EEEN 322
Communication Engineering

İpek Şen

Spring 2019

Week 2

Last Week

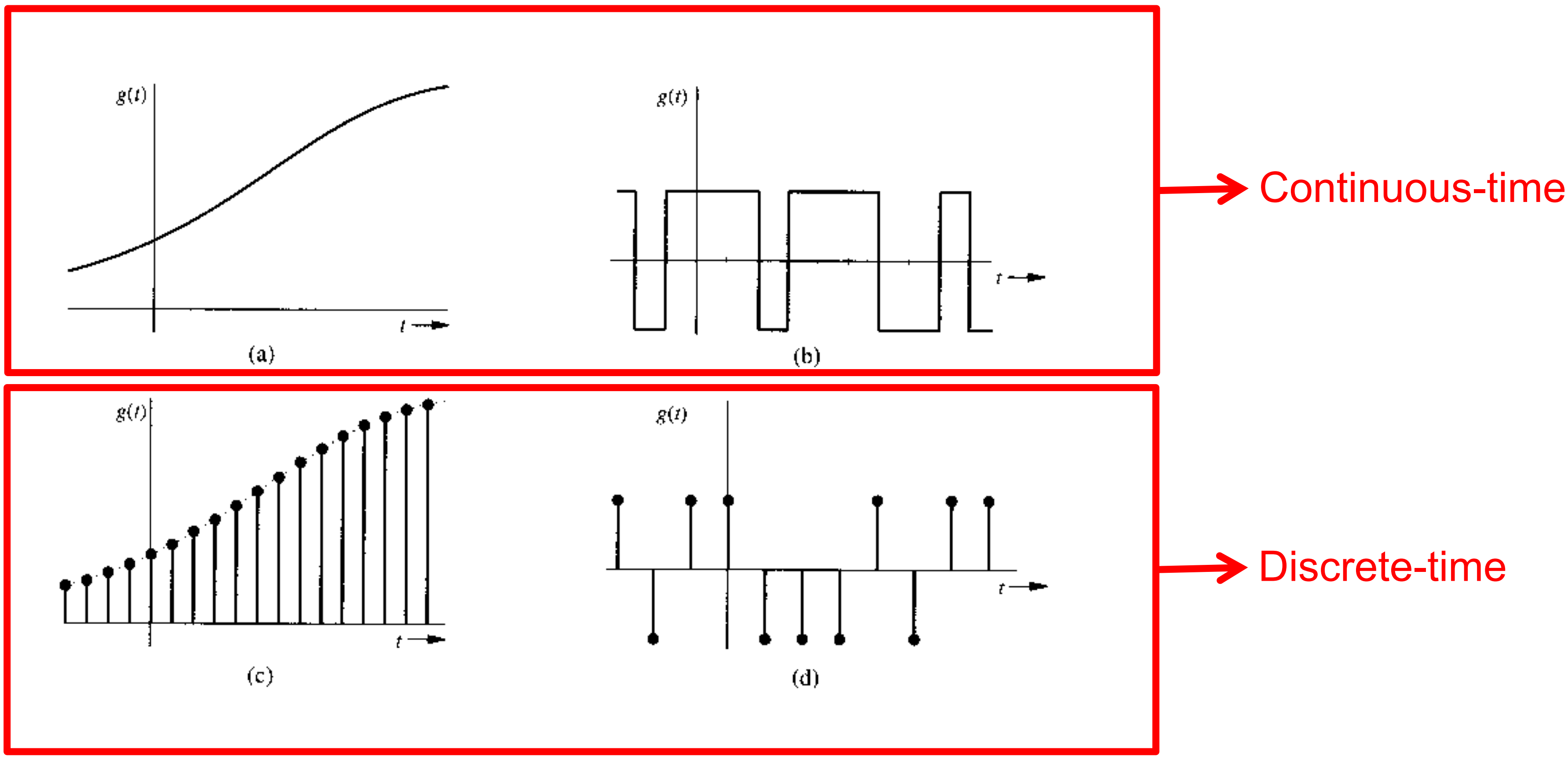
- Analog versus digital communication
- Wired versus wireless communication
- What is AM?
- What is FM? PM?
- Why do we need AM and FM (or PM)? *(instead of sending the the baseband signal directly)*

