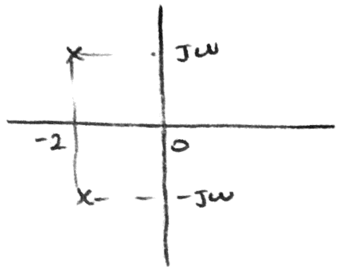
Tugberk Göçü

115200084

Problem 1)

a) It is a second order system.

$$\text{General Form} = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$$



$$P_1 = -2 + Jw$$

$$P_2 = -2 - Jw$$

$$G(s) = \frac{K}{(s-P_1).(s-P_2)} = \frac{K}{(s+2+Jw).(s+2-Jw)}$$

$$= \frac{K}{s^2+4s+4+\pi^2}$$

$$\frac{K}{s^2+4s+4+\pi^2} \longleftrightarrow \frac{K}{s^2+2\xi w_n s + w_n^2}$$

$$w_n = \sqrt{4+\pi^2}$$

$$\boxed{w_n = 3.72 \text{ rad/s}}$$

$$2\xi w_n = 4$$

$$\boxed{\xi = \frac{2}{w_n} = \frac{2}{3.72} = 0.537}$$

$0 < \xi < 1 \rightarrow$ It is an underdamped system.

b) $w_n = 3.72$ rad/s

$\xi = 0.537$

$$w_d = w_n\sqrt{1-\xi^2} = 3.72\sqrt{1-0.537}$$

$$= (3.72).(0.68)$$

$$\boxed{w_d = 2.53 \text{ rad/s}}$$

$$\boxed{T_p = \frac{\pi}{w_d} = \frac{\pi}{2.53} = 1.24\,sn}$$

$$\boxed{T_s = \frac{4}{\xi w_n} = \frac{4}{2} = 2s}$$

c) $$C_{ss} = \lim_{s \to 0} s \cdot \frac{K}{s \cdot (s^2 + 2\xi w_n + w_n^2)}$$

$$C_{ss} = 10 = \frac{K}{w_n^2}$$

$$K = 10 w_n^2$$

$$K = 10 \cdot K \cdot (3.72)^2$$

$$\boxed{K = 138.384}$$

System T.F. $$T(s) = \frac{C(s)}{R(s)} = \frac{138.384}{s^2 + 4s + 138.384}$$

d) peak values (cmax)

$$1 + m_p \longrightarrow m_p = e^{-\xi \pi / \sqrt{1-\xi^2}}$$

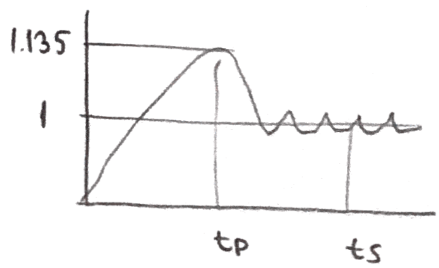
$$m_p = e^{-0.537\pi / \sqrt{1-0.537^2}}$$

$$m_p = 0.135$$

% overshoot = 13.5%

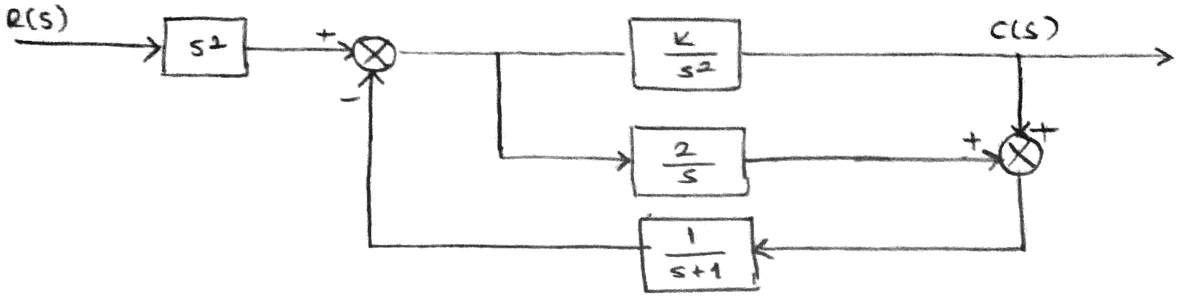
peak value (cmax) $\rightarrow$ $1 + 0.135 = 1.135$

