Ex 2

```
> restart: with(DEtools):with(plots):
> RD_eq:=diff(x(t),t)=-k*x(t);
```

$$RD\_eq := \frac{d}{dt} x(t) = -k\,x(t)$$

```
> sol:=dsolve({RD_eq,x(0)=x0},x(t));
```

$$sol := x(t) = x0\, e^{-kt}$$

```
> x_sol:=unapply(rhs(sol),t,x0,k);
```

$$x\_sol := (t, x0, k) \rightarrow x0\, e^{-kt}$$

```
> eq:=x_sol(2,3,k)=0.9;
```

$$eq := 3\, e^{-2k} = 0.9$$

```
> kk:=solve(eq,k);
```

$$kk := 0.6019864022$$

```
> T12:=ln(2)/kk;evalf(%);
```

$$T12 := 1.661167090 \ln(2)$$

$$1.151433285$$

Ex3

```
> T12_C14:=5730;
```

$$T12\_C14 := 5730$$

```
> k_C14:=ln(2)/T12_C14;
```

$$k\_C14 := \frac{1}{5730} \ln(2)$$

```
> evalf(k_C14);
```

$$0.0001209680943$$

```
> eq:=x_sol(t1,x0,k_C14)=91.57/100*x0
```

$$eq := x0\, e^{-\frac{1}{5730} \ln(2)\, t1} = 0.9157000000\, x0$$

```
> solve(eq,t1);
```

$$728.0141045$$

```
> eq:=x_sol(t2,x0,k_C14)=93.021/100*x0
```

$$eq := x0\, e^{-\frac{1}{5730} \ln(2)\, t2} = 0.9302100000\, x0$$

```
> solve(eq,t2);
```

$$598.0495293$$

Ex. 4

```
> eqd:=diff(T(t),t)=-k*(T(t)-Tout(t));
```

$$eqd := \frac{d}{dt} T(t) = -k\,(T(t) - Tout(t))$$

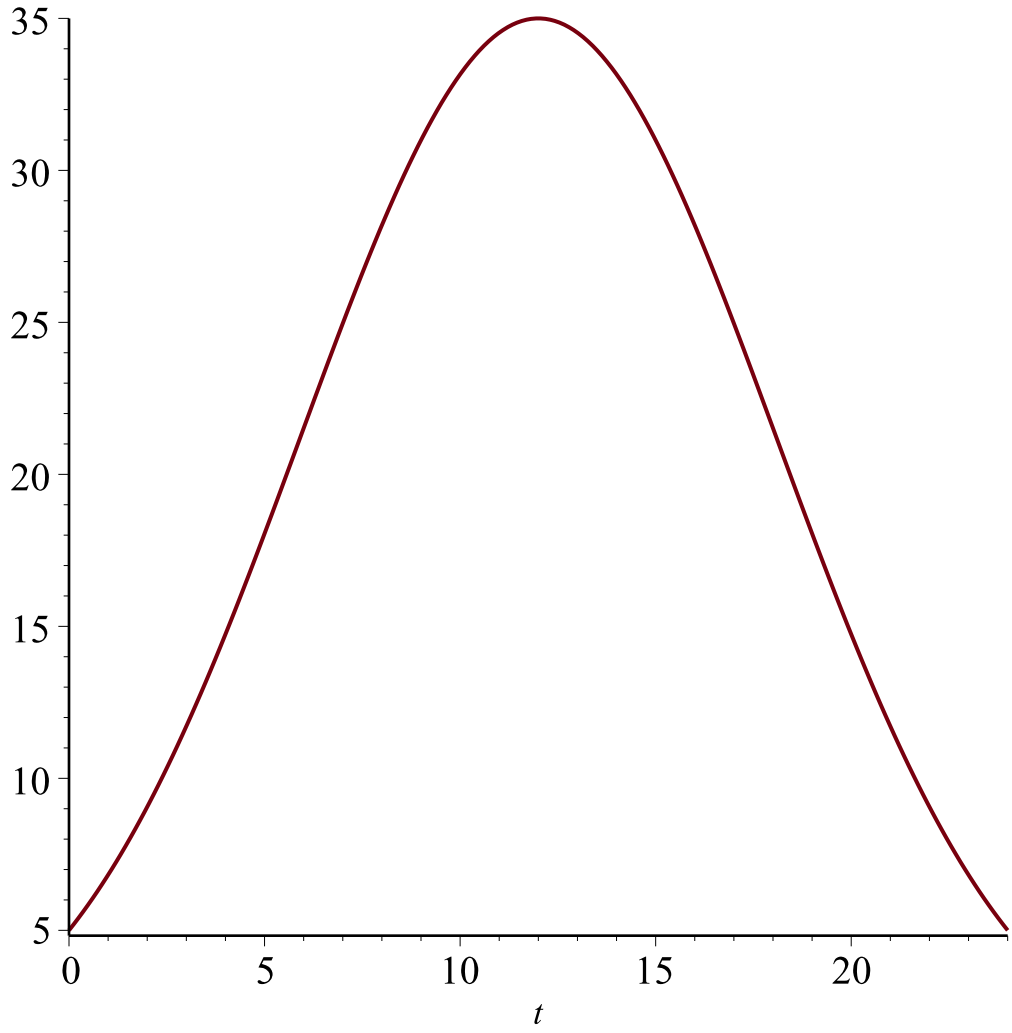
```
> s:=dsolve({eqd,T(0)=T0},T(t));
```

