



Problem	1	2	3	4	Total
Maximum score	25	25	25	25	100
Program output	2	1	2	2	1, 2

Problem 1)

The circuit shown in Figure P1 is at steady state before the switch opens at time $t = 0$. Determine the inductor current, $i(t)$ and the capacitor voltage $v(t)$ after the switch opens.

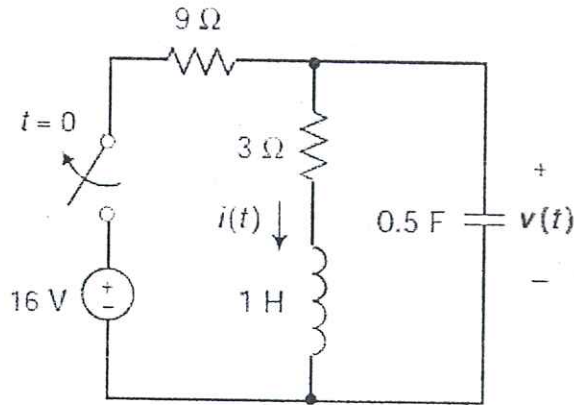


Figure P1

Problem 2)

The op-amp in the inverting amplifier circuit of Figure P2 has an input resistance of 600 kΩ, an output resistance of 20 kΩ and an open loop gain of 100,000. Assume that the amplifier is operating in its linear region.

- Draw the circuit in Figure P2 with the non-ideal op-amp model. Clearly state all the components. (5 points)
- Calculate the voltage gain $\frac{V_O}{V_S}$ of the amplifier. (15 points)
- Find the inverting terminal input voltage v_n when $V_S = 1$ V. (5 points)

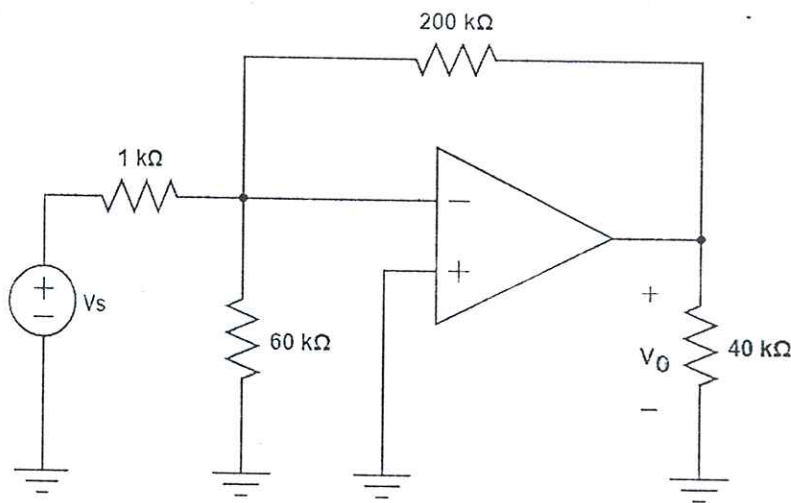


Figure P2

Problem 3)

Consider the circuit shown in Figure P3a and Figure P3b.

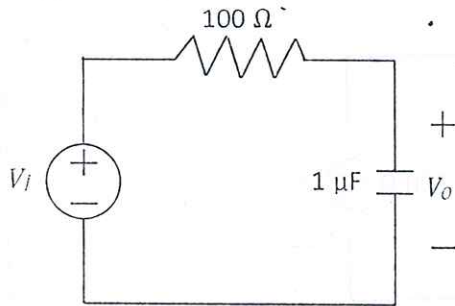


Figure P3a

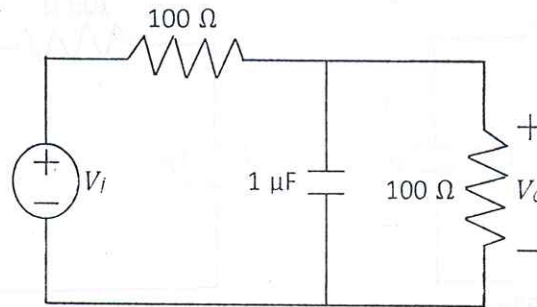


Figure P3b

- Derive the transfer function, $H(s) = V_o(s)/V_i(s)$, of the circuit shown in Figure P3a. (6 points)
- Determine the type of filter shown in Figure P3a. (3 points)
- Calculate the cutoff frequency of the filter shown in Figure P3a. (4 points)
- Suppose a $100\ \Omega$ load resistor is attached to the filter as shown in Figure P3b. Derive the transfer function, $H(s) = V_o(s)/V_i(s)$, of this loaded circuit shown in Figure P3b. (8 points)
- Calculate the cutoff frequency of the loaded circuit shown in Figure P3b. (4 points)

