
# Statistical and active shape models

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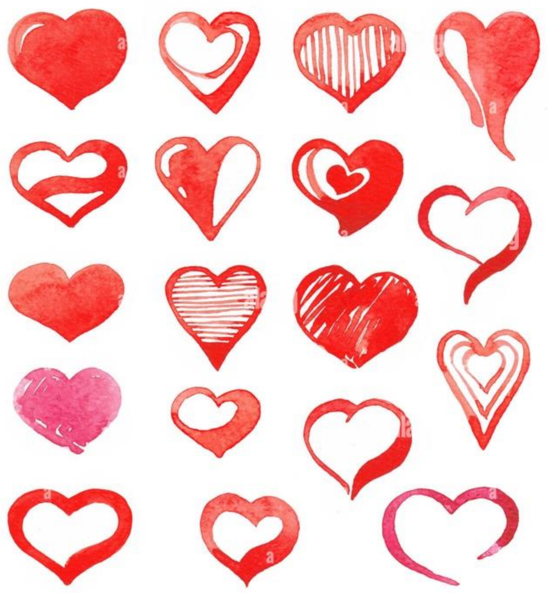
# Background and motivation

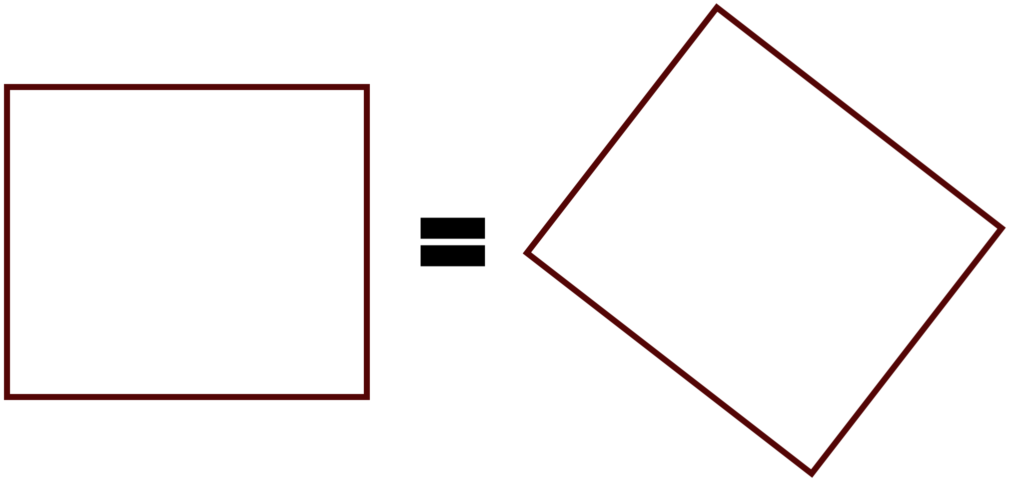
# What is a shape?

“We here define ‘shape’ informally as ‘all the geometrical information that remains when location, scale and rotational effects are filtered out from an object.” *David George Kendall*



# What else do we know about a shape?

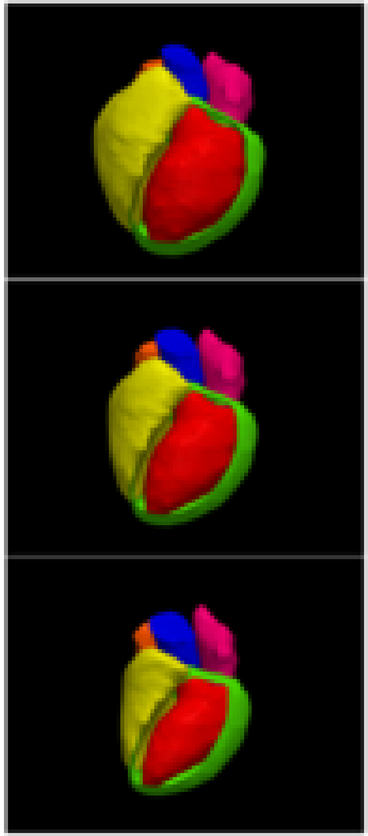
Two shapes are considered to be the same if one can translate, rotate, and scale one so that it exactly matches the other. For instance, a square remains a square if it is moved, scaled, or translated.



# Why are we interested in shape?

While size is important (this can be measured from segmentations.

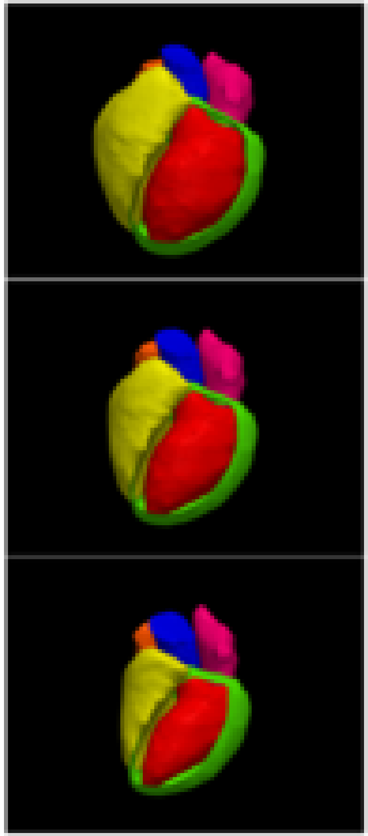
Differences in shape can also indicate disease



# Statistical shape model

A data-driven approach to gain knowledge about the mean shape of an object and how much the shape can vary between subjects

Mean shape



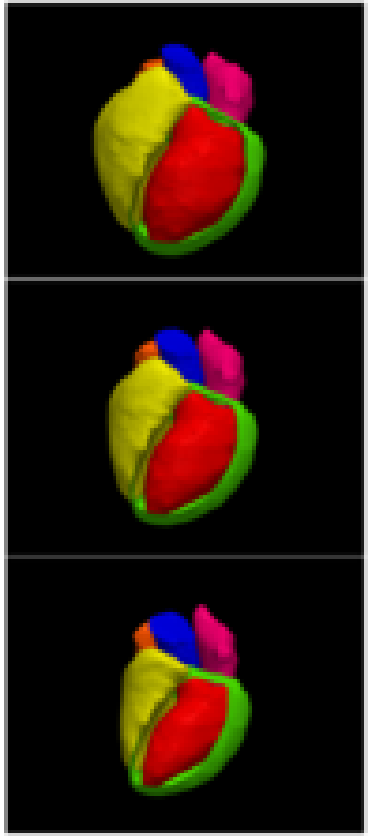
# Statistical shape model

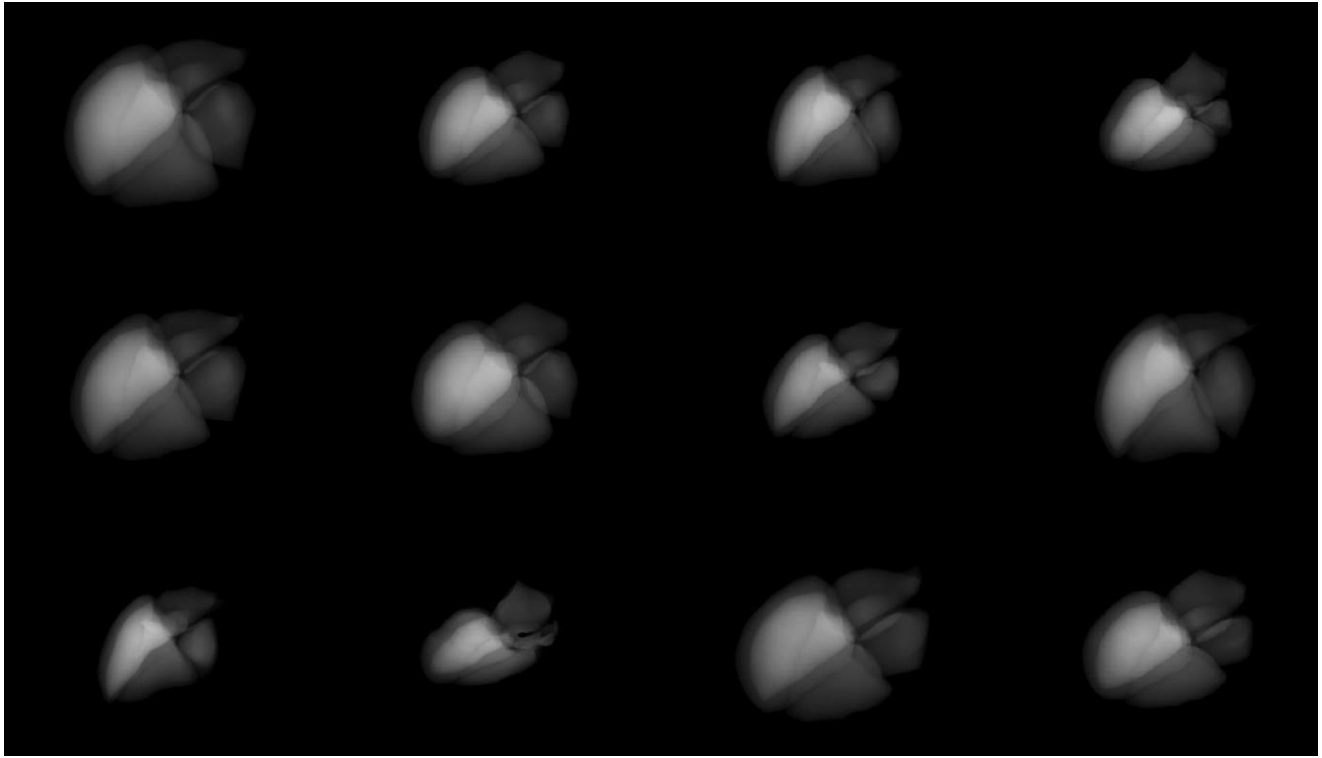
A data-driven approach to gain knowledge about the mean shape of an object and how much the shape can vary between subjects

