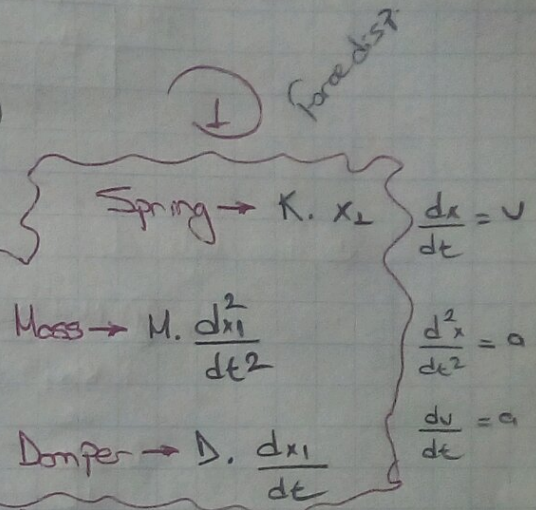
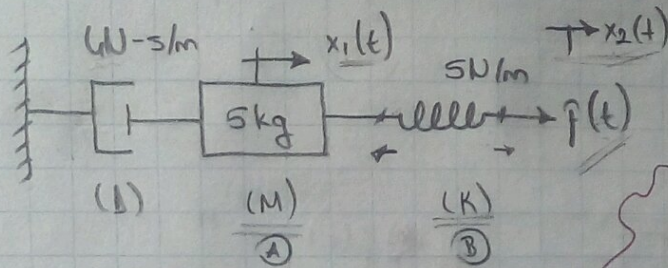


2-23) Find the transfer function, $G(s) = X_1(s)/F(s)$, for the translational mechanical system.



① Equations of motion.

$$\textcircled{A} \quad M \cdot \frac{d^2 x_1}{dt^2} + D \cdot \frac{dx_1}{dt} + K \cdot x_1 - K \cdot x_2(t) = 0$$

$$\textcircled{B} \quad -K \cdot x_1(t) + K \cdot x_2(t) = F(t)$$

$$\rightarrow 5s^2 X_1(s) + 4s X_1(s) + 5 X_1(s) - 5 X_2(s) = 0$$

$$\textcircled{A} (5s^2 + 4s + 5) X_1(s) - 5 X_2(s) = 0$$

$$\rightarrow \textcircled{B} -5 X_1(s) + 5 X_2(s) = F(s)$$

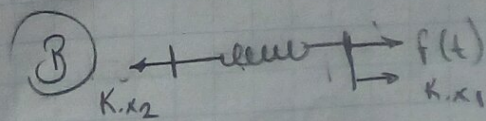
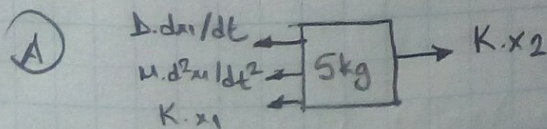
+

$$(5s^2 + 4s) X_1(s) = F(s)$$

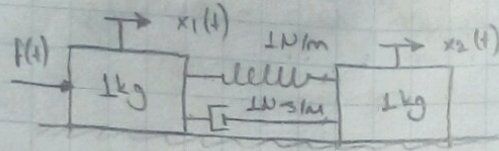
$$G(s) = X_1(s)/F(s) = \frac{1}{5s^2 + 4s}$$

②

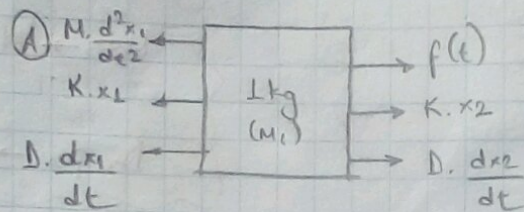
Free-body diagram



2-24) find the transfer f., $G(s) = x_2(s)/f(s)$, for the translational mechanical network.



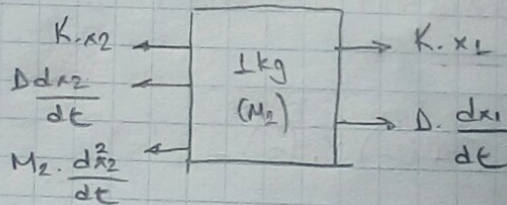
① free-body d.



$$\textcircled{A} \quad (1) \cdot \frac{d^2 x_1}{dt^2} + (1) \cdot \frac{dx_1}{dt} + (1) \cdot x_1 - (1) \cdot x_2 - (1) \cdot \frac{dx_2}{dt} = f(s)$$

$$\rightarrow (s^2 + s + 1) X_1(s) - (s + 1) X_2(s) = f(s)$$

② free-body d.



