EEEN 474
Wireless Communication

Spring 2020

Mobile Radio Propagation:

Small-Scale Fading and Multipath

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading

1. BW of signal $<$ BW of channel
2. Delay spread $<$ Symbol period

Frequency Selective Fading

1. BW of signal $>$ BW of channel
2. Delay spread $>$ Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading

1. High Doppler spread
2. Coherence time $<$ Symbol period
3. Channel variations faster than baseband signal variations

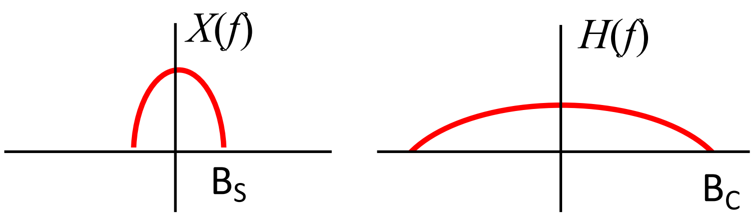
Slow Fading

1. Low Doppler spread
2. Coherence time $>$ Symbol period
3. Channel variations slower than baseband signal variations

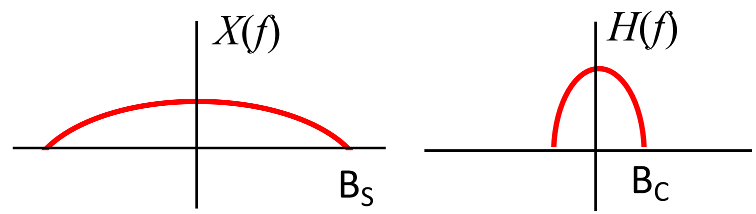
Figure 4.11
Types of small-scale fading.

(1) Fading Effects due to Multipath Time Delay Spread

Flat Fading



Frequency Selective Fading



Flat Fading:

$\boxed{B_S \ll B_C}$

$\boxed{T_S \gg \sigma_\tau}$

$\boxed{T_S \geq 10\sigma_\tau}$ rule of thumb

Frequency Selective Fading:

$\boxed{B_S > B_C}$

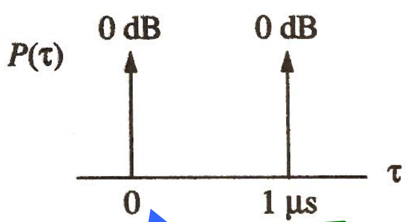
$\boxed{T_S < \sigma_\tau}$

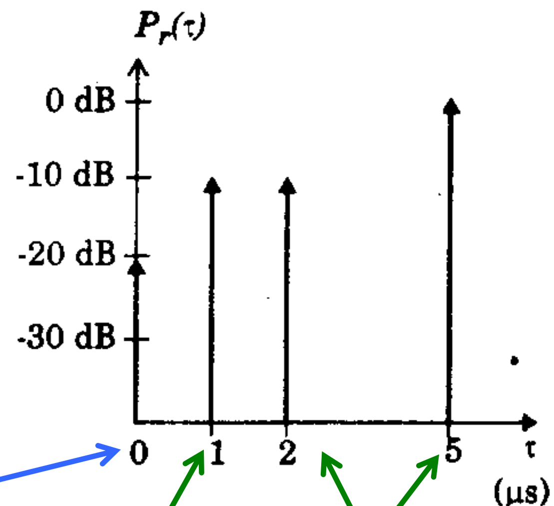
T_S: symbol period
B_S: bandwidth of the transmitted modulation ($B_S = 1/T_S$)
σ_τ: rms delay spread
B_C: coherence bandwidth ($B_C \propto 1/\sigma_\tau$)

time dispersion parameters

RMS Delay Spread (σ_τ)

- A **power delay profile** shows relative received power as a function of excess delay with respect to a fixed time delay reference
- Examples:





τ_0=0, excess time delay of the first arriving signal

τ_k: excess delay, relative delay of the k'th multipath component as compared to the first arriving signal