Variations

Mean shape

Variations





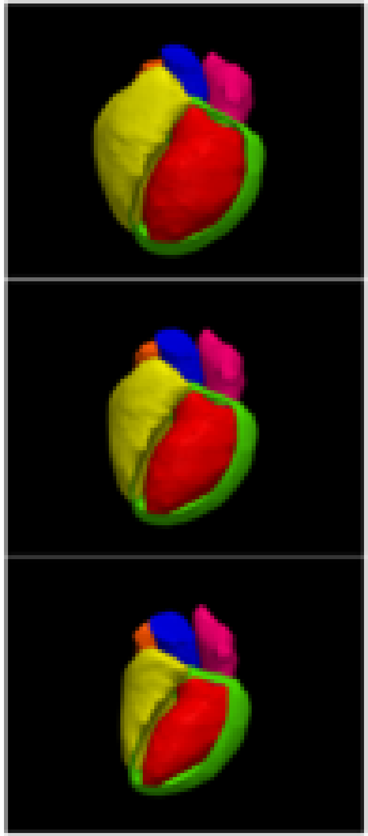
A training set of variations in a cardiac model

# Applications

**Classification**

to distinguish between different types of variations

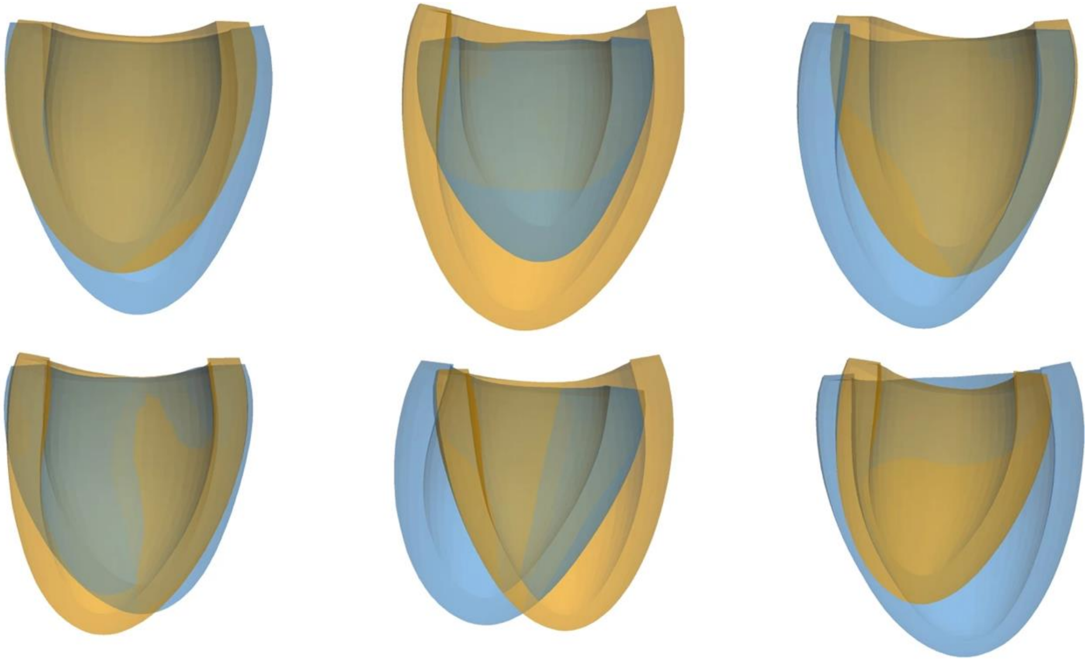
For example, you know that a certain change in shape is caused by a disease



# Applications

**Phantom generation**

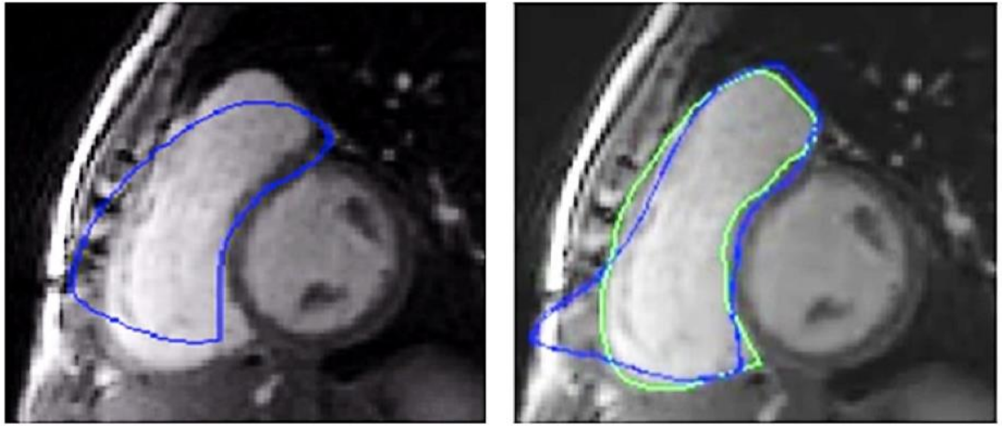
Use shape extrapolation to create realistic representations within given variation



# Applications

**Segmentation** (later)

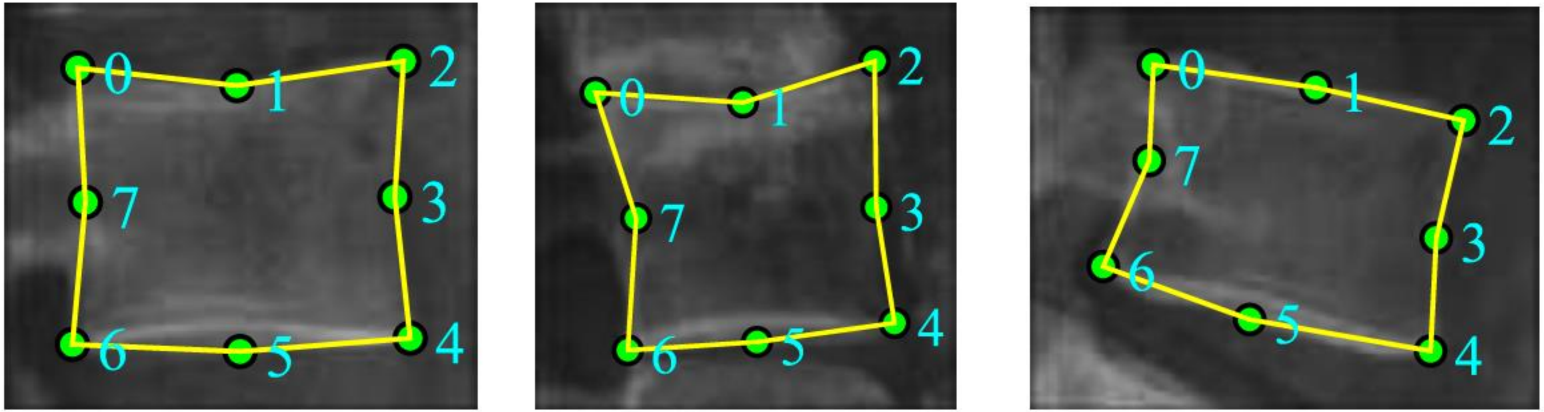
Use prior knowledge about the shape to segment images



# Algorithm

# Statistical shape modelling

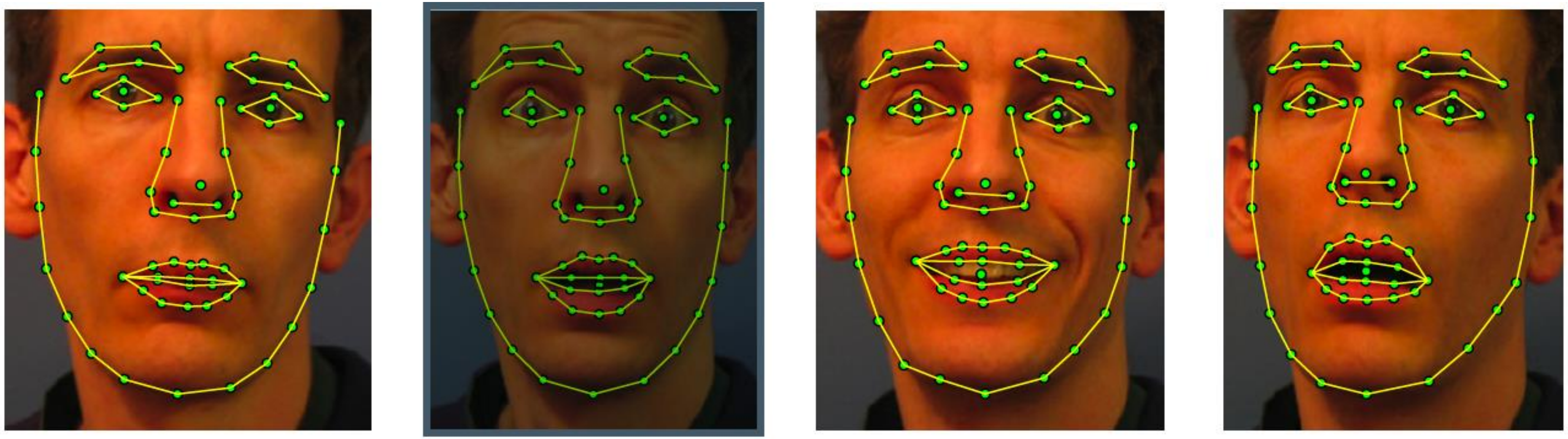
- Assume that shapes can be defined by a set of landmarks
- Assume that there is a correspondence between landmarks on different images



# Statistical shape modelling

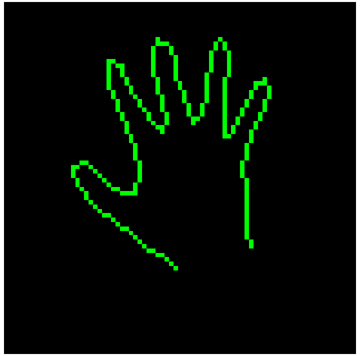
That is:

- shapes are represented by set of K points, e.g. in 2D: $(x_i, y_i)$
- and the same point corresponds to the same landmarks on different images



# Comparing shapes

To understand variations in shapes we need to know when shapes are the same and when shapes are different



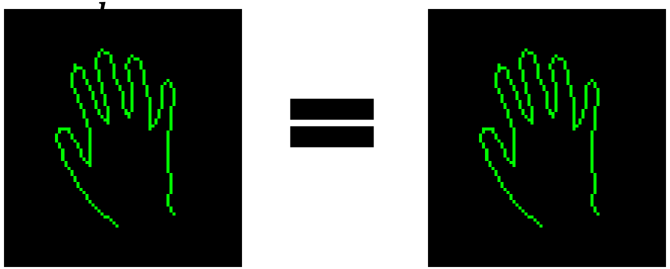
$\neq$

# Analyzing shapes

To compare two shapes we can compute the sum of square distances between their points.

