Conditioning and Bayes' Rule

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1 Conditioning

Conditional probability are the same as original probality, but when we have a theory suppose is A. We have the probability of A, then B occured. Now we have to reconsider the proability of A given the B is occured. It can be denoted as:

$$P(A|B)$$

To calculate this we consider the event that B occured as a sample space: As

you can see the red part in B it's the $P(A \cap B)$, and you consider B is a sample space of it. You can get this formula:

$$P(A|B) = \frac{P(A \cup B)}{P(B)}$$

Its like rescaling the sample space to B By the collary, We can obtain:

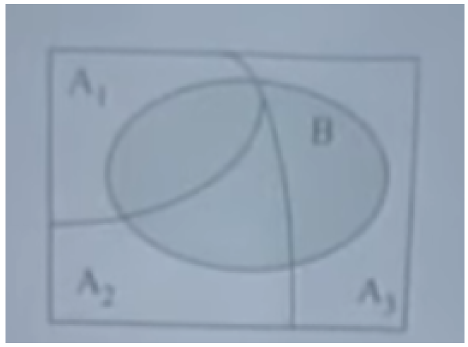
$$P(A \cup B) = P(A|B)P(B) = P(B|A)P(A)$$

1.1 The total probabily theorem

We have a total probabily theorem. It makes a way to calcuate $P(B)$. As you can see the B created by many parts of $A_i \cup B$

Therefore we have the formula:

$$P(B) = \sum P(A_i)P(B|A_i)$$



2 The bayes theorem

Bayes theorem is refine all the thing we have mentioned to more formal way:

$$P(A|B) = \frac{P(B|A)P(A)}{\sum P(A_i)P(B|A_i)}$$