EEEN 322 PS 7 QUESTIONS

Q1

Example 1 Sketch FM and PM waves for the modulating signal $m(t)$ shown in Fig 5.4a. The constants k_f and k_p are $2\pi \times 10^5$ and 10π, respectively, and the carrier frequency f_c is 100MHz

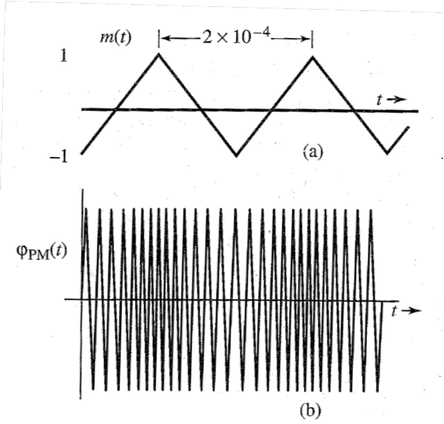

Figure 5.4 FM and PM waveforms.

* *

Q2

Example 2 Sketch FM and PM waves for the digital modulating signal $m(t)$ shown in Fig 5.5a. The constants k_f and k_p are $2\pi \times 10^5$ and $\pi/2$, respectively, and $f_c = 100$ MHz.

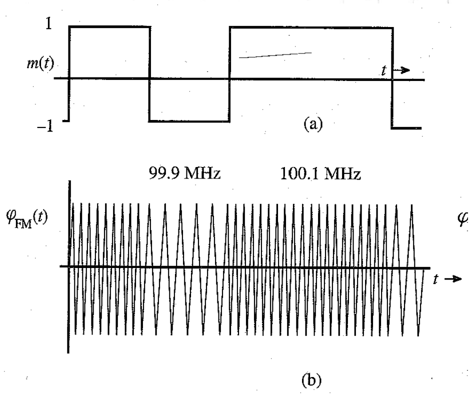

Figure 5.5 FM and PM waveforms.

* *

Q3

5.1-1 Sketch $\varphi_{FM}(t)$ and $\varphi_{PM}(t)$ for the modulating signal $m(t)$ shown in Fig. P5.1-1, given $\omega_c = 10^8$, $k_f = 10^5$, and $k_p = 25$.

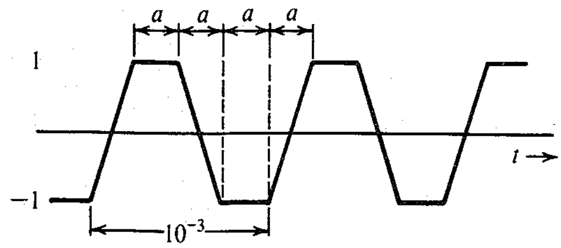

Figure P5.1-1

* *

Q4

5.1-2 A baseband signal $m(t)$ is the periodic sawtooth signal shown in Fig. P5.1-2. Sketch $\varphi_{FM}(t)$ and $\varphi_{PM}(t)$ for this signal $m(t)$ if $\omega_c = 2\pi \times 10^6$, $k_f = 2000\pi$, and $k_p = \pi/2$. Explain why it is necessary to use $k_p < \pi$ in this case.

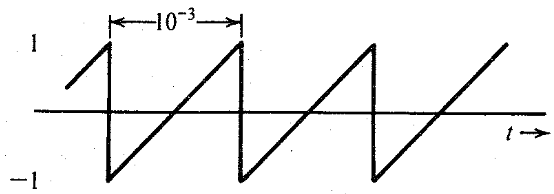

Figure P5.1-2

EEEN 322 PS 7 SOLUTIONS

Q1

For FM: $\omega_i(t) = \omega_c + k_f m(t)$

$$f_i(t) = \frac{\omega_i(t)}{2\pi} \Rightarrow f_i(t) = f_c + \frac{k_f}{2\pi} m(t)$$

$$= 10^8 + 10^5 m(t)$$

Since $m(t)$ varies between -1 and 1, $[m(t)]_{min} = -1$ and $[m(t)]_{max} = 1$. Therefore

$$[f_i(t)]_{min} = 10^8 + 10^5 [m(t)]_{min} = 10^8 - 10^5 = 99.9 \text{ MHz}$$

$$[f_i(t)]_{max} = 10^8 + 10^5 [m(t)]_{max} = 10^8 + 10^5 = 100.1 \text{ MHz}$$

Since $m(t)$ increases and decreases linearly with time, $f_i(t)$ increases linearly from 99.9 MHz to 100.1 MHz over the half cycle of $m(t)$ and decreases linearly from 100.1 MHz over the remaining half cycle of $m(t)$ (see Fig 5.4b)

