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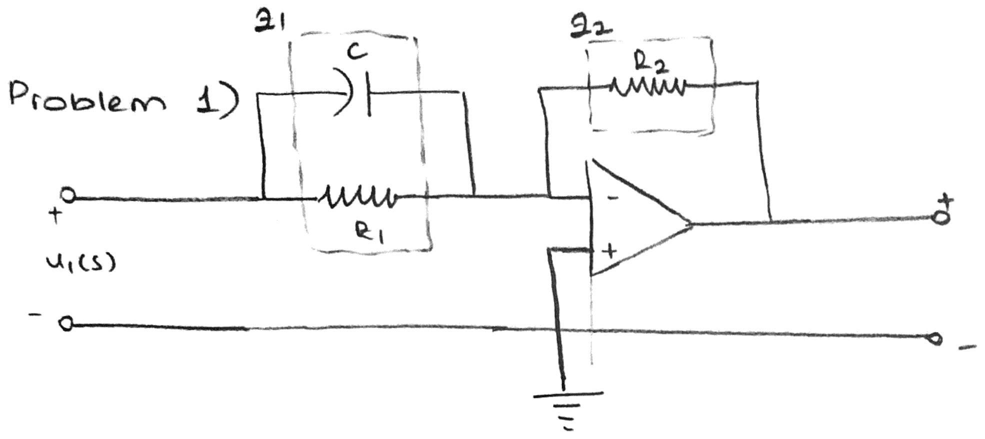
Problem 1)



a) KCL at inverting node;

$$\frac{C\,du_1}{dt} + \frac{u_1 - 0}{R_1} = \frac{u_2 - 0}{R_2}$$

$$\frac{C\,du_1}{dt} + \frac{u_1}{R_1} = \frac{u_2}{R_2}$$

$$\frac{C\,du_1}{dt} + \frac{u_1}{R_1} + \frac{u_2}{R_2} = 0$$

b) $Z_1 = R_1 \parallel 1/sC$

$$= \frac{R_1 \cdot 1/sC}{R_1 + 1/sC}$$

$$= \frac{R_1/sC}{\frac{sRC + 1}{sC}}$$

$$= \frac{R_1}{sR_1C + 1}$$

Apply KCL at inverting node;

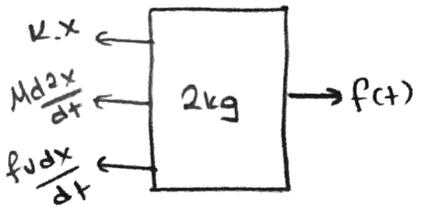
$$\frac{0 - u_1(s)}{Z_1} + \frac{0 - u_2(s)}{Z_2} = 0$$

$$\frac{-u_1(s)}{Z_1} = \frac{u_2(s)}{Z_2} \longrightarrow \boxed{\frac{u_2(s)}{u_1(s)} = -\frac{Z_2}{Z_1}}$$

$$\boxed{\frac{u_2(s)}{u_1(s)} = \frac{-(sR_1R_2C + R_2)}{R_1}}$$

Problem 2)

a)



$f_v = 4.56$ N-s/m

$K = 12.5$ N/m

$$\frac{Md^2x(t)}{dt^2} + \frac{f_v \cdot dx(t)}{dt} + K.x(t) = f(t)$$

$$\boxed{\frac{2d^2x(t)}{dt^2} + 4.56\frac{dx(t)}{dt} + 12.5.x(t) = f(t)}$$

$$(2s^2 + 4.56s + 12.5)\,X(S) = F(S)$$

b)

$$G(s) = \frac{X(S)}{F(S)} \rightarrow \boxed{\frac{X(S)}{F(S)} = \frac{1}{2s^2 + 4.56s + 12.5}}$$

c) Assume $f(t) = 12.5$ N $\xrightarrow{\mathcal{L}}$ $F(S) = \frac{12.5}{S}$

$$X(S) = \frac{F(S) \cdot \frac{1}{M}}{s^2 + \frac{f_v}{M}s + \frac{K}{M}}$$

Divide $M \rightarrow \frac{1/M}{s^2 + \frac{f_v}{M}s + \frac{K}{M}} = \frac{X(S)}{F(S)}$

$$= \frac{\frac{12.5}{S} \cdot \frac{1}{2}}{s^2 + \frac{4.56}{2}s + \frac{12.5}{2}}$$

$$\boxed{X(S) = \frac{\frac{6.25}{S}}{s^2 + 2.28s + 6.25}}$$

In order to find Steady state value $\lim_{S \to 0} s.X(S)$

$$\boxed{\lim_{S \to 0} \cancel{s} \cdot \frac{6.25}{\cancel{s}.(\underbrace{\cancel{s^2}}_{0} + \underbrace{2.28\cancel{s}}_{0} + 6.25)} = \frac{6.25}{6.25} = 1}$$