Classification of Signals

1. Continuous-time and discrete-time signals
2. Analog and digital signals
3. Periodic and aperiodic signals
4. Energy and power signals
5. Deterministic and random signals

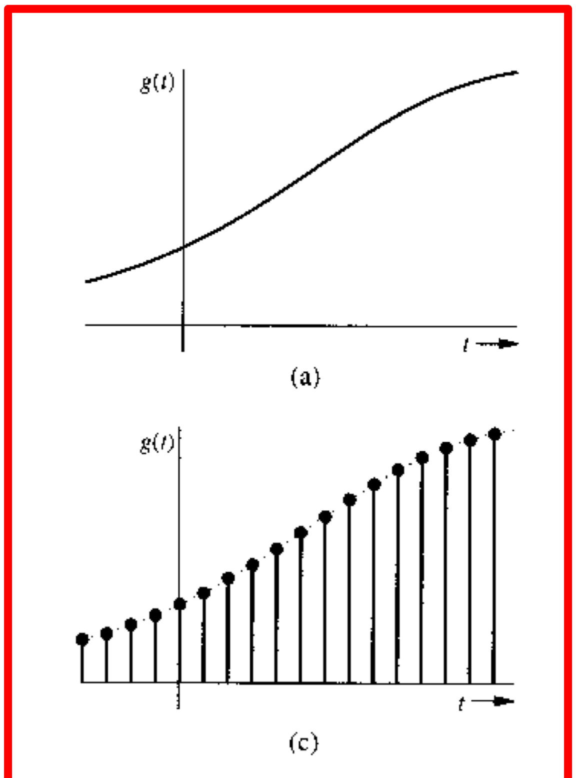
Classification of Signals

1. Continuous-time and discrete-time signals

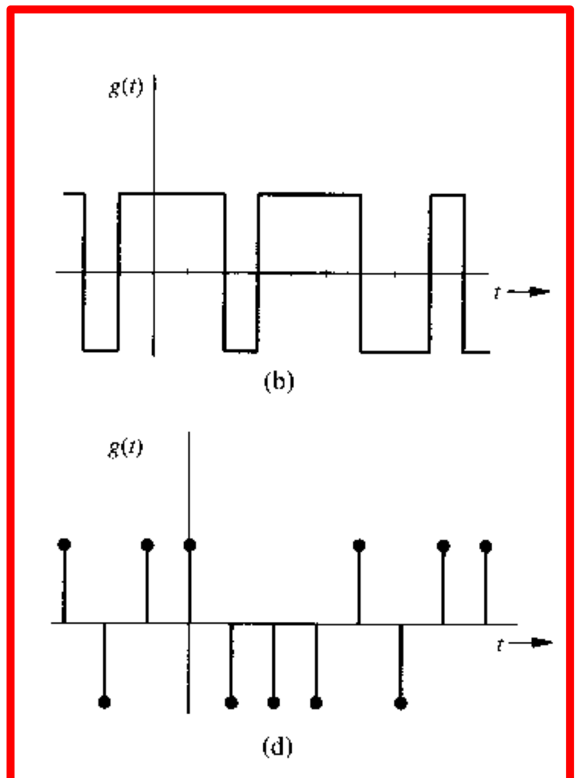


Classification of Signals

2. Analog and digital signals



Analog

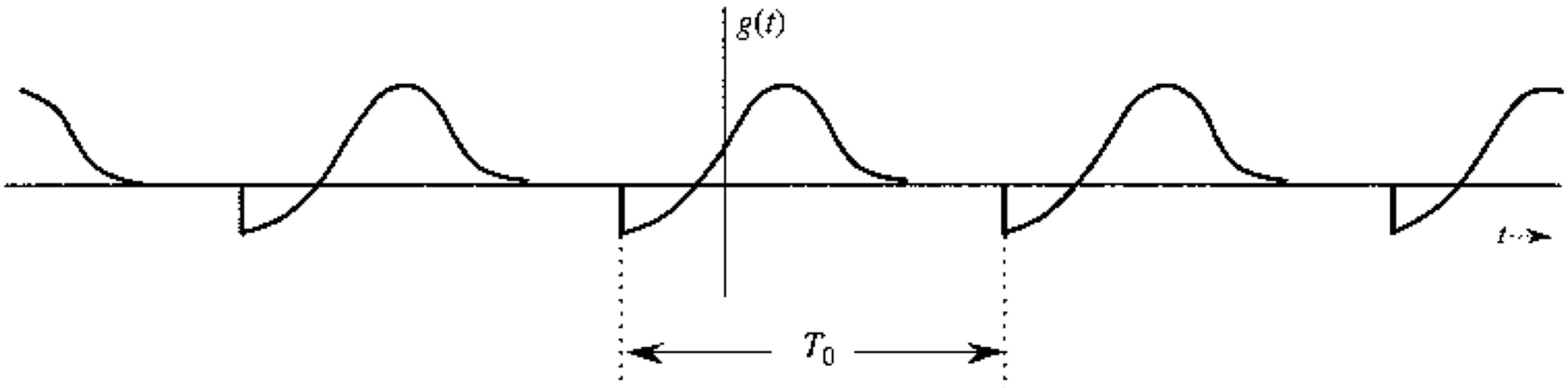


Digital

Classification of Signals

3. Periodic and aperiodic signals

A signal $g(t)$ is said to be periodic if there exists some constant T_0 such that $g(t+T_0) = g(t)$, $\forall t$



The smallest value of T_0 that satisfies the above condition is called the **period** (or, **fundamental period**)

If $g(t)$ is periodic with period T_0, it is also periodic with mT_0, where m is an integer. That is, $g(t+mT_0) = g(t)$, $\forall t$, $m \in \mathbb{Z}$

If a signal does not satisfy the above condition (if it is **not periodic**), it is called **aperiodic**.

Size* of a Signal

*The size of any entity is a quantity that indicates its strength

Assume that $g(t)$ is the voltage across a one-ohm resistor.

- Signal Energy

We define **signal energy** E_g of the signal $g(t)$ as the energy that the voltage $g(t)$ dissipates on the resistor.

$$E_g = \int_{-\infty}^{\infty} g(t)g^*(t)dt = \int_{-\infty}^{\infty} |g(t)|^2 dt$$

$$E_g = \int_{-\infty}^{\infty} g^2(t)dt \quad \text{if } g(t) \text{ is real}$$

- Signal Power

If the energy of a signal is not finite, a more meaningful measure of signal size is the time average of the energy (if it exists), i.e., the **average power** P_g, defined as