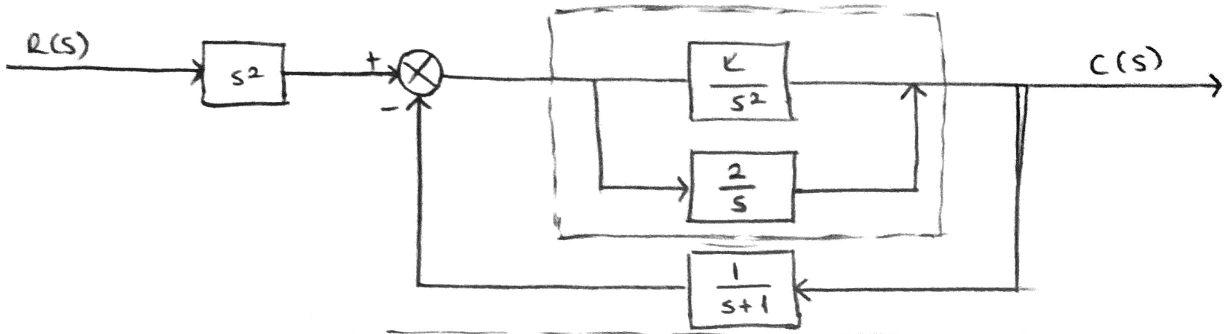
$c_{max} = 1.135$

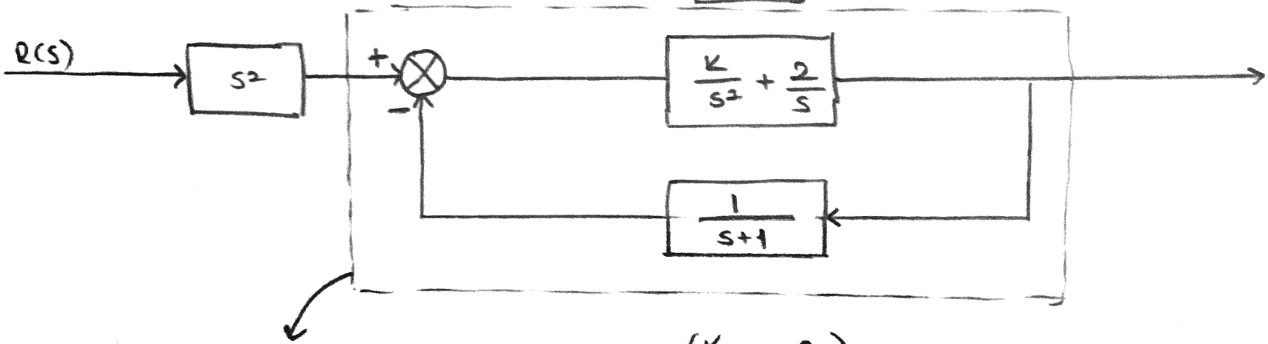


Problem 2)

a)







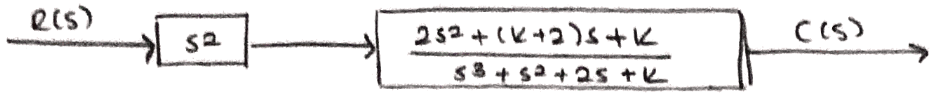
$$T(s) = \frac{G(s)}{1+G(s).H(s)} = \frac{\left(\frac{K}{s^2}+\frac{2}{s}\right)}{1+\left(\frac{K}{s^2}+\frac{2}{s}\right)\cdot\frac{1}{s+1}}$$