$$\textcircled{B} \quad (1) \cdot \frac{d^2 x_2}{dt^2} + (1) \cdot \frac{dx_2}{dt} + (1) \cdot x_2 - (1) \cdot x_1 - (1) \cdot \frac{dx_1}{dt} = 0$$

$$\rightarrow (s^2 + s + 1) X_2(s) - (s + 1) X_1(s) = 0$$

$$\textcircled{A} \quad (s^2 + s + 1) X_1(s) - (s + 1) X_2(s) = f(s)$$

$$\textcircled{B} \quad (s^2 + s + 1) X_2(s) - (s + 1) X_1(s) = 0$$

\rightarrow

$$X_1(s) = X_2(s) \frac{(s^2 + s + 1)}{(s + 1)} \rightarrow \underline{\underline{A}}$$

$$(s^2 + s + 1) \cdot \frac{(s^2 + s + 1)}{(s+1)} \cdot X_2(s) - (s+1) \cdot X_2(s) = F(s)$$

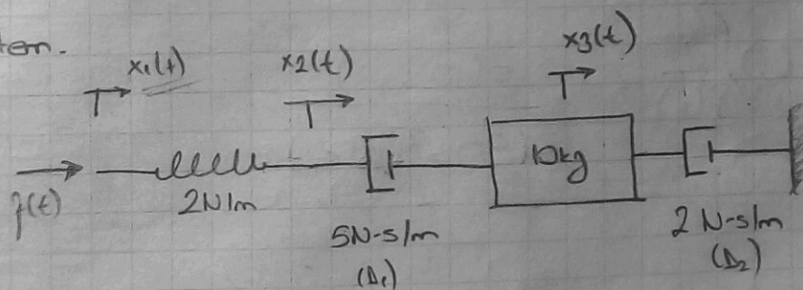
$$\rightarrow \left[\frac{s^4 + s^3 + s^2 + s^3 + s^2 + s + s^2 + s + 1}{s+1} - (s+1) \cdot (s+1) \right] X_2(s) = F(s)$$

$$\rightarrow \left(\frac{s^4 + 2s^3 + 3s^2 + 2s + 1}{s+1} - s^2 - 2s - 1 \right) X_2(s) = F(s)$$

$$\frac{(s^4 + 2s^3 + 2s^2)}{(s+1)} X_2(s) = F(s)$$

$$G(s) = X_2(s)/F(s) = \frac{s+1}{s^4 + 2s^3 + 2s^2}$$

2-25) Find the tra. f., $G(s) = X_2(s)/F(s)$, for the translational system.



① $Kx_1 \leftarrow \text{spring} \rightarrow f(t) \rightarrow Kx_2$ $\rightarrow Kx_1 - Kx_2 = f(t)$
 $2x_1(s) - 2x_2(s) = F(s)$

② $D_1 \frac{dx_2}{dt} \leftarrow \text{damper} \rightarrow D_2 \frac{dx_3}{dt}$ $\rightarrow (5) \frac{dx_2}{dt} + 2 \cdot x_2 - 2 \cdot x_1 - 5 \frac{dx_3}{dt} = 0$
 $\rightarrow (5s+2)x_2(s) - 2x_1(s) - 5sx_3(s) = 0$

③ $M \frac{d^2x_3}{dt^2} \leftarrow \text{spring} \rightarrow D_1 \frac{dx_2}{dt}$ $\rightarrow (10) \frac{d^2x_3}{dt^2} + (5) \frac{dx_3}{dt} + (2) \frac{dx_3}{dt}$
 $- (2) \frac{dx_2}{dt} = 0$

$$\rightarrow (10s^2 + 7s) \cdot X_3(s) - (2s)x_2(s) = 0$$

$$(1) \quad x_1(s) = \frac{f(s) + 2x_2(s)}{2}$$

$$(3) \quad x_3(s) = \frac{5s x_2(s)}{10s^2 + 7s}$$

$$(2) \quad -2 \left[\frac{f(s) + 2x_2(s)}{2} \right] + (5s+2) \cdot x_2(s) - (5s) \cdot \left(\frac{5s \cdot x_2(s)}{10s^2 + 7s} \right) = 0$$

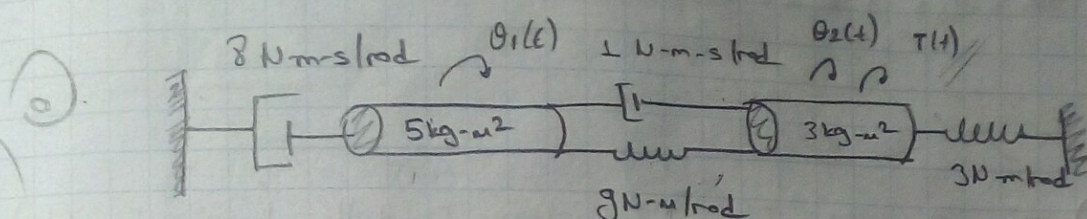
$$f(s) = -2x_2(s) + (5s+2) \cdot x_2(s) - \frac{(25s^2) \cdot x_2(s)}{10s^2 + 7s}$$

$$f(s) = x_2(s) \left[(5s+2) - 2 \right] - \frac{(25s^2) \cdot x_2(s)}{10s^2 + 7s}$$

$$f(s) = x_2(s) \left[\frac{50s^3 + 35s^2 - 25s^2}{10s^2 + 7s} \right]$$

$$G(s) = \frac{x_2(s)}{f(s)} = \frac{\cancel{s}(10s+7)}{\cancel{s}(50s^2+10s)} = \frac{10s+7}{50s^2+10s}$$

