Applied Macroeconomic Modelling

Simulation of the 3-equation New Keynesian Model

Gauthier Vermandel

https://vermandel.fr/applied-macroeconomic-modelling/

Objectives

- Understanding the basic concepts of the New Keynesian Model;
- Simulating the 3-equation New Keynesian Model.

Additional reading list

- Galí, Jordi. Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications. Princeton University Press, 2015.
- Adjemian, Stéphane, et al. "Dynare: Reference manual, version 5." (2024).

Some history of modern macroeconomics

- In the 1960s, Keynesian economics was the dominant view;
- Macroeconometrics, policy planning, and forecasting were addressed through the lens of large-scale macroeconomic models. Refer to the description of the 80-equation model by Rasche and Shapiro (1968) for an example;
- These models were real (no nominal effects), such that inflation and its expectations would not play any role in driving aggregate fluctuations.

Some history of modern macroeconomics

US Change in GDP deflator (annualized)

Some history of modern macroeconomics

- On top of that, their forecasting performances were poor compared to simple time series models [Nelson, 1972].
- Emergence of a rich set of time series models: AR, MA, VAR, ARCH, ... up to DSGE models (that we will estimate later on).
- Influential revolution, including in macroeconomic theory: pushed toward much more parsimonious models (part of Bob Lucas' intellectual revolution in macroeconomics).

Some history of modern macroeconomics

- Finally, those large macro models were not "structural" and therefore subject to Lucas Jr (1976) critique.
- To illustrate, one can estimate IS effect, based on equation (2) of Rasche and Shapiro (1968):

$$I_t = \gamma_1 Y_t + \gamma_2 R_t + \gamma_3 C_t$$

- - Assume that real rate is $R = r - \pi^e$, the Fed decides on r through a rule, a change in the determination (policy reform) of r_t would change the determination of investment in response (i.e. elasticities $\hat{\gamma}_1$, $\hat{\gamma}_2$, $\hat{\gamma}_3$).
  - Agents adapt rationally to this new environment, reshape their expectations π^e, leading in turn to erroneous policy recommendation if $\hat{\gamma}_2$ used for forecast under alternative monetary regime.

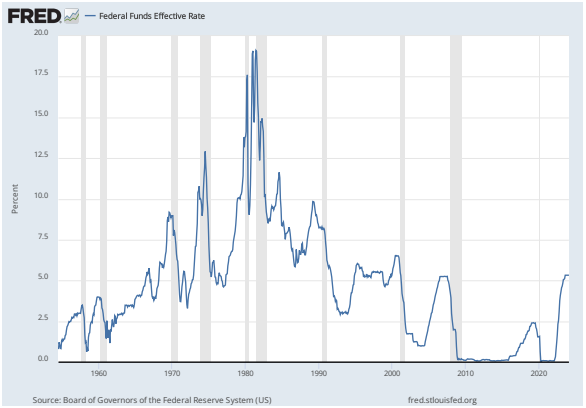
Some history of modern macroeconomics

- A crystal clear description of the research agenda is given in Lucas (1977) that provides the core basis of modern macroeconomics: all business cycles are alike, fluctuations described as an AR(1) process, thinking at the aggregate level is sufficiently accurate as sectoral effects wash out, etc.;
- The first (Mickey Mouse) prototype of business cycles in Lucas Jr (1975);
- The first prototype of a full-fledged macro model in Kydland and Prescott (1982).
- However, these setups are still real models, with nominal effects neglected. This literature is referred to as Real Business Cycle (RBC) theory: macroeconomics based only on quantities with a single source of fluctuations;
- Still an extremely influential framework.

Some history of modern macroeconomics

- What created so much inflation in the 70s? This remains an open question in macroeconomics: Was it the role of money supply or expectations/credibility or wage negotiations?
- All these conclusions led to a major rethinking of the operational framework of central banks: money supply is no longer considered a core instrument.
- Volcker's Fed disinflation period was characterized by (i) a large increase in real rate, and (ii) anchoring of inflation expectations to a certain target.
- However, this new operational framework, referred to as inflation targeting, was not supported by any theoretical model.
- Monetary policymaking came to be recognized more as an "art" than a science.

