# LESSON 2
GEOMETRY

Computation of Slopes

Slope formulas are not given on the SAT. You should make sure that you know the following.

$$\text{Slope} = m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: Lines with positive slope have graphs that go upwards from left to right. Lines with negative slope have graphs that go downwards from left to right. If the slope of a line is zero, it is horizontal. Vertical lines have **no** slope (this is different from zero slope). We may also use the expressions **undefined** or **infinite** to describe the slope of vertical lines.

positive slope

negative slope

zero slope

no slope
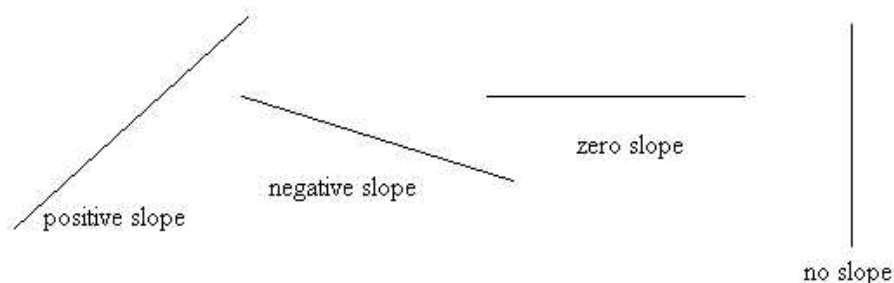
The **slope-intercept form of an equation of a line** is $y = mx + b$ where m is the slope of the line and b is the y-coordinate of the y-intercept, i.e. the point $(0, b)$ is on the line. Note that this point lies on the y-axis.

Technical note: The SAT sometimes contains an abuse of language with regard to intercepts. A problem may talk about the y-intercept b. Technically a y-intercept is a point of the form $(0, b)$, but many people identify this point with the number b.

The **point-slope form of an equation of a line** is $y - y_0 = m(x - x_0)$ where m is the slope of the line and (x_0, y_0) is any point on the line.

Try to answer the following question using this strategy together with the strategy of picking numbers from Lesson 1. **Do not** check the solution until you have attempted this question yourself.

28

LEVEL 4: GEOMETRY

1. If $a > 1$, what is the slope of the line in the xy-plane that passes through the points (a^2, a^4) and (a^3, a^6)?

   (A) $-a^3 + a^2$
   (B) $-a^3 - a^2$
   (C) $a^3 - a^2$
   (D) $a^3 + a^2$

Solution by picking a number: Let's pick a number for a, say $a = 2$. So the two points are $(4,16)$ and $(8,64)$. The slope of the line passing through these two points is

$$m = \frac{64-16}{8-4} = \frac{48}{4} = \textbf{12}$$

Put a nice big, dark circle around the number 12. We now plug $a = 2$ into each answer choice.

   (A) $-8 + 4 = -4$
   (B) $-8 - 4 = -12$
   (C) $8 - 4 = 4$
   (D) $8 + 4 = 12$

Since choices (A), (B), and (C) all came out incorrect, the answer is (D).

Remark: We could have also gotten the slope geometrically by plotting the two points, and noticing that to get from $(4,16)$ to $(8,64)$ we need to travel up 48 units and right 4 units. So the slope is

$$m = \frac{rise}{run} = \frac{48}{4} = \textbf{12}.$$

Before we go on, try to solve this problem directly (without plugging in numbers).

* Algebraic solution: Using the slope formula we have

$$m = \frac{a^6-a^4}{a^3-a^2} = \frac{a^4(a^2-1)}{a^2(a-1)} = \frac{a^2(a+1)(a-1)}{a-1} = a^2(a+1) = a^3 + a^2.$$

This is choice (D).

Note: Do not worry if you have trouble following this solution. The algebra performed here will be reviewed later in the book.

Plug in the Given Point

If the graph of a function or other equation passes through certain points, plug those points into the equation to eliminate answer choices.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

LEVEL 4: GEOMETRY

2.   Which of the following is an equation of the line in the xy-plane that passes through the point $(4, -2)$ and is perpendicular to the line $y = -4x + 7$?

        (A) $y = -4x - 3$
        (B) $y = -4x + 3$
        (C) $y = \frac{1}{4}x - 3$
        (D) $y = \frac{1}{4}x + 6$

* Solution by plugging in the point: Since the point $(4, -2)$ lies on the line, if we substitute 4 in for x, we should get $-\mathbf{2}$ for y. Let's substitute 4 in for x in each answer choice.

        (A) $-4 * 4 - 3 = -16 - 3 = -19$
        (B) $-4 * 4 + 3 = -16 + 3 = -13$
        (C) $(1/4) * 4 - 3 = 1 - 3 = -2$
        (D) $(1/4) * 4 + 6 = 1 + 6 = 7$

We can eliminate choices (A), (B) and (D) because they did not come out to -2. The answer is therefore choice (C).

Important note: (C) is **not** the correct answer simply because y came out to -2. It is correct because all 3 of the other choices did **not** give -2 for y.

Before we go on, try to solve this problem using geometry.

Geometric solution: Note that the given line has a slope of -4. Since **perpendicular lines have slopes that are negative reciprocals of each other**, $m = \frac{1}{4}$.

