MATH 233
Fall 2018
Quiz #2 B Solutions

<u>Duration:</u> 50 minutes.
<u>Remark:</u> Show your thinking/work. Do not just write a number as a result.

1. A cell divides into two in every minute. Assume we have a single cell in a laboratory tube.

   - Find a **recurrence relation** for the number of cells after n minutes.

minute	cells	# of celss
0	1	1
1	1,1	2
2	1,1,1,1	4
3	1,1,1,1,1,1,1,1	8

Let C_n be the number of cells at minute n. The recurrence relation is:
$C_n = 2 \cdot C_{n-1}$

   - What is/are the **initial condition(s)**?

$C_0 = 1$

   - What is the number of cells after an hour?

From the recurrence relation, we see that $C_n = 2^n$

Thus, $C_{60} = 2^{60}$

2. Two fair dice and a fair coin are tossed.

a) What is the **experiment**?

Two fair dice and a fair coin are tossed.

b) What is the **sample space**?

Sample Space = {{1,1,H}, {1,1,T}, {1,2,H}, {1,2,T}, ….. {6,6,T}}.

c) What is the **size** of the sample space?

|Sample Space| = 6.6.2 = 72

d) What is the probability that a head occurs? (Describe the event E_H)

E_H= {{1,1,H}, {1,2,H}, … {6,6,H}}

$|E_H|$= 6.6 = 36

$P(E_H)= |E_H| / |\text{Sample Space}| = 36/72 = 0.5$

e) What is the probability that a 6 occurs? (Describe the event E_6)

E_6= {{1,6,H}, {1,6,T}, {2,6,H}, …. , {6,6,T}}

Consider the complementary event, i.e. a 6 does not occur at all. Let call this event E_{6c}

Size of E_{6c} = 5.5.2 = 50

$|E_6|$ = 72-50 = 22

$P(E_6)= |E_6| / |\text{Sample Space}| = 22 / 72 = 11/36 = 0.305$

f) What is the probability that the total number on the dice is more than the number of heads? (Describe the event E_{more})

E_{more}= Sample Space

The total number on the dice is any number between [2,12]. The number of heads is at most 1. Thus all outcomes are in E_{more}.

$P(E_{more})= |E_{more}| / |\text{Sample Space}| = 72 / 72 = 1$