İstanbul Bilgi Üniversitesi

İstanbul Bilgi University
Faculty of Engineering and Natural Sciences
Department of Electrical and Electronics Engineering

Course: EEEN 322 – Communication Engineering
Instructor: İpek Şen
Exam / Date: Midterm Exam / 04.04.2019 13:00
Duration: 100 min.

Name & surname: ....

ID number: ....

Question	1	2	3	4	Total
Score					
Maximum score	20	25	35	30	100
CLO	1,2	1,2,4	1,2,4	1,3,4,5	1,2,3,4,5

Question 1 (20 points)

Suppose that the message signal to be modulated is $m(t) = \text{sinc}(\pi t)$, where $\text{sinc}(x) = \begin{cases} 1, & x = 0 \\ \frac{\sin x}{x}, & x \neq 0 \end{cases}$

a) Find the total energy of $m(t)$. (*15p*)

b) Is $m(t)$ a power signal or energy signal? State your reasoning. (*5p*)

*BONUS: Suppose that we perform DSB+C (AM) modulation to modulate $m(t)$ given in (a). Find the minimum carrier amplitude (*hint: A in the mathematical expression*) that will allow envelope detection for demodulation. (*10p*)

Question 2 (25 points)

Suppose that we perform DSB+C (AM) modulation to modulate $m(t)$ given in Question 1, using a carrier with frequency 1kHz. Let the carrier amplitude (*hint: A in the mathematical expression*) be equal to 2.

a) Write the mathematical expression of $\varphi_{AM}(t)$. (5p)

b) Write the mathematical expression of $\Phi_{AM}(\omega)$. (10p)

c) Sketch $\Phi_{AM}(\omega)$ (label the axes carefully). (10p)

Question 3 (35 points)

Suppose you perform DSB-SC modulation and the modulated signal is $\varphi_{DSB-SC}(t) = e^{-|t|}\cos(1000\pi t)$.

a) Give the mathematical expression of the message signal $m(t)$. (5p)

b) Give the value of the carrier frequency in Hz. (5p)

c) Sketch $\varphi_{DSB-SC}(t)$ (label the axes carefully). (10p)

d) Can we use an envelope or rectifier detector circuit for demodulation here (for the specific message signal of this question)? Why? Give a brief and neat explanation. (5p)

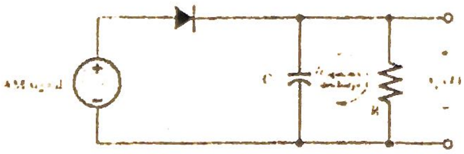
e) Write the mathematical expression of the spectrum $\Phi_{DSB-SC}(\omega)$. (10p)

Question 4 (30 points)

Suppose that DSB+C (AM) modulation is being performed to modulate the message signal $m(t) = 0.25\cos 5000\pi$ using a carrier with frequency 10MHz.

a) If the power efficiency of the modulation is 10%, what is the amplitude of the carrier? (10p)

b) To be able to demodulate the message signal by using the below circuit, what is the acceptable range of values for the carrier amplitude? (5p)



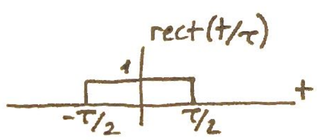
c) For the output of the above circuit to follow the envelope with acceptable performance, what is the acceptable range of values for R, if C=22 pF? (10p)

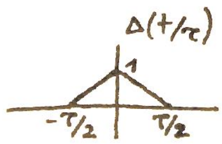
d) For the case that $m(t)$ can be demodulated by using the circuit in (b), what is the carrier amplitude that would yield the maximum power efficiency? (5p)

Table 1: Some Fourier Transform pairs

	$g(t)$	$G(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\dfrac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\dfrac{\omega\tau}{2}\right)$	
18	$\dfrac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\dfrac{\omega}{2W}\right)$	
19	$\Delta\left(\dfrac{t}{\tau}\right)$	$\dfrac{\tau}{2}\operatorname{sinc}^2\left(\dfrac{\omega\tau}{4}\right)$	
20	$\dfrac{W}{2\pi}\operatorname{sinc}^2\left(\dfrac{Wt}{2}\right)$	$\Delta\left(\dfrac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\dfrac{2\pi}{T}$