Steady state value of Displacement of $x(t)$.

d) $$\frac{X(s)}{F(s)} = \frac{1}{2s^2 + 4.56s + 12.5}$$

$$= \frac{1}{2\left[s^2 + \frac{4.56}{2}s + \frac{12.5}{2}\right]}$$

$$w_n^2 = \frac{12.5}{2} \rightarrow w_n = \sqrt{\frac{12.5}{2}} = 2.5$$

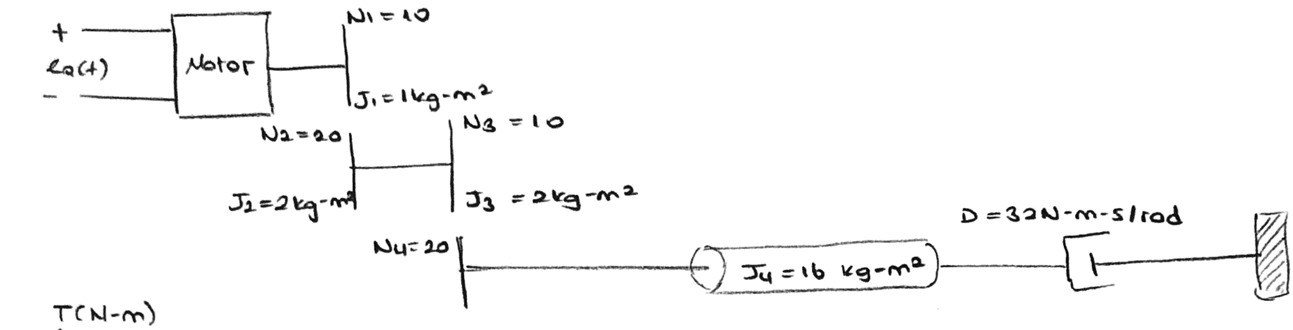
$$2\xi w_n = \frac{4.56}{2} \rightarrow 2\xi \times (2.5) = \frac{4.56}{2}$$

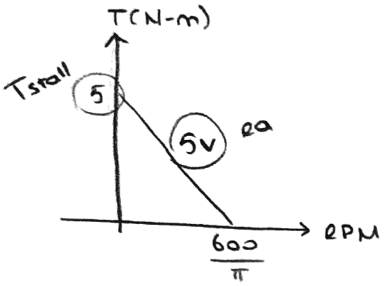
$$\xi = 0.456$$

$$e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} = e^{\frac{-0.456 \times 3.14}{\sqrt{1-(0.456)^2}}} = 0.2$$

$$\boxed{\begin{aligned} \text{Maximum value} &= (1 + 0.2) \\ &= 1.2 \end{aligned}}$$ (Steady-State value)

Problem 3)





we have three equations;

① $$\frac{\theta_M(s)}{E_a(s)} = \frac{K_t/(R_a \cdot J_M)}{s\cdot\left[s+\frac{1}{J_M}\cdot\left(D_M+\frac{K_t\cdot K_b}{R_a}\right)\right]}$$

② $$\frac{K_t}{R_a} = \frac{T_{stall}}{E_a}$$

③ $$K_b = \frac{e_a}{\omega_{no\text{-}load}}$$

a)

Firstly, we find K_t/R_a :

* $T_{stall} = 5$, $e_a = 5$

$$\boxed{\frac{K_t}{R_a} = \frac{5}{5} = 1}$$

Secondly, we find K_b ;

* $$\overbrace{\underbrace{\frac{600}{\pi}\cdot\frac{1}{60}}_{f\,(Hz)}\cdot 2\pi}^{\omega_{no\text{-}load}} \rightarrow \omega_{no\text{-}load} = 20$$

$$\boxed{K_b = \frac{e_a}{\omega_{no\text{-}load}} = \frac{5}{20} = \frac{1}{4}}$$

Now, we find J_M and D_M,

$$J_M = J_1 + J_2\left(N_1/N_2\right)^2 + J_3\cdot\left(N_1/N_2\right)^2 + J_4\cdot\left(N_3/N_4\right)^2\cdot\left(N_1/N_2\right)^2$$

$$= 1 + 2\cdot(1/2)^2 + 2\cdot(1/2)^2 + 16\cdot(1/2)^2\cdot(1/2)^2$$

$$= 3$$

$$D_M = D\cdot\left(N_3/N_4\right)^2\cdot\left(N_1/N_2\right)^2$$

$$= 32\cdot(1/4)\cdot(1/4)$$

$$= 2$$

We put K_t/R_a, K_b, J_M, D_M to the ① equation;

$$\frac{\theta_M(s)}{E_a(s)} = \frac{1/3}{s.\left[s + 1/3 \cdot (2 + 1 \cdot 1/4)\right]}$$

