Problem 1)

$$G(s) = \frac{K}{s.(s^2+4s+8)}$$

Root locus:

Poles: $s^2+4s+8=0$; $\boxed{s=0}$

$$s = \frac{-4 \pm \sqrt{16-4(8)}}{2.1}$$

$$= \frac{-4 \pm \sqrt{16-32}}{2}$$

$$= \frac{-4 \pm \sqrt{-16}}{2} = \boxed{-2 \pm 2i}$$

$\therefore$ poles : $s=0$, $s=-2-2i$, $s=-2+2i$

Number of poles $= 3$ (P) Number of zeros $= 0$ (Z)

Number of root locus branches $= N = P = 3$

Number of asymptotic lines $= n = P - Z = 3$

Thus, root locus will have three branches. The negative real axis will have branch starting from $s=0$, along it. The other branches start from the remaining poles.

$$\text{Angle of Asymptotes} = \frac{\pm 180(2K+1)}{3} \quad (K = 0,1,2)$$

$$= \begin{cases} \pm 60 \text{ (for } K=0) \\ \pm 180 \text{ (for } K=1) \\ \pm 300 \text{ (for } K=2) \end{cases}$$

Those asymptotes will intersect the real axis at centroid σ_c, and is given by;

$$\sigma_c = \frac{0 + (-2+2j) + (-2-2j)}{3}$$

$$= -\frac{4}{3}$$

$$\boxed{\sigma_c = -1.33}$$

Now, consider the characteristics equations for the systems.

$$1 + G(s)\,H(s) = 0$$

$$1 + \frac{K}{s(s^2+4s+8)} = 0$$

$$s^3 + 4s^2 + 8s + K = 0 \quad (1)$$

Applying Routh's Hurwitz criterion.

The routh array is given by:

s^3	1	8
s^2	4	K
s^1	$\frac{32-K}{4}$	0
s^0	K	

$$\frac{32-K}{4} = 0 \rightarrow K = 32$$

The auxiliary equations for this case;

$$4s^2 + K = 0$$

$$4s^2 + 32 = 0$$

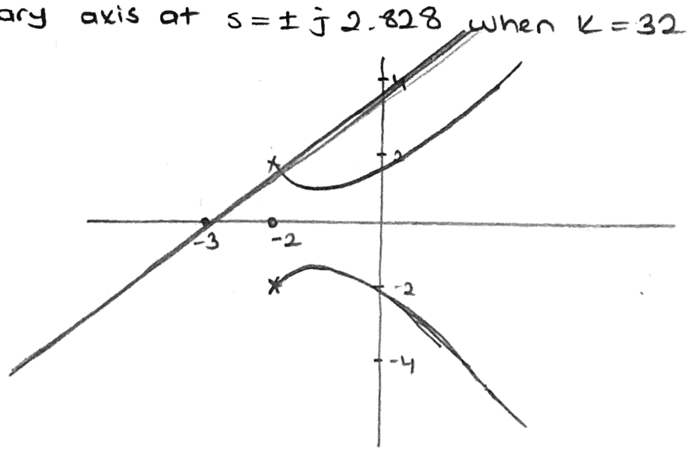
$$s = \sqrt{-\frac{32}{4}} \qquad s = \pm j\,2.828$$

The root locus crosses the imaginary axis at $s = \pm j\,2.828$ when $K = 32$.

$$\theta_1 + 90° + 135° = 180°$$

$$\theta_1 = 180 - 90 - 135$$

$$= -45°$$



If we consider $\boxed{K = 2;}$

$$(1): \quad s^3 + 4s^2 + 8s + K = 0$$

$$s^3 + 4s^2 + 8s + 2 = 0$$

The roots of this equation give the poles of the closed loop system.

$$\rightarrow \boxed{s = \begin{cases} -1.8557 + j\,1.8669 \\ -1.8557 - j\,1.8669 \\ -0.2887 \end{cases}}$$

Problem 2) $G(s) = \dfrac{K(s+9)}{s(s^2+4s+11)}$, $H(s) = 1$

Open loop T.F. is $= G(s) \cdot H(s) = \dfrac{K(s+9)}{s(s^2+4s+11)}$

$s^2 + 4s + 11 = 0$

$\Delta = b^2 - 4ac = 16 - 4 \cdot 1 \cdot (11) = -28$

$$s_{1,2} = \frac{-4 \pm \sqrt{-28}}{2} = -2 \pm j2.64$$

Open loop poles at $s = 0, -2 \pm j2.64$ # of open loop poles $= 3$ (P)

Open loop zeros at $s = -9$ # of open loop zeros $= 1$ (Z)

# of asymptotic lines $= 3 - 1 = 2$ (P-Z)

$$\theta = \frac{(2k+1)\,180^\circ}{P-Z} \; ; \quad k = P - Z - 1$$

$P - Z = 2$, so $k = 0, 1$

when $k = 0$; $\theta_0 = 180^\circ/2 = 90^\circ$

when $k = 1$; $\theta_1 = 3 \cdot 180^\circ/2 = 270^\circ$

Center of asymptotes is; centroid $= \dfrac{\overbrace{-4}^{\text{poles}} - \overbrace{(-9)}^{\text{zeros}}}{\underbrace{2}_{(P-Z)}} = 2.5$