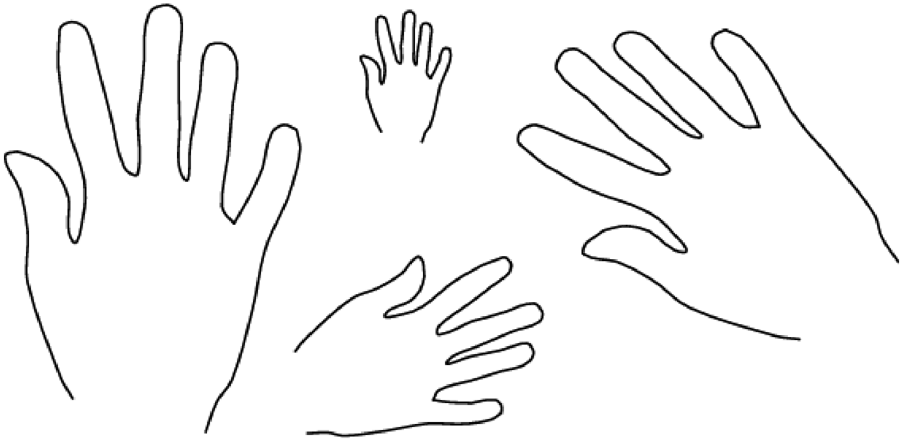
If $x_i$ is point i from shape x, then the sum of square distances between two shapes, x and z, is given by:

$$d = \sum_i |x_i - z_i|^2 = \|x - z\|_2^2$$



# But….

Translation, rotation, and scaling effects are not important when analyzing shape



# Alignment

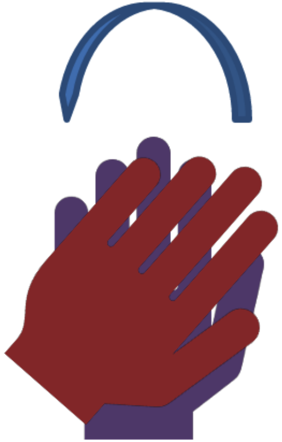
So to compare two shapes x and z. It makes sense to first align shape x to z.

This involves finding the parameters t to minimize

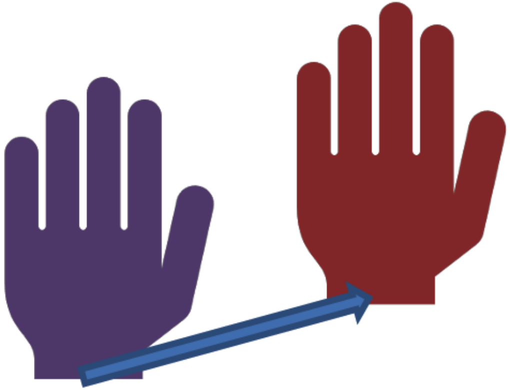
$$Q(t) = \|z - T(x;t)\|_2^2$$

# Restrict to similarity transformations

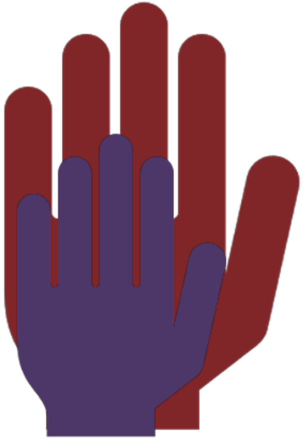
$$\boldsymbol{p}' = s\boldsymbol{R}\boldsymbol{p} + \boldsymbol{t}$$



Rotation $\boldsymbol{R}$



Translation $\boldsymbol{t}$

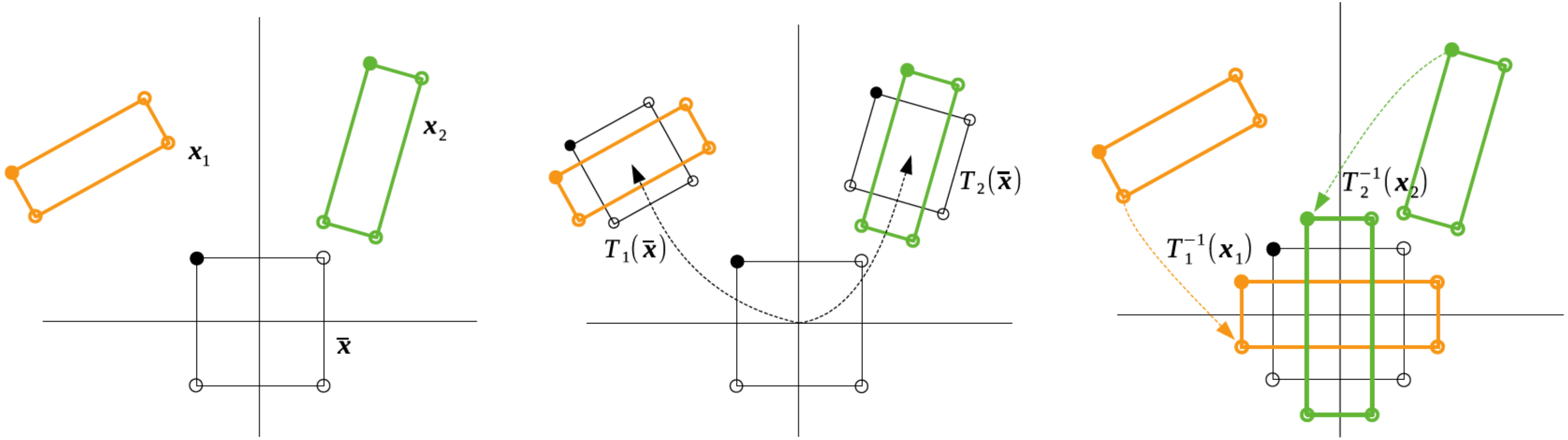


Scaling $s$

# Generalized Procrustes analysis

Iteratively determine a mean shape to which all shapes in the training set have minimal distance (includes alignment and scaling).

# Align a set of shapes




(a) Two shapes $\mathbf{x}_1, \mathbf{x}_2$ and reference $\bar{\mathbf{x}}$

(b) Find $T_i$ to best map $\bar{\mathbf{x}}$ to each $\mathbf{x}_i$

(c) Apply $T_i^{-1}$ to map each $\mathbf{x}_i$ to the reference frame

# Algorithm

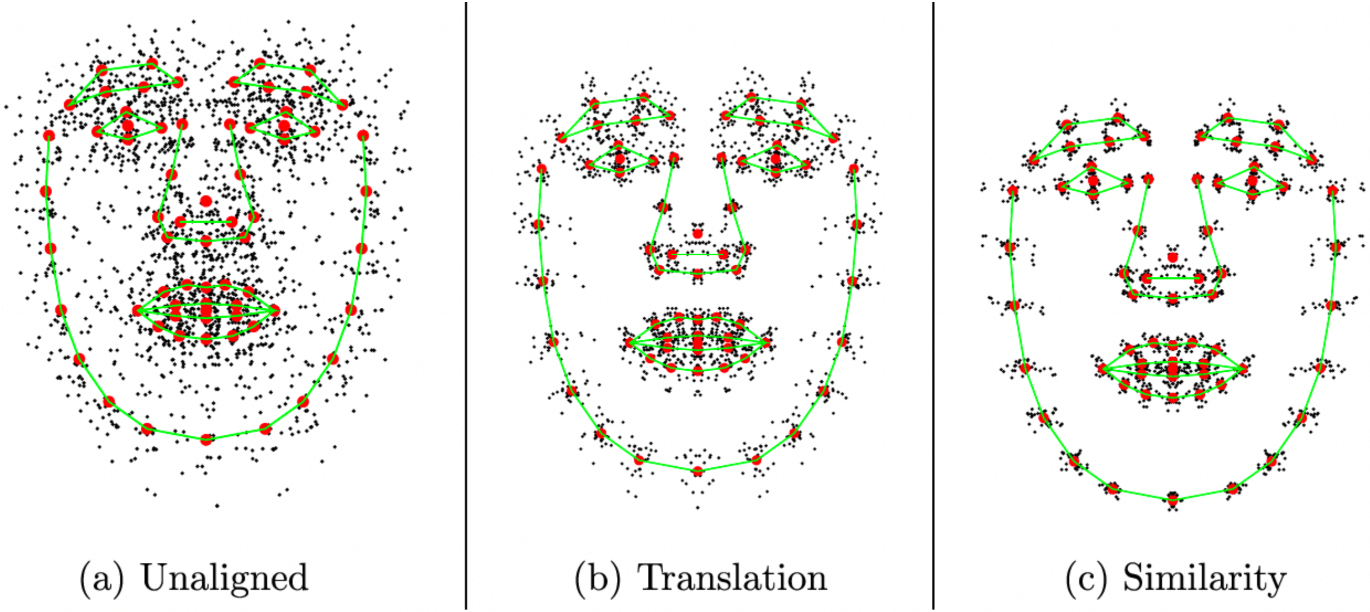
1. Choose one shape to define the orientation, say $x_r = x_0$.
2. Center $x_r$ so its center of gravity is at the origin.
3. Scale $x_r$ to have unit size ($|x_r| = 1$).
4. Set initial estimate of mean $\bar{x} = x_r$ .
5. Repeat until convergence:

   For each shape compute $T_j$ to minimize $|T_j(\bar{x}) - x_j|^2$.

   Update estimate of mean: $\bar{x} = \frac{1}{\mathrm{N}} \sum_{j=1}^{N} T_j^{-1}(x_j)$

   Align $\bar{x}$ to $x_r$ to fix rotation, then scale so that $|\bar{x}| = 1$.

# Result



(a) Unaligned (b) Translation (c) Similarity

# How to model the variation?

## Building a statistical shape model

Once aligned, use Principal Component Analysis (PCA) is to extract the principal modes of variation by computing the eigenvectors of the covariance matrix.

