Laboratory 4: Modelling with first order differential equations

1. Find the decay constant for a radioactive substance for the given half-life value

   (a) $T_{1/2} = 5730$ years for C^{14}

   (b) $T_{1/2} = 4,468 \cdot 10^9$ years for U^{238}

   (c) $T_{1/2} = 706 \cdot 10^6$ years for U^{235}

2. In two years 3 g of radioisotope decay to $0,9$ g. Find the half-life and the decay constant.

3. (Carbon dating of Shroud from Turin) In 1988 three independent dating tests reveald that the quantity of C^{14} in the shroud was between 91.57% and 93.021%. Using the decay constant for C^{14} found it in the previous exercise determine when shroud was made.

4. Find room temperature variation in a summer day knowing that the outside temperature variation is given by the function $T_{out}(t) = 35 \cdot e^{-\frac{(t-12)^2}{74}}$ (the time variable is measured in hours, $t = 0$ means the midnight, notice that at $t = 12$, the midday, we have the highest outside temperature of 35^oC and at the midnight we have the lowest outside temperature, aprox. 5^oC). Suppose that the initial room temperature at $t = 0$ is $T_0 = 15^oC$ and the room thermic coeficient is $k = 0.2 \cdot hours^{-1}$. Plot the solution on a day interval $[0; 24]$ and estimate the time when the room temperature is highest.

5. Let's consider the Gompertz model used in the growth of tumors

$$\begin{cases} x'(t) &= r \cdot x(t) \cdot \ln\left(\frac{K}{x(t)}\right) \\ x(0) &= x_0 \end{cases}$$

   where $x_0 > 0$ and $r > 0$.

   (a) Find the model solution;

   (b) Make numerical simulation.

6. Let's consider the mathematical model used in the growth of cells

$$\begin{cases} x'(t) &= \frac{bx(t)}{1+x(t)} - dx(t) \\ x(0) &= x_0 \end{cases}$$

   where b is the cells birth rate, d is the cells death rate. The factor $\frac{1}{1+x(t)}$ simulates the crowding effect.

   (a) Find the equilibrium solutions and study their stability.

   (b) Make numerical simulation.