MATH 233
Fall 2018
Quiz #2 A Solutions

***Duration:* 50 minutes.**
***Remark:* Show your thinking/work. Do not just write a number as a result.**

1. A person can take one stair, two stairs or three stairs at a time when climbing a stairway.

- Find a **recurrence relation** for the number of ways to climb n stairs.

# of stairs	climbing ways	# of climbs
1	1	1
2	1-1, 2	2
3	1-1-1, 2-1, 1-2, 3	4
4	1-1-1-1, 2-1-1, 1-2-1, 3-1, 1-1-2, 2-2, 1-3	7

As can be seen from column 2, the different climbing ways for n stairs is the sum of:

1) climbing ways for n-1 stairs and a final one step
2) climbing ways for n-2 stairs and a final 2 stair-step
3) climbing ways for n-3 stairs and a final 3-stair step

If W_n is the number of ways to slimb n stairs, then the recurrence relation is:
$W_n = W_{n-1} + W_{n-2} + W_{n-3}$

- What are the **initial conditions**?

$W_1 = 1$ $W_2 = 2$ and $W_3 = 4$

- In how many ways can the person climb a 10-stair stairway?

$W_{10} = W_9 + W_8 + W_7 = (W_8 + W_7 + W_6) + W_8 + W_7 = 2W_8 + 2W_7 + W_6$
$= 4W_7 + 3W_6 + 2W_5 = 7W_6 + 6W_5 + 4W_4 = 13W_5 + 11W_4 + 7W_3 =$
$24W_4 + 20W_3 + 13W_2 = 24.7 + 20.4 + 13.2 = 274$

2. A fair dice and two fair coins are tossed.

a) What is the **experiment**?

A fair dice and two fair coins are tossed.

b) What is the **sample space**?

Sample Space = { {1,H,H}, {1,H,T}, {1,T,H}, {1,T,T},
{2,H,H}, {2,H,T}, {2,T,H}, {2,T,T},
….
{6,H,H}, {6,H,T}, {6,T,H}, {6,T,T}}.

c) What is the **size** of the sample space?

| Sample Space | = 6.2.2 = 24

d) What is the probability that a head occurs? (Describe the event E_H)

E_H = The event that a head occurs in the outcome.

It is easier to think about the complement event, the event that a head does not occur in the outcome (i.e. both coints show tails).

$\overline{E_H}$ = The event that a head does **not** occur in the outcome

$\overline{E_H}$ = {{1,T,T},{2,T,T},{3,T,T},{4,T,T},{5,T,T},{6,T,T}}

$|\overline{E_H}|$ = 6 and therefore $|E_H|$ = 24 - 6 = 18

$P(E_H) = |E_H| / |\text{Sample Space}| = 18 / 24 = 0.75$

e) What is the probability that a 6 occurs? (Describe the event E_6)

E_6 = {{6,H,H}, {6,H,T}, {6,T,H}, {6,T,T}}.

$|E_6| = 4$

$P(E_6) = |E_6| / |\text{Sample Space}| = 4 / 24 = 1/6 = 0.167$

f) What is the probability that the number on the dice is equal to the number of heads or tails? (Describe the event E_{same})

What outcomes are in E_{same}?

E_{same}= {{1,H,T}, {1,T,H}, {2,H,H},{2,T,T}}

$P(E_{same}) = |E_{same}| / |\text{Sample Space}| = 4/24 = 0.167$