# Principal Component Analysis (PCA) - Recap

**Goal:** Finding the principal components that describe our data.
= finding the directions in which the data shows most variation

Useful for dimensionality reduction

- E.g. find low-dimensional classification boundaries

Results in better generalization!

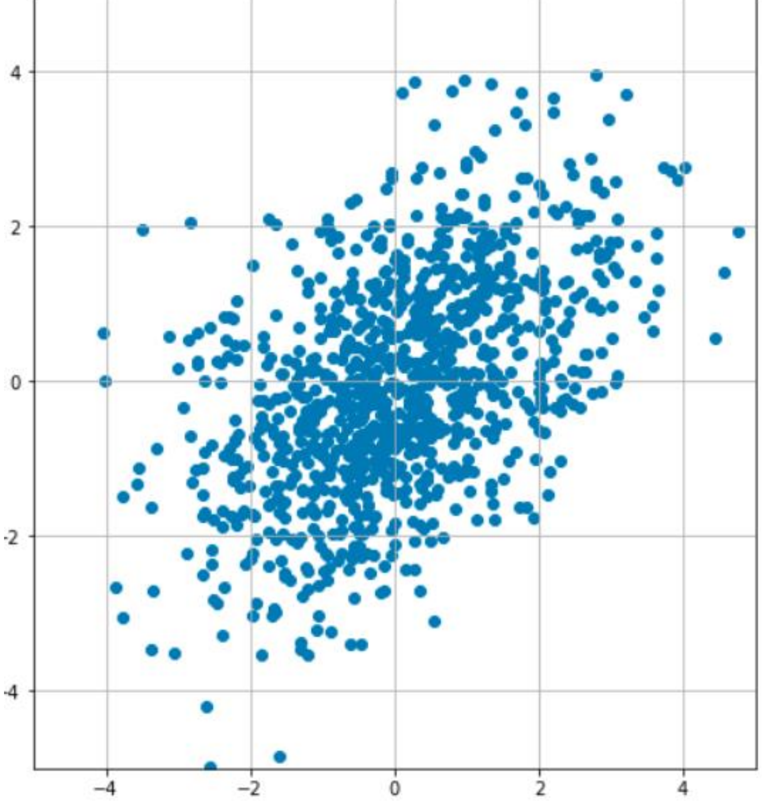
# Principal Component Analysis

**Example data set**

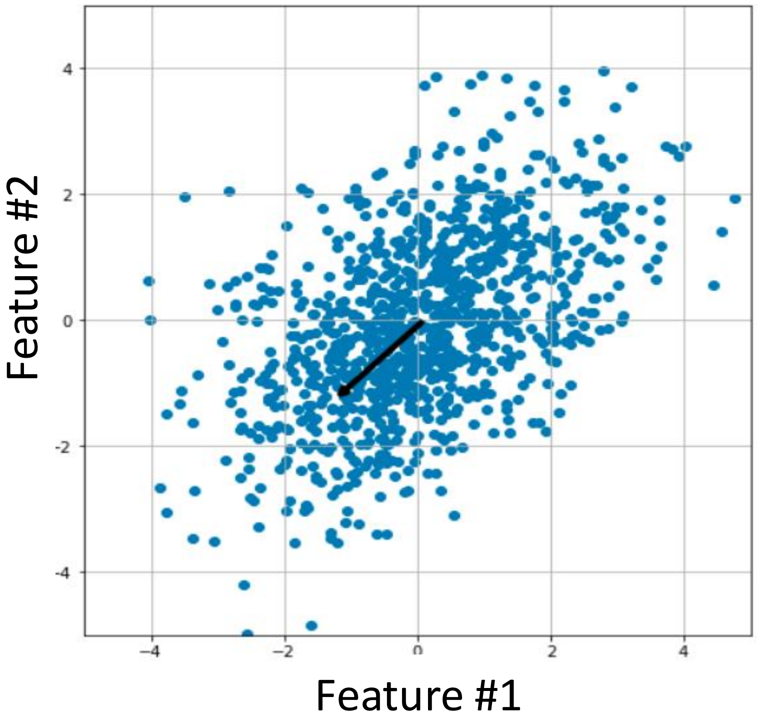
$M$-by-2 matrix $\boldsymbol{X}$ containing $M$ points

Sampled from 2D Gaussian distribution

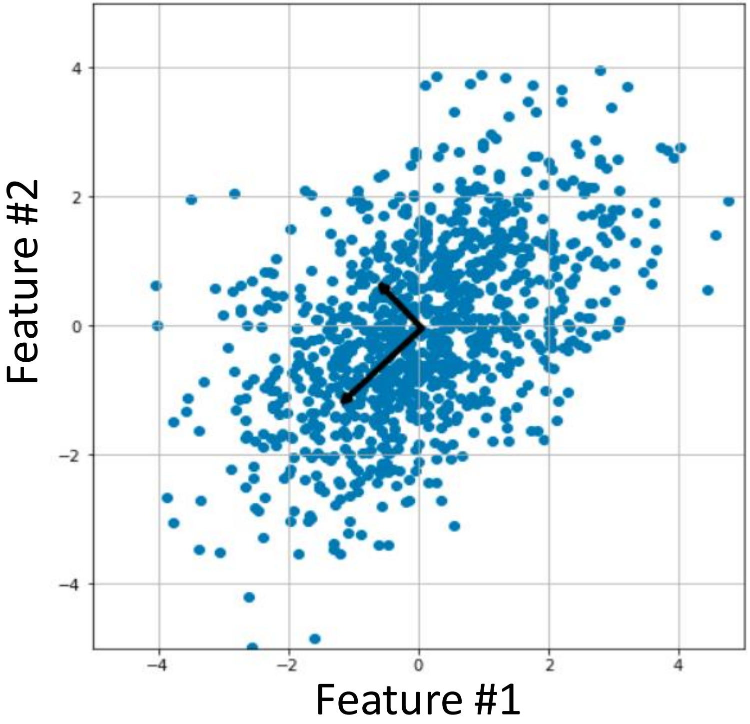
$$\mu_1 = 0$$
$$\mu_2 = 0$$
$$\Sigma = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$



# Principal Component 1



# Principal component are orthogonal to one another



# Finding the principal components

- Center data by subtracting mean of each variable

$$\widehat{\boldsymbol{X}} = \boldsymbol{X} - \overline{\boldsymbol{X}}$$

- Calculate covariance matrix

$$\Sigma = \frac{1}{M-1} \boldsymbol{X}^T \boldsymbol{X}$$

- Singular value decomposition (SVD) to find a matrix $\boldsymbol{U}$ that contains eigenvectors, ordered by largest to smallest variance

  → **principal components**

- Multiply $\hat{X}$ with $\boldsymbol{U}$ to obtain $\boldsymbol{X_{pca}}$

- Model:

$$\boldsymbol{x} = \overline{\boldsymbol{x}} + \boldsymbol{Ub}$$

# Modes of variation

Typically not all modes of variation are used.

To get the principal modes of variation of the statistical shape model, the first $m$ eigenvectors are chosen. What is $m$?

# Modes of variation

The total variance about the mean in the training set is given by the sum of all eigenvalues:

$$v_M = \sum \lambda_j$$

A widely used approach is to choose $m$ to represent a particular proportion of the total variance $p_m$ –> e.g. 0.95

$$m = \arg\min_{m\prime} \sum_{j=1}^{m\prime} \lambda_j > p_m v_M$$

# Statistical shape model

Any shape in the training set can be approximated using the mean shape and a weighted sum of the variations obtained by the first $m$ modes.

$$\boldsymbol{x} \approx \overline{\boldsymbol{x}} + \boldsymbol{U}_{\boldsymbol{m}} \boldsymbol{b}_{\boldsymbol{m}}$$

Where $\boldsymbol{U}_{\boldsymbol{m}}$ is restricted to the first $m$ modes.

# Applications

# Using statistical shape models

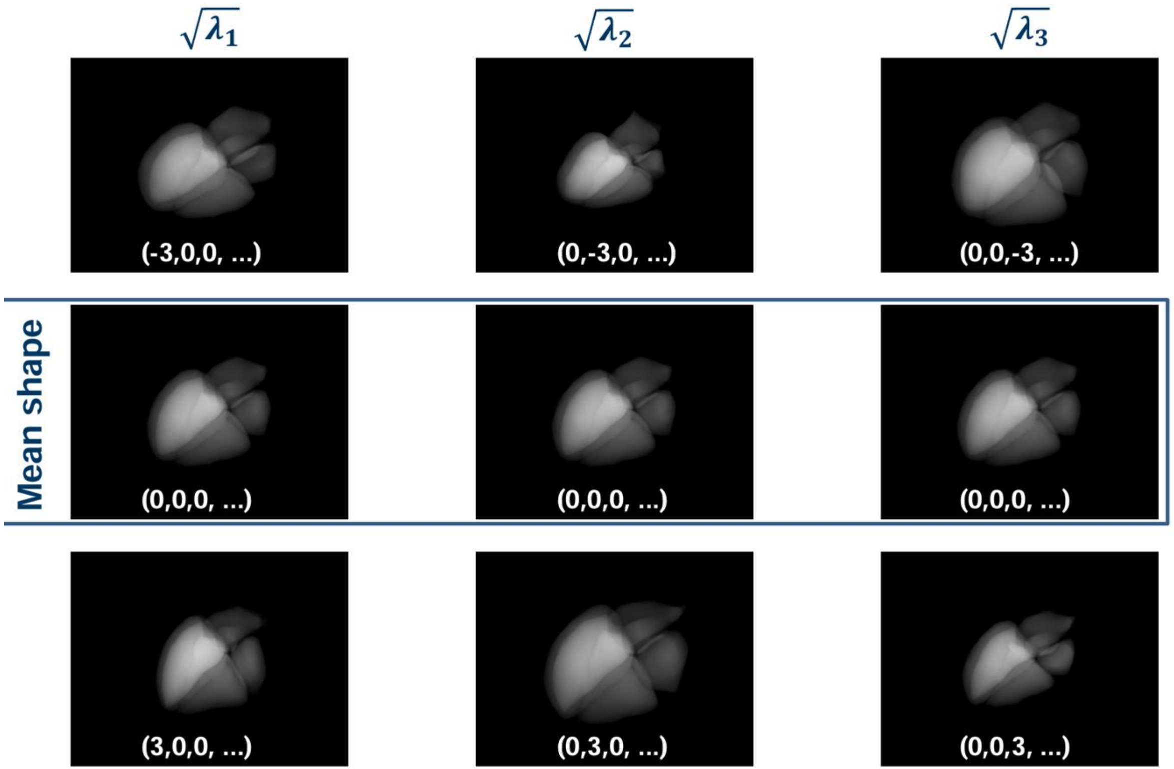
Varying the weighting of modes to explore the effect of this variation on the modes.