- Then: $\boxed{\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}}$ where $\boxed{\overline{\tau^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}}$ and $\boxed{\bar{\tau} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}}$

rms delay spread

Microseconds for outdoor radio channels

Nanoseconds for indoor radio channels

mean excess delay

Table 4.1 Typical Measured Values of RMS Delay Spread

Environment	Frequency (MHz)	RMS Delay Spread (σ_τ)	Notes	Reference
Urban	910	1300 ns avg. 600 ns st. dev. 3500 ns max.	New York City	[Cox75]
Urban	892	10-25 μs	Worst case San Francisco	[Rap90]
Suburban	910	200-310 ns	Averaged typical case	[Cox72]
Suburban	910	1960-2110 ns	Averaged extreme case	[Cox72]
Indoor	1500	10-50 ns 25 ns median	Office building	[Sal87]
Indoor	850	270 ns max.	Office building	[Dev90a]
Indoor	1900	70-94 ns avg. 1470 ns max.	Three San Francisco buildings	[Sei92a]

Coherence Bandwidth (B_C)

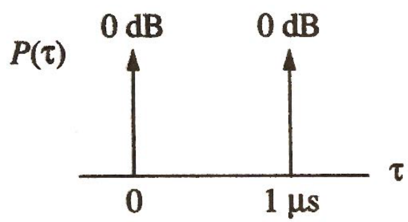
Frequency bandwidth over which the correlation function of two samples of channel response taken at the **same time** but at **different frequencies** falls below a certain threshold

$$\boxed{B_C = \frac{1}{50\sigma_\tau}} \qquad \text{(Threshold is 0.9 correlation)}$$

$$\boxed{B_C = \frac{1}{5\sigma_\tau}} \qquad \text{(Threshold is 0.5 correlation)}$$

Example 5.4
Compute the RMS delay spread for the following power delay profile:

(a)



(b) If BPSK modulation is used, what is the maximum bit rate that can be sent through the channel without needing an equalizer?

Note: In BPSK, you send one bit per symbol

Solution

(a) $$\bar{\tau} = \frac{(1)(0)+(1)(1)}{1+1} = \frac{1}{2} = 0.5\mu s$$

$$\overline{\tau^2} = \frac{(1)(0)^2+(1)(1)^2}{1+1} = \frac{1}{2} = 0.5\mu s^2$$

$$\sigma_\tau = \sqrt{\overline{\tau^2}-(\bar{\tau})^2} = \sqrt{0.5-(0.5)^2} = \sqrt{0.25} = 0.5\mu s$$

(b) $$\frac{\sigma_\tau}{T_s} \leq 0.1$$

$$T_s \geq \frac{\sigma_\tau}{0.1}$$

$$T_s \geq \frac{0.5\mu s}{0.1}$$

$$T_s \geq 5\mu s$$

$$R_s = \frac{1}{T_s} = 0.2\times 10^6 \text{sps} = 200\text{ksps}$$

$$R_b = 200\text{kbps}$$

Example 4.4

Calculate the mean excess delay, rms delay spread, ~~and the maximum excess delay (10 dB)~~ for the multipath profile given in the figure below. Estimate the 50% coherence bandwidth of the channel. Would this channel be suitable for AMPS or GSM service without the use of an equalizer?

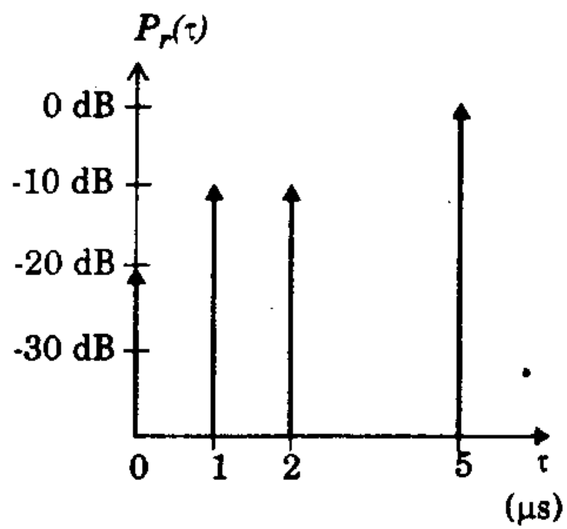


Figure E4.4

Notes:
In AMPS, B_S=30kHz
In GSM, B_S=200kHz

Solution to Example 4.4

The rms delay spread for the given multipath profile can be obtained using equations (4.35) — (4.37). The delays of each profile are measured relative to the first detectable signal. The mean excess delay for the given profile

$$\bar{\tau} = \frac{(1)(5) + (0.1)(1) + (0.1)(2) + (0.01)(0)}{[0.01 + 0.1 + 0.1 + 1]} = 4.38\mu s$$

The second moment for the given power delay profile can be calculated as

$$\overline{\tau^2} = \frac{(1)(5)^2 + (0.1)(1)^2 + (0.1)(2)^2 + (0.01)(0)}{1.21} = 21.07\mu s^2$$

Therefore the rms delay spread, $\sigma_\tau = \sqrt{21.07 - (4.38)^2} = 1.37\mu s$

The coherence bandwidth is found from equation (4.39) to be

$$B_c \approx \frac{1}{5\sigma_\tau} = \frac{1}{5(1.37\mu s)} = 146 \text{ kHz}$$

Since B_c is greater than 30 kHz, AMPS will work without an equalizer. However, GSM requires 200 kHz bandwidth which exceeds B_c, thus an equalizer would be needed for this channel.