Some history of modern macroeconomics



US Federal Funds rate

Plan

The 3-equation model

- Microfoundation of price stickiness as discussed in Calvo (1983) and Rotemberg (1982) leads to the New Keynesian Phillips curve in a linear form:

$$\text{AS curve: } \widehat{\pi}_t = \beta \mathbb{E}_t \widehat{\pi}_{t+1} + \kappa \hat{y}_t + \varepsilon_t^S.$$

  where $\widehat{\pi}_t$ represents inflation, β the discount factor, κ the slope of the curve, the output gap $\hat{y}_t$, and ε_t^S a stochastic AR(1) process (supply shifter).

- This represents a dynamic reinterpretation, with a forward-looking term, of the Phillips curve that relates inflation to economic activity (here, represented by the output gap).

- An expected imbalance in the goods market, indicated by a positive output gap ($\sum_{s=0}^{\infty} \beta^s \kappa \hat{y}_{t+s} > 0$), leads to a rise in inflation $\widehat{\pi}_t$.

The 3-equation model

- The Euler equation (i.e., the indifference curve between consumption now and tomorrow) combined with the resource constraint ($\hat{y}_t = \hat{c}_t$) yields the Aggregate Demand (AD) curve:

$$\text{AD curve: } \hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}\left(\hat{r}_t - \mathbb{E}_t \widehat{\pi}_{t+1}\right) + \varepsilon_t^D.$$

  where $\hat{y}_t$ represents aggregate demand, σ is the risk aversion coefficient, $\hat{r}_t$ the central bank interest rate, and ε_t^D a stochastic AR(1) demand shifter process.

- The determination of aggregate spending relies on the forward sum of (log) real interest rates:

$$\sum_{s=0}^{\infty} \left[\frac{-1}{\sigma}\left(\hat{r}_{t+s} - \mathbb{E}_t \widehat{\pi}_{t+1+s}\right) + \varepsilon_{t+s}^D\right].$$

- The term structure of interest rates, influenced by both the central bank and private sector expectations, determines the extent of household spending now.

The 3-equation model

- The last equation, Taylor (1993)'s rule, represents the setting of the nominal rate by the central bank:

$$\text{MP curve: } \hat{r}_t = \rho \hat{r}_{t-1} + (1-\rho)\left(\phi^{\pi}\hat{\pi}_t + \phi^{y}\hat{y}_t\right) + \varepsilon_t^R.$$

  where $\hat{r}_t$ is the nominal rate, $\rho \in [0, 1)$ the smoothing coefficient, $\phi^{\pi} \geq 1$ the inflation stance, ϕ^{y} the output gap stance, and ε_t^R a stochastic AR(1) monetary policy shock.

- For stability, it is required that $\phi^{\pi} > 1$. This means the Monetary Policy (MP) must respond proportionally more to a change in inflation to ensure the stability of the system and avoid inflation spirals. In other words, the real interest rate must rise following an increase in inflation to anchor expectations.

The 3-equation model

The baseline New Keynesian model emerged in the 90s around three core linearized equations:

$$\text{AS curve: } \widehat{\pi}_t = \beta \mathbb{E}_t \widehat{\pi}_{t+1} + \kappa \hat{y}_t + \varepsilon_t^S. \tag{1}$$

$$\text{AD curve: } \hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}\left(\hat{r}_t - \mathbb{E}_t \widehat{\pi}_{t+1}\right) + \varepsilon_t^D. \tag{2}$$

$$\text{MP curve: } \hat{r}_t = \rho \hat{r}_{t-1} + (1-\rho)\left(\phi^{\pi} \widehat{\pi}_t + \phi^{y} \hat{y}_t\right) + \varepsilon_t^R. \tag{3}$$

$$\text{supply shock: } \varepsilon_t^S = \rho^S \varepsilon_{t-1}^S + \eta_t^S. \tag{4}$$

$$\text{demand shock: } \varepsilon_t^D = \rho^D \varepsilon_{t-1}^D + \eta_t^D. \tag{5}$$

$$\text{monetary policy shock: } \varepsilon_t^R = \rho^R \varepsilon_{t-1}^R + \eta_t^R. \tag{6}$$

A first order Taylor expansion

- Macroeconomics still dominated by linearized models.
- The model can be written in matrix form:

$$F\mathbb{E}_t\{\hat{z}_{t+1}\} + G\hat{z}_t + H\hat{z}_{t-1} + M\eta_t = 0$$

where

- F, G, H and M are matrices stacking coefficients related to the dynamic system.
- $\hat{z}_t$ stacks in a vector the endogenous variables $\hat{z}_t = [\hat{y}_t, \hat{\pi}_t, \hat{r}_t, \varepsilon_t^S, \varepsilon_t^D, \varepsilon_t^R]$.
- η_t stacks the innovations $\eta_t = [\eta_t^S, \eta_t^D, \eta_t^R]$.