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t)g^*(t)dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt$$

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(t)dt \quad \text{if } g(t) \text{ is real}$$

RMS Value

- Note that the average power P_g is the average (mean) of the square of the signal (→ mean-square)

$$P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} g^2(t)dt$$

- **RMS value** is the square-root of the average power (→ root-mean-square)

$$g_{RMS} = \sqrt{P_g} = \sqrt{\lim_{T\to\infty} \underbrace{\boxed{\frac{1}{T}\int_{-T/2}^{T/2}}}_{\text{mean}} \underbrace{\boxed{g^2}}_{\text{square}}(t)dt}$$

root

RMS value of g(t) is the DC value that would deliver the same power to the circuit as g(t)

Classification of Signals (*continued*)

4. Energy and power signals

A signal with finite energy ($0 < E_g < \infty$) is an **energy signal**
A signal with finite power ($0 < P_g < \infty$) is a **power signal**

$$E_g = \int_{-\infty}^{\infty} g^2(t)dt$$

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(t)dt$$

Classification of Signals (*continued*)

4. Energy and power signals

A signal with finite energy ($0 < E_g < \infty$) is an **energy signal**
A signal with finite power ($0 < P_g < \infty$) is a **power signal**

$$E_g = \int_{-\infty}^{\infty} g^2(t)dt$$

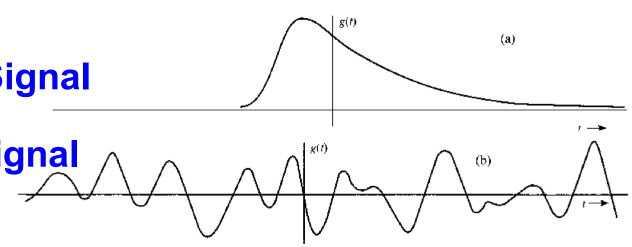
$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(t)dt$$

An energy signal must have finite duration. (A physical signal is always an energy signal.)
A power signal must have infinite duration.