See below for the definitions of these signals (17–19)





$$\operatorname{sinc}(x)=\begin{cases}1, & x=0\\ \dfrac{\sin x}{x}, & x\neq 0\end{cases}$$

Table 2: Some properties of Fourier Transform

Operation	$g(t)$	$G(\omega)$
Addition	$g_1(t) + g_2(t)$	$G_1(\omega) + G_2(\omega)$
Scalar multiplication	$kg(t)$	$kG(\omega)$
Symmetry (Duality)	$G(t)$	$2\pi g(-\omega)$
Scaling	$g(at)$	$\frac{1}{\lvert a\rvert} G\left(\frac{\omega}{a}\right)$
Time shift	$g(t - t_0)$	$G(\omega)e^{-j\omega t_0}$
Frequency shift	$g(t)e^{j\omega_0 t}$	$G(\omega - \omega_0)$
Time convolution	$g_1(t) * g_2(t)$	$G_1(\omega)G_2(\omega)$
Frequency convolution	$g_1(t)g_2(t)$	$\frac{1}{2\pi} G_1(\omega) * G_2(\omega)$
Time differentiation	$\frac{d^n g}{dt^n}$	$(j\omega)^n G(\omega)$
Time integration	$\int_{-\infty}^{t} g(x)\, dx$	$\frac{G(\omega)}{j\omega} + \pi G(0)\delta(\omega)$

Some definitions & identities:

Parseval's Theorem: $\int_{-\infty}^{\infty} |g(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(\omega)|^2 d\omega$

$$\eta = \frac{P_{sidebands}}{P_{sidebands} + P_{carrier}}$$

$$2\sin\alpha\cos\beta = \sin(\alpha - \beta) + \sin(\alpha + \beta)$$

$$2\sin\alpha\sin\beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$$

Hints:

$$\frac{1}{\omega_c} << RC << \frac{1}{2\pi B}$$

$$m(t)\cos\omega_c t$$

$$|A + m(t)|\cos\omega_c t$$

İstanbul
Bilgi University
LAUREATE INTERNATIONAL UNIVERSITIES

2018 - 2019 ACADEMIC YEAR

Complete the following sections 1-7

Aşağıdaki Bölümleri Doldurunuz 1-7

1. First Names / Ad	2. Surname / Soyad
İPEK ŞEN	

3. Department / Bölüm	4. Student ID / Öğrenci No

5. Course Name / Ders Adı	6. Academic Year / Akademik Yıl
EEEN 322	2018 - 2019 (BAHAR)

7. Date of Exam / Sınav Tarihi
04.04.2019 (MdT)

For Examiner's only / Öğretim Elemanı İçin	
Question Number / Soru Numarası	Marks / Puan
Toplam Puan / Total Marks	

No. of Question / Soru No

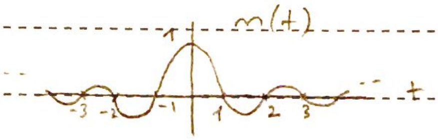
Do not write in this margin / Bu alana yazmayınız

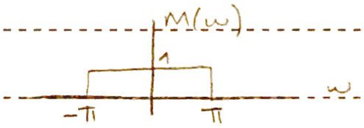
Q1

a) $m(t) = \text{sinc}(\pi t)$

$$E_m = \int_{-\infty}^{\infty} |m(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |M(\omega)|^2 d\omega$$

$$m(t) = \text{sinc}(\pi t) \overset{\mathcal{F}}{\longleftrightarrow} M(\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right) \quad \text{(see Table 1)}$$





$$\Rightarrow E_m = \frac{1}{2\pi}\int_{-\infty}^{\infty} |M(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} 1^2 d\omega = \frac{1}{2\pi}\cdot 2\pi = 1$$ 15

b) Since $0 < E_m < \infty$, $m(t)$ is an energy signal 5

* BONUS For envelope detection we should have $A + m(t) \geq 0 \ \forall t$, i.e. after shifting by A, the signal should be above 0-level.