30

Also, we are given that the point $(x_0, y_0) = (4, -2)$ is on the line. We use the point-slope form for the equation of a line $\boldsymbol{y - y_0 = m(x - x_0)}$ to get $y - (-2) = \frac{1}{4}(x - 4)$. Let's solve this equation for y.

$$y + 2 = \frac{1}{4}(x - 4)$$
$$y + 2 = \frac{1}{4}x - 1$$
$$y = \frac{1}{4}x - 3$$

Therefore, the answer is choice (C).

Note: To get the reciprocal of a number we interchange the numerator and denominator. The number -4 has a "hidden" denominator of 1, so the reciprocal of -4 is $-\frac{1}{4}$. Now to get the negative reciprocal, we simply change the sign of the reciprocal. Thus, the negative reciprocal of -4 is $\frac{1}{4}$.

Now try to solve each of the following problems by plugging in the given points if possible. Then, if possible, solve each problem another way. The answers to these problems, followed by full solutions are at the end of this lesson. **Do not** look at the answers until you have attempted these problems yourself. Please remember to mark off any problems you get wrong.

LEVEL 1: GEOMETRY
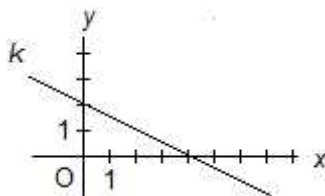
3. What is the equation of line k in the figure above?

     (A) $y = -2x + 2$
     (B) $y = -2x + 4$
     (C) $y = -\frac{1}{2}x + 2$
     (D) $y = -\frac{1}{2}x + 4$

LEVEL 2: GEOMETRY

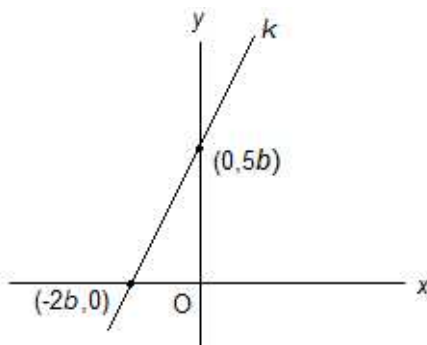4.   A line in the xy-plane passes through the origin and has a slope of $-\frac{2}{3}$. Which of the following points lies on the line?

   (A) $(-6,4)$
   (B) $(3,-3)$
   (C) $(3,2)$
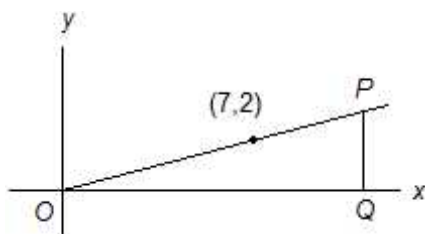   (D) $(0,\frac{2}{3})$

LEVEL 3: GEOMETRY

5.   Which of the following equations represents a line that passes through the point $(0,-5)$ and is parallel to the line with equation $y = -4x + 7$?

   (A) $4x + y = -7$
   (B) $4x + y = -5$
   (C) $x + 4y = 20$
   (D) $x + 4y = 28$

$$y = -4x + b$$
$$b = -5$$

6.   In the figure above, what is the slope of line k ?

32

7. Line k (not shown) passes through O and intersects $\overline{PQ}$ between P and Q. What is one possible value of the slope of line k?

8. The line m in the xy-plane contains points from each of Quadrants I, III, and IV, but no points from Quadrant II. Which of the following must be true?

    (A) The slope of line m is positive.
    (B) The slope of line m is negative.
    (C) The slope of line m is zero.
    (D) The slope of line m is undefined.

9. In the xy-coordinate plane, line n passes through the points $(0,5)$ and $(-2,0)$. If line m is perpendicular to line n, what is the slope of line m?

    (A) $-\dfrac{5}{2}$

    (B) $-\dfrac{2}{5}$

    (C) $\dfrac{2}{5}$

    (D) $\dfrac{5}{2}$

LEVEL 4: GEOMETRY

10. Which of the following is the equation of a line in the xy-plane that is perpendicular to the line with equation $y = 3$?

    (A) $y = -3$
    (B) $y = -\dfrac{1}{3}$
    (C) $x = -2$
    (D) $y = -3x$

11. In the xy-plane, the line determined by the points $(c, 5)$ and $(10, 2c)$ passes through the origin. Which of the following could be the value of c ?

   (A) 0
   (B) 5
   (C) 10
   (D) 25

[handwritten: $5\cancel{Q} = ac + 5$; $2c = \cancel{a} \cdot 10 + b$; $0 = ac + b$]

LEVEL 5: GEOMETRY

12. In the xy-plane, the points $(5, e)$ and $(f, 7)$ are on a line that is perpendicular to the graph of the line $y = -\frac{1}{5}x + 12$. Which of the following represents e in terms of f?

   (A) $5f + 32$
   (B) $-5f + 32$
   (C) $-\frac{1}{5}f + 32$
   (D) $\frac{1}{5}f + 32$

[handwritten: -5]

Answers

1. D	5. B	9. B
2. C	6. 5/2 or 2.5	10. C
3. C	7. $0 < m < .259$	11. B
4. A	8. A	12. B

Full Solutions

5.

* Solution by plugging in the point: Since the point $(0, -5)$ lies on the line, if we substitute 0 in for x and -5 in for y we should get a true equation.

   (A) $0 + (-5) = -7$ False
   (B) $0 + (-5) = -5$ True
   (C) $0 + 4(-5) = 20$ False
   (D) $0 + 4(-5) = 28$ False

We can eliminate choices (A), (C) and (D) because they came out False. The answer is therefore choice (B).