Weights are usually restricted to only allow “reasonable” shapes to be constructed.

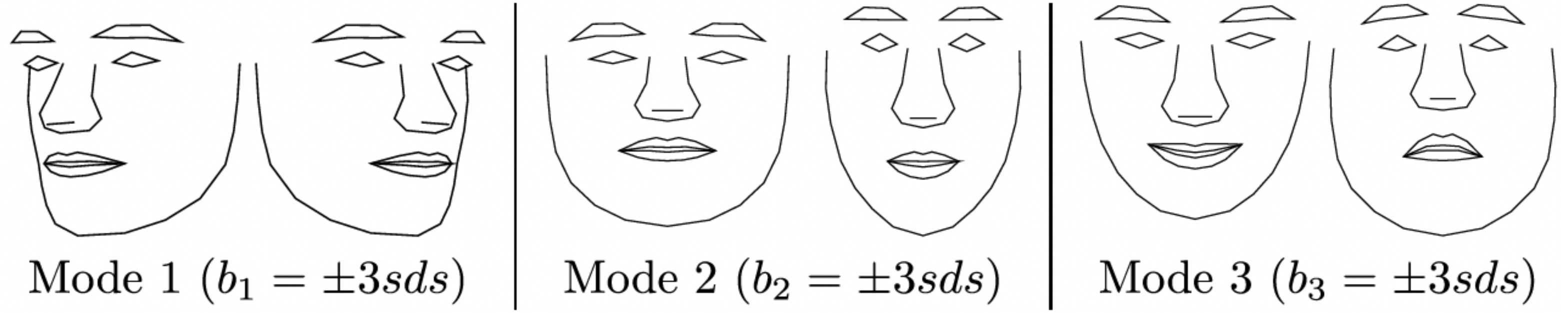
Assuming the data follows a normal distribution, feature weights $b_j$ are bounded within a certain range of the standard deviation:

$$-3\sqrt{\lambda_j} \leq b_j \leq 3\sqrt{\lambda_j}, \qquad j \in \{1, \dots, m\}$$

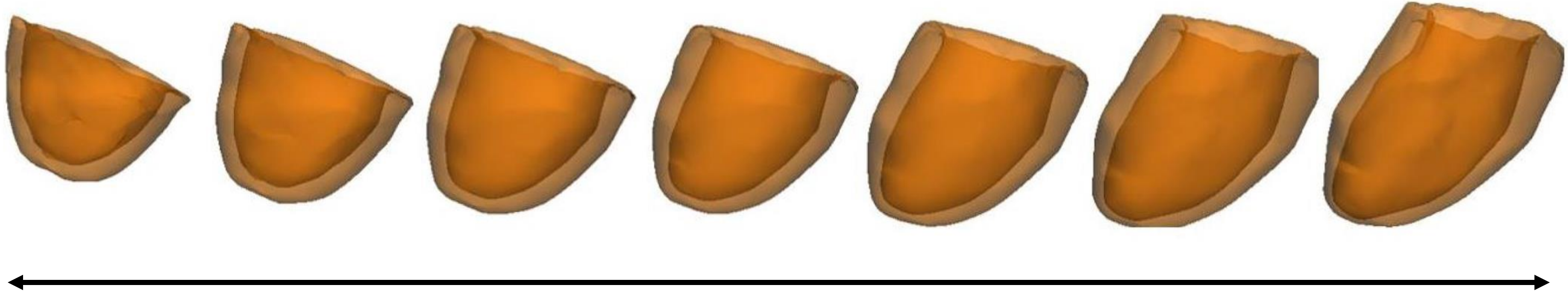
# Statistical shape model of the heart



# Interpretable

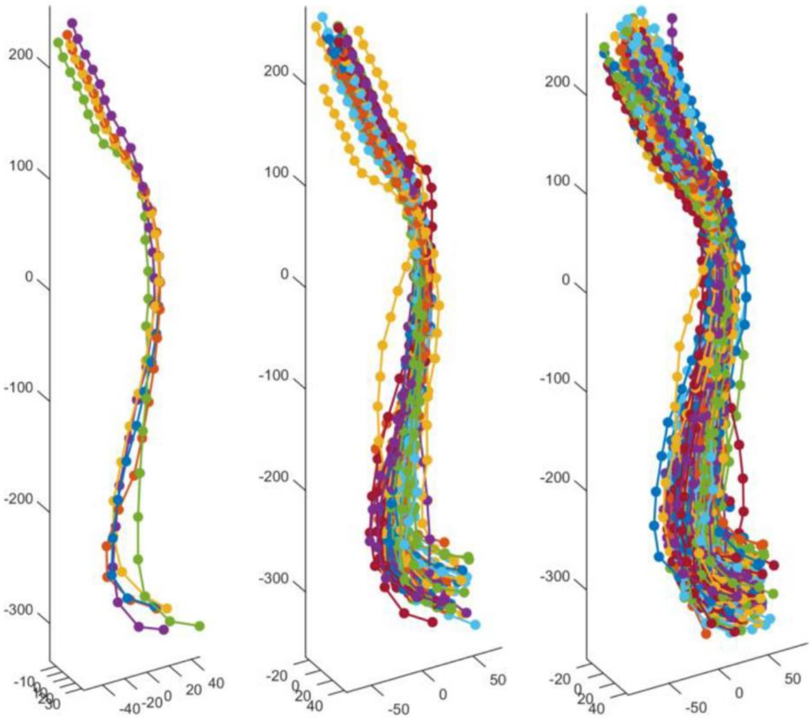


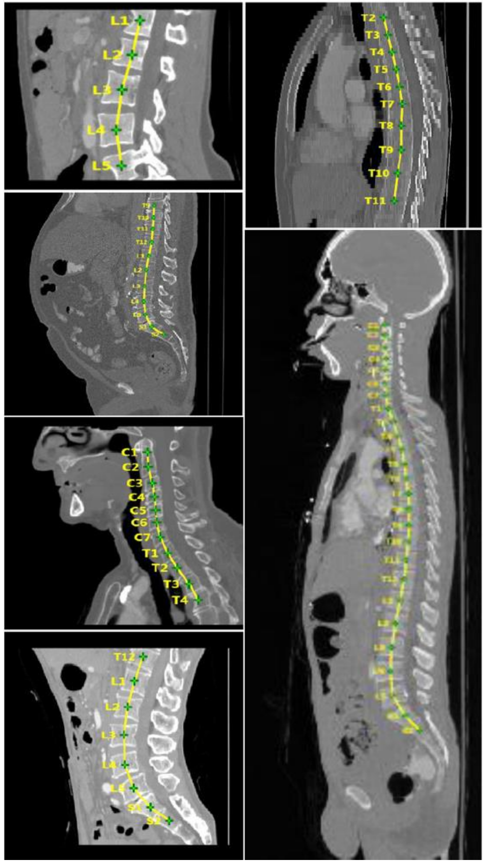
# Smooth variation



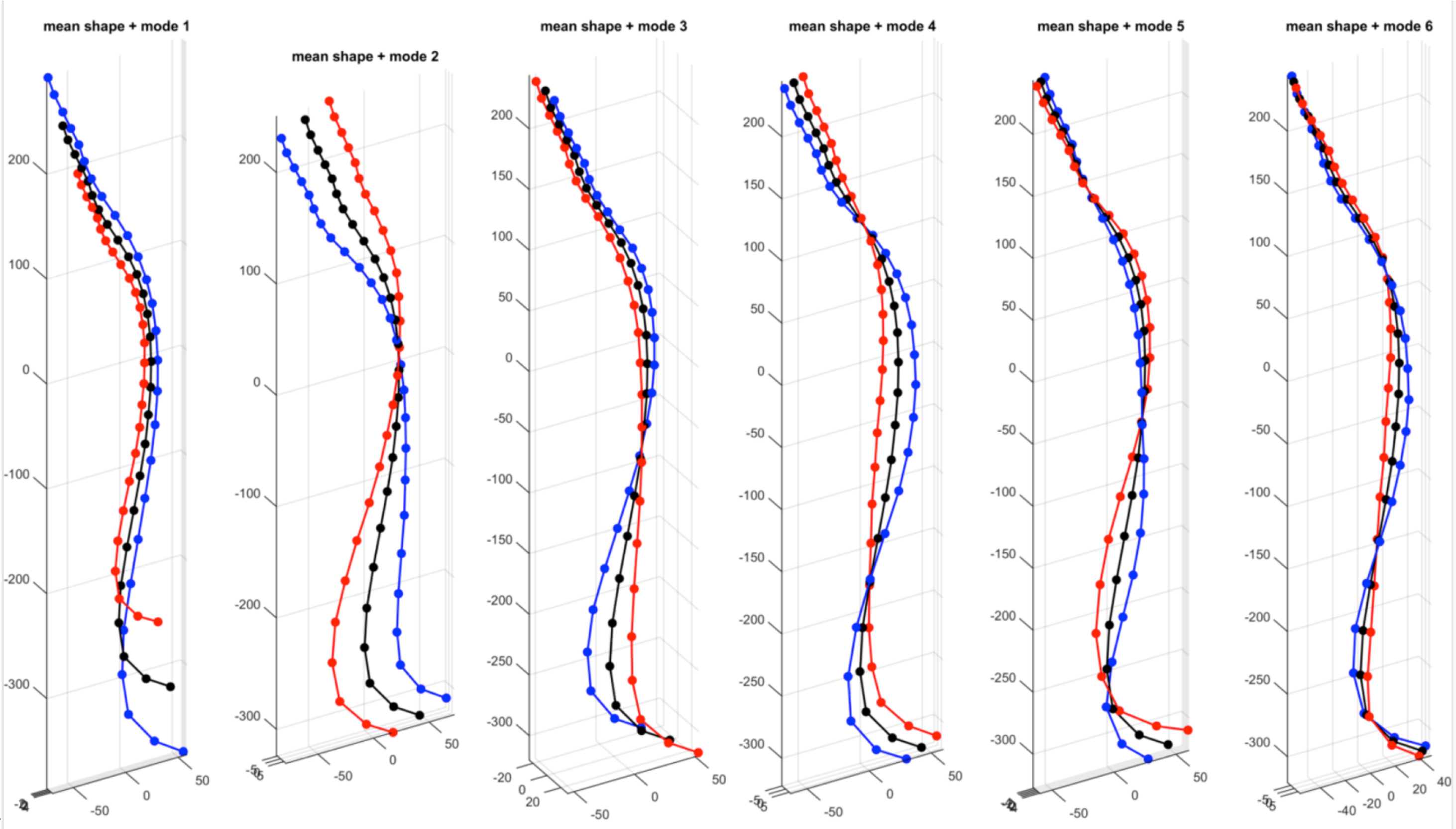
$-3\sqrt{\lambda_j}$ $-2\sqrt{\lambda_j}$ $-\sqrt{\lambda_j}$ $0$ $\sqrt{\lambda_j}$ $2\sqrt{\lambda_j}$ $3\sqrt{\lambda_j}$

