MATH 233
Fall 2018
Class Worksheet for Week #4

0. Solutions of Quiz#1 (A and B sections; posted on http://learn.bilgi.edu.tr).

1. One hundred tickets, numbered 1, 2, 3, . . . , 100, are sold to 100 different people for a drawing. Four different prizes are awarded, including a grand prize (a trip toTahiti).How many ways are there to award the prizes if

a) there are no restrictions?
b) the person holding ticket 47 wins the grand prize?
c) the person holding ticket 47 wins one of the prizes?
d) the person holding ticket 47 does not win a prize?
e) the people holding tickets 19 and 47 both win prizes?
f) the people holding tickets 19, 47, and 73 all win prizes?
g) the people holding tickets 19, 47, 73, and 97 all win prizes?
h) none of the people holding tickets 19, 47, 73, and 97 wins a prize?
i) the grand prize winner is a person holding ticket 19, 47, 73, or 97?
j) the people holding tickets 19 and 47 win prizes, but the people holding tickets 73 and 97 do not win prizes?

Solution: Let the 100 tickets be T1, T2, …. T100. And let the four prizes be P1, P2, P3, P4.

a) First prize can be given to 100 different people. 2nd prize to 99, … and 4th to 96. The result is 4-permutations from a set of 100 items. I.e., P(100,4)

b) T47 wins the grand prize. Then remaining 3 prizes is P(99,3) because it is choosing 3-permutations from a set of 99 items.

c) There are four ways that T47 wins one of the prizes. And in each case the remaining prizes can be in P(99,3) ways. Therefore, the answer is 4.P(99,3)

d) If T47 does not win, then we consider a set with 99 elements, where 4-permutations are choosen. Thus, P(99,4)

e) T19 and T47 can win prizes in P(4, 2) ways. The remaining 98 can win in P(98,2) ways. Therefore, the answer is P(4,2) P(98, 2)

f) P(4,3) is the number of ways T19 and T47 and T73 wins 3 of the 4 prizes. The remaining prizes can be won by any of the 97 tickets. The asnwer is P(4,3) 97

g) P(4, 4) = 4! (number of ways of arranging 4 distinct items)

h) In this case, the problem is finding 4-permutations out of 96 people: P(96, 4)

i) Grand prize winner is T19 then, P(99,3). 3 other such cases. Therefore, the asnwer is 4 P(99,3)

j) P(4,2) (as in e) is the number of ways T19 and T47 wins prizes. P(96, 2) is the number of ways remaining 96 people win prizes. Thus, the asnwer is P(4,2) P(96, 2)

2. A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?

Solution: Let 40 questions be Q1, Q2, …. Q40. Each Qi can be true or false.
17 of the Qi's are true.

The number of ways these 17 true eustions can be choosen is equivalent to:

Number of 40-bit strings with 17 ones. The answer is C(40,17)