Breakaway points; $G(s) \cdot H(s) = \dfrac{K \cdot (s+9)}{s \cdot (s^2+4s+11)}$

Characteristic eqn; $1 + G(s)H(s) = 1 + \dfrac{K \cdot (s+9)}{s \cdot (s^2+4s+11)} = 0$

$$K = -\frac{s^3 + 4s^2 + 11s}{s+9}$$

$$\frac{dK}{ds} = -\frac{(3s^2+8s+11)(s+9) - (s^3+4s^2+11s)}{(s+9)^2} = 0$$

$$= -\frac{2s^3 + 31s^2 + 72s + 99}{(s+9)^2} = 0$$

$-(2s^3 + 31s^2 + 72s + 99) = 0$ $\quad s = -1.235 \pm j1.507 \;,\; -13.03$

Breakaway point can not be imaginary, and according to our characteristic equation, -13.03 can not be breakaway point. So, there is no breakaway point.

Routh array with using characteristic equation;

$$1 + \frac{K(s+9)}{s(s^2+4s+11)} = 0$$

$$s.(s^2+4s+11) + K(s+9) = 0$$

$$s^3+4s^2+(11+K)s+9K = 0$$

Routh Array;

s^3	1	$(11+K)$
s^2	4	$9K$
s^1	$11-1.25K$	0
s^0	$9K$	

$\rightarrow$ $11-1.25K > 0$

$K < 8.8$

The stable range for K is: $0 < K < 8.8$

Crossing $j\omega$ axis can be found out by s^2 row;

$$4s^2+9K = 0$$

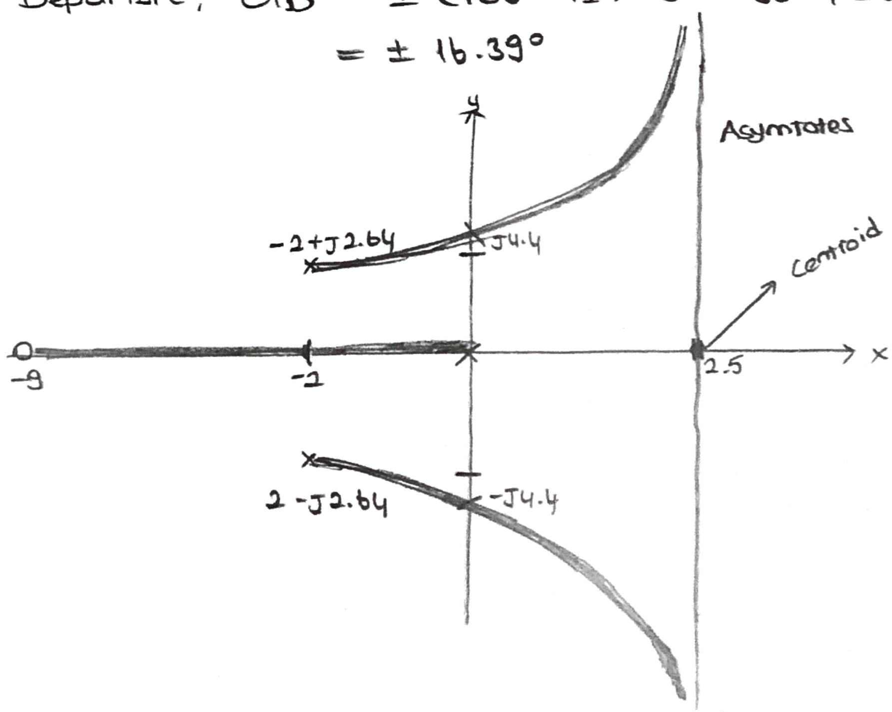
if $K = 8.8 \rightarrow 4s^2 + 9.(8.8) = 0$

$$s = \pm j4.449$$

So, root locus cross imaginary axis at $\pm j4.449$.

Angle of Departure; $\theta_D = \pm(180° - 127.09° - 90° + 20.70°)$

$= \pm 16.39°$



When we locate the closed loop poles on root loci where damping ratio $\delta = 0.5$. For this, we will draw root locus with help of mathlab and we locate poles.

```
num = [0 0 1 9];
den = [1 4 11 0];
rlocus (num, den);
sgrid (0.5, []);
```

According to our mathlab figure, root loci placed at;