For PM: PM for $m(t)$ is FM for $\dot{m}(t)$ (see Fig 5.4c for $\dot{m}(t)$)

$$f_i(t) = f_c + \frac{k_p}{2\pi} \dot{m}(t)$$

$$= 10^8 + \frac{10\pi}{2\pi} \dot{m}(t)$$

$$= 10^8 + 5 \dot{m}(t)$$

$$= \begin{cases} 10^8 + 5(20000) = 10^8 + 10^5 = 100.1 \text{ MHz} & \text{when } \dot{m}(t) = 20000 \\ 10^8 + 5(-20000) = 10^8 - 10^5 = 99.9 \text{ MHz} & \text{when } \dot{m}(t) = -20000 \end{cases}$$

So, during the time interval where $m(t)$ increases linearly from -1 to 1 (i.e., where $\dot{m}(t) = 20000$), the instantaneous frequency is constant and equal to 100.1 MHz, and during the time interval where $m(t)$ decreases linearly from 1 to -1 (i.e., where $\dot{m}(t) = -20000$), the instantaneous frequency is constant and equal to 99.9 MHz.

(See Fig 5.4d)

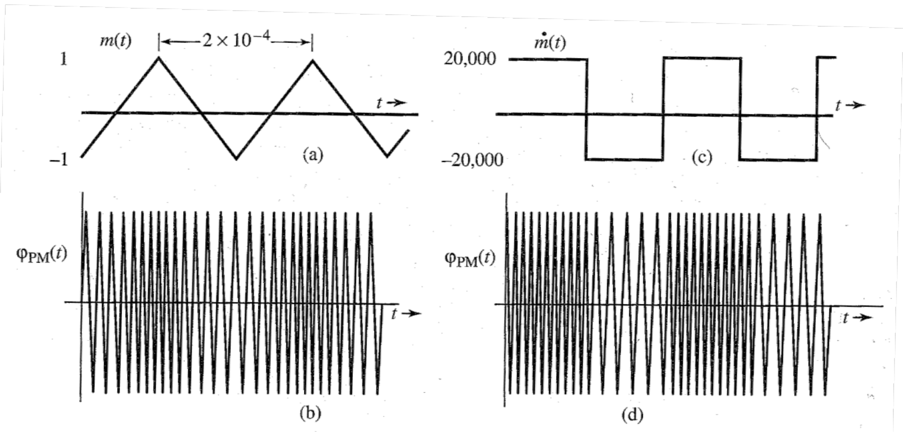

Figure 5.4 FM and PM waveforms.

* *

Q2

For FM: $f_i(t) = f_c + \frac{k_f}{2\pi} m(t) = 10^8 + 10^5 m(t)$

$$\Rightarrow f_i(t) = \begin{cases} 10^8 - 10^5 = 99.9\,\text{MHz} & \text{when } m(t) = -1 \\ 10^8 + 10^5 = 100.1\,\text{MHz} & \text{when } m(t) = 1 \end{cases}$$

(See Fig 5.5b)

Note: This scheme of carrier frequency modulation by a digital signal is called frequency-shift-keying (FSK) because information digits are transmitted by shifting the carrier frequency.

For PM: $f_i(t) = f_c + \frac{k_p}{2\pi} \dot{m}(t) = 10^8 + \frac{1}{4} \dot{m}(t)$

See Fig 5.5c for $\dot{m}(t)$. Now, how to change the instantaneous frequency by an infinite amount and then change it back to the original frequency in zero time?

$\Rightarrow$ The indirect method of sketching PM (using $\dot{m}(t)$ to frequency-modulate a carrier) works as long as $m(t)$ is a continuous signal. If $m(t)$ is discontinuous, $\dot{m}(t)$ contains impulses, and this method fails. Use the direct approach if this is the case (if $m(t)$ has discontinuities)

$$\varphi_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$$
$$= A\cos\left[2\pi\times10^8 t + \frac{\pi}{2} m(t)\right]$$
$$= \begin{cases} A\cos[2\pi\times10^8 t + \pi/2] & \text{when } m(t)=1 \\ A\cos[2\pi\times10^8 t - \pi/2] & \text{when } m(t)=-1 \end{cases}$$
$$= \begin{cases} -A\sin(2\pi\times10^8 t) & \text{when } m(t)=1 \\ A\sin(2\pi\times10^8 t) & \text{when } m(t)=-1 \end{cases}$$

(See Fig 5.5d)

Note: This scheme of carrier PM by a digital signal is called phase-shift-keying (PSK) because information digits are transmitted by shifting the carrier phase.