Problem 4)

Design a passive bandpass filter with 19 krad/sec bandwidth and a lower cutoff frequency ω_{C1} of 81 krad/sec using $40\ \mu\text{F}$ capacitor.

- Calculate the quality factor, center frequency and the higher cut-off frequency ω_{C2} . (5 points)
- Draw your circuit by labeling the component values and the output voltage. (10 points)
- Derive the transfer function of the bandpass filter. (10 points)

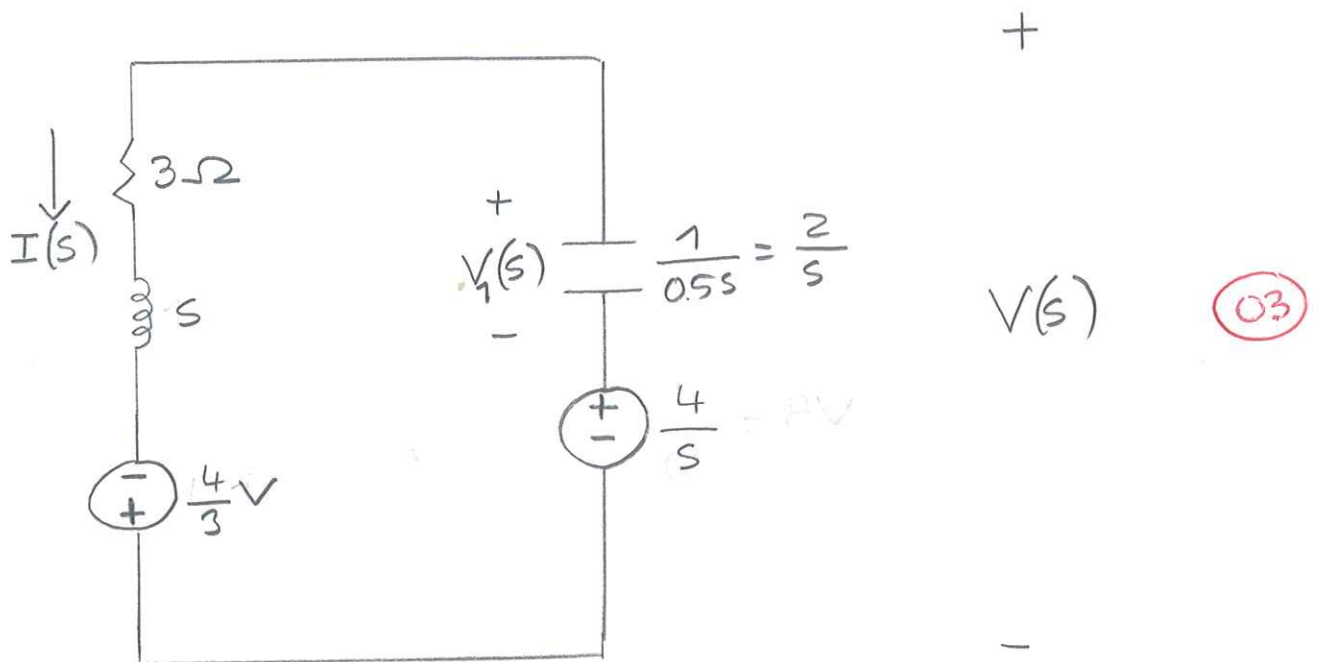
ELEN 202
Midterm Exam Solution
Key

Problem 1) At $t = 0^-$, we find that

$$i(0^-) = \frac{16}{s+3} = \frac{4}{3} \text{ A} = i(0) = i(0^+) \quad (03)$$

$$v(0^-) = \frac{4}{3} \cdot 3 = 4 \text{ V} = v(0) = v(0^+) \quad (03)$$

then for $t > 0$, we consider the s-domain equivalent circuit



$$I(s) \left(3 + s + \frac{2}{s} \right) - \frac{4}{3} + \frac{4}{s} = 0$$

