Laboratory 6: Equilibrium points. Stability

Exerciţiul 1 *Draw the phase portrait of the following linear systems and specify the stability of the origin* $(0;0)$

$$(a)\ \begin{cases} x'(t) = & 2x(t) + y(t) \\ y'(t) = & x(t) + 2y(t) \end{cases} \qquad (e)\ \begin{cases} x'(t) = & x(t) + 4y(t) \\ y'(t) = & x(t) + y(t) \end{cases}$$

$$(b)\ \begin{cases} x'(t) = & -3x(t) + 4y(t) \\ y'(t) = & -2x(t) + 3y(t) \end{cases} \qquad (f)\ \begin{cases} x'(t) = & 2x(t) - y(t) \\ y'(t) = & x(t) + 2y(t) \end{cases}$$

$$(c)\ \begin{cases} x'(t) = & -x(t) - y(t) \\ y'(t) = & x(t) - 3y(t) \end{cases} \qquad (g)\ \begin{cases} x'(t) = & -y(t) \\ y'(t) = & x(t) \end{cases}$$

$$(d)\ \begin{cases} x'(t) = & -2x(t) \\ y'(t) = & -4x(t) - 2y(t) \end{cases} \qquad (h)\ \begin{cases} x'(t) = & x(t) - 4y(t) \\ y'(t) = & 5x(t) - 3y(t) \end{cases}$$

Exerciţiul 2 *Find the equilibrium points of the following nonlinear systems and study their stability. Draw in each case the corresponding phase portrait:*

$$(a)\ \begin{cases} x'(t) = & y(t) \\ y'(t) = & x(t) \cdot (1 - x^2(t)) + y(t) \end{cases}$$

$$(b)\ \begin{cases} x'(t) = & -2x(t) + y(t) + 2 \\ y'(t) = & x(t) \cdot y(t) \end{cases}$$

$$(c)\ \begin{cases} x'(t) = & y^2(t) \\ y'(t) = & x(t) \end{cases}$$

$$(d)\ \begin{cases} x'(t) = & x^2(t) - y^2(t) \\ y'(t) = & x(t) \cdot y(t) - 1 \end{cases}$$

Exerciţiul 3 *Let' consider the following two competion species system:*

$$\begin{cases} x' = & r_1 x \left(1 - \frac{x}{K_1}\right) - \frac{b_{12}}{K_1} xy \\ y' = & r_2 y \left(1 - \frac{y}{K_2}\right) - \frac{b_{21}}{K_2} xy \end{cases}$$

(b) Find the equilibrium points and study their stability.

(c) Make some numerical simulations.