(2) Fading Effects due to Doppler Spread

- Slow Fading

$$\boxed{T_S \ll T_C}$$

$$\boxed{B_S \gg B_D}$$

- Fast Fading

$$\boxed{T_S > T_C}$$

$$\boxed{B_S < B_D}$$

Then signal envelope follows:
- Rayleigh distribution
- Ricean distribution
- Nakagami-*m* distribution

T_S: symbol period
B_S: bandwidth of the transmitted modulation ($B_S = 1/T_S$)
T_C: coherence time ($T_C \propto 1/f_m$ where f_m is the maximum Doppler shift, $f_m = v/\lambda$)
B_D: Doppler spread ($B_D = f_d$, where f_d is the Doppler shift)

frequency dispersion parameters

Doppler Effect

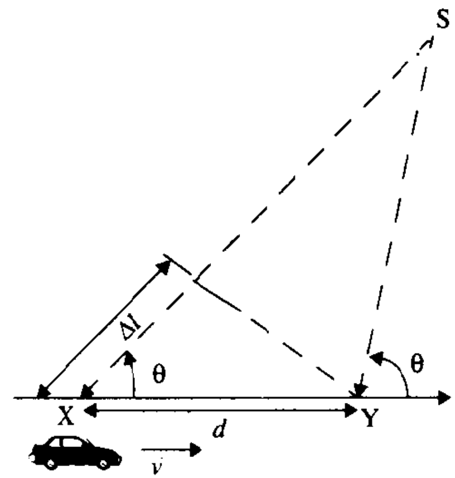


Figure 4.1
Illustration of Doppler effect.

Phase change in the received signal due to the difference in path lengths:

$$\Delta\phi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t}{\lambda} cos\theta$$

Hence, the apparent change in frequency (Doppler shift):

$$f_d = \frac{1}{2\pi} \cdot \frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda} \cdot cos\theta$$

Example 4.1

Consider a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving 60 mph, compute the received carrier frequency if the mobile is moving (a) directly towards the transmitter, (b) directly away from the transmitter, (c) in a direction which is perpendicular to the direction of arrival of the transmitted signal.

Solution to Example 4.1

Given:

Carrier frequency $f_c = 1850\,\text{MHz}$

Therefore, wavelength $\lambda = c/f_c = \dfrac{3\times 10^8}{1850\times 10^6} = 0.162\,\text{m}$

Vehicle speed $v = 60$ mph $= 26.82$ m/s

(a) The vehicle is moving directly towards the transmitter.
The Doppler shift in this case is positive and the received frequency is given by equation (4.2)

$$f = f_c + f_d = 1850\times 10^6 + \frac{26.82}{0.162} = 1850.00016\text{ MHz}$$

(b) The vehicle is moving directly away from the transmitter.
The Doppler shift in this case is negative and hence the received frequency is given by

$$f = f_c - f_d = 1850\times 10^6 - \frac{26.82}{0.162} = 1849.999834\text{ MHz}$$

(c) The vehicle is moving perpendicular to the angle of arrival of the transmitted signal.
In this case, $\theta = 90°$, $\cos\theta = 0$, and there is no Doppler shift.
The received signal frequency is the same as the transmitted frequency of 1850 MHz.

Coherence Time (T_C)

Time duration after which the correlation function of two samples of channel response taken at the **same frequency** but at **different time instances** falls below a certain threshold

$$T_C \approx \frac{1}{f_m}$$

$$T_C \approx \frac{9}{16\pi f_m}$$

$$\boxed{T_C = \sqrt{\frac{9}{16\pi f_m^2}}}$$

rule of thumb

Doppler Spread (B_D)

$$\boxed{B_D = f_d}$$

Example

If a baseband binary message with a bit rate R_b=100 kbps is modulated by an RF carrier using BPSK

a) Find the range of values required for the rms delay spread of the channel such that the received signal is a flat fading signal

b) If the modulation carrier frequency is 900 MHz, what is the coherence time of the channel assuming a vehicle speed of 100 km/hour?

c) For your answer in (b), is the channel fast or slow fading?