# Application to the spine



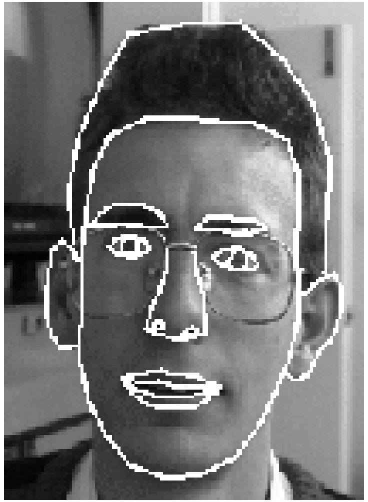


# Application to the spine



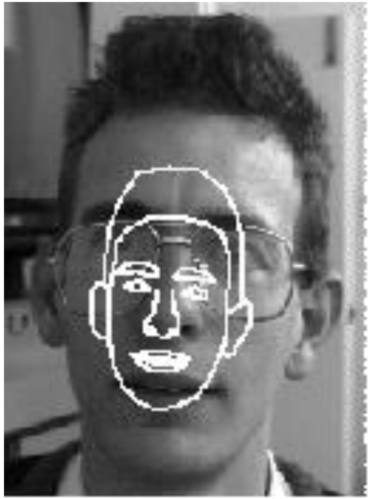
# Segmentation – active shape models

Having generated statistical shape model, we can use them to find new examples of these modelled shapes in images (segmentation).



# Segmentation – active shape models

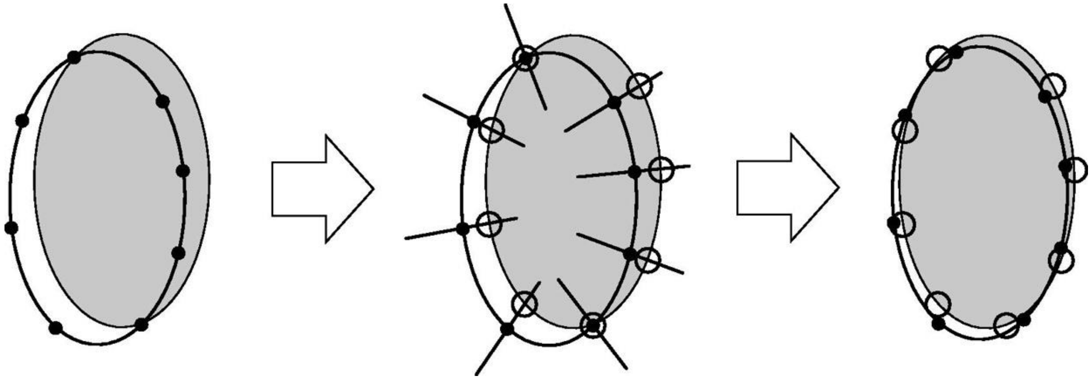
Given an initial estimate of the position of the shape, this is an iterative approach that aims to find a set of adjustments to move each point towards a better position.



# Active shape models

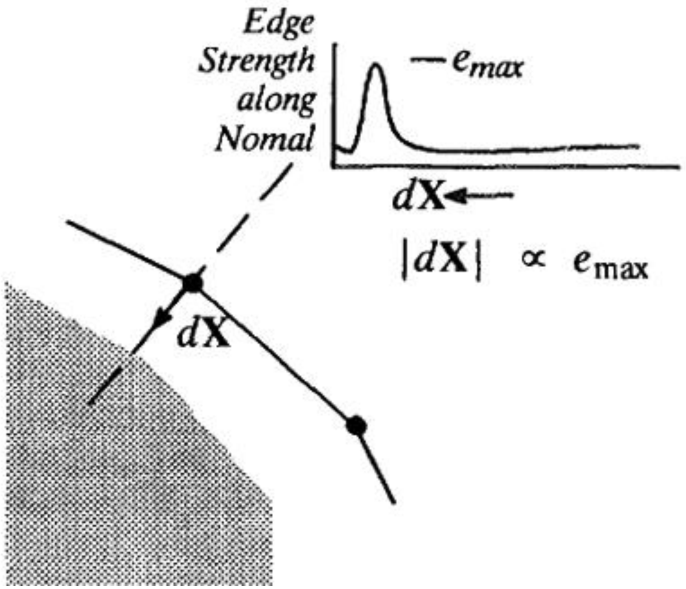
Commonly the points of the statistical shape model represent the boundaries of an object

So we need to move them towards edges in the image



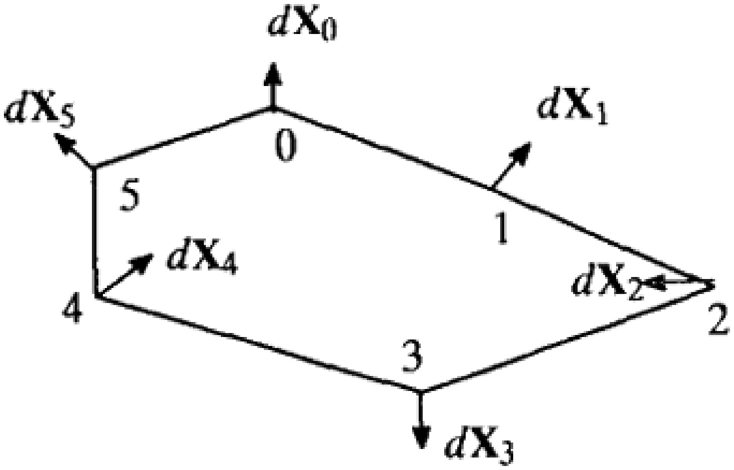
# Active shape models

E.g. move in direction that is normal to the model boundary towards the strongest image edge, taking a step that is proportional to the strength of the image edge.



# Model fitting

Iterate….



# Example

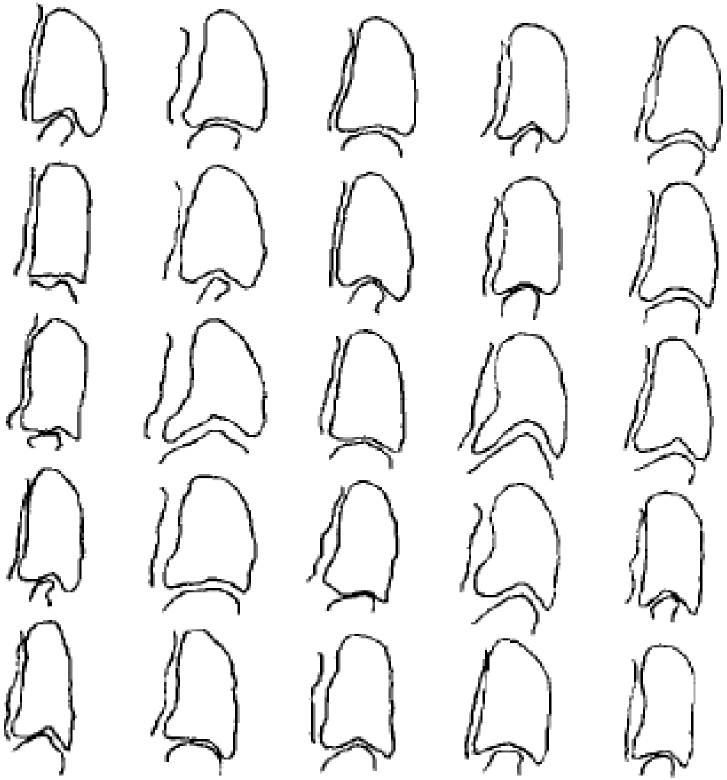


FIG. 10. Examples of heart ventricle shapes, each containing 96 points.

