

```
> eqd:=diff(y(x),x)=-(x+y(x))/y(x);
```

$$eqd := \frac{d}{dx} y(x) = -\frac{x+y(x)}{y(x)}$$

```
> dsolve(eqd,y(x));
```

$$y(x) = \frac{1}{2} \sqrt{3} x \tan\left(\text{RootOf}\left(\sqrt{3} \ln\left(\frac{3}{4} x^2 + \frac{3}{4} x^2 \tan(_Z)^2\right) + 2 \sqrt{3} _CI - 2 _Z\right)\right) - \frac{1}{2} x$$

```
> ans:=dsolve(eqd,y(x),implicit);
```

$$ans := -\frac{1}{2} \ln\left(\frac{x^2 + x y(x) + y(x)^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2 y(x) + x) \sqrt{3}}{x}\right) - \ln(x) - _CI = 0$$

```
> lhs(ans);
```

$$-\frac{1}{2} \ln\left(\frac{x^2 + x y(x) + y(x)^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2 y(x) + x) \sqrt{3}}{x}\right) - \ln(x) - _CI$$

```
> ans2:=subs(y(x)=y,lhs(ans));
```

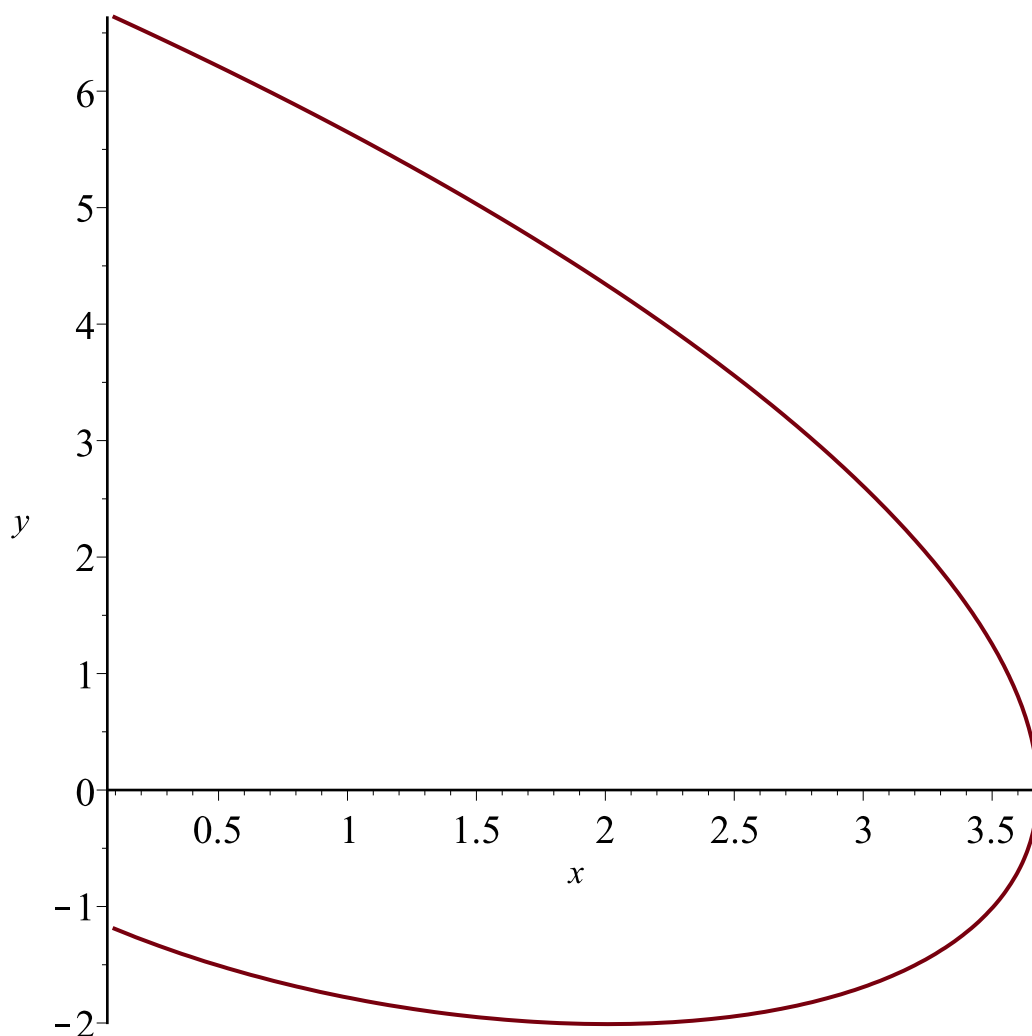
$$ans2 := -\frac{1}{2} \ln\left(\frac{x^2 + x y + y^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2 y + x) \sqrt{3}}{x}\right) - \ln(x) - _CI$$

```
> fsol:=unapply(ans2,x,y,_C1);
```

