2020年12月15日 星期二 08:10

$$\begin{cases} \min f(x) = \frac{1}{2}x^TQx + q^Tx \quad \left(= \frac{1}{2}\|Dx-C\|^2\right) \\ \text{s.t. } Ax = b \end{cases}$$

$L(x,\lambda) = f(x) - \lambda^T(Ax-b)$, $L_\rho(x,\lambda) = L(x,\lambda) + \frac{\rho}{2}\|Ax-b\|^2$

convex

$(x^k,\lambda^k) \to (x^{k+1},\lambda^{k+1})$ $\begin{cases} \lambda^{k+1} = \lambda^k - \gamma\rho(Ax^k - b) \\ x^{k+1} = \operatorname{Argmin}_x L_\rho(x,\lambda^{k+1}) \end{cases}$ $\gamma \in \left(0, \frac{1+\sqrt{5}}{2}\right)$ $\to$ $\lambda^{k+1} = \operatorname{Argmax}_\lambda \left\{ L_\rho(x^k,\lambda) - \frac{1}{2\gamma\rho}\|\lambda - \lambda^k\|^2 \right\}$

$\|\lambda^{k+1} - \lambda^k\| \to 0$

(x^*,λ^*)

$L_\rho(x,\lambda) \le \beta, \ \forall (x,\lambda)$

$\nabla f(x^*) - A^T\lambda^* = 0$

$Ax^* - b = 0$

$1 < k < T$

$$\begin{cases} x^{k+1} = \operatorname{Argmin}_x L_\rho(x,\lambda^k) \\ \lambda^{k+1} = \lambda^k - \gamma\rho(Ax^{k+1} - b) \end{cases}$$

$\gamma\rho(Ax^k - b) = \lambda^k - \lambda^{k+1}$

$\nabla f(x^{k+1}) - A^T\lambda^{k+1} + \rho A^T(Ax^{k+1} - b) = 0$

$\nabla f(x^{k+1}) - A^T\lambda^{k+1} + \rho A^T\left(\frac{1}{\gamma\rho}\right)(\lambda^k - \lambda^{k+1}) = 0$

$\nabla f(x^{k+1}) - A^T\lambda^{k+1} + \frac{1}{\gamma}A^T(\lambda^k - \lambda^{k+1}) = 0$

$$\begin{cases} \nabla f(x^{k+1}) - A^T\lambda^{k+1} = \frac{1}{\gamma}A^T(\lambda^{k+1} - \lambda^k) \to 0 \\ (Ax^{k+1} - b) + \underbrace{A(x^k - x^{k+1})}_{\to 0} = \frac{1}{\gamma\rho}\underbrace{(\lambda^k - \lambda^{k+1})}_{\to 0} \end{cases}$$

PADMM

ManPG

Proximal Gradient

BB-gradient

α_{BB}

Nes. 加速

ADMM Alternating Direction method of multipliers

$$\begin{cases} \min \theta(x,y) = f(x) + g(y) \\ \text{s.t. } Ax + By - b = 0 \end{cases}$$

$x \in X, y \in Y$ → convex set

$\|x\|_0$ 非零元个数

$Ax - b = 0$, $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, $x \in \mathbb{R}^n$

$n \gg m$

前沿

$\begin{cases} \min \|x\|_0 \\ \text{s.t. } Ax - b = 0 \end{cases} \Leftrightarrow \begin{cases} \min \|x\|_1 \\ \text{s.t. } Ax - b = 0 \end{cases}$

$x \ge 0, \ \sum x_i = c$

$\Rightarrow \begin{cases} \min \|x\|_1 \\ \text{s.t. } Ax - b = 0 \end{cases} \Rightarrow \min \tau\|x\|_1 + \frac{1}{2}\|Ax - b\|^2$

$\Rightarrow \begin{cases} \min \theta_1(x) + \theta_2(y) \\ \text{s.t. } Ax - y = b \end{cases}$ $y = Ax - b$

$\theta_1(x) = \tau\|x\|_1$, $\theta_2(y) = \frac{1}{2}\|y\|^2$

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$L(x,y,\lambda) = f(x) + g(y) - \lambda^T(Ax + By - b)$

$L_\rho(x,y,\lambda) = L(x,y,\lambda) + \frac{\rho}{2}\|Ax + By - b\|^2$

$\rho > \|\lambda^*\|_\infty$, $\min_{(x,y)} L_\rho(x,y,\lambda^k)$

$$\begin{cases} \lambda^{k+1} = \lambda^k - \gamma\rho(Ax^k + By^k - b) \\ (x^{k+1}, y^{k+1}) = \operatorname{Argmin}_{x,y} L_\rho(x,y,\lambda^{k+1}) \end{cases}$$

$\Downarrow$

$$\begin{cases} x^{k+1} = \operatorname{Argmin}_x L_\rho(x, y^k, \lambda^{k+1}) \\ y^{k+1} = \operatorname{Argmin}_y L_\rho(x^{k+1}, y, \lambda^{k+1}) \end{cases}$$

ALM

$\frac{\gamma}{2}\|V - V^k\|^2$

失言

$$\begin{cases} \min_{x,y} xy \\ x \in [0,1] \\ y \in [0,1] \end{cases}$$

$f(x,y) = x \cdot y$