$$\Rightarrow I(s) = \frac{\frac{4}{3} + \frac{4}{s}}{3 + s + \frac{2}{s}} = \frac{\frac{4}{3}s + 4}{s^2 + 3s + 2} = \frac{1}{3} \frac{4s + 12}{(s+1)(s+2)} \quad (04)$$

$$= \frac{C_1}{s+1} + \frac{C_2}{s+2}$$

$$\Rightarrow C_1 = \frac{1}{3} \frac{-4+12}{-1+2} = \frac{8}{3}, \quad C_2 = \frac{1}{3} \frac{-8+12}{-2+1} = -\frac{4}{3}$$

$$\mathcal{L}^{-1}\{I(s)\} \triangleq i(t) = \frac{8}{3} e^{-t} - \frac{4}{3} e^{-2t}, \quad t \geq 0 \quad (04)$$

Moreover;

$$V_1(s) = -I(s) \cdot \frac{2}{s} + \frac{4}{s}$$

$$= -\frac{2}{3} \frac{4s+12}{s(s+1)(s+2)} + \frac{4}{s}$$

(04)

$$= \frac{D_1}{s} + \frac{D_2}{s+1} + \frac{D_3}{s+2}$$

$$\Rightarrow D_1 = -\frac{2}{3} \frac{12}{1 \cdot 2} = -4, \quad D_2 = -\frac{2}{3} \frac{-4+12}{(-1) \cdot (1+2)} = \frac{16}{3},$$

$$\Rightarrow D_3 = -\frac{2}{3} \frac{-8+12}{(-2) \cdot (-2+1)} = -\frac{4}{3}$$

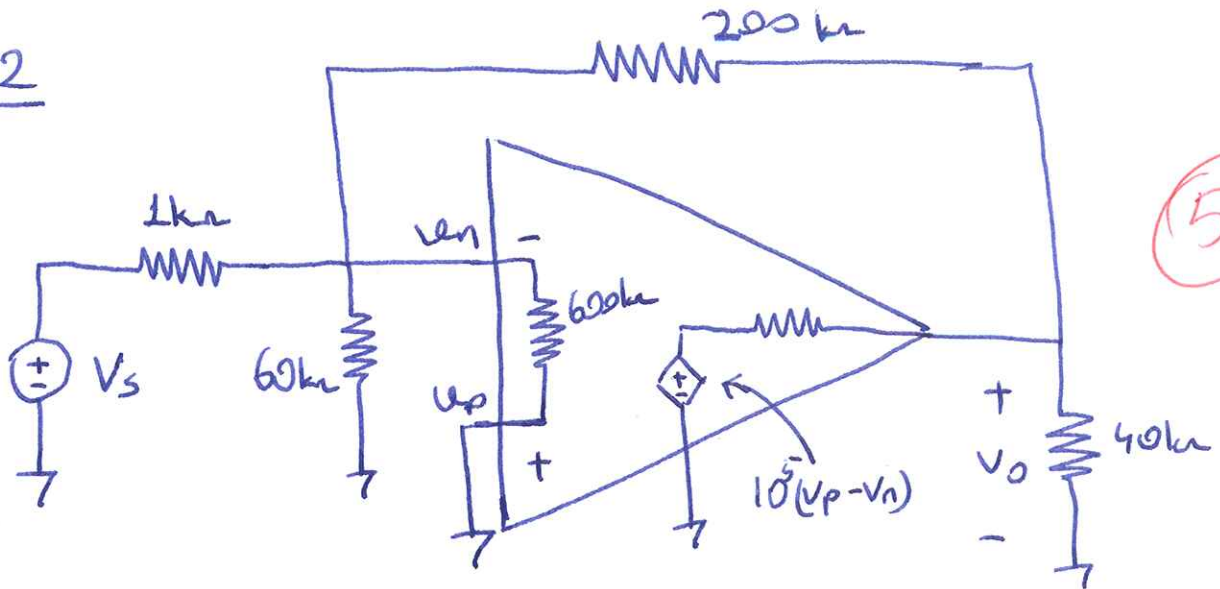
$$\text{Hence, } V(s) = V_1(s) + \frac{4}{s}$$