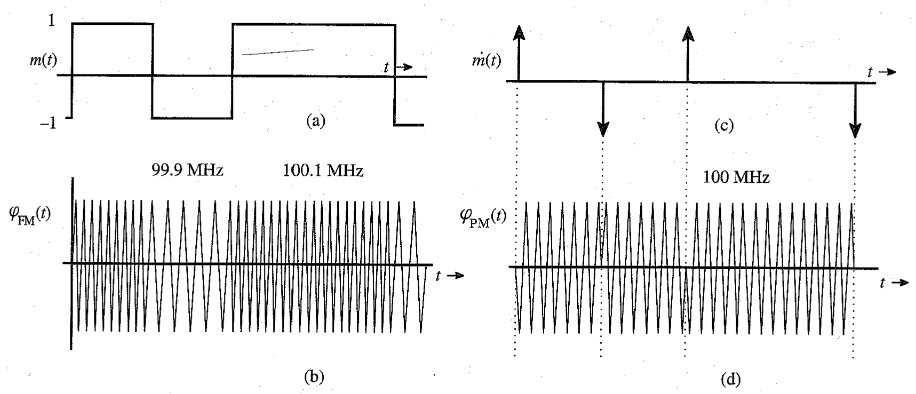

Figure 5.5 FM and PM waveforms.

Note: PSK can also be viewed as DSB-SC by $m(t)$.

Note that the phase shifts by π instantaneously in this example.

RULE: The phase deviation $k_p m(t)$ must be restricted to a range $(-\pi, \pi)$ in order to avoid ambiguity in demodulation.

E.g. if $k_p = 3\pi/2$ in this example, then

$$\varphi_{PM}(t) = A\cos\left[\omega_c t + \frac{3\pi}{2} m(t)\right]$$
$$= \begin{cases} A\sin\omega_c t & \text{when } m(t)=1 \text{ or } -1/3 \\ -A\sin\omega_c t & \text{when } m(t)=-1 \text{ or } 1/3 \end{cases}$$ ← ambiguity

$\Rightarrow$ k_p should be small enough to restrict the phase change $k_p m(t)$ to the range $(-\pi, \pi)$.

* *

Q3

5.1-1 In this case $f_c = 10$ MHz, $m_p = 1$ and $m'_p = 8000$.

For FM :

$\Delta f = k_f m_p/2\pi = 2\pi \times 10^5/2\pi = 10^5$ Hz. Also $f_c = 10^7$. Hence, $(f_i)_{max} = 10^7 + 10^5 = 10.1$ MHz and $(f_i)_{min} = 10^7 - 10^5 = 9.9$ MHz. The carrier frequency increases linearly from 9.9 MHz to 10.1 MHz over a quarter (rising) cycle of duration a seconds. For the next a seconds, when $m(t) = 1$, the carrier frequency remains at 10.1 MHz. Over the next quarter (the falling) cycle of duration a, the carrier frequency decreases linearly from 10.1 MHz to 9.9 MHz., and over the last quarter cycle, when $m(t) = -1$, the carrier frequency remains at 9.9 MHz. This cycles repeats periodically with the period $4a$ seconds as shown in Fig. S5.1-1a.

For PM :

$\Delta f = k_p m'_p/2\pi = 50\pi \times 8000/2\pi = 2 \times 10^5$ Hz. Also $f_c = 10^7$. Hence, $(f_i)_{max} = 10^7 + 2 \times 10^5 = 10.2$ MHz and $(f_i)_{min} = 10^7 - 2 \times 10^5 = 9.8$ MHz. Figure S5.1-1b shows $\dot{m}(t)$. We conclude that the frequency remains at 10.2 MHz over the (rising) quarter cycle, where $\dot{m}(t) = 8000$. For the next a seconds, $\dot{m}(t) = 0$, and the carrier frequency remains at 10 MHz. Over the next a seconds, where $\dot{m}(t) = -8000$, the carrier frequency remains at 9.8 MHz. Over the last quarter cycle $\dot{m}(t) = 0$ again, and the carrier frequency remains at 10 MHz. This cycles repeats periodically with the period $4a$ seconds as shown in Fig. S5.1-1.

