Power Distribution Systems

Lecture 4

Alternating Current (A.C.) Distribution

Part I

(*A.C. Distribution Calculations*)

Introduction

Although in the beginning of the electrical age, electricity was generated, transmitted and distributed as a direct current (d.c.), with the development of transformer by George Westinghouse, alternating current (a.c.) system has become so predominant.

Now-a-days, electrical energy is generated, transmitted and distributed in the form of alternating current as an economical proposition. The electrical energy produced at the power station is transmitted at very high voltages by 3-phase, 3- wire system to step-down sub-stations for distribution.

A.C. Distribution Calculations

A.C. distribution calculations differ from those of d.c. distribution in the following respects :

a) In case of d.c. system, the voltage drop is due to resistance alone. However, in a.c. system, the voltage drops are due to the combined effects of *resistance*, *inductance* and *capacitance*.

b) In a d.c. system, additions and subtractions of currents or voltages are done arithmetically but in case of a.c. system, these operations are done *vectorially*.

c) In an a.c. system, *power factor (p.f.)* has to be taken into account. Loads tapped off form the distributor are generally at different power factors. There are two ways of referring power factor namely;
   i. It may be referred to supply or receiving end voltage which is regarded as the reference vector.
   ii. It may be referred to the voltage at the load point itself.

Methods of Solving A.C. Distribution Problems

In a.c. distribution calculations, power factors of various load currents have to be considered since currents in different sections of the distributor will be the vector sum of load currents and not the arithmetic sum. The power factors of load currents may be given

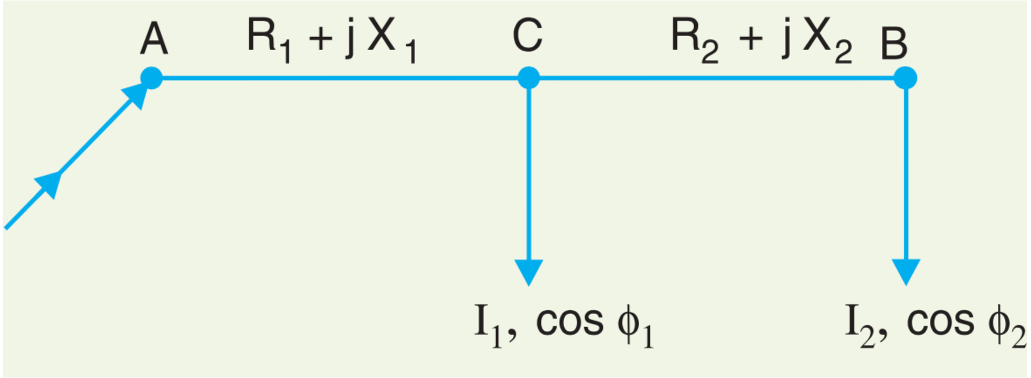
a) With respect to receiving or sending end voltage
b) With respect to load voltage itself.

Power Factors Referred to Receiving End Voltage

Consider an a.c. distributor AB with concentrated loads of I_1 and I_2 tapped off at points C and B as shown in figure.

Taking the receiving end voltage V_B as the reference vector, let lagging power factors at C and B be cos ϕ_1 and cos ϕ_2 with respect to V_B.

Let R_1, X_1 and R_2, X_2 be the resistance and reactance of sections AC and CB of the distributor.



Power Factors Referred to Receiving End Voltage

Impedance of section AC, $\overrightarrow{Z_{AC}} = R_1 + j\,X_1$

Impedance of section CB, $\overrightarrow{Z_{CB}} = R_2 + j\,X_2$

Load current at point C, $\vec{I_1} = I_1\,(\cos\phi_1 - j\,\sin\phi_1)$

Load current at point B, $\vec{I_2} = I_2\,(\cos\phi_2 - j\,\sin\phi_2)$

Current in section CB, $\overrightarrow{I_{CB}} = \vec{I_2} = I_2\,(\cos\phi_2 - j\,\sin\phi_2)$

Current in section AC, $\overrightarrow{I_{AC}} = \vec{I_1} + \vec{I_2}$

$$= I_1\,(\cos\phi_1 - j\,\sin\phi_1) + I_2\,(\cos\phi_2 - j\,\sin\phi_2)$$

Voltage drop in section CB, $\overrightarrow{V_{CB}} = \overrightarrow{I_{CB}}\,\overrightarrow{Z_{CB}} = I_2\,(\cos\phi_2 - j\,\sin\phi_2)\,(R_2 + j\,X_2)$