$$s := T(t) = \left(\int_0^t k\, Tout(\_z1)\, e^{k\_z1}\, d\_z1 + T0 \right) e^{-kt}$$

```
> Tout:=t->35*exp(-(t-12)^2/74);
```

$$Tout := t \to 35\, e^{-\frac{1}{74}(t-12)^2}$$

```
> plot(Tout(t),t=0..24);
```




```
> Tout(12);
```

$$35$$

```
> T0:=15;k:=0.2;
```

$$T0 := 15$$

$$k := 0.2$$

```
> s:=dsolve({eqd,T(0)=T0},T(t));
```

$$s := T(t) = \left(\frac{7}{2}\sqrt{\pi}\, e^{\frac{157}{50}} \sqrt{74}\, \operatorname{erf}\left(\frac{1}{74}\sqrt{74}\, t - \frac{97}{370}\sqrt{74} \right) + 15 + \frac{7}{2}\sqrt{\pi}\, e^{\frac{157}{50}} \sqrt{74}\, \operatorname{erf}\left(\frac{97}{370}\sqrt{74} \right) \right) e^{-\frac{1}{5}t}$$

```
> ans:=rhs(s);
```

$$ans := \left(\frac{7}{2}\sqrt{\pi}\, e^{\frac{157}{50}} \sqrt{74}\, \operatorname{erf}\left(\frac{1}{74}\sqrt{74}\, t - \frac{97}{370}\sqrt{74} \right) + 15

$$+\frac{7}{2}\sqrt{\pi}\,e^{\frac{157}{50}}\sqrt{74}\,\mathrm{erf}\left(\frac{97}{370}\sqrt{74}\right)\Bigg)e^{-\frac{1}{5}t}$$

```
> Tsol:=unapply(ans,t)
```

$$Tsol := t \rightarrow \left(\frac{7}{2}\sqrt{\pi}\,e^{\frac{157}{50}}\sqrt{74}\,\mathrm{erf}\left(\frac{1}{74}\sqrt{74}\,t-\frac{97}{370}\sqrt{74}\right)+15 + \frac{7}{2}\sqrt{\pi}\,e^{\frac{157}{50}}\sqrt{74}\,\mathrm{erf}\left(\frac{97}{370}\sqrt{74}\right)\right)e^{-\frac{1}{5}t}$$

