ANGLE MODULATION

In AM modulation, the amplitude of a carrier is modulated by a signal $m(t)$, hence, the information content is in the amplitude variations of the carrier.

In angle modulation (or, exponential modulation) the phase (in PM) or frequency (in FM) is modulated by $m(t)$, hence, the information content is in the phase or frequency.

Consider a generalized sinusoidal signal $\varphi(t)$

$$\varphi(t) = A \cos \theta(t)$$

$\theta(t)$: generalized angle (it is a function of time, t)

The curve in Fig 5.1 is an example of $\theta(t)$. The line in Fig 5.1 is the generalized angle of a conventional sinusoid $A\cos(\omega_c t+\theta_0)$, i.e., $\omega_c t + \theta_0$, a line with slope ω_c and intercept θ_0.

Note that this line is tangent to the curve at some instant t.

If $t_1 < t < t_2$ and $\Delta t = t_2 - t_1 \to 0$, over this small time interval Δt the generalized sinusoidal signal $\varphi(t) = A\cos\theta(t)$ is identical with the conventional sinusoid $A\cos(\omega_c t+\theta_0)$ $\Rightarrow$ over this small time interval Δt the frequency of $\varphi(t)$ is ω_c $\Rightarrow$ the "instantaneous frequency" of $\varphi(t)$ at time t is ω_c.

The instantaneous frequency ω_i at any instant t is the slope of $\theta(t)$ at t. Thus, for $\varphi(t)$ defined above:

$$\omega_i(t) = \frac{d\theta(t)}{dt}$$

$$\Rightarrow \quad \theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\, d\alpha$$

Now suppose we transmit the information of $m(t)$ by varying the angle $\theta(t)$ of a carrier → angle (exponential) modulation

Two simple possibilities → phase modulation (PM)
→ frequency modulation (FM)

Phase Modulation (PM)

The angle $\theta(t)$ is varied linearly with the modulating signal $m(t)$

$$\theta(t) = w_c t + \theta_0 + k_p m(t)$$

k_p : constant

w_c : carrier freq.

We may assume $\theta_0 = 0$ without loss of generality

$$\boxed{\theta(t) = w_c t + k_p m(t)}$$

The resulting PM wave:

$$\boxed{\varphi_{PM}(t) = A\cos[w_c t + k_p m(t)]}$$

The instantaneous frequency:

$$\boxed{w_i(t) = \frac{d\theta(t)}{dt} = w_c + k_p \dot{m}(t)}$$

$\dot{m}(t)$: the first derivative of $m(t)$

★ In PM, the <u>instantaneous frequency</u> varies linearly with the <u>derivative</u> of $m(t)$

Frequency Modulation (FM)

The instantaneous frequency $w_i(t)$ is varied linearly with the modulating signal $m(t)$

$$\boxed{w_i(t) = w_c + k_f m(t)}$$

k_f : constant

w_c : carrier freq.

The angle:

$$\theta(t) = \int_{-\infty}^{t} w_i(\alpha)\,d\alpha = \int_{-\infty}^{t} [w_c + k_f m(\alpha)]\,d\alpha$$

$$\Rightarrow \boxed{\theta(t) = w_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha}$$

The resulting FM wave:

$$\varphi_{FM}(t) = A\cos\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]$$

★ In FM, the angle $\theta(t)$ varies linearly with the integral of $m(t)$

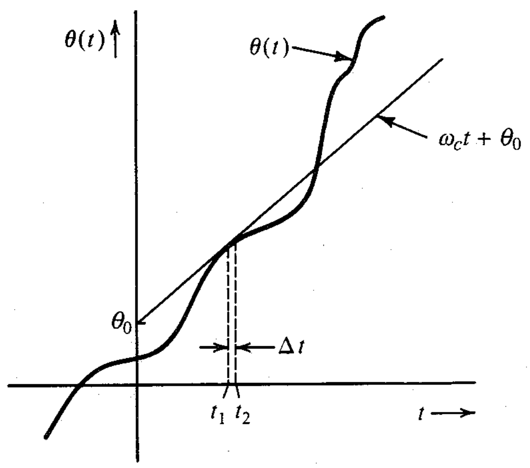

Figure 5.1 Concept of instantaneous frequency.