# Example

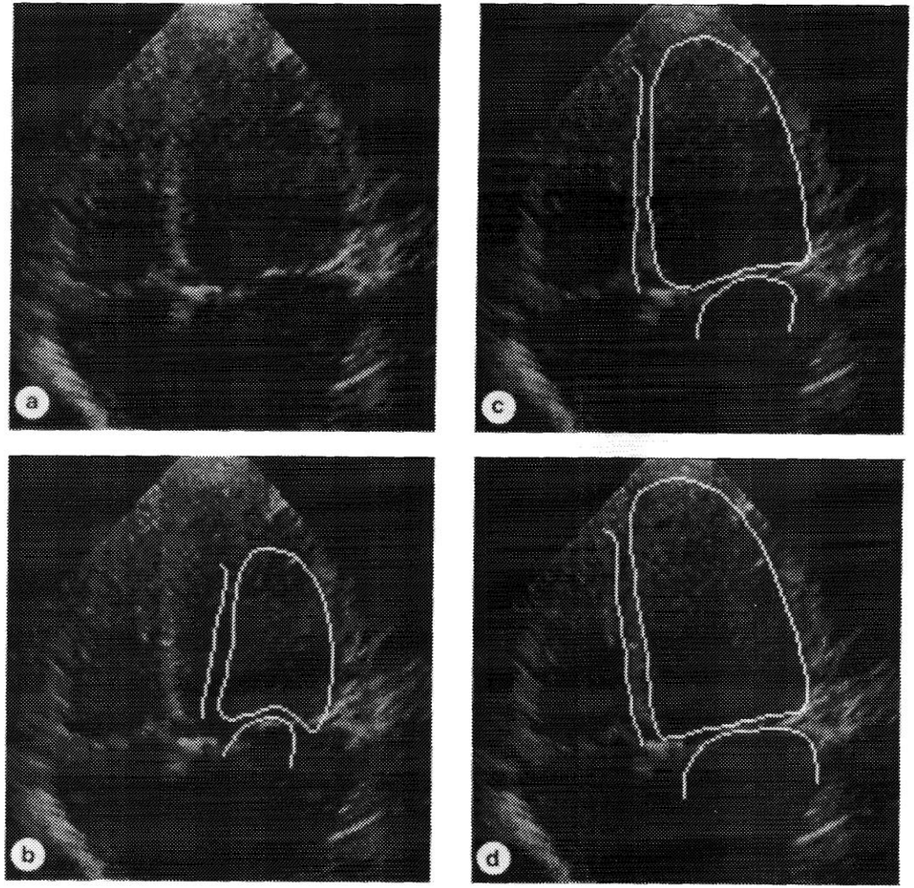
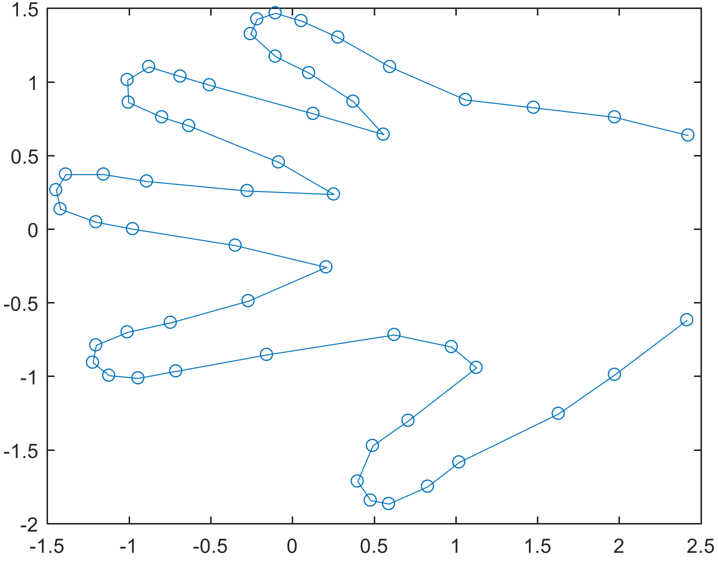
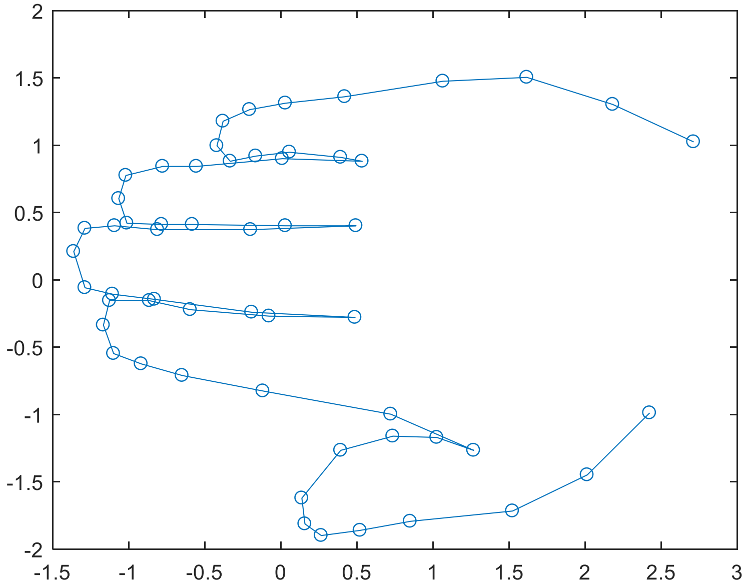



FIG. 24. Echocardiogram image with heart chamber boundary model superimposed, showing its initial position and its location after 80 and 200 iterations.

# Full example for active shape models



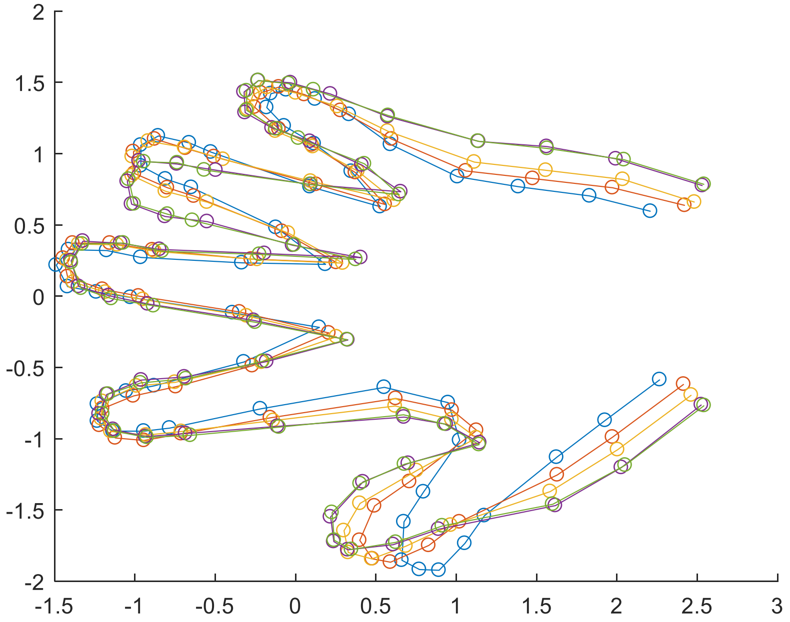



# Active shape models: idea behind the model

Not all shapes are probable, for example the points at the tips of the fingers will vary together

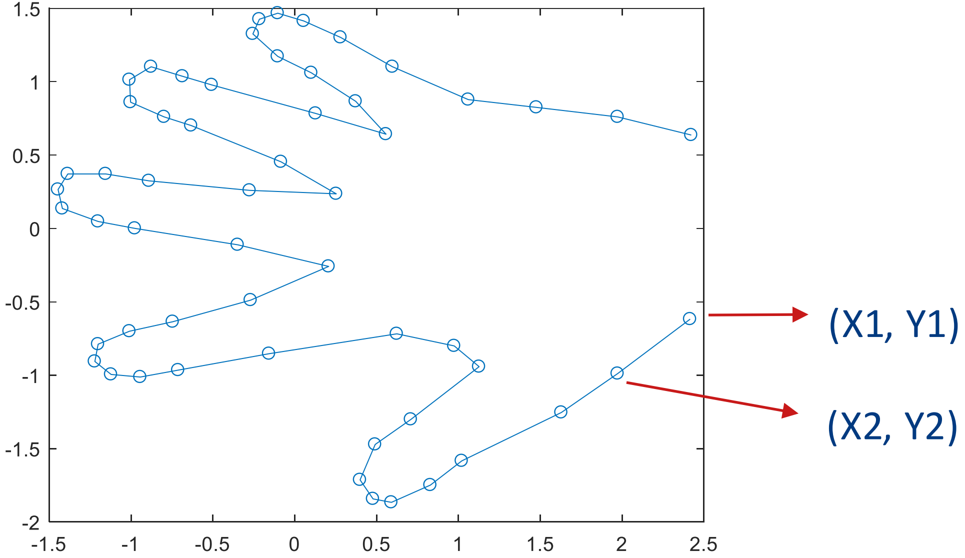
“Length of fingers” is not a feature in our 2K dimensional space, but a combination of features

We can store our model in less than 2K parameters



## Active shape models: idea behind the model

Use Principal Component Analysis (PCA) to find main modes of variation




| X1 | X2 | … | Y1 | … |
|---|---|---|---|---|
| -0.1 | 0.2 | | 1 | |
| -0.2 | 0.1 | | 1.1 | |
| -0.1 | 0.2 | | 0.9 | |
| -0.2 | 0.2 | | 1 | |

# Active shape model: assumption

Any allowed shape can be approximated / described as mean + linear combination of eigenvectors

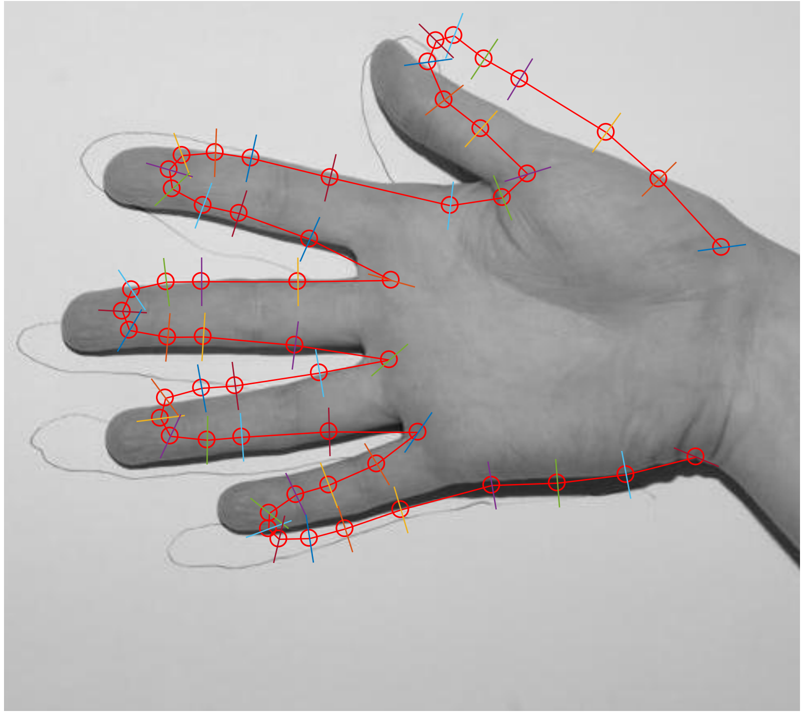
$$\mathbf{x} = \bar{\mathbf{x}} + \mathbf{U}_m \mathbf{b}$$