ⓐ
$$\boxed{\frac{\theta_M(s)}{E_a(s)} = \frac{1/3}{s.(s+0.75)}}$$

$$\theta_2(s).\ (N_3/N_4) \cdot (N_1/N_2) = \theta_M(s)$$

$$\theta_2(s) \cdot (10/20) \cdot (10/20) = \theta_M(s)$$

ⓑ
$$\boxed{\theta_M(s) = 1/4\ \theta_2(s)}$$

put ⓑ to ⓐ :

$$G_1(s) = \boxed{\frac{\theta_2(s)}{E_a(s)} = \frac{1/12}{s.(s+0.75)}}$$

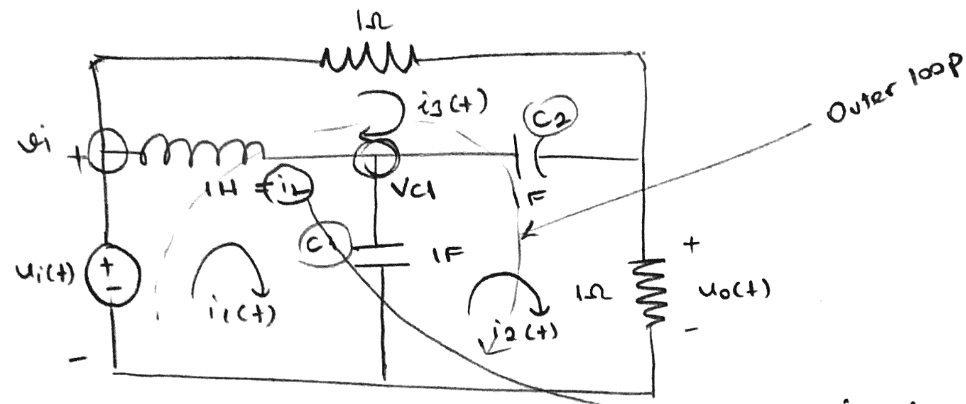
b)

$$G_1(s) = \frac{\omega_2(s)}{E_a(s)}$$

$$\frac{s.\theta_1(s)}{E_a(s)} = \frac{\omega_1(s)}{E_a(s)} = \frac{\omega_2(s)}{E_a(s)} = \frac{\cancel{s}.\ 1/12}{\cancel{s}.(s+0.75)}$$

$$\boxed{G_1(s) = \frac{\omega_2(s)}{E_a(s)} = \frac{1/12}{s+0.75}}$$

Problem 4)



$\rightarrow i_1 - i_3 = i_L$ inductor current

a)

$$\left.\begin{aligned} \frac{di_L}{dt} &= u_i - u_{c1} \\ \frac{du_{c1}}{dt} &= i_1 - i_2 \\ \frac{dU_{c2}}{dt} &= i_2 - i_3 \end{aligned}\right\}$$

Thus, our state vector is;

$$x = \begin{bmatrix} i_L \\ v_{c1} \\ u_{c2} \end{bmatrix}$$

KVL for i_2's loop: $u_{c1} - u_{c2} - i_2 = 0$

$$\boxed{i_2 = u_{c1} - u_{c2}} \quad (1)$$

KVL for outer loop: $i_3 + i_2 - u_i = 0$

$$\boxed{i_3 = u_i - i_2 = u_i - u_{c1} + u_{c2}} \quad (2)$$

If $i_1 - i_3 = i_L$, then;

$$\boxed{i_1 = i_L + i_3 = i_L + u_i - v_{c1} + u_{c2}} \quad (3)$$

Now, we need to have equations respect to $i_1 - i_2$ and $i_2 - i_3$.

FIRST: (3) - (1) $\rightarrow$ $i_1 - i_2 = i_L - 2u_{c1} + 2U_{c2} + u_i$

SECOND: (1) - (2) $\rightarrow$ $i_2 - i_3 = 2u_{c1} - 2u_{c2} - u_i$

AND, we have $\frac{di_L}{dt} = u_i - u_{c1}$

Use the third one

$$\dot{x} = \begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_{c1}}{dt} \\ \frac{dv_{c2}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & -2 & 2 \\ 0 & 2 & -2 \end{bmatrix} \underbrace{\begin{bmatrix} i_L \\ v_{c1} \\ v_{c2} \end{bmatrix}}_{x} + \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} v_i$$

First one

Second one

Also, $v_o = i_2 \cdot \underbrace{1}_{R}$

$$v_o = v_{c1} - v_{c2} = i_2$$

$$y = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_L \\ v_{c1} \\ v_{c2} \end{bmatrix}$$