$m(t) = \text{sinc}(\pi t) = \frac{\sin \pi t}{\pi t}$ has lobes that get smaller in magnitude as t gets larger

$m(-1.5) = \frac{\sin 1.5\pi}{1.5\pi} = \frac{-1}{1.5\pi} = -0.212$ (the peak amplitude of the first negative lobe) is the negative peak of $m(t)$ $\Rightarrow$ the minimum value of A is 0.212 10

(See the above sketch of the signal. It crosses $m(t)=0$ line @ $t = \pm 1, \pm 2, \pm 3, \ldots$ The first negative peak is the one with the largest magnitude and occurs @ $t = \pm 1.5$.)

No. of Question / Soru No

Do not write in this margin / Bu alana yazmayınız

Q2

$f_c = 1\,kHz \Rightarrow \omega_c = 2\pi \times f_c = 2\pi \times 1000 = 2000\pi$ rad/s

$A = 2$

a) $\varphi_{AM}(t) = [A + m(t)] \cos \omega_c t$

$= [2 + \text{sinc}(\pi t)] \cos 2000\pi t$ 5

b) $\varphi_{AM}(t) = 2\cos 2000\pi t + \text{sinc}(\pi t) \cos 2000\pi t$

$\cos \omega_c t \overset{\mathcal{F}}{\longleftrightarrow} \pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$

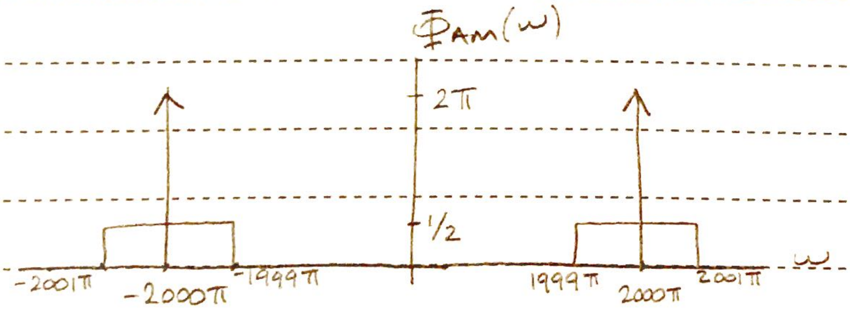
$m(t)\cos \omega_c t \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}[M(\omega - \omega_c) + M(\omega + \omega_c)]$

$m(t) = \text{sinc}(\pi t) \overset{\mathcal{F}}{\longleftrightarrow} M(\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right)$

$\Rightarrow \text{sinc}(\pi t)\cos 2000\pi t \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}\left[\text{rect}\left(\frac{\omega - 2000\pi}{2\pi}\right) + \text{rect}\left(\frac{\omega + 2000\pi}{2\pi}\right)\right]$

$\Rightarrow \Phi_{AM}(\omega) = 2\pi[\delta(\omega - 2000\pi) + \delta(\omega + 2000\pi)] + \frac{1}{2}\left[\text{rect}\left(\frac{\omega - 2000\pi}{2\pi}\right) + \text{rect}\left(\frac{\omega + 2000\pi}{2\pi}\right)\right]$ 10

c)



10

No. of Question / Soru No

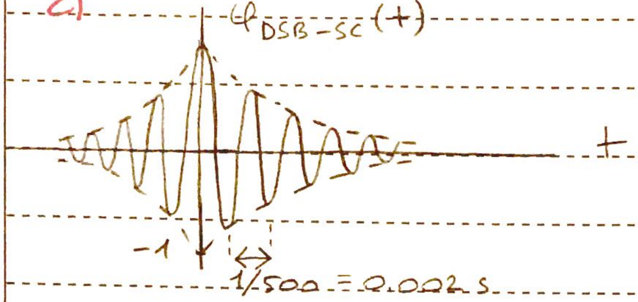
Do not write in this margin / Bu alana yazmayınız