$$fsol := (x, y, _CI) \rightarrow -\frac{1}{2} \ln\left(\frac{x^2 + x y + y^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2 y + x) \sqrt{3}}{x}\right) - \ln(x) - _CI$$

```
> with(plots):
```

```
> implicitplot(fsol(x,y,-1)=0,x=-10..10,y=-10..10,numpoints=50000);
```



```
> fsol(x,y,-1);
```

$$-\frac{1}{2} \ln\left(\frac{x^2 + x y(x) + y(x)^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2 y(x) + x) \sqrt{3}}{x}\right) - \ln(x) + 1 = 0$$

```
> fsol(1,0,-1);
```

$$1 + \frac{1}{18} \sqrt{3} \pi = 0$$

```
> ans3:=dsolve({eqd,y(1)=3},y(x),implicit);
```

$$\begin{aligned} ans3 := & -\frac{1}{2} \ln\left(\frac{x^2 + x y(x) + y(x)^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2 y(x) + x) \sqrt{3}}{x}\right) - \ln(x) \\ & + \frac{1}{2} \ln(13) - \frac{1}{3} \sqrt{3} \arctan\left(\frac{7}{3} \sqrt{3}\right) = 0 \end{aligned}$$

```
> ans3:=subs(y(x)=y,ans3);
```

$$\begin{aligned} ans3 := & -\frac{1}{2} \ln\left(\frac{x^2 + x y + y^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2 y + x) \sqrt{3}}{x}\right) - \ln(x) + \frac{1}{2} \ln(13) \\ & - \frac{1}{3} \sqrt{3} \arctan\left(\frac{7}{3} \sqrt{3}\right) = 0 \end{aligned}$$

```
> f:=(x,y)->-(1/2)*ln((x^2+x*y+y^2)/x^2)+(1/3)*sqrt(3)*arctan((1/3)
```

```
* (2*y+x)*sqrt(3)/x)-ln(x)+(1/2)*ln(13)-(1/3)*sqrt(3)*arctan((7/3)*sqrt(3));
```

$$f := (x, y) \rightarrow -\frac{1}{2} \ln\left(\frac{x^2 + xy + y^2}{x^2}\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \frac{(2y+x)\sqrt{3}}{x}\right) - \ln(x) + \frac{1}{2} \ln(13) - \frac{1}{3} \sqrt{3} \arctan\left(\frac{7}{3} \sqrt{3}\right)$$

```
> f(1,1);
```

$$-\frac{1}{2} \ln(3) + \frac{1}{9} \sqrt{3} \pi + \frac{1}{2} \ln(13) - \frac{1}{3} \sqrt{3} \arctan\left(\frac{7}{3} \sqrt{3}\right)$$

```
> evalf(%)
```

0.5709129577

```
> eqd:=diff(y(x),x)+k/x*y(x)=x^3;
```

$$eqd := \frac{d}{dx} y(x) + \frac{ky(x)}{x} = x^3$$

```
> ans:=dsolve({eqd,y(1)=0},y(x));
```