A signal cannot be both an energy and a power signal (If it is one, it cannot be the other). A signal with finite energy has zero average power, a signal with finite average power has infinite energy.

Some signals are neither energy nor power signals.

Is an energy signal?
- Yes → **Energy Signal**
- No → Is a power signal?
  - Yes → **Power Signal**
  - No → **Neither**
    e.g., ramp signal



Classification of Signals (*continued*)

5. Deterministic and random signals

- A signal whose physical description is known completely
  - either in a mathematical form
  - or in a graphical form

  is a **deterministic** signal

- A signal that is known only in terms of a probabilistic description
  - such as a mean value
  - mean square value
  - distributions

  rather than its full mathematical or graphical description is a **random** signal

Classification of Signals (*continued*)

5. Deterministic and random signals

- A signal whose physical description is known completely
  - either in a mathematical form
  - or in a graphical form

  is a **deterministic** signal

- A signal that is known only in terms of a probabilistic description
  - such as a mean value
  - mean square value
  - distributions

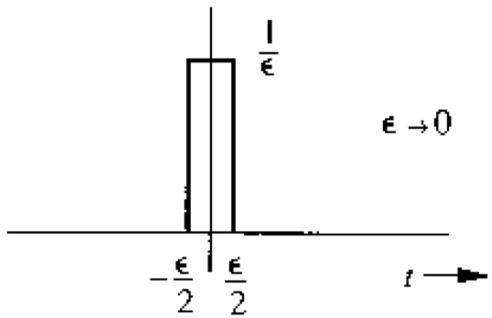
  rather than its full mathematical or graphical description is a **random** signal

Most of the noise signals encountered in practice are random signals.

All message signals are random signals, because a signal, to convey information, must have some uncertainty (randomness).

Unit Impulse Signal (Dirac Delta) $\delta(t)$

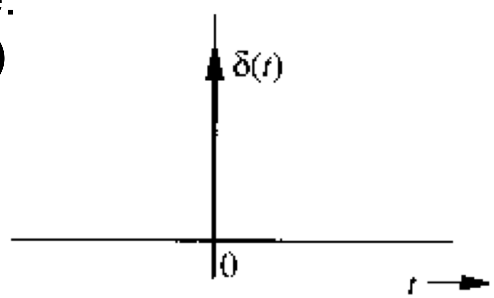
Consider:



In the limit as $\epsilon \to 0$

- the height goes to infinity
- the width goes to zero
- the area remains constant at unity

Unit Impulse:
(Dirac Delta)



$$\delta(t) = 0,\ t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

- Multiplication of a function by the unit impulse

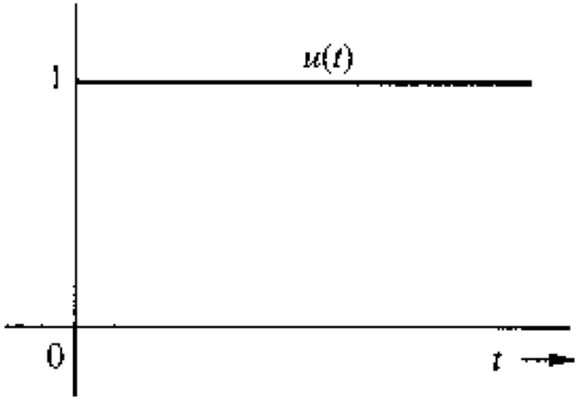
$$\phi(t)\delta(t) = \phi(0)\delta(t) \quad \text{(provided } \phi(t) \text{ is defined at } t = 0\text{)}$$

$$\phi(t)\delta(t - t_0) = \phi(t_0)\delta(t - t_0) \quad \text{(provided } \phi(t) \text{ is defined at } t = t_0\text{)}$$

