

Probability Models and Axioms

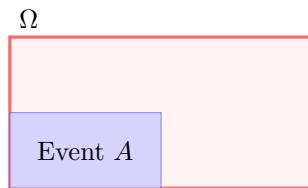
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1 Sample Space, Likelihood, and Events

The **Sample Space** (denoted as Ω) is the set of all possible outcomes. We assign a **probability** to these outcomes to represent their likelihood.

An **Event** is a subset of the sample space. Think of the sample space as a cake and an event as cream cheese spread on it. If the cream cheese spreads over the entire cake, the probability is 1.



2 Union

A **union** (\cup) is the merging of subsets. Like mixing two flavors of cream cheese, the union includes regions with only flavor A, only flavor B, and the region where they overlap.

2.1 Axiom of Additivity

If events A_1, A_2, \dots, A_n are **disjoint** (they do not overlap), the probability of their union is the sum of their individual probabilities:

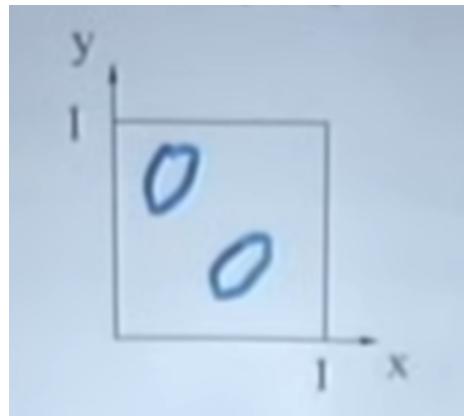
$$P(A_1 \cup A_2 \cup \dots \cup A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$$



3 Discrete vs. Continuous Uniform Law

In a Discrete Uniform Law, every individual outcome has the same probability (like a fair die). In a Continuous Uniform Law, the sample space is infinite (like a 2D plane).

In the continuous case:



- **Uniform Law:** Probability is proportional to the area.

- $P(A) = \frac{\text{Area}(A)}{\text{Total Area of } \Omega}$

If two different regions have the same area, they have the same probability.