$\delta = 0.5$, $s = -1.52 + j2.6$

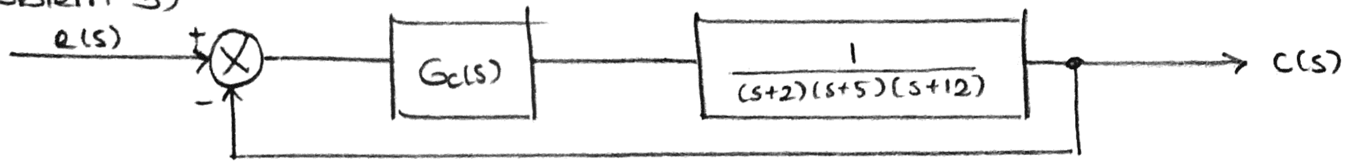
$$K = -\frac{s^3 + 4s^2 + 11s}{(s+9)} \quad (\text{when } s = -1.52 + j2.6$$

$$K = -\frac{(-1.52+j2.6)^3 + 4(-1.52+j2.6)^2 + 11(-1.52+j2.6)}{((-1.52+j2.6)+9)}$$

$$= \left|\frac{7.94}{7.93}\right|$$

$$\boxed{K \cong 1}$$

Problem 3)



a) $G_c(s) = K$

$\therefore$ open loop Transfer Function $G_c(s)\,H(s) = \dfrac{K}{(s+2)(s+5)(s+12)}$

poles $= -2, -5, -12$

Number of Asymptotes $= P - Z = 3 - 0 = 3$

Angle of Asymptotes $= \dfrac{(2q+1).180}{(P-Z)}$ $\quad (q = 0, \pm 1, \pm 2)$

$= 60^\circ, 180^\circ, 300^\circ$

Position of Asymptote $= \sigma_a = \dfrac{-2-5-12-0}{3} = -19/3 = -6.33$

$\sigma_a = -6.33$

Breakaway point;

Characteristic Equation is: $1 + G(s)H(s) = 0$

$$1 + \frac{K}{(s+2)(s+5)(s+12)} = 0$$

$$K = -(s+2)(s+5)(s+12)$$
$$= -(s+2)(s^2+17s+60)$$
$$= -(s^3+19s^2+94s+120)$$

$$\frac{dK}{ds} = -(3s^2+38s+94) = 0$$

For stability; $19 \times 94\ (120 + K) > 0$

$K < 1666$

$120 + K > 0$

$K > -120$

So, $\boxed{-120 < K < 1666}$

Poles intersection on $j\omega$ axis $= +9.695j$ & $-9.695j$

The value of Gain K at this point $\boxed{K = 1666}$

Range of K for stability $-120 < K < 1666$

c) $\therefore \quad G_c(s)\,H(s) = \dfrac{K}{(s+2)(s+5)(s+12)}$

Number of pole at origin for open loop T.F. $= 0$

$\therefore$ System is $\boxed{\text{type '0'}}$

d) when $G_c(s) = 162$

Open Loop T.F. $= \dfrac{162}{(s+2)(s+5)(s+12)}$

Error for unit step $= \dfrac{1}{1+K_p}$

$$K_p = \lim_{s\to 0} G_c(s)H(s) = \lim_{s\to 0} \frac{162}{(s+2)(s+5)(s+12)}$$

$$\boxed{K_p = \frac{162}{2.5.12} = 1.35}$$

$$\boxed{\text{Unit Step Error} = \frac{1}{1+1.35} = 0.425}$$

Error for ramp input $= \dfrac{1}{K_v}$

$$K_v = \lim_{s\to 0} s.G(s)H(s)$$

$$= \lim_{s\to 0} s.\frac{162}{(s+2)(s+5)(s+12)}$$

$$= 0$$

$$\boxed{\text{Ramp input error} = \frac{1}{0} = \infty}$$

e) $K = 162$, $s = -2.62 \pm j\,3.67$

2nd order approximation :

$$s^3 + 19s^2 + 94s + 120 + \overbrace{K}^{162} = (s+a)\left((s+2.62)^2 + 3.67^2\right)$$

$$a = 13.867$$

3rd closed loop pole $= -13.67$

$$T_s = \frac{4}{\delta \omega_n} = \frac{4}{2.62} = 1.527 \text{ sec}$$

f) $T_s = 0.763$ sec $\quad \delta = 0.59$

Desired Poles : $-5.242 \pm j\,7.174$

$$G_{PD}(s) = K_p + K_d s$$

$$G_{PD}(s) \cdot G_p(s) = \frac{162(K_p + K_d s)}{(s+2)(s+5)(s+12)}$$

Characteristic Eqn: $s^3 + 19s^2 + (94 + 162K_d)s + 120 + 162K_p = 0$

2nd order approximation : $s^3 + 19s^2 + (94 + 162K_d)s + 120 + 162K_p = (s+a)\left((s+5.242)^2 + 7.174^2\right)$

$$19 = a + 10.484 \rightarrow a = 8.516$$

$$94 + 162K_d = 78.945 + 10.484a \rightarrow K_d = 0.458$$

$$120 + 162K_p = 78.945a \rightarrow K_p = 3.41$$

$$G_{PI}(s) = K_p + \frac{K_I}{s}$$

$$e_{ss\,step} = 0$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{1 + G_{PI}(s)G_{PD}(s)G_p(s)}$$

$$= \frac{1}{4.6K_I}$$

g) $$G_{PID}(s) = \frac{80(s+0.4)(s+10)}{s} = 80s + 832 + \frac{320}{s}$$

$K_p = 832 \qquad K_d = T_d = 80 \qquad K_i = \frac{1}{T_i} = 320 \rightarrow T_i = 0.003125$