Q3

a) $m(t) = e^{-|t|}$ (5)

b) $\omega_c = 1000\pi \Rightarrow f_c = \frac{\omega_c}{2\pi} = \frac{1000\pi}{2\pi} = 500$ Hz (5)

c)



(10)

d) Since $m(t) \geq 0 \;\forall t$, yes we can use an envelope detector for demodulation (observe that the envelope of $\varphi_{DSB-SC}(t)$ given in (c) is the message itself) (5)

e) $$m(t)\cos\omega_c t \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}\left[M(\omega-\omega_c) + M(\omega+\omega_c)\right]$$

$$m(t) = e^{-|t|} \overset{\mathcal{F}}{\longleftrightarrow} M(\omega) = \frac{2}{1+\omega^2}$$ (see Table 1)

$$\Rightarrow \Phi_{DSB-SC}(\omega) = \frac{1}{2}\left[\frac{2}{1+(\omega-1000\pi)^2} + \frac{2}{1+(\omega+1000\pi)^2}\right]$$ (10)

$$= \frac{1}{1+(\omega-1000\pi)^2} + \frac{1}{1+(\omega+1000\pi)^2}$$

No. of Question / Soru No

Do not write in this margin / Bu alana yazmayınız

Q4 $m(t) = 0.25 \cos 5000\pi t$, $\omega_c = 2\pi \times 10^7$ rad/s

a) $\eta = \dfrac{P_{sidebands}}{P_{carrier} + P_{sidebands}} = 10\% = 0.1$

$$\varphi_{AM}(t) = [A + 0.25 \cos 5000\pi t] \cos 2\pi \times 10^7 t$$

$$= \underbrace{A \cos 2\pi \times 10^7 t}_{carrier} + \underbrace{0.25 \cos 5000\pi t \cos 2\pi \times 10^7 t}_{sidebands}$$

$$= \overbrace{A \cos 2\pi \times 10^7 t} + \overbrace{0.125 \cos \omega_x t + 0.125 \cos \omega_y t}$$

where $\omega_x = 2\pi \times 10^7 - 5000\pi$

and $\omega_y = 2\pi \times 10^7 + 5000\pi$

3

$P_{carrier} = \dfrac{A^2}{2}$, $P_{sidebands} = \dfrac{(0.125)^2}{2} + \dfrac{(0.125)^2}{2} = (0.125)^2$ 4

$$\Rightarrow \eta = \frac{(0.125)^2}{\frac{A^2}{2} + (0.125)^2} = 0.1$$

$$\Rightarrow 0.1 \frac{A^2}{2} + 0.1 (0.125)^2 = (0.125)^2$$

$$0.1 \frac{A^2}{2} = 0.9 (0.125)^2$$

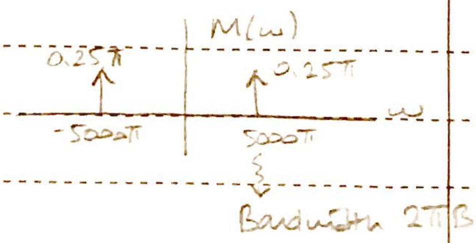
$A^2 = 18 (0.125)^2 \Rightarrow A = 0.53$ 3

b) $m(t) = 0.25 \cos 5000\pi t$

$\Rightarrow m_p = 0.25$

For envelope det., $A \geq m_p \Rightarrow A \geq 0.25$ 5

c) $\dfrac{1}{\omega_c} << RC << \dfrac{1}{2\pi B}$



$\omega_c = 2\pi \times 10^7$ rad/s 3

$2\pi B = 5000\pi$ rad/s 3

$C = 22$ pF

No. of Question / Soru No

Do not write in this margin / Bu alana yazmayınız

$$\frac{1}{2\pi \times 10^7 \times 22 \times 10^{-12}} \ll R \ll \frac{1}{5000\pi \times 22 \times 10^{-12}}$$

$$723.4\,\Omega \ll R \ll 2.894\,M\Omega$$ 3

$$\Rightarrow 7.234\,k\Omega < R < 289.4\,k\Omega$$ 1

d) Max power efficiency is obtained with the maximum allowable carrier amplitude $\Rightarrow A = 0.25$ 5