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A first order Taylor expansion

- One looks for a recursive solution that would have the following form:

$$\hat{z}_t = P\hat{z}_{t-1} + Q\eta_t$$

  where P and Q are two unknown matrices.

- To solve the problem, one must determine the expectation scheme. Lucas implements 'rational' expectations, i.e. model-based expectations: $E_t\{\hat{z}_{t+1}\} = P\hat{z}_t + Q\eta_{t+1}$, with conditional expectation $E_t\{\eta_{t+1}\} = 0$ as $\eta_t \sim \mathcal{N}(0, \Sigma)$.

- One can find P and Q by substituting $E_t\{\hat{z}_{t+1}\}$ and $\hat{z}_t$:

$$\underbrace{\left[FP^2 + GP + H\right]}_{\text{determining } P}\hat{z}_{t-1} + \underbrace{(FPQ + GQ + M)}_{\text{determining } Q}\eta_t = 0$$

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- Presence of quadratic matrix equation $FP^2 + GP + H = 0$ with two solutions. The usual pratice is to calculate solutions for P, pick P that have eigenvalues <1 absolute terms. If no stable solution, have a look at Blanchard and Kahn (1980).
- Puzzling aspect of linear rational expectations model: one path of the economy not stable.
- Once solved, a linear DSGE model has similar features as VAR(1) model (useful when it comes to its estimation).

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A static illustration of central bank conduct to shocks

- To illustrate the core mechanisms of the New Keynesian model, consider a static model such as:

$$\rho^S = \rho^D = \rho^R = \rho = 0$$

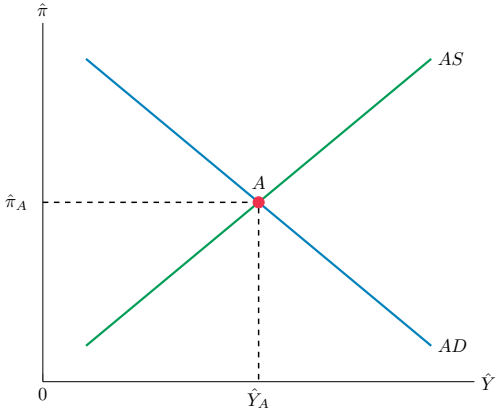
- If no persistence (no state variable), the solution of the system is $\hat{z}_t = Q\eta_t \Rightarrow$ expectations are fixed $\hat{z}_{t+1} = Q\eta_{t+1} = 0$
- The system boils down to

$$\text{AS curve: } \widehat{\pi}_t = \kappa \hat{y}_t + \eta_t^S. \tag{7}$$

$$\text{AD curve: } \hat{y}_t = -\frac{1}{\sigma}\hat{r}_t + \eta_t^D. \tag{8}$$

$$\text{MP curve: } \hat{r}_t = \phi^\pi \widehat{\pi}_t + \phi^y \hat{y}_t + \eta_t^R. \tag{9}$$

- In absence of shocks, each variable is at its steady state (point A)
- What is the conduct of monetary policy in the wake of demand or supply shock?



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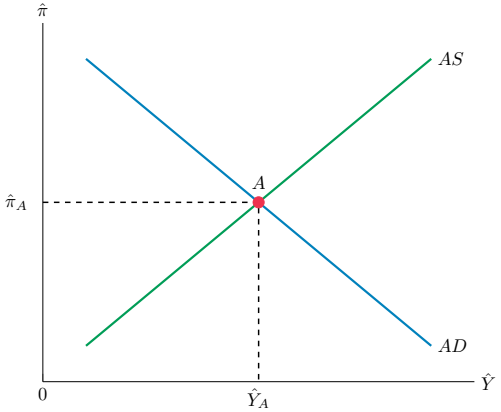
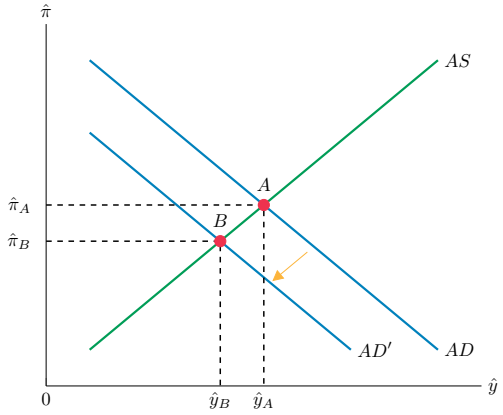