2-30) For each of the rotational mechanical sys., write, but do not solve the equations of motion.

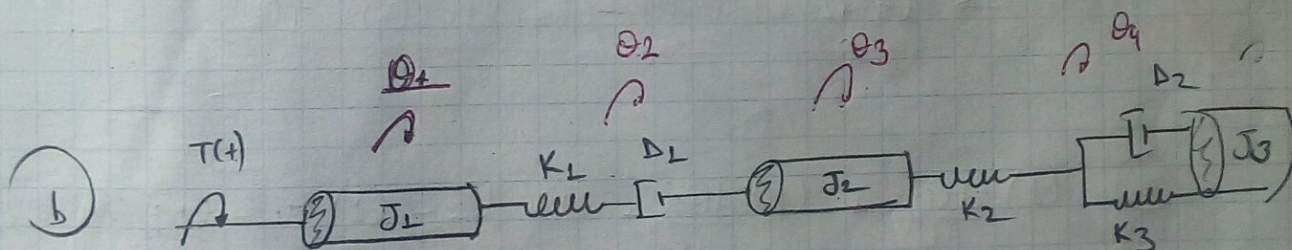


① $5 \frac{d^2 \theta_1}{dt^2} + 8 \frac{d\theta_1}{dt} + 1 \cdot \frac{d\theta_1}{dt} + 9 \cdot \theta_1 - 1 \cdot \frac{d\theta_2}{dt} - 9 \cdot \theta_2 = 0$

$(5s^2 + 8s + 1) \theta_1(s) - (s + 9) \theta_2(s) = 0$

② $3 \frac{d^2 \theta_2}{dt^2} + 3 \cdot \theta_2 + 9 \cdot \theta_2 + 1 \cdot \frac{d\theta_2}{dt} - 1 \cdot \frac{d\theta_1}{dt} - 9 \cdot \theta_1 = T(t)$

$(3s^2 + 12 + s) \theta_2(s) - (s + 9) \theta_1(s) = T(s)$



① $J_1 \frac{d^2 \theta_1}{dt^2} + K_L \theta_1 - K_L \theta_2 = T(t)$

$\rightarrow (J_1 s^2 + K_L) \theta_1(s) - K_L \theta_2(s) = T(s)$

② $K_L \theta_2 + D_L \frac{d\theta_2}{dt} - K_L \theta_1 - D_L \frac{d\theta_3}{dt} = 0$

$\rightarrow (D_L s + K_L) \theta_2(s) - K_L \theta_1(s) - D_L s \theta_3(s) = 0$

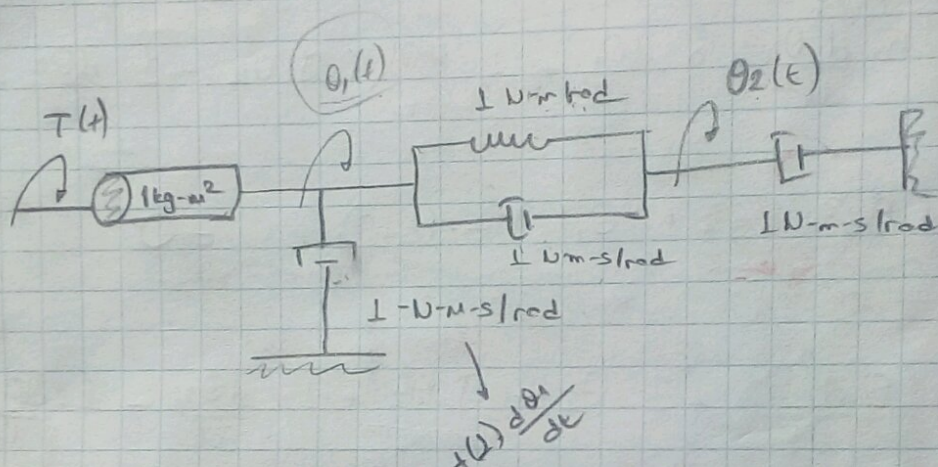
③ $J_2 \frac{d^2 \theta_3}{dt^2} + D_2 \frac{d\theta_3}{dt} + K_2 \theta_3 - D_2 \frac{d\theta_2}{dt} - K_2 \theta_2 = 0$