$$\begin{aligned} \mathcal{L}^{-1}\{V(s)\} &\triangleq v(t) = -4 + \frac{16}{3}e^{-t} - \frac{4}{3}e^{-2t} + 4 \\ &= \frac{16}{3}e^{-t} - \frac{4}{3}e^{-2t}, \quad t \geq 0 \end{aligned}$$

(04)

Soln 2

a-)



b-) $V_p = 0$

KCL at inverting terminal

$$\frac{V_n - V_s}{1k} + \frac{V_n}{60k} + \frac{V_n - V_o}{200k} + \frac{V_n}{600k} = 0$$

(600) (10) (3) (1)

$$614V_n - 600V_s - 3V_o = 0 \quad \text{--- (1)}$$

KCL at output terminal

$$\frac{V_o}{40k} + \frac{V_o - 10^5(V_p - V_n)}{20k} + \frac{V_o - V_n}{200k} = 0$$

(5) (10) (1)

$$V_n = \frac{-16}{99999} V_o \quad \text{--- (2)}$$

$$\text{(2)} \Rightarrow \text{(1)} \quad 614 \left(\frac{-16V_o}{99999} \right) - 600V_s - 3V_o = 0$$

$$V_o(-0,00982 - 3) = 600V_s$$

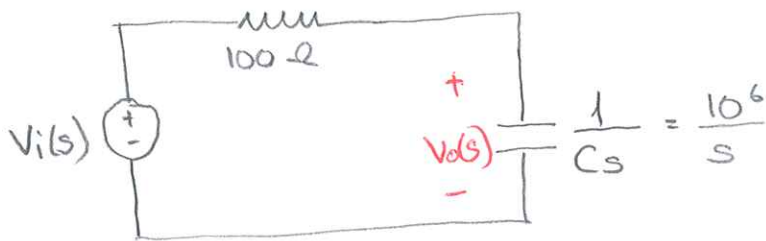
$$\frac{V_o}{V_s} = \underline{\underline{-199,34 \text{ V/V}}}$$

$$\text{c-) } V_s = 1V \Rightarrow V_o = -199,34V$$

$$V_n = \frac{-16}{99999} (-199,34) \Rightarrow \underline{\underline{V_n = 3,18V}}$$

Problem 3)

* s-domain equivalent circuit:



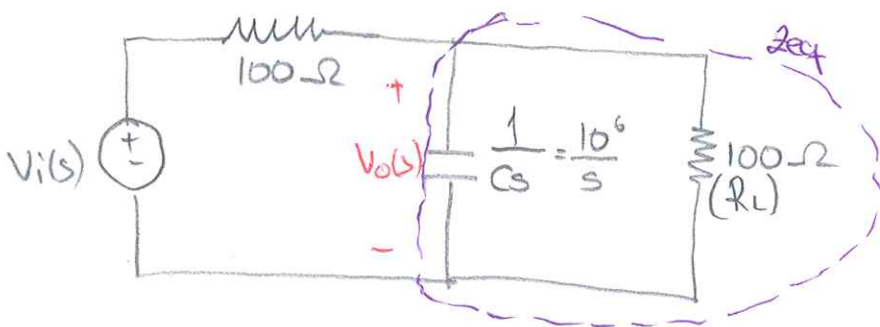
$$a) H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{RCs + 1} = \frac{1/RC}{s + 1/RC} = \frac{10^4}{s + 10^4}$$