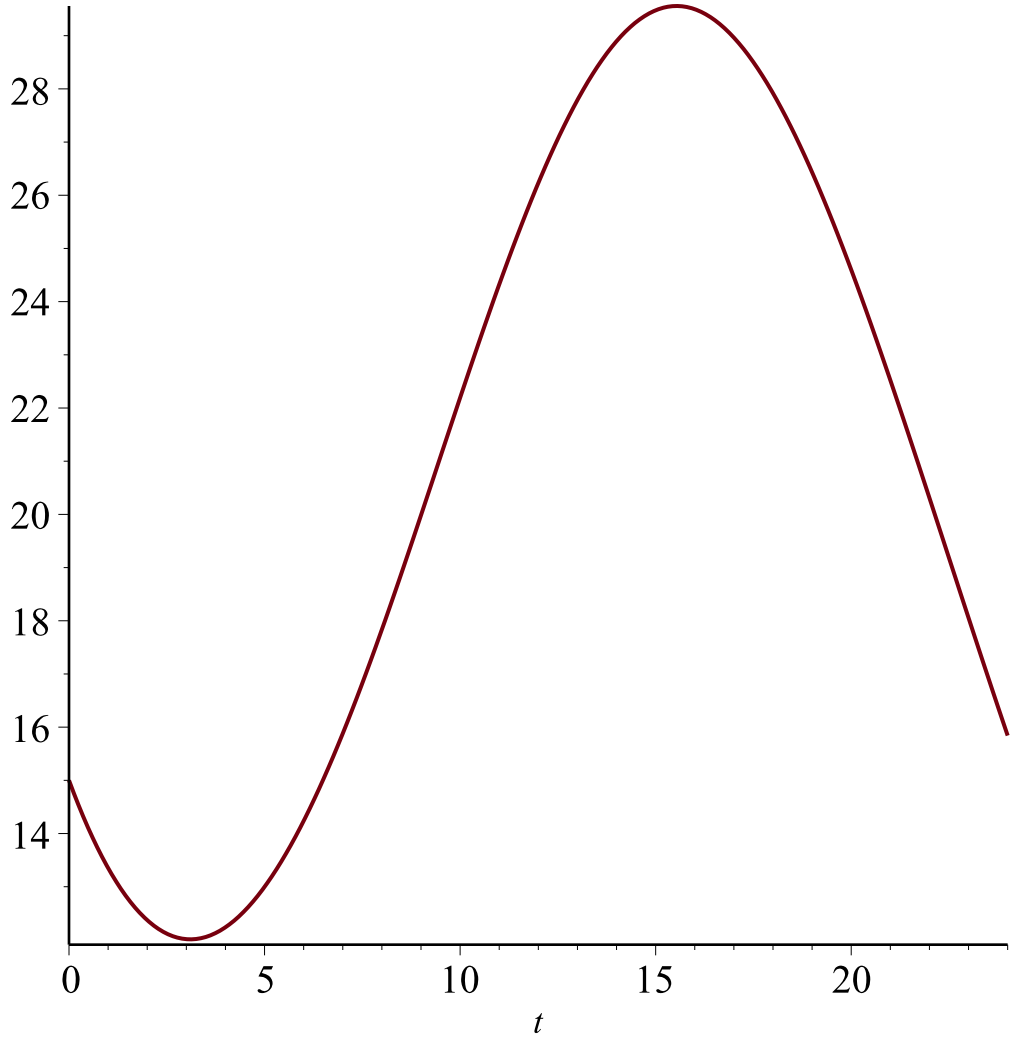
```
> Tsol(5);
```

$$\left(-\frac{7}{2}\sqrt{\pi}\,e^{\frac{157}{50}}\sqrt{74}\,\mathrm{erf}\left(\frac{36}{185}\sqrt{74}\right)+15+\frac{7}{2}\sqrt{\pi}\,e^{\frac{157}{50}}\sqrt{74}\,\mathrm{erf}\left(\frac{97}{370}\sqrt{74}\right)\right)e^{-1}$$

```
> evalf(%);
```

$$12.99774351$$

```
> plot(Tsol(t),t=0..24);
```




```
> eq2:=diff(Tsol(t),t)=0;
```