A static illustration of central bank conduct to shocks

- Consider the impact of a negative demand shock, indicated by $\eta_t^D < 0$.
- Without policy intervention, the Aggregate Demand (AD) curve shifts downward, leading to new equilibrium B.
- The CB responds by decreasing the interest rate to stimulate the economy to shift back to A.
- This situation illustrates the concept of "Divine coincidence" (as termed by Blanchard), where both prices and quantities move in the same direction, simplifying policy responses.



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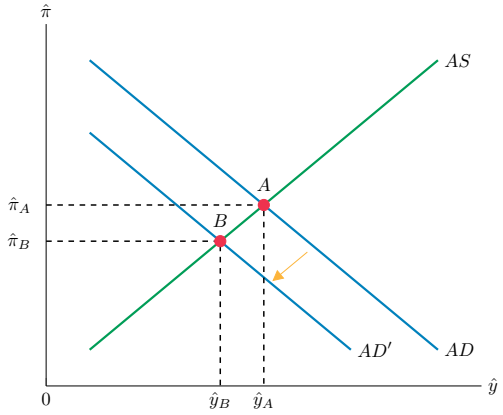

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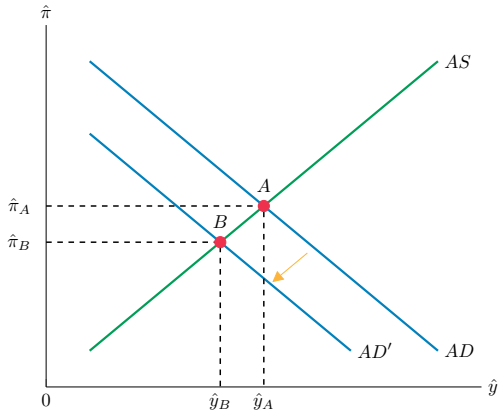

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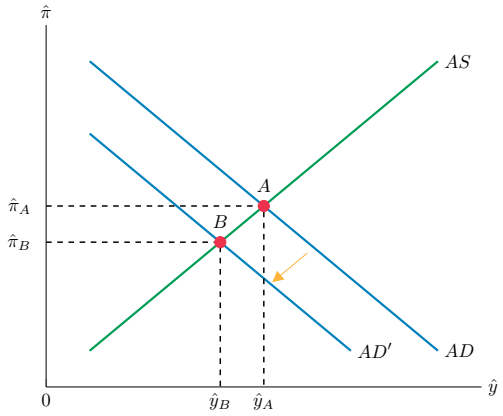
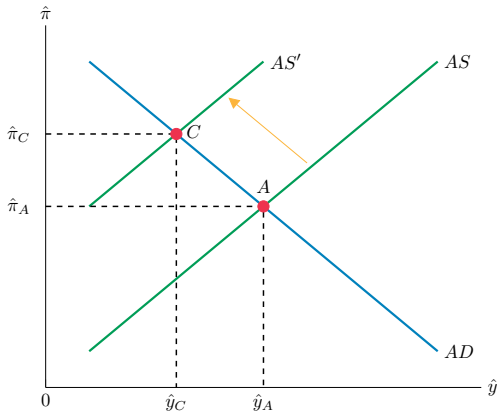


A static illustration of central bank conduct to shocks

- Consider if the fluctuation originates from a supply increase, represented as $\eta_t^S > 0$.
- In the absence of policy intervention, the Aggregate Supply (AS) curve shifts upward, leading to a new equilibrium at point C.
- Since quantities and prices move in opposite directions, it is not possible to mitigate both types of fluctuations simultaneously.