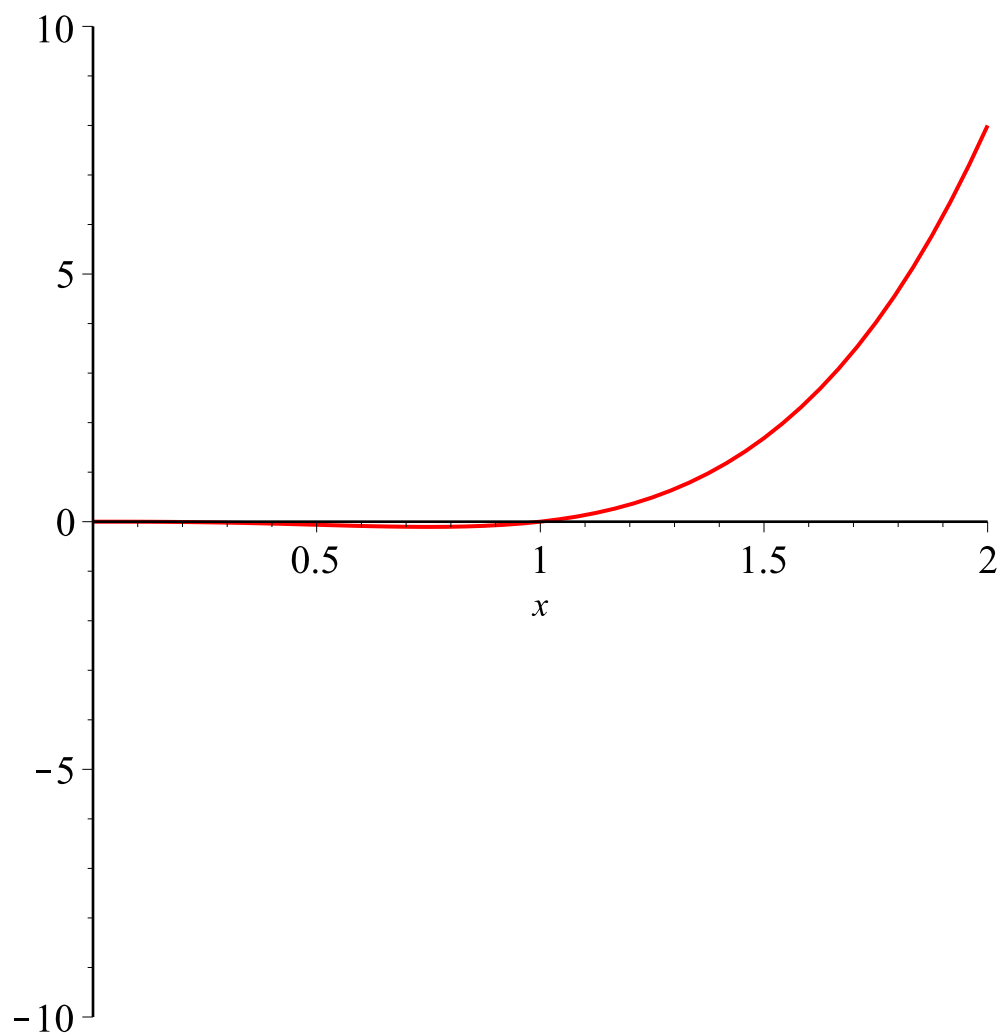
$$ans := y(x) = \frac{x^4}{4+k} - \frac{x^{-k}}{4+k}$$

```
> ysol:=unapply(rhs(ans),x,k);
```

$$ysol := (x, k) \rightarrow \frac{x^4}{4+k} - \frac{x^{-k}}{4+k}$$

```
> with(plots):
```

```
> animate(ysol(x,k),x=0..2,k=-3..3,view = [0..2, -10 .. 10]);
```



```
> eqd:=diff(y(x),x,x)-diff(y(x),x)-2*y(x)=0;
```

$$eqd := \frac{d^2}{dx^2} y(x) - \left( \frac{d}{dx} y(x) \right) - 2 y(x) = 0$$

```
> ans:=dsolve({eqd,y(0)=a,D(y)(0)=2},y(x));
```

$$ans := y(x) = \left( -\frac{2}{3} + \frac{2}{3} a \right) e^{-x} + \left( \frac{1}{3} a + \frac{2}{3} \right) e^{2x}$$

```
> ysol:=unapply(rhs(ans),x,a);
```

$$ysol := (x, a) \rightarrow \left( -\frac{2}{3} + \frac{2}{3} a \right) e^{-x} + \left( \frac{1}{3} a + \frac{2}{3} \right) e^{2x}$$

```
> limit(ysol(x,a),x=infinity);
```

$$\text{signum}(a + 2) \infty$$

```
> limit(ysol(x,-2),x=infinity);
```

$$0$$

```
> yy:=x->sqrt(1-x^2);
```

$$yy := x \rightarrow \sqrt{1 - x^2}$$

```
> plot(yy(x),x=-1..1);
```

