LESSON 6
GEOMETRY

Reminder: Before beginning this lesson remember to redo the problems from Lesson 2 that you have marked off. Do not "unmark" a question unless you get it correct.

Equations of Lines in General Form

The **general form of an equation of a line** is $\boldsymbol{ax + by = c}$ where a, b and c are real numbers. If $b \neq 0$, then the slope of this line is $m = -\frac{a}{b}$. If $b = 0$, then the line is vertical and has no slope.

Let us consider 2 such equations.

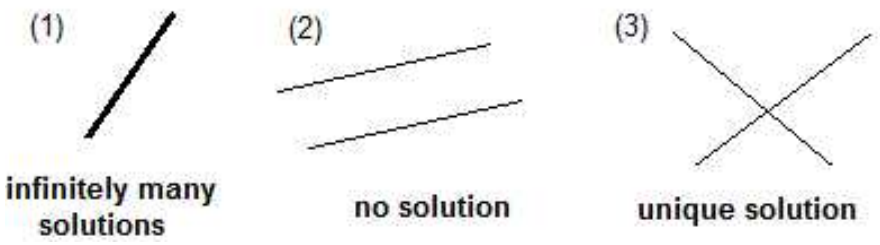
$$\boldsymbol{ax + by = c}$$
$$\boldsymbol{dx + ey = f}$$

(1) If there is a number r such that $ra = d$, $rb = e$, and $rc = f$, then the two equations represent the **same line**. Equivalently, the two equations represent the same line if $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$. In this case the system of equations has **infinitely many solutions**.

(2) If there is a number r such that $ra = d$, $rb = e$, but $rc \neq f$, then the two equations represent **parallel** but distinct lines. Equivalently, the two equations represent parallel but distinct lines if $\frac{a}{d} = \frac{b}{e} \neq \frac{c}{f}$. In this case the system of equations has **no solution**.

(3) Otherwise the two lines intersect in a single point. In this case $\frac{a}{d} \neq \frac{b}{e}$, and the system of equations has a **unique solution**.

These three cases are illustrated in the figure below.



Example: The following two equations represent the same line.

$$\boldsymbol{2x + 8y = 6}$$
$$\boldsymbol{3x + 12y = 9}$$

To see this note that $\frac{2}{3} = \frac{8}{12} = \frac{6}{9}$.(or equivalently, let $r = \frac{3}{2}$ and note that $\left(\frac{3}{2}\right)(2) = 3$, $\left(\frac{3}{2}\right)(8) = 12$, and $\left(\frac{3}{2}\right)(6) = 9$).

The following two equations represent parallel but distinct lines.

$$\boldsymbol{2x + 8y = 6}$$
$$\boldsymbol{3x + 12y = 10}$$

This time $\frac{2}{3} = \frac{8}{12} \neq \frac{6}{10}$.

The following two equations represent a pair of intersecting lines.

$$\boldsymbol{2x + 8y = 6}$$
$$\boldsymbol{3x + 10y = 9}$$

This time $\frac{2}{3} \neq \frac{8}{10}$.

Try to answer the following question. **Do not** check the solution until you have attempted this question yourself.

LEVEL 4: GEOMETRY

$$2x - 7y = 12$$
$$kx + 6y = -17$$

1. For which of the following values of k will the system of equations above have no solution?

(A) $-\frac{144}{17}$

(B) $-\frac{12}{7}$

(C) $\frac{12}{7}$

(D) $\frac{144}{17}$

$\frac{2}{k} \neq \frac{-7}{6}$ $\quad k \neq -\frac{12}{7}$

As mentioned above, the system of equations

$$ax + by = c$$
$$dx + ey = f$$