$$eq2 := 7\,e^{\frac{157}{50}}\,e^{-\left(\frac{1}{74}\sqrt{74}\,t-\frac{97}{370}\sqrt{74}\right)^2}e^{-\frac{1}{5}t}-\frac{1}{5}\left(\frac{7}{2}\sqrt{\pi}\,e^{\frac{157}{50}}\sqrt{74}\,\mathrm{erf}\left(\frac{1}{74}\sqrt{74}\,t\right.\right.$$

$$-\frac{97}{370}\sqrt{74}\Big)+15+\frac{7}{2}\sqrt{\pi}\,e^{\frac{157}{50}}\sqrt{74}\,\mathrm{erf}\Big(\frac{97}{370}\sqrt{74}\Big)\Big)e^{-\frac{1}{5}t}=0$$

```
> solve(eq2,t);
```

Warning, solutions may have been lost

$$RootOf\left(7\sqrt{\pi}\,e^{\frac{157}{50}}\sqrt{74}\,\mathrm{erf}\left(\frac{97}{370}\sqrt{74}\right)+7\sqrt{\pi}\,e^{\frac{157}{50}}\sqrt{74}\,\mathrm{erf}\left(\frac{1}{74}\sqrt{74}\,\_Z-\frac{97}{370}\sqrt{74}\right)-70\,e^{\frac{157}{50}}\,e^{-\frac{1}{1850}(5\_Z-97)^2}+30\right)$$

```
> evalf(allvalues(%));
```

$$3.103567586,\ -1.070032994-11.69265473\,I,\ -1.070032994+11.69265473\,I,\ 15.53621149$$

```
> Tsol(15.53621149);evalf(%);
```

$$-0.1565344243\sqrt{\pi}\,e^{\frac{157}{50}}\sqrt{74}\,\mathrm{erf}\left(0.0522133583\sqrt{74}\right)+0.6708618184+0.1565344243\sqrt{\pi}\,e^{\frac{157}{50}}\sqrt{74}\,\mathrm{erf}\left(\frac{97}{370}\sqrt{74}\right)$$

$$29.55829424$$

Ex. 5

```
> restart;
> eqd:=diff(x(t),t)=r*x(t)*ln(K/x(t));
```

$$eqd:=\frac{d}{dt}x(t)=r\,x(t)\ln\left(\frac{K}{x(t)}\right)$$

```
> assume(x0>0);assume(K>0);
> dsolve({eqd,x(0)=x0},x(t));
```

$$x(t)=\frac{K}{e^{\frac{\ln\left(\frac{K}{x0}\right)+2I\pi\_Z2\sim}{e^{t}r}}}$$

```
> eq:=dsolve({eqd,x(0)=x0},x(t),implicit);
```

$$eq:=t+\frac{\ln\left(\ln\left(\frac{K}{x(t)}\right)\right)}{r}-\frac{\ln\left(\ln\left(\frac{K}{x0}\right)\right)}{r}=0$$

```
> solve(eq,x(t),real);
```

$$\frac{K}{e^{\frac{\ln\left(\frac{K}{x0}\right)}{e^{t}r}}}$$

```
> evalc(%);
```

$$x(t)=e^{\frac{\ln(K\sim)\,e^{t}r-\ln(K\sim)+\ln(x0\sim)}{e^{t}r}}\cos\left(\frac{2\,e^{tr}\pi\_Z7\sim-2\,\pi\_Z7\sim+2\,\pi\_Z8\sim}{e^{tr}}\right)$$

$$+ \mathrm{I}\, \mathrm{e}^{\frac{\ln(K\sim)\, \mathrm{e}^{t r} - \ln(K\sim) + \ln(x0\sim)}{\mathrm{e}^{t r}}} \sin\left(\frac{2\, \mathrm{e}^{t r}\, \pi\_Z7\sim - 2\, \pi\_Z7\sim + 2\, \pi\_Z8\sim}{\mathrm{e}^{t r}} \right)$$

```
> r:=0.1;K:=200;x0:=10;
```

$$r := 0.1$$
$$K := 200$$
$$x0 := 10$$

```
> dsolve({eqd,x(0)=x0},x(t));
```

$$x(t) = 200\ 10^{\mathrm{e}^{-\frac{1}{10} t}\, \mathrm{e}^{-\mathrm{e}^{-\frac{1}{10} t}\, (-2\, \mathrm{I}\, \pi\_Z9\sim + 4\, \mathrm{I}\, \pi\_Z10\sim + 6\, \mathrm{I}\, \pi\_Z11\sim + 3 \ln(2))}}\ 25^{-\mathrm{e}^{-\frac{1}{10} t}\, t}$$

