$$\begin{cases}\min f(x)\\ s.t\ \ c_i(x)=0,\ i\in\mathcal{E}\\ \quad\ c_i(x)\ge 0,\ i\in\mathcal{I}\end{cases}$$ CNP

$$\Omega=\{x\in\mathbb{R}^n:\ c_i(x)\ge 0,\ i\in\mathcal{I};\ c_i(x)=0,\ i\in\mathcal{E}\}$$

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$$L(x,\lambda)=f(x)-\sum_{i\in\mathcal{E}}\lambda_i c_i(x)-\sum_{i\in\mathcal{I}}\lambda_i c_i(x)$$

$\nabla f(x^*)=\sum\lambda_i^*\nabla c_i(x^*)$ cone

$i\in A(x^*)$ $\{\nabla c_i(x^*)\}$

KKT conditions:

x^* solves (CNP)

Then, $\exists\lambda^*$ such that

① $\nabla_x L(x^*,\lambda^*)=\nabla f(x^*)-\sum_{i\in\mathcal{E}\cup\mathcal{I}}\lambda_i^*\nabla c_i(x^*)=0$

② $c_i(x^*)\ge 0,\ i\in\mathcal{I};\quad c_i(x^*)=0,\ i\in\mathcal{E}$

③ $\lambda_i^*\ge 0,\quad \lambda_i^* c_i(x^*)=0,\quad i\in\mathcal{I}$

$c_i(x^*)>0\Rightarrow\lambda_i^*=0$

$c_i(x^*)=0$

① $T_\Omega(x^*)\subseteq F(x^*)$

② LiCQ: $F(x^*)=T_\Omega(x^*)$

$x^*\in\Omega,\ i\in A(x^*)$ $\{\nabla c_i(x^*)\}$ Li.

Farkas:

$\forall g\in\mathbb{R}^n$, 一真一假

① $g\in K$

② $\exists d\in\mathbb{R}^n,\ g^Td<0,\ B^Td\ge 0,\ C^Td=0$

Not both.

$K=\{v\in\mathbb{R}^n:\ v=By+Cw,\ y\ge 0\}$

$B=(b_1,b_2,\cdots b_s),\quad C=(c_1,c_2,\cdots c_t)$

$b_i,c_i\in\mathbb{R}^n,\quad b_i=\nabla c_i(x),\ i\in\mathcal{I}$

$c_i=\nabla c_i(x),\ i\in\mathcal{E}$

$$v=[\nabla c_i(x)]_{i\in\mathcal{I}}\,y+[\nabla c_i(x)]_{i\in\mathcal{E}}\,w,\quad y\ge 0$$

$$=\sum_{i\in\mathcal{I}}y_i\nabla c_i(x)+\sum_{i\in\mathcal{E}}w_i\nabla c_i(x)$$

($y_i\to\lambda_i$, $w_i\to\lambda_i$)

① $g=\nabla f(x^*)=\sum_{i\in\mathcal{I}}\lambda_i^*\nabla c_i(x^*)+\sum_{i\in\mathcal{E}}\lambda_i^*\nabla c_i(x^*)$ ($\lambda_i^*\ge 0$)

② g: $\nabla f(x^*)^Td<0$ TP等

$d\in\{B^Td\ge 0,\ C^Td=0\}$ 可行

Th2. x^* solution, $\exists\lambda^*$, KKT conditions hold.

$C(x^*,\lambda^*)$, $\forall w\in C(x^*,\lambda^*)$

$w^T\nabla_{xx}^2L(x^*,\lambda^*)w\ge 0$

Thm. x^* ... $w^T\nabla_{xx}^2L(x^*,\lambda^*)w>0$

$$\begin{cases}\min f(x)\\ s.t\ \ x\in\Omega\end{cases}$$ ①

$T_\Omega(x)$, $N_\Omega(x)$

$N_\Omega(x)=\{v:\ v^Tw\le 0,\ \forall w\in T_\Omega(x)\}$

x^* solves ①, then

$-\nabla f(x^*)\in N_\Omega(x^*)$

$\nabla f(x^*)\in T_\Omega(x^*)=F(x^*)$ LiCQ

$-\nabla f(x)\in N_\Omega(x)$

x^*, ← $x^*+\Delta x$

leader follower

Duality

$G\in S_n^{++}$

$$①\begin{cases}\min f(x)=\frac{1}{2}x^TGx+C^Tx\\ s.t\ \ \ Ax-b\ge 0\end{cases}\quad b\in\mathbb{R}^m$$ (Prime)

$L(x,\lambda)=\frac{1}{2}x^TGx+C^Tx-\lambda^T(Ax-b)$

$\nabla_x L$: $Gx+C-A^T\lambda=0$

$x=G^{-1}(A^T\lambda-C)$

$q(\lambda)=\inf_x L(x,\lambda)$

$=\frac{1}{2}(G^{-1}(A^T\lambda-C))^TG(G^{-1}(A^T\lambda-C))+C^T(G^{-1}(A^T\lambda-C))-\lambda^T(A(G^{-1}(A^T\lambda-C))-b)$

$=\frac{1}{2}(A^T\lambda-C)G^{-1}GG^{-1}(A^T\lambda-C)$

$$\begin{cases}\max\ q(\lambda)\\ s.t\ \ \lambda\ge 0\quad A^T\lambda\ge C\end{cases}$$ (Dual)

三元记，备考主题