- Sampling (or, sifting) property of the unit impulse

$$\int_{-\infty}^{\infty} \phi(t)\delta(t - t_0)dt = \int_{-\infty}^{\infty} \phi(t_0)\delta(t - t_0)dt = \phi(t_0) \int_{-\infty}^{\infty} \delta(t - t_0)dt = \phi(t_0)$$

Unit Step Signal $u(t)$

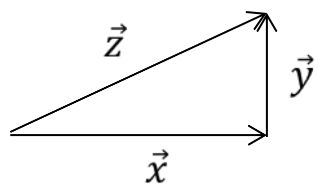


$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Note that: $u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$

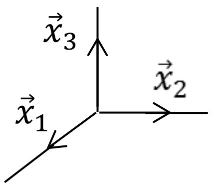
From this result, it follows that: $\delta(t) = \frac{du(t)}{dt}$

Vector Space / Signal Space

	Vector space	**Signal space**
Inner product	$\vec{x}\cdot\vec{y} = <\vec{x},\vec{y}> = \|\|\vec{x}\|\|\ \|\|\vec{y}\|\|\ cos\theta$ $= \sum_i x_i y_i$	$<x(t), y(t)> = \int_{t_1}^{t_2} x(t)y^*(t)dt$
Orthogonality	$\vec{x}\cdot\vec{y} = <\vec{x},\vec{y}> = \sum_i x_i y_i = 0$	$<x(t), y(t)> = \int_{t_1}^{t_2} x(t)y^*(t)dt = 0$
Norm	$\|\|\vec{x}\|\|^2 = \vec{x}\cdot\vec{x} = <\vec{x},\vec{x}> = \sum_i x_i^2$ *This is the square of the norm*	$\|\|x(t)\|\|^2 = <x(t), x(t)> = \int_{t_1}^{t_2} \|x(t)\|^2 dt = E_x$ *This is the square of the norm*
Norm of the sum of orthogonal vectors/signals	 $\|\|\vec{z}\|\|^2 = \|\|\vec{x}\|\|^2 + \|\|\vec{y}\|\|^2$	If *x(t)* and *y(t)* are orthogonal signals and *z(t)=x(t)+y(t)*, we have: $\|\|z(t)\|\|^2 = \|\|x(t)\|\|^2 + \|\|y(t)\|\|^2$ $E_z = E_x + E_y$

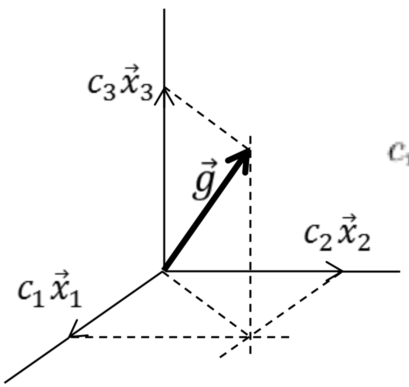
Orthogonal Vector Space / Orthogonal Signal Space

Vector space



<u>Orthogonal</u> set of basis vectors

$$< \vec{x}_m, \vec{x}_n >= \begin{cases} 0, & m \neq n \\ ||\vec{x}_m||^2, & m = n \end{cases}$$



$$\vec{g} = c_1\vec{x}_1 + c_2\vec{x}_2 + c_3\vec{x}_3$$

$$c_n = \frac{< \vec{g}, \vec{x}_n >}{||\vec{x}_n||^2}, \quad n = 1, 2, 3$$

If also $||\vec{x}_n||^2 = 1, \forall n$, then the set is **orthonormal** and we have $c_n =< \vec{g}, \vec{x}_n >, \quad n = 1, 2, 3$

Signal space

Set of signals <u>orthogonal</u> over (t_1, t_2)

$$< x_m(t), x_n(t) >= \int_{t_1}^{t_2} x_m(t)x_n^*(t)dt = \begin{cases} 0, & m \neq n \\ E_m, & m = n \end{cases}$$

If $\{x_n(t)\}$ form a **complete set** for $g(t)$, (i.e., is a **set of basis** signals), then g(t) can be written with zero error as

$$\boxed{g(t) = \sum_{n=1}^{\infty} c_n x_n(t), \quad t_1 \leq t \leq t_2}$$

Generalized Fourier Series

where $c_n = \dfrac{< g(t), x_n(t) >}{||x_n(t)||^2} = \dfrac{1}{E_n} \displaystyle\int_{t_1}^{t_2} g(t)x_n^*(t)dt$