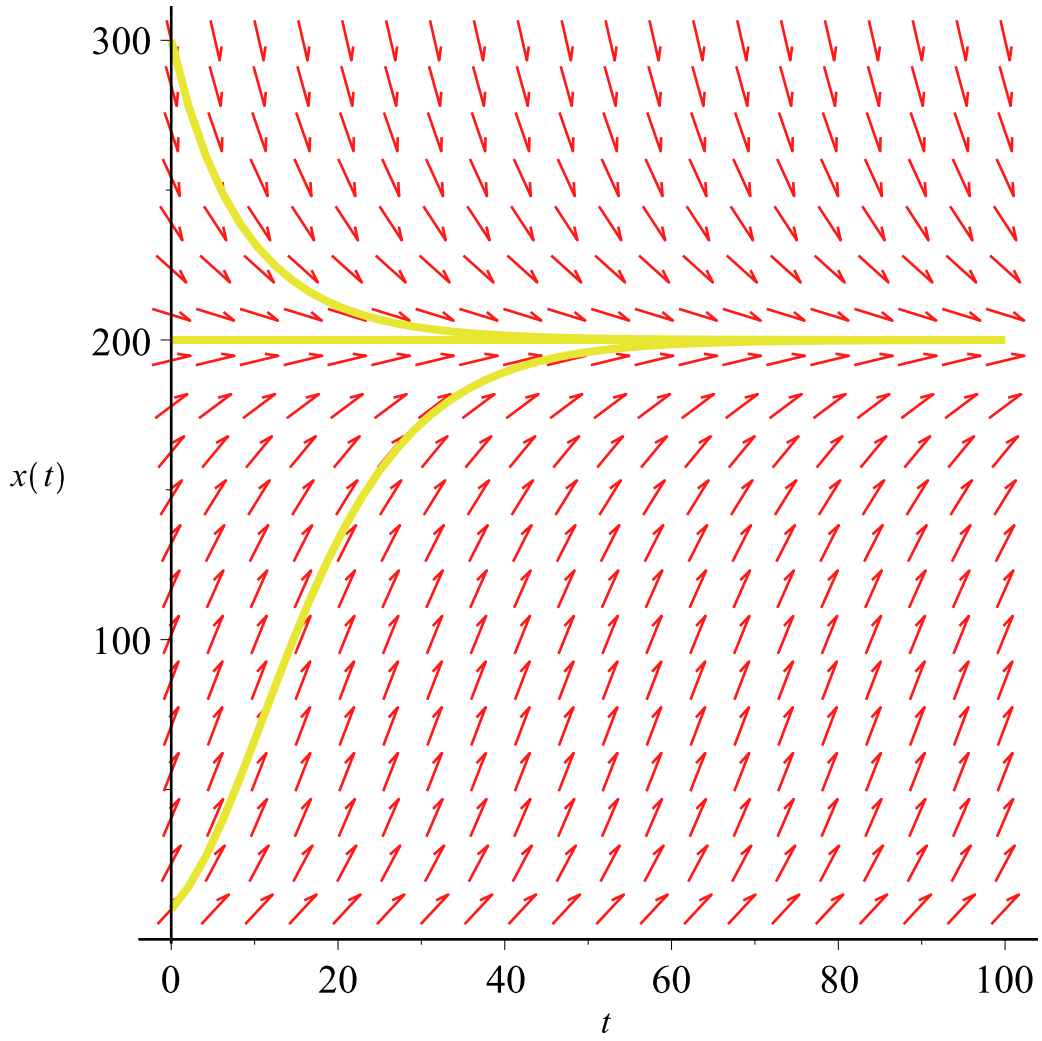
```
> evalc(%);
```

$$x(t) = 200\ 10^{\mathrm{e}^{-\frac{1}{10} t}\, \mathrm{e}^{-3\, \mathrm{e}^{-\frac{1}{10} t}\, \ln(2)} \cos\left(\mathrm{e}^{-\frac{1}{10} t}\, (4\, \pi\_Z10\sim + 6\, \pi\_Z11\sim - 2\, \pi\_Z9\sim) \right)}\ 25^{-\mathrm{e}^{-\frac{1}{10} t}\, t}$$
$$- 200\, \mathrm{I}\, 10^{\mathrm{e}^{-\frac{1}{10} t}\, \mathrm{e}^{-3\, \mathrm{e}^{-\frac{1}{10} t}\, \ln(2)} \sin\left(\mathrm{e}^{-\frac{1}{10} t}\, (4\, \pi\_Z10\sim + 6\, \pi\_Z11\sim - 2\, \pi\_Z9\sim) \right)}\ 25^{-\mathrm{e}^{-\frac{1}{10} t}\, t}$$

```
> simplify(%,trig);
```

$$x(t) = 200\ 10^{\mathrm{e}^{-\frac{1}{10} t}\, 8^{-\mathrm{e}^{-\frac{1}{10} t}}\, 25^{-\mathrm{e}^{-\frac{1}{10} t}\, t} \left(-\mathrm{I} \sin\left(2\, \mathrm{e}^{-\frac{1}{10} t}\, \pi\, (2\, \_Z10\sim + 3\, \_Z11\sim - \_Z9\sim) \right) \right.}$$
$$\left. + \cos\left(2\, \mathrm{e}^{-\frac{1}{10} t}\, \pi\, (2\, \_Z10\sim + 3\, \_Z11\sim - \_Z9\sim) \right) \right)$$

```
> DEplot(eqd,x(t),t=0..100,[[x(0)=x0],[x(0)=K],[x(0)=300]]);
```




```