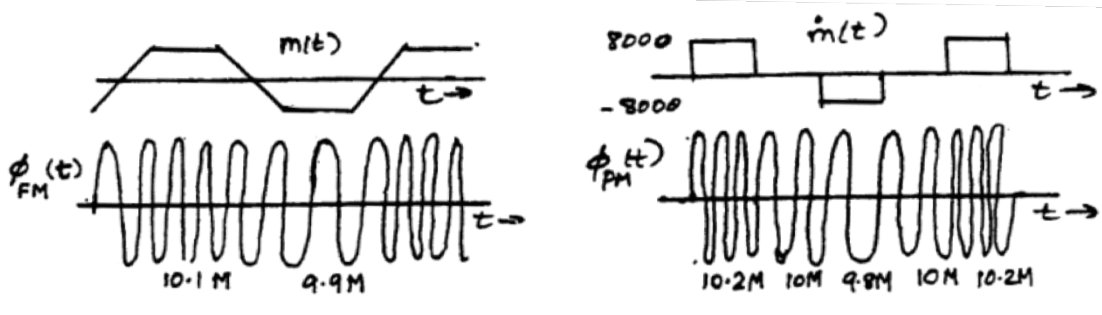

Fig. S5.1-1

* *

Q4

5.1-2 In this case $f_c = 1$ MHz, $m_p = 1$ and $m'_p = 2000$.

For FM :

$\Delta f = k_f m_p/2\pi = 20,000\pi/2\pi = 10^4$ Hz. Also $f_c = 1$ MHz. Hence, $(f_i)_{max} = 10^6 + 10^4 = 1.01$ MHz and $(f_i)_{min} = 10^6 - 10^4 = 0.99$ MHz. The carrier frequency rises linearly from 0.99 MHz to 1.01 MHz over the cycle (over the interval $-\frac{10^{-3}}{2} < t < \frac{10^{-3}}{2}$). Then instantaneously, the carrier frequency falls to 0.99 MHz and starts rising linearly to 10.01 MHz over the next cycle. The cycle repeats periodically with period 10^{-3} as shown in Fig. S5.1-2a.

For PM :

Here, because $m(t)$ has jump discontinuities, we shall use a direct approach. For convenience, we select the origin for $m(t)$ as shown in Fig. S5.1-2. Over the interval $\frac{10^{-3}}{2}$ to $\frac{10^{3}}{2}$, we can express the message signal as $m(t) = 2000t$. Hence,

$$
\begin{aligned}
\varphi_{PM}(t) &= \cos\left[2\pi(10)^6 t + \frac{\pi}{2} m(t)\right] \\
&= \cos\left[2\pi(10)^6 t + \frac{\pi}{2} 2000t\right] \\
&= \cos\left[2\pi(10)^6 t + 1000\pi t\right] = \cos\left[2\pi\left(10^6 + 500\right) t\right]
\end{aligned}
$$

At the discontinuity, the amount of jump is $m_d = 2$. Hence, the phase discontinuity is $k_p m_d = \pi$. Therefore, the carrier frequency is constant throughout at $10^6 + 500$ Hz. But at the points of discontinuities, there is a phase discontinuity of π radians as shown in Fig. S5.1-2b. In this case, we must maintain $k_p < \pi$ because there is a discontinuity of the amount 2. For $k_p > \pi$, the phase discontinuity will be higher than 2π giving rise to ambiguity in demodulation.

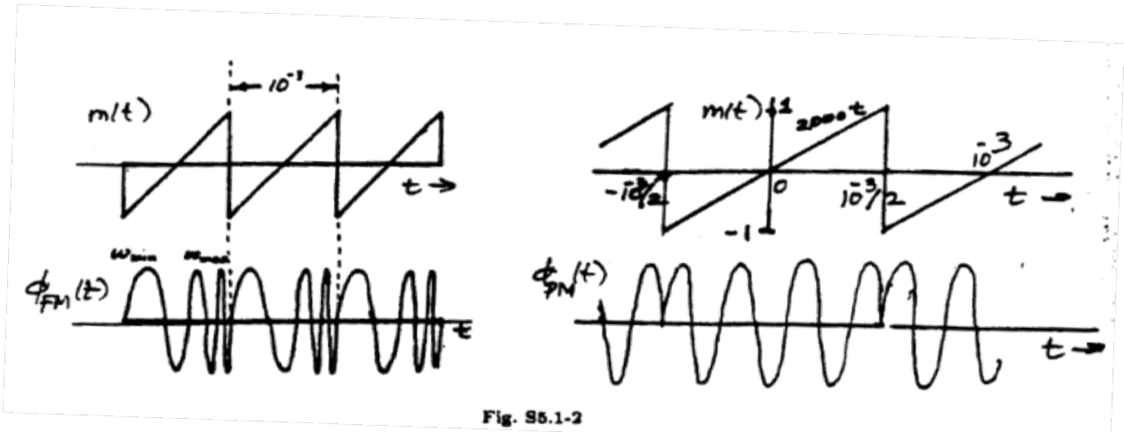

Fig. S5.1-2