If $E_n = 1, \forall n$, then the set is **orthonormal**

Orthogonal Signal Space

$$\boxed{g(t) = \sum_{n=1}^{\infty} c_n x_n(t), \quad t_1 \le t \le t_2}$$

Generalized Fourier Series

where

$$\boxed{c_n = \frac{1}{E_n} \int_{t_1}^{t_2} g(t) x_n^*(t) dt}$$

Coefficients of the generalized Fourier Series

Since $\{x_n(t)\}$ are orthogonal, we have

$$\boxed{E_g = \sum_{n=1}^{\infty} |c_n|^2 E_n}$$

Parseval's Theorem

For a periodic signal $g(t)$ with period T_0, complex exponentials $\{e^{jw_0 nt}\}_{n=-\infty}^{\infty}$ form an orthogonal basis set over any interval of duration T_0.
(Note that $w_0 = \frac{2\pi}{T_0}$)

Then

$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} c_n e^{jw_0 nt}}$$

Fourier Series Expansion of periodic signal *g(t)*

Synthesis equation

where

$$\boxed{c_n = \frac{1}{T_0} \int_{T_0} g(t) e^{-jw_0 nt} dt}$$

Fourier Series coefficients

Analysis equation

$$w_0 = \frac{2\pi}{T_0}$$

$$\boxed{\frac{1}{T_0} \int_{T_0} |g(t)|^2 dt = \sum_{n=-\infty}^{\infty} |c_n|^2}$$

Parseval's Theorem
(note that this is the exact equivalent of the one on the left)

WHY?

Dirichlet Conditions

Let $g(t)$ be a periodic signal with period T_0. If the following (Dirichlet) conditions are satisfied:

1. $\int_{T_0} |g(t)|\, dt < \infty$
2. The number of maxima and minima in each period is finite
3. The number of discontinuities of $g(t)$ in a period is finite

then $g(t)$ can be expanded in terms of complex exponential signals:

$$g(t) = \sum_{n=-\infty}^{\infty} c_n e^{jw_0 n t} \qquad \text{where} \qquad c_n = \frac{1}{T_0}\int_{T_0} g(t) e^{-jw_0 n t} dt \qquad \left(w_0 = \frac{2\pi}{T_0} \right)$$

Dirichlet conditions are “sufficient” conditions, not “necessary” conditions

$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} c_n e^{jw_0 nt}}$$

$$\boxed{c_n = \frac{1}{T_0} \int_{T_0} g(t) e^{-jw_0 nt} dt}$$

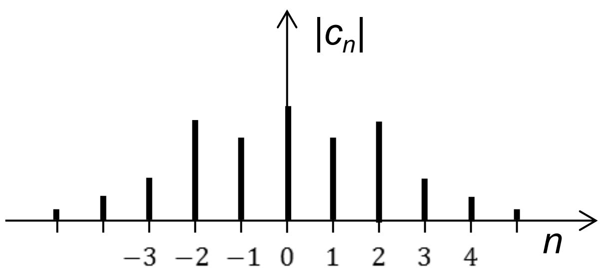
$$w_0 = \frac{2\pi}{T_0}$$

c_n is a complex number in general, therefore has a magnitude $|c_n|$ and phase $\angle c_n$

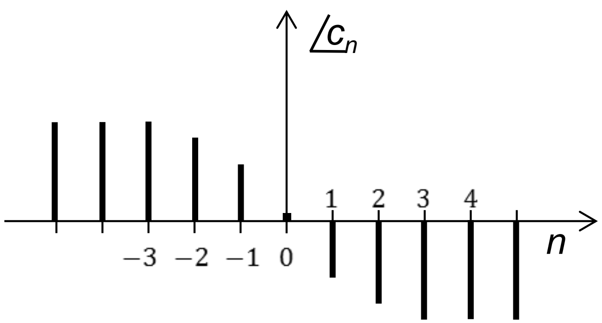
Plot of $|c_n|$ versus n → Magnitude spectrum