$\rightarrow (J_2 s^2 + D_2 s + K_2) \theta_3(s) - D_2 s \theta_2(s) - K_2 \theta_2(s) = 0$

$$(1) \quad J_3 \frac{d^2 \theta_1}{dt^2} + D_2 \frac{d\theta_1}{dt} + K_3 \theta_1 + K_2 \theta_4 - K_2 \theta_3 = 0$$

$$\rightarrow (J_3 s^2 + D_2 s + (K_3 + K_4)) \theta_1(s) - K_2 \theta_3(s) = 0$$

2-31) For the rotational mech. system, find the transfer \hat{f} , $G(s) = \theta_2(s)/T(s)$



$$(1) \quad (1) \cdot \frac{d^2 \theta_1}{dt^2} + (1) \cdot \theta_1 + (1) \cdot \frac{d\theta_1}{dt} - (1) \cdot \theta_2 + (1) \cdot \frac{d\theta_2}{dt} = T(s)$$

$$\rightarrow (s^2 + 2s + 1) \theta_1(s) - (s + 1) \theta_2(s) = T(s)$$

$$(2) \quad (1) \cdot \frac{d\theta_2}{dt} + (1) \cdot \frac{d\theta_2}{dt} + (1) \cdot \theta_2 - (1) \cdot \theta_1 - (1) \cdot \frac{d\theta_1}{dt} = 0$$

$$\rightarrow (s + s + 1) \theta_2(s) - (s + 1) \theta_1(s) = 0$$

↓

$$\theta_1(s) = \frac{(2s + 1) \theta_2(s)}{(s + 1)}$$

→ (1)

$$(s^2 + 2s + 1) \cdot \frac{(2s + 1) \theta_2(s)}{(s + 1)} - (s + 1) \cdot \theta_2(s) = T(s)$$

$$\frac{[(2s^3 + s^2 + 4s^2 + 2s + 2s + 1) - (s^2 + 2s + 1)] \theta_2(s)}{s+1} = T(s)$$

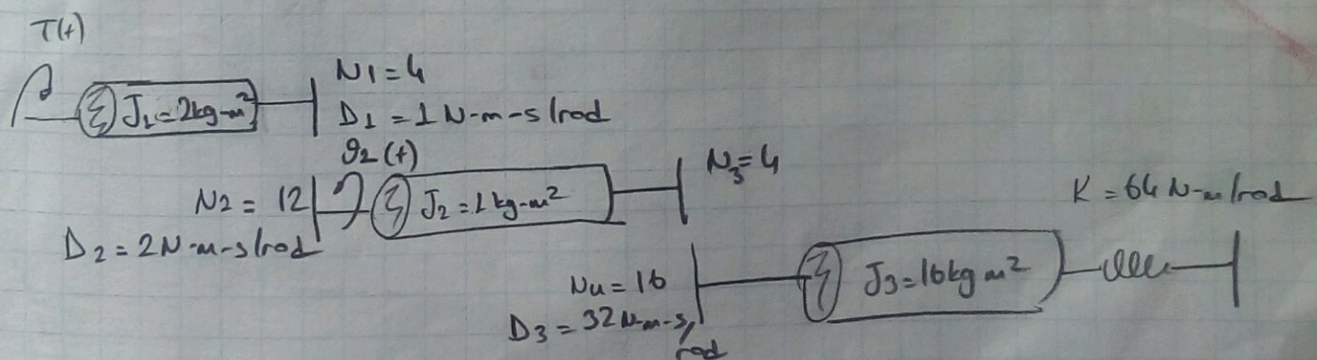
$$\frac{(2s^3 + 4s^2 + 2s)}{s+1} \theta_2(s) = T(s)$$

$$\frac{2s(s^2 + 2s + 1)}{s+1} \theta_2(s) = T(s)$$

$$= \frac{2s(s+1)^2}{s+1} \theta_2(s) = T(s)$$

$$G(s) = \theta_2(s)/T(s) = \frac{1}{2s(s+1)}$$

2-33) for the rotational system, find the transfer f., $G(s) = \theta_2(s)/T(s)$



$$\left\{ s^2 \left[\left(\frac{N_2}{N_1} \right)^2 J_1 + J_2 + \left(\frac{N_3}{N_u} \right)^2 J_3 \right] + s \left[D_2 + D_1 \left(\frac{N_2}{N_1} \right)^2 + D_3 \left(\frac{N_3}{N_u} \right)^2 \right] + \left[K \cdot \left(\frac{N_3}{N_u} \right)^2 \right] \right\} \theta_2(s) = T(s) \cdot \left(\frac{N_2}{N_1} \right)$$

$$\frac{\theta_2(s)}{T(s)} = \frac{3}{20s^2 + 13s + 4}$$