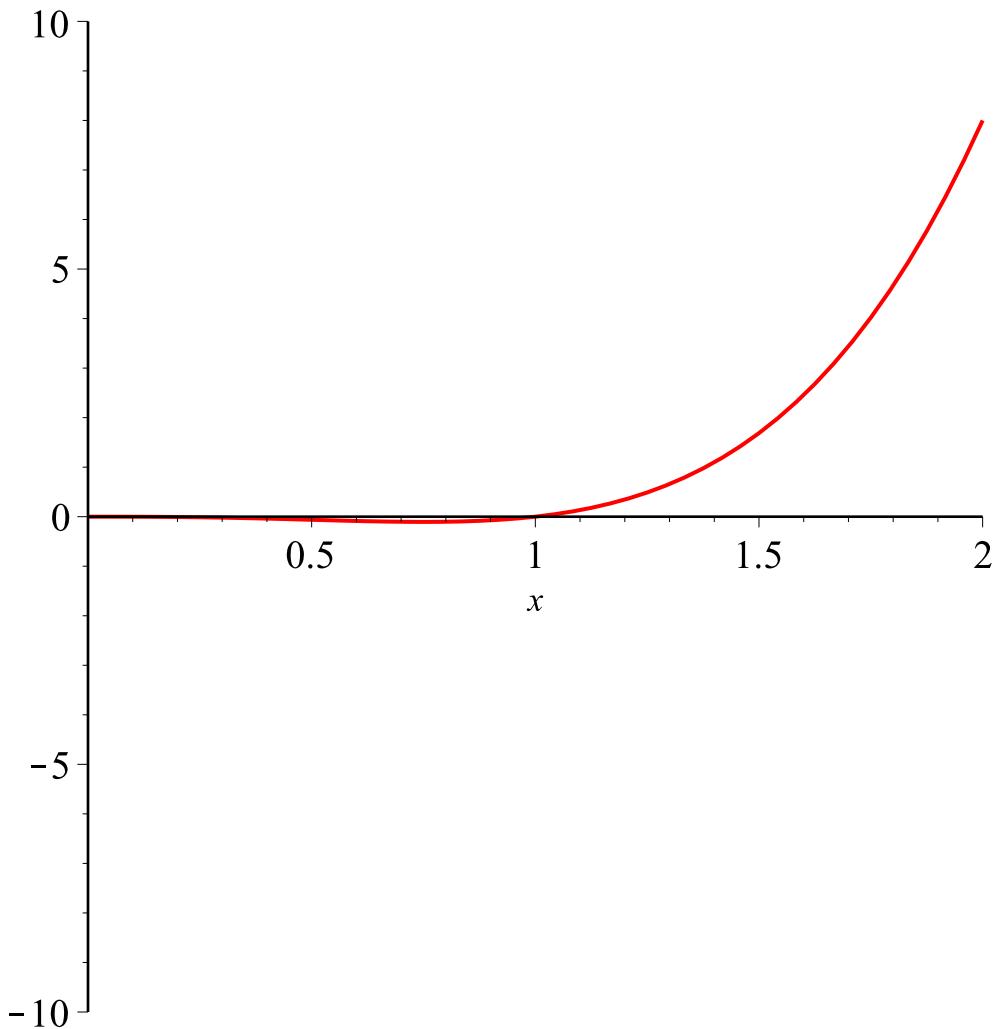
> f:=(x,y)->-(1/2)*ln((x^2+x*y+y^2)/x^2)+(1/3)*sqrt(3)*arctan((1/3)

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*(2*y+x)*sqrt(3)/x)-ln(x)+(1/2)*ln(13)-(1/3)*sqrt(3)*arctan((7/3)
*sqrt(3));
f:=(x,y)→ - $\frac{1}{2} \ln\left(\frac{x^2 + xy + y^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2y+x)\sqrt{3}}{x}\right) - \ln(x) + \frac{1}{2} \ln(13)$ 
- $\frac{1}{3} \sqrt{3} \arctan\left(\frac{7}{3} \sqrt{3}\right)$ 
> f(1,1);
- $\frac{1}{2} \ln(3) + \frac{1}{9} \sqrt{3} \pi + \frac{1}{2} \ln(13) - \frac{1}{3} \sqrt{3} \arctan\left(\frac{7}{3} \sqrt{3}\right)$ 
> evalf(%)
0.5709129577
> eqd:=diff(y(x),x)+k/x*y(x)=x^3;
eqd :=  $\frac{d}{dx} y(x) + \frac{ky(x)}{x} = x^3$ 
> ans:=dsolve({eqd,y(1)=0},y(x));
ans :=  $y(x) = \frac{x^4}{4+k} - \frac{x^{-k}}{4+k}$ 
> ysol:=unapply(rhs(ans),x,k);
ysol := (x, k) →  $\frac{x^4}{4+k} - \frac{x^{-k}}{4+k}$ 
> with(plots):
> animate(ysol(x,k),x=0..2,k=-3..3,view = [0..2, -10 .. 10]);

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> eqd:=diff(y(x),x,x)-diff(y(x),x)-2*y(x)=0;
           $eqd := \frac{d^2}{dx^2} y(x) - \left( \frac{d}{dx} y(x) \right) - 2 y(x) = 0$ 
=> ans:=dsolve({eqd,y(0)=a,D(y)(0)=2},y(x));
           $ans := y(x) = \left( -\frac{2}{3} + \frac{2}{3} a \right) e^{-x} + \left( \frac{1}{3} a + \frac{2}{3} \right) e^{2x}$ 
=> ysol:=unapply(rhs(ans),x,a);
           $ysol := (x, a) \rightarrow \left( -\frac{2}{3} + \frac{2}{3} a \right) e^{-x} + \left( \frac{1}{3} a + \frac{2}{3} \right) e^{2x}$ 
=> limit(ysol(x,a),x=infinity);
          signum(a+2) ∞
=> limit(ysol(x,-2),x=infinity);
          0
=> yy:=x->sqrt(1-x^2);
           $yy := x \rightarrow \sqrt{1 - x^2}$ 
=> plot(yy(x),x=-1..1);

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