> f:=x->r*x*ln(K/x);
```

$$f := x \to r\,x \ln\left(\frac{K}{x}\right)$$

```
> solve(f(x)=0,x);
```

$$200.$$

```
> D(f)(200);
```

$$-0.1$$

```
> f:=x->b*x/(1+x)-d*x;
```

$$f := x \to \frac{b\,x}{x+1} - d\,x$$

```
> deq:=diff(x(t),t)=f(x(t));
```

$$deq := \frac{\mathrm{d}}{\mathrm{d}t}\,x(t) = \frac{b\,x(t)}{x(t)+1} - d\,x(t)$$

```
> dsolve({deq,x(0)=x0},x(t));
```

$$x(t) = 1 \Big/ d\left(e^{RootOf\left(-d\,e^{\frac{(\ln(10)\,d - b\ln(11\,d-b))\,b}{d\,(b-d)}}\,e^{t\,b}\,e^{\frac{b\_Z}{d}} + e^{\_Z}\,e^{\frac{\ln(10)\,d - b\ln(11\,d-b)}{b-d}}\,e^{d\,t}\right.}$$

$$+ b\,e^{\frac{\ln(10)\,d - b\ln(11\,d-b)}{b-d}}\,e^{d\,t} - d\,e^{\frac{\ln(10)\,d - b\ln(11\,d-b)}{b-d}}\,e^{d\,t}\Big) + b - d\Big)$$

```
> allvalues(%);
```

$$x(t) = \frac{e^{RootOf\left(e^{\_Z} + b - d - d\,e^{\frac{d\,t\,b - d^2\,t + \ln(10)\,d - b\ln(11\,d-b) + b\_Z}{d}}\right)} + b - d}{d}$$

```
> dsolve({deq,x(0)=x0},x(t), implicit);
```

$$t - \frac{\ln(x(t))}{b-d} + \frac{b\ln(d\,x(t) - b + d)}{(b-d)\,d} + \frac{\ln(10)}{b-d} - \frac{b\ln(11\,d-b)}{(b-d)\,d} = 0$$