Voltage drop in section AC, $\overrightarrow{V_{AC}} = \overrightarrow{I_{AC}}\,\overrightarrow{Z_{AC}} = \left(\vec{I_1} + \vec{I_2}\right) Z_{AC}$

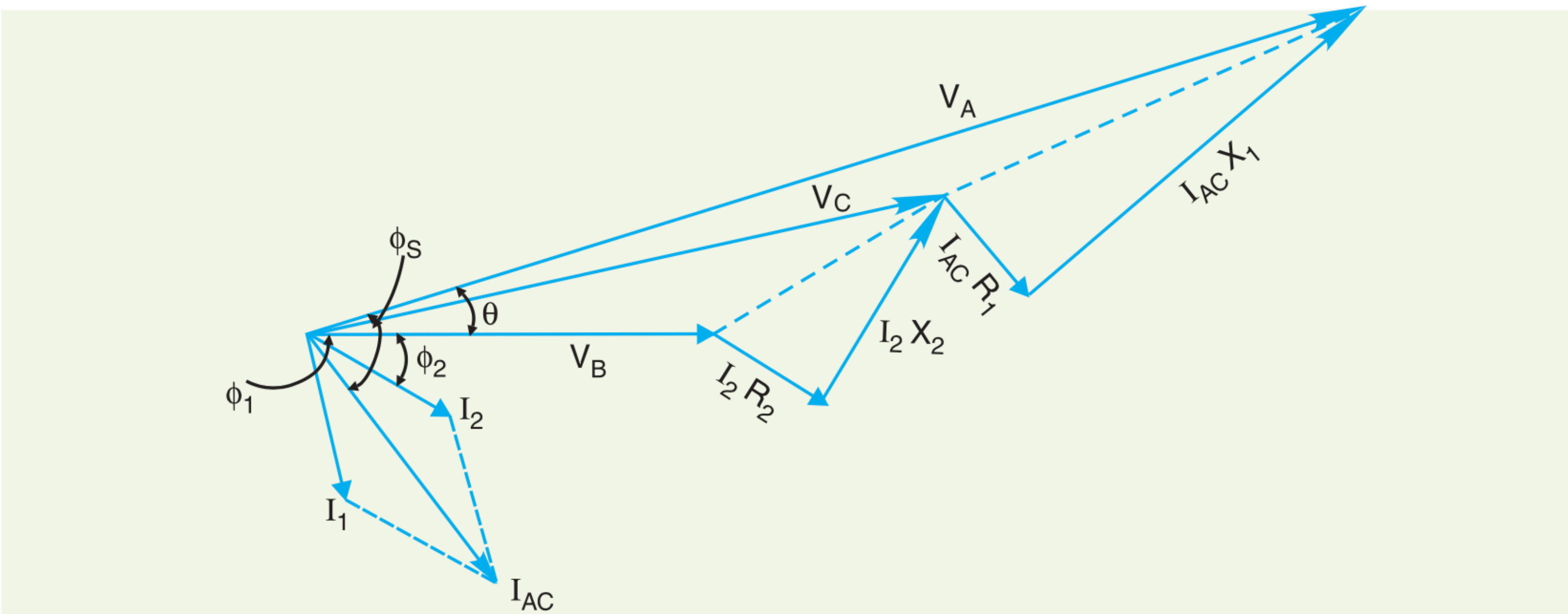
$$= [I_1(\cos\phi_1 - j\,\sin\phi_1) + I_2\,(\cos\phi_2 - j\,\sin\phi_2)]\,[R_1 + jX_1]$$

Sending end voltage, $\vec{V_A} = \vec{V_B} + \overrightarrow{V_{CB}} + \overrightarrow{V_{AC}}$

Sending end current, $\vec{I_A} = \vec{I_1} + \vec{I_2}$

Power Factors Referred to Receiving End Voltage

The vector diagram of the a.c. distributor under these conditions is shown in figure below. Here, the receiving end voltage V_B is taken as the reference vector.



Power Factors Referred to Receiving End Voltage

Example 4.1 *A single phase a.c. distributor AB 300 meters long is fed from end A and is loaded as under :*

a) 100 A at 0.707 p.f. lagging 200 m from point A
b) 200 A at 0.8 p.f. lagging 300 m from point A

The load resistance and reactance of the distributor is 0.2 Ω and 0.1 Ω per kilometer. Calculate the total voltage drop in the distributor. The load power factors refer to the voltage at the far end.

