Optimization with Karnaugh method

- We first consider the adjacent terms

Definition 7. Let $f: S^n \to S$. If 2^k terms of the sum of minterms form of f involves $(n-k)$ common terms and if these $(n-k)$ terms are factored out such that the terms inside the paranthesis includes all the minterms of a k-variable Boolean function, then these 2^k terms are said to be k^{th} degree adjacent as $k = 1, 2, \dots, n$.

example.

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' \overbrace{(x_2' x_4' + x_2' x_4 + x_2 x_4' + x_2 x_4)}^{1}$$
$$+ x_1 x_2 x_3 x_4$$

- we say that the first four terms are 2^{nd} degree adjacent

When can we use the method?

- The Karnaugh method is used for $n \leq 6$
- No lists and prime implicants are considered

Karnaugh matrix

- an $n \times n$ matrix with 2^n cells
- the Karnaugh matrix possess two properties:

Property 1. Each cell in the matrix correspond to one of the 2^n minterms/maxterms. If the function is 1 (0) at some point then the corresponding cell element is also 1 (0). In other words, the matrix is another representation form for the function.

Property 2. If two cells of the matrix are adjacent, then the corresponding minterms/maxterms are also adjacent. In general, if 2^k terms are k^{th} degree adjacent, then each corresponding cell is adjacent to k terms among (2^k-1) ones.

Karnaugh matrix for n=4

- it can be given as follows

X_{12} \ X_{34}	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

- the cells that correspond to every minterms/maxterms are shown on the matrix
- any two cells with common border line are adjacent

Moreover;

- the cells a_{1j} and a_{4j}, $j=1,2,3,4$

PLUS

- the cells a_{j1} and a_{j4}, $j=1,2,3,4$

$\hookrightarrow$ are also ADJACENT

1^{st} degree adjacent terms

- the necessary and sufficient condition for any two minterms/maxterms to be adjacent
  ↳ the corresponding cells have common borderline on the torus obtained from that matrix

2^{nd} degree adjacent terms

- assume that the Karnaugh matrix is constructed on a torus
- the necessary and sufficient condition for any four minterms/maxterms to be adjacent
  ↳ the corresponding cells form a square or a row on that torus

3^{rd} degree adjacent terms

- eight minterms/maxterms are 3^{rd} degree adjacent
  ↳ iff the corresponding eight cells construct two adjacent rows or columns

Example. Let us consider the optimization of the following function

$$f(x_1,x_2,x_3,x_4) = \sum m(2,4,6,8,9,10,12,13,15)$$

by using the Karnaugh method.

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00				1
01	1			1
11	1	1	1	
10	1	1		1

Sum of products form (SOP)

$(8,9,12,13): \; x_1 x_3'$

$(13,15): \; x_1 x_2 x_4$

$(4,6): \; x_1' x_2 x_4'$

$(2,10): \; x_2' x_3 x_4'$

$$f(x_1,x_2,x_3,x_4) = x_1x_3' + x_1x_2x_4 + x_1'x_2x_4' + x_2'x_3x_4'$$

Product of sums form (POS)

- we consider the minterms of $f'(x_1,x_2,x_3,x_4)$

$$f'(x_1,x_2,x_3,x_4) = \sum m(0,1,3,5,7,11,14)$$

- and place these minterms onto a Karnaugh map

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	1	1	1	
01		1	1	
11				1
10			1	

$(1,3,5,7): \quad x_1' x_4$

$(0,1): \quad x_1' x_2' x_3'$

$(3,11): \quad x_2' x_3 x_4$

$14: \quad x_1 x_2 x_3 x_4'$

$$f'(x_1,x_2,x_3,x_4) = x_1' x_4 + x_1' x_2' x_3' + x_2' x_3 x_4 + x_1 x_2 x_3 x_4'$$

$$\Rightarrow f''(x_1,x_2,x_3,x_4) = (x_1 + x_4')(x_1 + x_2 + x_3)(x_2 + x_3' + x_4') \cdot (x_1' + x_2' + x_3' + x_4)$$

4.5

5.5