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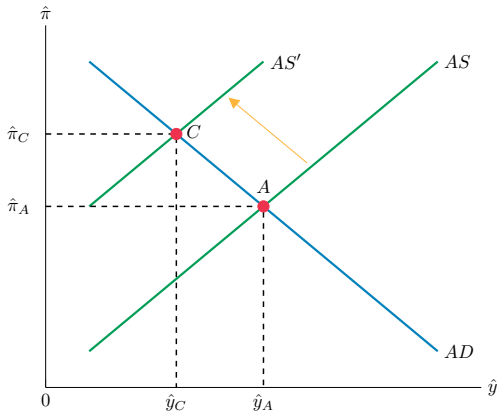

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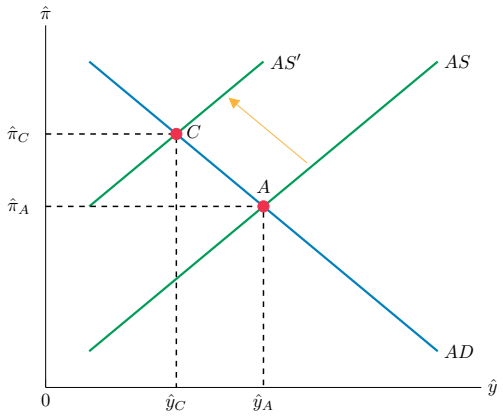
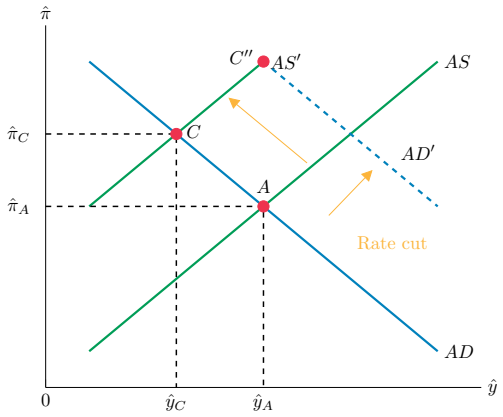


A static illustration of central bank conduct to shocks

- A dovish central banker will aim to stabilize output.
- How? By cutting interest rates, albeit at the risk of fueling inflation.
- This demonstrates a trade-off between stabilizing inflation and stabilizing output.
- The origin of economic fluctuations is crucial in guiding the conduct of monetary policy!



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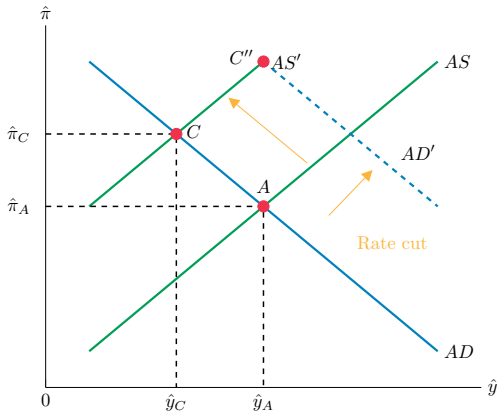

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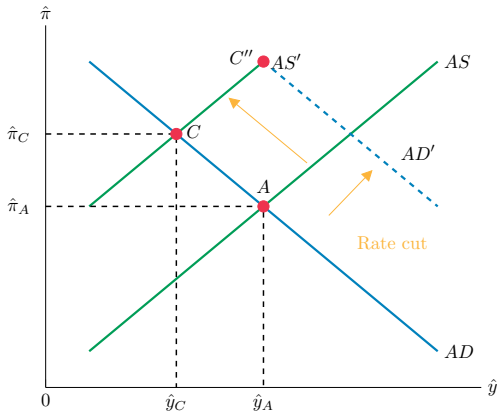

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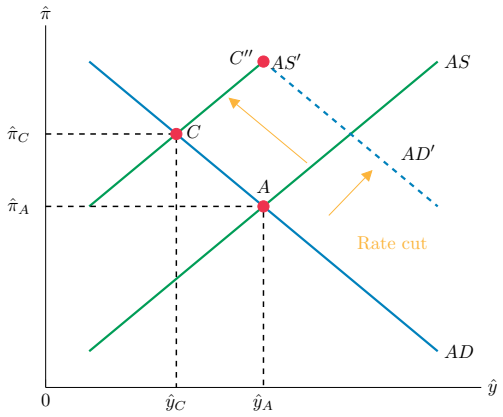

Concluding remarks

- Since the 1990s, the New Keynesian model has been providing theoretical foundations for the modern conduct of monetary policy.
- The New Keynesian framework has become akin to a universal language among central bankers, essential for characterizing and discussing the operational aspects of monetary policy.
- Following the insights from Clarida et al. (1999), monetary policy has evolved to be recognized more as a mature science than an art.

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