Plot of $\angle c_n$ versus n → Phase spectrum

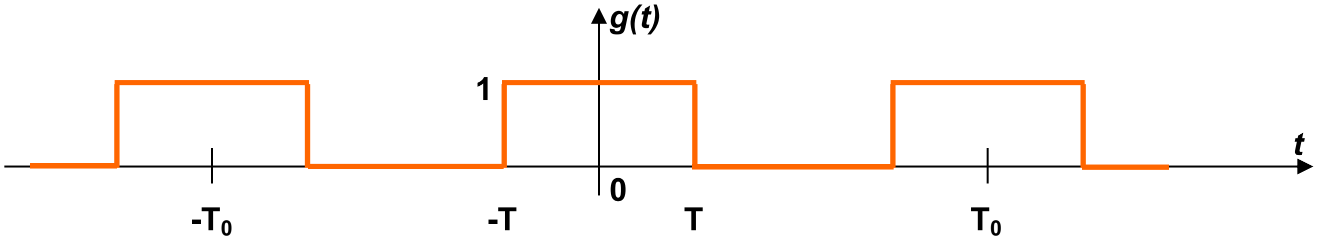
Magnitude Spectrum



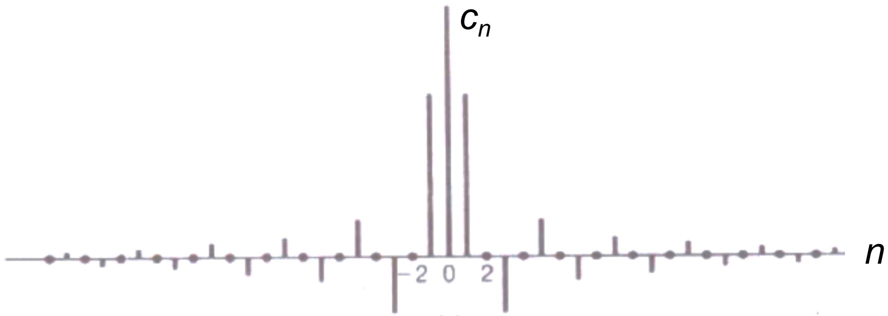
Phase Spectrum



Rectangular wave (periodic)

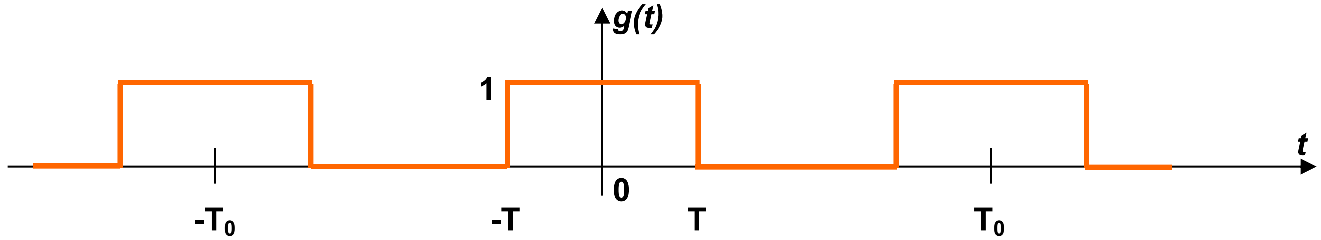


$T_0 = 4T$

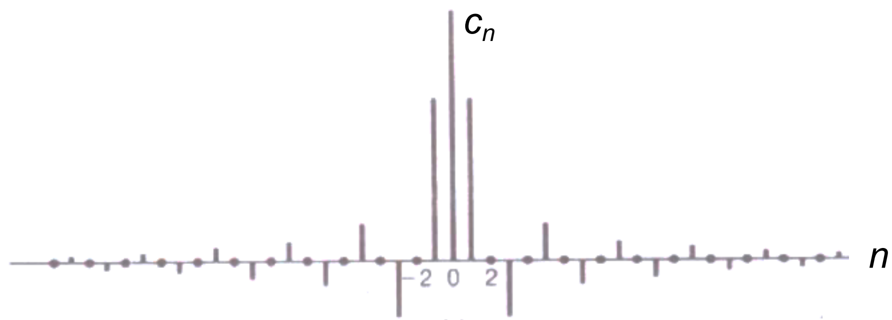


(From EEEN 321)