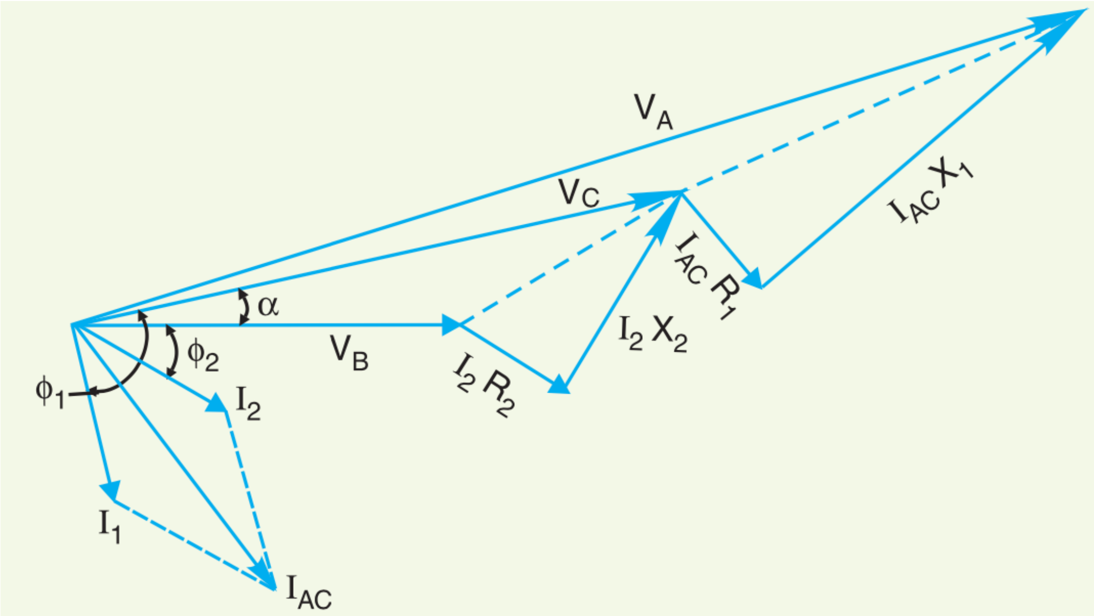
Power Factors Referred to Receiving End Voltage

Example 4.2 *A single phase distributor 2 kilometers long supplies a load of 120 A at 0.8 p.f. lagging at its far end and a load of 80 A at 0.9 p.f. lagging at its mid-point. Both power factors are referred to the voltage at the far end. The resistance and reactance per km (go and return) are 0.05 Ω and 0.1 Ω respectively. If the voltage at the far end is maintained at 230 V, calculate :*

a) voltage at the sending end
b) phase angle between voltages at the two ends.

Power Factors Referred to Respective Load Voltage

Suppose the power factors of loads in the previous *AB* distributor are referred to their respective load voltages. Then ϕ_1 is the phase angle between *VC* and I_1 and ϕ_2 is the phase angle between *VB* and I_2. The vector diagram under these conditions is shown in the following figure.



Power Factors Referred to Respective Load Voltage

$$\text{Voltage drop in section } CB = \vec{I_2}\,\overrightarrow{Z_{CB}} = I_2(\cos\phi_2 - j\sin\phi_2)(R_2 + jX_2)$$

$$\text{Voltage at point } C = \vec{V_B} + \text{Drop in section } CB = V_C\angle\alpha \text{ (say)}$$

Now $\vec{I_1} = I_1\angle-\phi_1$ *w.r.t.* voltage V_C

$\therefore$ $\vec{I_1} = I_1\angle-(\phi_1-\alpha)$ *w.r.t.* voltage V_B

i.e. $\vec{I_1} = I_1[\cos(\phi_1-\alpha) - j\sin(\phi_1-\alpha)]$

Now $\overrightarrow{I_{AC}} = \vec{I_1} + \vec{I_2}$

$$= I_1[\cos(\phi_1-\alpha) - j\sin(\phi_1-\alpha)] + I_2(\cos\phi_2 - j\sin\phi_2)$$

$$\text{Voltage drop in section } AC = \overrightarrow{I_{AC}}\,\overrightarrow{Z_{AC}}$$

$\therefore$ $\text{Voltage at point } A = V_B + \text{Drop in } CB + \text{Drop in } AC$

Power Factors Referred to Respective Load Voltage

<u>Example 4.3</u> *A single phase distributor one km long has resistance and reactance per conductor of 0.1 Ω and 0.15 Ω respectively. At the far end, the voltage VB = 200 V and the current is 100 A at a p.f. of 0.8 lagging. At the mid-point M of the distributor, a current of 100 A is tapped at a p.f. of 0.6 lagging with reference to the voltage VM at the mid-point. Calculate :*

a) *voltage at mid-point*
b) *sending end voltage* V_A
c) *phase angle between* V_A *and* V_B