Laboratory 2: Solutions of the Difference Equations

1. Find the solution for the following initial value problems. Plot your data to observe patterns in the solutions.

   (a) $a_{n+1} = -1.2a_n + 50$, $a_0 = 1000$;

   (b) $a_{n+1} = 0.8a_n - 100$, $a_0 = 500$;

   (c) $a_{n+1} = 0.8a_n - 100, a_0 = -500$;

   (d) $a_{n+1} = a_n - 100, a_0 = 1000$;

2. Consider the simple interest formula

$$S_{n+1} = S_n + pS_0$$

   and the compound interest formula

$$S_{n+1} = S_n + \frac{p}{r}S_n.$$

   There are three options to earn interest. Company A offers simple interest at a rate of 6%. Company B offers compound interest at a 4% rate with a conversion period of one month. Company C offers compound interest at a 4% rate with a conversion period of three months.

   (a) Calculate for the three cases the amount on deposit after 5, 10, 15, and 20 years for any principal S_0.

   (b) Which interest offer maximizes the amount on deposit after 5, 10, 15,and 20 years?

3. The loan on a house is \$200,000. The mathematical model that describes the repayment of the loan is:

$$S_{n+1} = S_n + \frac{p}{r}S_n - R$$

   (a) Calculate the monthly repayment R needed to have the loan repaid after 30 years. The annualy interest rate is 5%.

   (b) Calculate the total amount paid back on the loan.

4. Find the solutions of the following difference equations and plot them graphically.

   (a) $x_{n+2} - 5x_{n+1} + 6x_n = 0$, $x_0 = 1$, $x_1 = 1$;

   (b) $x_{n+3} - 6x_{n+2} + 11x_{n+1} - 6x_n = 0$, $x_0 = 2$, $x_1 = 2$, $x_2 = 4$;

   (c) $x_{n+2} - 3x_{n+1} + 2x_n = 3^n\,(2n^2 + 4n)$, $x_0 = 2$, $x_1 = 1$;

   (d) $x_{n+2} - 3x_{n+1} + 2x_n = 2n^2 + 6n$, $x_0 = 1$, $x_1 = 2$;