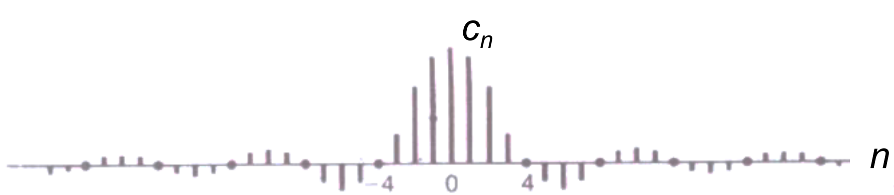
Rectangular wave (periodic)



$T_0 = 4T$

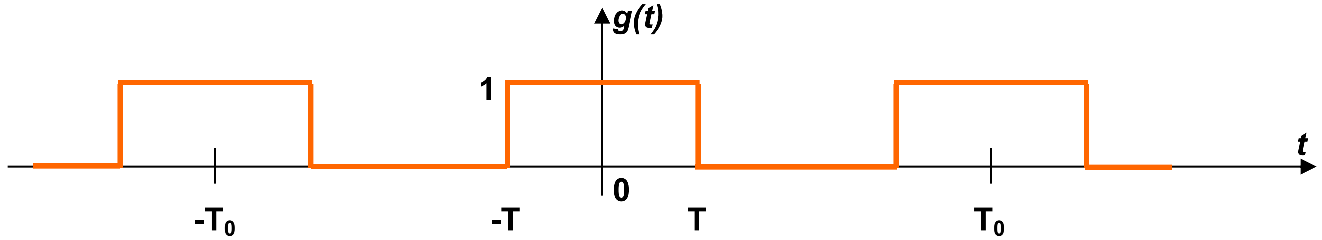


$T_0 = 8T$

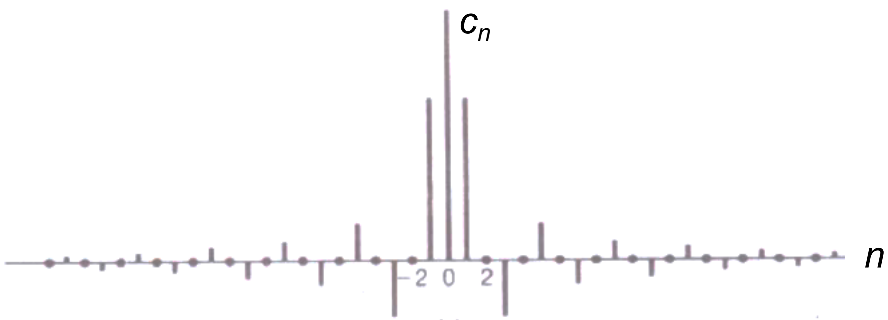


(From EEEN 321)

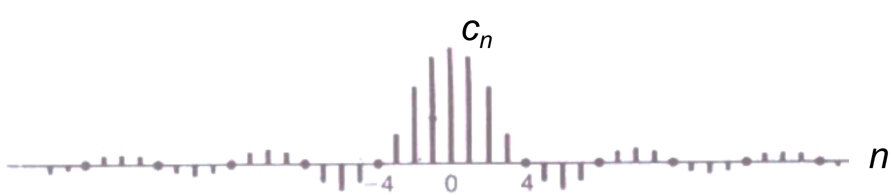
Rectangular wave (periodic)



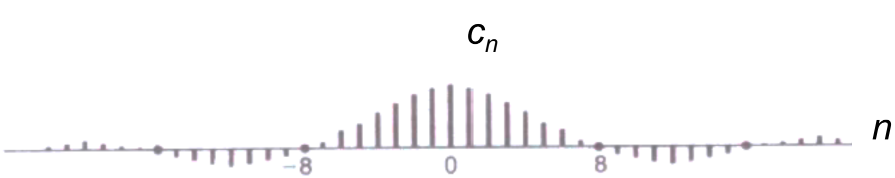
$T_0 = 4T$



$T_0 = 8T$



$T_0 = 16\ T$



(From EEEN 321)