例1.6.4　企管三甲 A10627004 周鎬軒

依 $E(X_i)=\mu \quad V(X_i)=\sigma^2=E(X_i^2)-\mu^2$

則 $E(\bar{x})=\mu \quad V(\bar{x})=\frac{\sigma^2}{n}=E(\bar{x})^2-\mu^2$

$E(\hat{\theta}_1)=E\left[\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}\right]=\frac{1}{n}\left(\sum_{i=1}^{n}x_i^2-n\bar{x}^2\right)$

$=\frac{1}{n}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)$

$=\sigma^2+\mu^2-\frac{\sigma^2}{n}-\mu^2$

$=\sigma^2-\frac{\sigma^2}{n}$

$=\frac{n-1}{n}\sigma^2 \Rightarrow E(\hat{\theta}_1)$ 未滿足 σ^2 之不偏估計量

$\Rightarrow$ 偏誤估計量

$E(\hat{\theta}_2)=E\left[\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}\right]=\frac{1}{n-1}E\left[\sum_{i=1}^{n}(x_i-\bar{x})^2\right]$

$=\frac{1}{n-1}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)$

$=\frac{n\sigma^2}{n-1}-\frac{\sigma^2}{n-1}$

$=\sigma^2 \Rightarrow E(\hat{\theta}_2)$ 滿足 σ^2 之不偏估計量

$\Rightarrow$ 不偏估計量