$\omega_c = 1/RC$

b) Low-Pass Filter

c) From $H(s)$: $\omega_c = 1/RC = 10^4 \text{ rad/s} = 10 \text{ krad/s}$

d) s-domain equivalent circuit:



$$Z_{eq} = R_L \parallel \frac{1}{Cs}$$

$$Z_{eq} = \frac{R_L / Cs}{R_L + 1/Cs}$$

$$Z_{eq} = \frac{R_L}{R_L Cs + 1} = \frac{1/C}{s + 1/R_L C}$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_{eq}}{R + Z_{eq}} = \frac{\frac{R_L}{R_L Cs + 1}}{R + \frac{R_L}{R_L Cs + 1}} = \frac{R_L}{RR_L Cs + (R + R_L)}$$

$$H(s) = \frac{1/RC}{s + \frac{(R + R_L)}{RR_L C}} = \frac{10^4}{s + (2 \times 10^4) \omega_c}$$

e) From $H(s)$ of loaded filter: $\omega_c = \frac{R + R_L}{RR_L C} = 2 \times 10^4 \text{ rad/s} = 20 \text{ krad/s}$

Prob4 Solution

a) $\beta = 19 \text{ krad/s}$, $\omega_1 = 81 \text{ krad/s}$

$$\beta = \omega_{c2} - \omega_{c1} \Rightarrow \omega_{c2} = 19 + 81 = 100 \text{ krad/s} \quad (1)$$

$$\omega_0 = \sqrt{\omega_{c1} \omega_{c2}} = \sqrt{81 \times 100} = 90 \text{ krad/s} \quad (2)$$

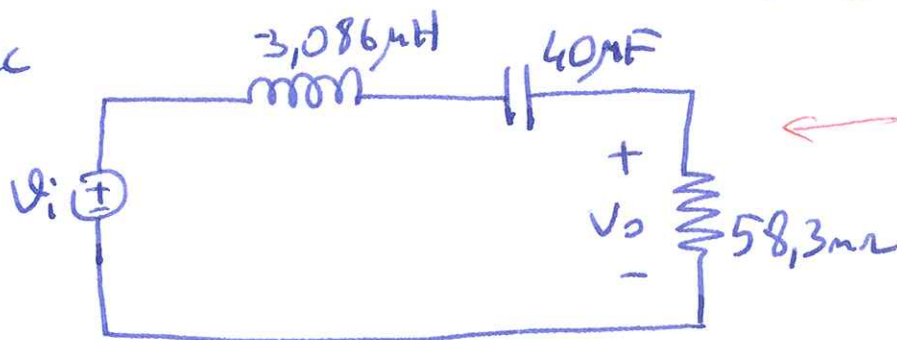
$$Q = \frac{\omega_0}{\beta} = \frac{90}{19} = 4,736 \quad (2)$$

b) $\omega_0 = \sqrt{\frac{1}{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} = \frac{1}{(90 \times 10^3)^2 40 \times 10^{-6}} = 3,086 \mu\text{H}$ (2)

$$\beta = \frac{R}{L} \Rightarrow R = \beta L = 19 \times 10^3 \times 3,086 \times 10^{-6}$$

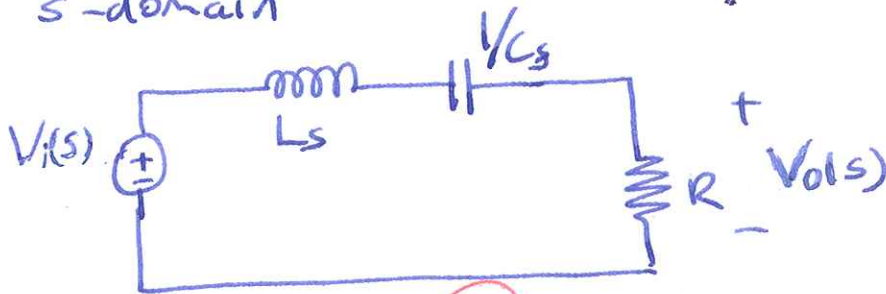
$$\Rightarrow R = 58,3 \text{ m}\Omega \quad (2)$$

\Rightarrow for series RLC



(5) $+1$ if values are correct

c-) s-domain



(3) for s-domain values.

$$\frac{V_o(s)}{V_i(s)} = \frac{R}{R + Ls + \frac{1}{Cs}} \quad (4) = \frac{RCs}{RCs + LCs^2 + 1} = \frac{RCs/LC}{\frac{RCs}{LC} + s^2 + \frac{1}{LC}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R/L s}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \quad (3) = \frac{\beta s}{s^2 + \beta s + \omega_0^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$\beta = R/L$$