**b** is a vector of weights, each weight corresponds to how much variation we want along that eigenvector

# Active shape model: application

Goal: Find 2K coordinates in a test image, such that these coordinates can be described by our shape model

Our model: weights **b** (shape), but also rotation/scaling of the shape (pose)

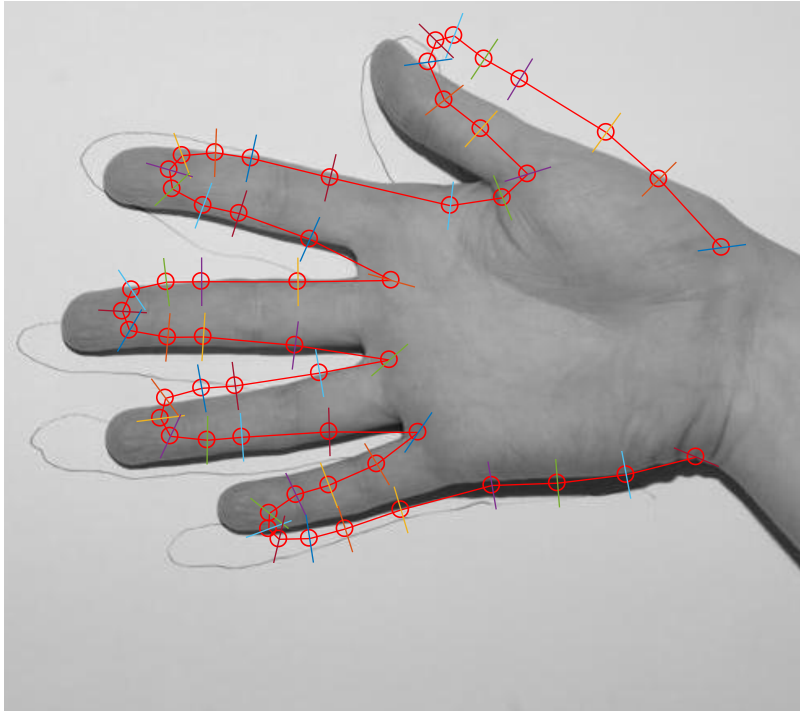


# Active shape model: application

1) Start with initial position of points, X (2K vector)

2) Find translation vector dX that moves each point to a better position (close to an edge)

This is possible by looking at the intensity profile along the normal vector at each point

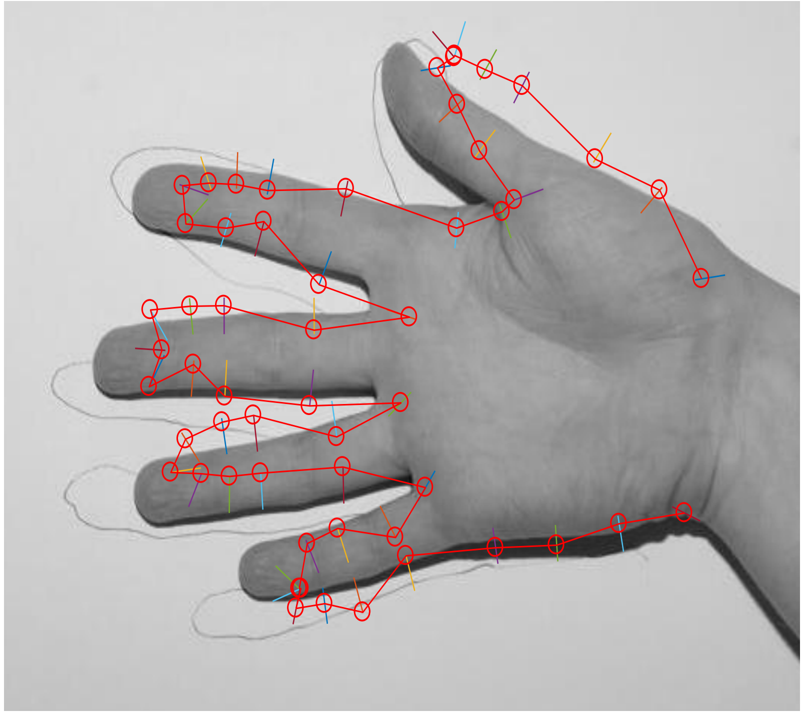


# Active shape model: application

Just moving each point to a better position is not enough! X+dX is not a valid shape, so:

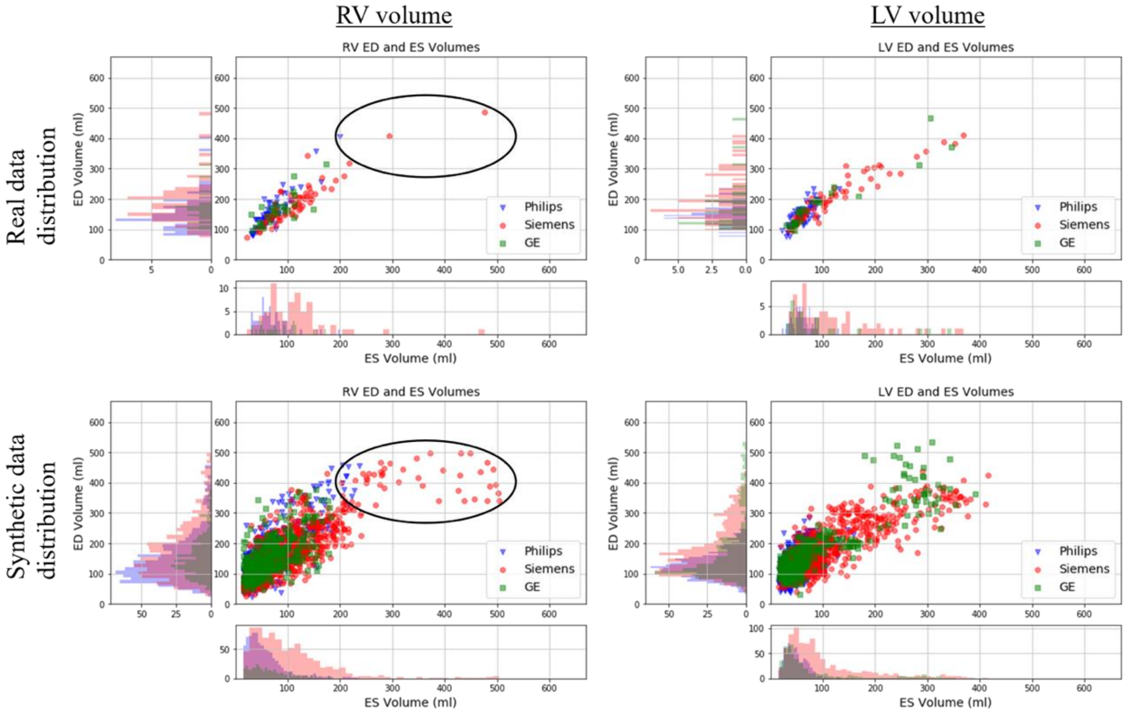
3) Find shape and pose, such that model(shape, pose) is close to X+dX

4) Repeat steps 2 and 3 until (almost) no change in dX



# Phantom generation

Targeted synthesis to increase the outlier cases



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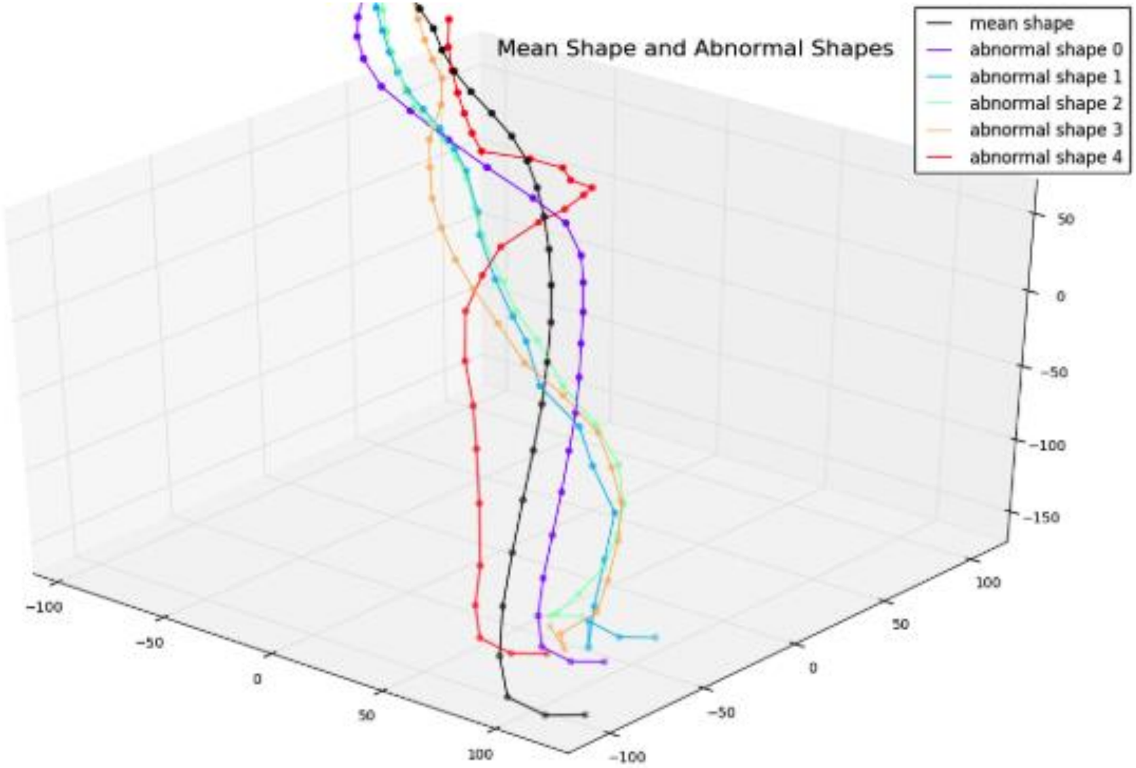
# Anomaly detection



# Questions

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