Solution

a) $R_s = R_b$ in BPSK. $T_s = 1/R_s = 1/(10^5) = 10^{-5}$ s = 10 μs

For flat fading we should have $T_s > 10\sigma_\tau$, therefore, $\sigma_\tau < 1$ μs

b) λ=0.33 m for 900 MHz.

Maximum Doppler frequency: $$f_m = \frac{v}{\lambda} = \frac{10^5\,\mathrm{m}}{3600\mathrm{s}} \cdot \frac{1}{0.33\mathrm{m}} = 84.17\mathrm{Hz}$$

Coherence time: $$T_C = \sqrt{\frac{9}{16\pi f_m^2}} = 5\mathrm{ms}$$

c) T_C = 5 ms, T_S = 10 μs => $T_C >> T_S$ => Slow fading

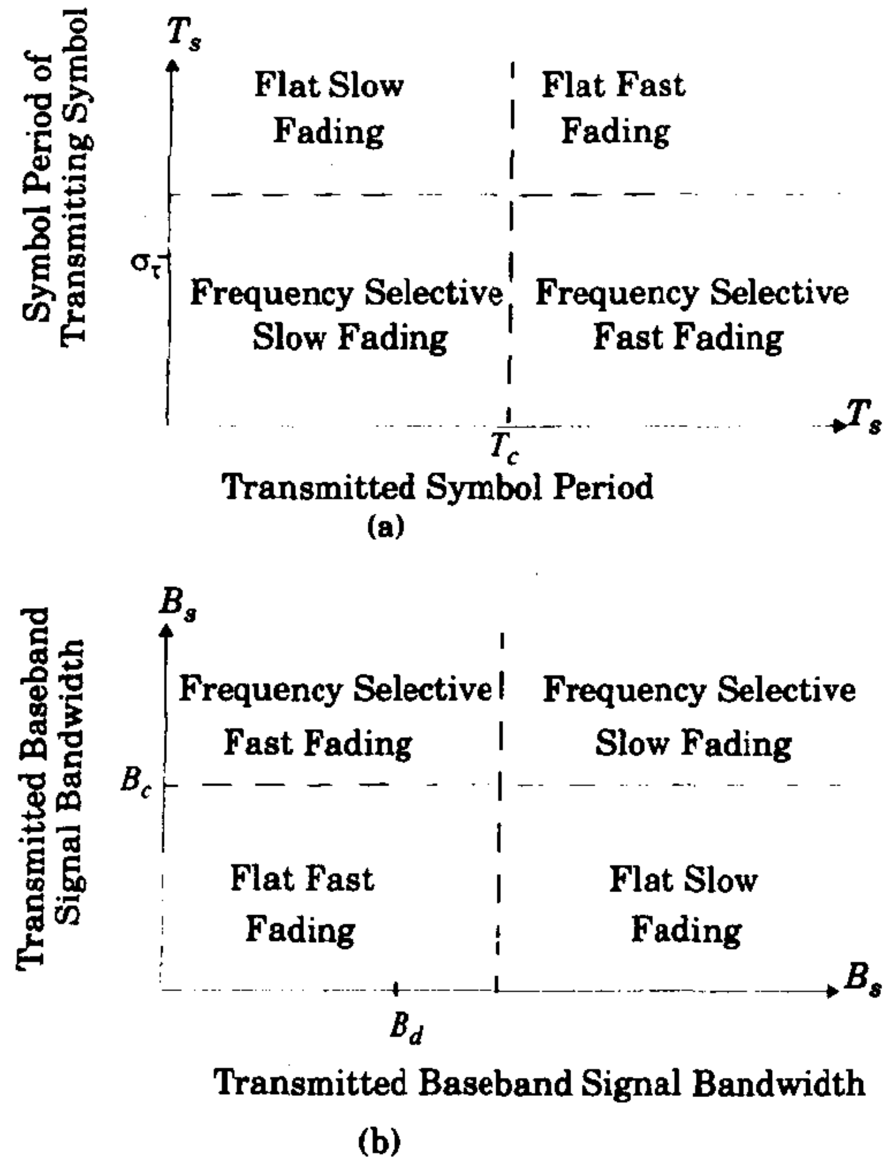


Figure 4.14

Matrix illustrating type of fading experienced by a signal as a function of
(a) symbol period
(b) baseband signal bandwidth.