$$= \frac{\frac{K+2s}{s^2}}{1+\frac{K}{s^2(s+1)}+\frac{2}{s(s+1)}}$$

$$= \frac{\frac{K+2s}{s^2}}{\frac{s^2(s+1)+K+2s}{s^2(s+1)}}$$

$$= \frac{\frac{K+2s}{s^2}}{\frac{s^3+s^2+K+2s}{s^2(s+1)}}$$

$$T(s) = \frac{(K+2s).(s+1)}{s^3+s^2+K+2s} = \frac{2s^2+(K+2)s+K}{s^3+s^2+2s+K}$$



We are going to multiply these two;

$$\boxed{\frac{C(s)}{R(s)} = T(s) = \frac{2s^4+(K+2)s^3+Ks^2}{s^3+s^2+2s+K} = \frac{(K+2s)(s+1)(s^2)}{s^3+s^2+2s+K}}$$

b) Applying Routh-Hurwitz criteria.

s^3	1	2
s^2	1	K
s^1	$\frac{2-K}{1}$	0
s^0	K	0

For stability; $2-K>0$

$K<2$ and $K>0$

Thus, the range of K will be $\boxed{0<K<2}$

c) From above, there are two poles in $j\omega$ axis. Those two poles obtained by s^2, if s^1 is row of zeros.

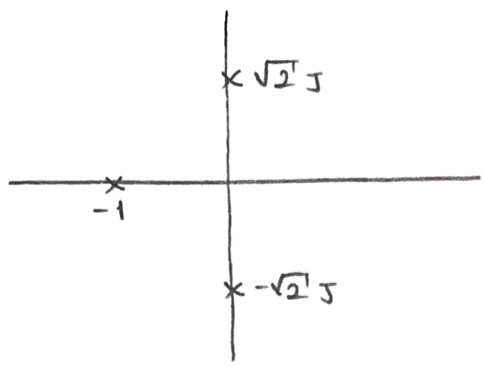
If $K=2$, then s^1 row gets zero.

$s^2+K=0$ (Substitute 2 for K.)

$s^2+2=0$

$s^2=-2$

$\boxed{s=\pm j\sqrt{2}}$



d) For oscillating system, the system should be marginally stable and for that from part (b) using Routh-Hurwitz table;

$2-K=0$

$K_{critical}=2$

Now, from even equation (from the table)

$s^2+K=0$ (putting $s=j\omega$)

$(j\omega)^2+K=0$ (putting $K=K_{critical}=2$)

$-\omega^2+K=0$

$\boxed{\omega=\sqrt{2}\ \text{rad/sec.}}$ This is the frequency value that system oscillates.

Problem 3)

a) $G(s) = \dfrac{K.(s^2-2s+2)}{s(s+1)(s+2)}$

i) Poles are ; $0, -1, -2$

Zeros are ; $s^2-2s+2=0$

$(s-1)^2+1=0 \longrightarrow s = 1 \pm j$

ii) Number of root locus Branch max (Pole, zero)

$= \max(3,2)$

$= 3$

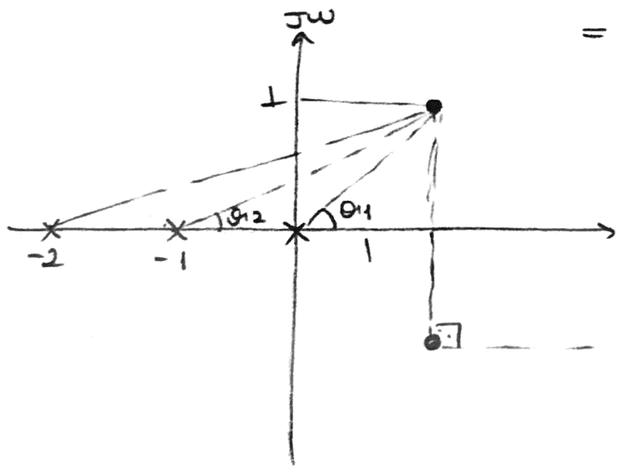
iii) Number of asymptotes $= P - Z = 3-2 = 1$

iv.) Centroid $= \dfrac{\Sigma P - \Sigma Z}{P-Z} = \dfrac{-3-2}{1} = (-5, 0)$

v.) Angle of asymptotes $= 180°$

vi.) Angle of arrival of the root locus at zero

$= \pm[180 - \varphi_P]$



s plane

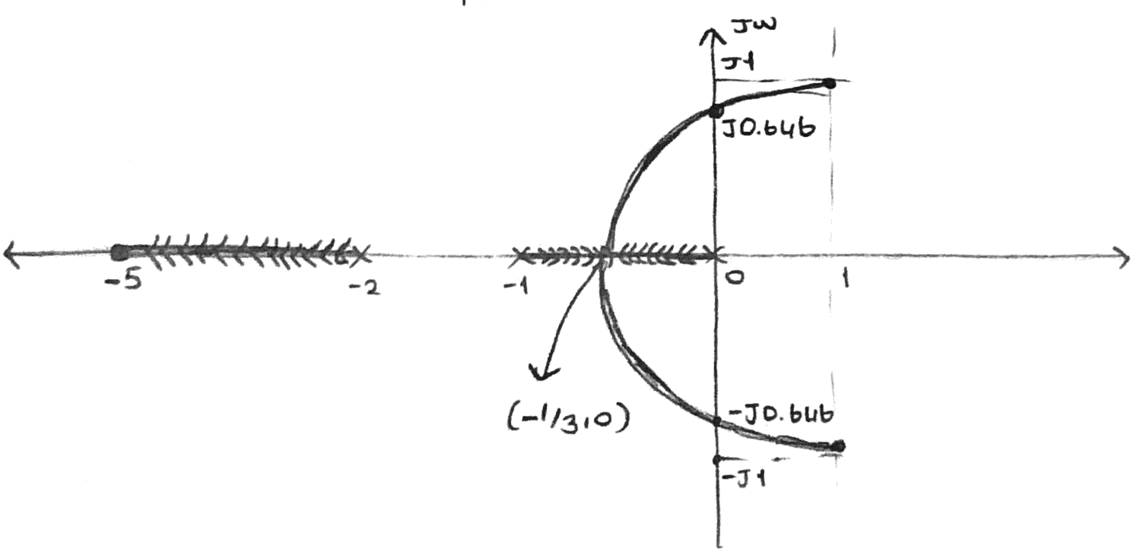
$\theta_1 = \tan^{-1}(1/1) = 45°$

$\theta_2 = \tan^{-1}(1/2) = 26.56°$

$\theta_3 = \tan^{-1}(1/3) = 18.43$

Arrival Angle $= \pm[180 - (90 - 45 - 26.56 - 18.43)]$

$\varphi_A = \pm 90°$



b) Characteristic Equation

$$1 + G(s) = 0$$

$$s.(s^2+3s+2) + Ks^2 - 2sK + 2k = 0$$

$$s^3 + (K+3)s^2 + (2-2K)s + 2k = 0$$

Routh–Hurwitz Table

s^3	1	$2-2k$
s^2	$k+3$	$2k$
s^1	$\frac{(k+3).(2-2k)-2k}{k+3}$	0
s^0	$2k$	

For jw intersection $s^1 = 0$.

$$(k+3)(2-2k) = 2k$$

$$(k+3).(1-k) = k$$

$$k - k^2 + 3 - 3k = k$$

$$k^2 + 3k - 3 = 0$$

$$k = 0.791, -3.792$$

c)

k can not be negative, so $k = 0.791$

Thus, system is stable for $\boxed{0 < k < 0.791}$ Answer for c

Jw points : $-(0.791+3)w^2 + 2.(0.791) = 0$

$$\boxed{w = 0.64599 \text{ rad/sec}}$$

Answer for b