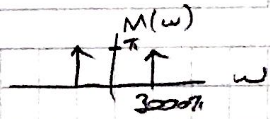
EEEN HW #2 SOL'N

Pr 1 $m(t) = 5\cos 3000\pi t$



a) NBFM $\rightarrow$ $B_{FM} = 2(\Delta f + B) \approx 2B$ since $\Delta f \ll B$

$B = 3000\pi / 2\pi = 1500\ Hz \Rightarrow B_{FM} \approx 2B = 3000\ Hz = 3\ kHz$

b) For $B_{FM} \approx 2\Delta f$, we should have $\Delta f \gg B$

$$\Delta f = \frac{k_f m_p}{2\pi} \gg B = 1500\ Hz$$

$$m_p = 5 \Rightarrow \frac{k_f 5}{2\pi} \gg 1500\ Hz \Rightarrow k_f \gg \frac{3000\pi}{5} = 600\pi$$

$$\Rightarrow k_f > 6000\pi$$

Therefore, the smallest k_f can be 6000π

c) $k_f = 6000\pi \Rightarrow \Delta f = \frac{k_f m_p}{2\pi} = \frac{6000\pi \times 5}{2\pi} = 15000\ Hz$

$$\Rightarrow B_{FM} \approx 2\Delta f = 30000\ Hz = 30\ kHz$$

Pr 2 $m(t) = 6\cos 2000\pi t - 2\sin 4000\pi t$

$\omega_c = 10^6\ rad/s$

$A = 5$

$k_f = 10^5\pi$

a)
$$\varphi_{FM}(t) = A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]$$
$$= 5\cos\left[10^6 t + 10^5\pi \int_{-\infty}^{t} (6\cos 2000\pi\alpha - 2\sin 4000\pi\alpha)\,d\alpha\right]$$
$$= 5\cos\left[10^6 t + \frac{6\times10^5\pi}{2000\pi}\sin 2000\pi t + \frac{2\times10^5\pi}{4000\pi}\cos 4000\pi t\right]$$
$$= 5\cos\left[10^6 t + 300\sin 2000\pi t + 50\cos 4000\pi t\right]$$

b) $B_{FM} = 2(\Delta f + B)$

$\Delta f = \frac{k_f m_p}{2\pi}$, $m_p = 8 \Rightarrow \Delta f = \frac{10^5\pi \times 8}{2\pi} = 400\ kHz$

$B = 4000\pi / 2\pi = 2000\ Hz = 2\ kHz$

$\Rightarrow B_{FM} = 2(400 + 2) = 804\ kHz$

c) $\Delta f = \frac{k_p m_p'}{2\pi} = 400\text{ kHz}$ for $B_{PM} = 804\text{ kHz}$

$\dot{m}(t) = -12000\pi \sin 2000\pi t - 8000\pi \cos 4000\pi t$

$\Rightarrow m_p' = 12000\pi + 8000\pi = 20000\pi$

$\Rightarrow \frac{k_p \times 20000\pi}{2\pi} = 400\text{ kHz} \Rightarrow k_p = \frac{400000 \times 2\pi}{20000\pi} = 40$

d) $\varphi_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$

$= 5\cos[10^6 t + 240\cos 2000\pi t - 80\sin 4000\pi t]$

Pr3 $\quad k_f = 20000\pi$, $k_p = 10\pi$, $m_p = 2$, $m_p' = 4000$, $B = 2500\text{Hz}$

a) $B_{FM} = 2(\Delta f + B)$

$\Delta f = \frac{k_f m_p}{2\pi} = \frac{20000\pi \times 2}{2\pi} = 20\text{ kHz}$

$B = 2500\text{ Hz} = 2.5\text{ kHz}$

$\Rightarrow B_{FM} = 2(20 + 2.5) = 45\text{ kHz}$

$B_{PM} = 2(\Delta f + B)$

$\Delta f = \frac{k_p m_p'}{2\pi} = \frac{10\pi \times 4000}{2\pi} = 20000 = 20\text{ kHz}$

$B = 2500\text{ Hz} = 2.5\text{ kHz}$

$\Rightarrow B_{PM} = 2(20 + 2.5) = 45\text{ kHz}$

b) $x(t) = m^2(t)$

If $|m(t)| \le m_p = 2$

$|x(t)| = |m(t)|^2 \le m_p^2 = 4 \Rightarrow x_p = 4$

$\dot{x}(t) = \frac{d}{dt} x(t) = \frac{d}{dt}[m^2(t)] = 2m(t)\frac{d}{dt}m(t) = 2m(t)\dot{m}(t)$

If $|\dot{m}(t)| \le m_p' = 4000$

$|\dot{x}(t)| = 2|m(t)||\dot{m}(t)| \le 2m_p m_p' = 2\times 2\times 4000 = 16000$

$\Rightarrow x_p' = 16000$

Finally, If the bandwidth of $m(t)$ is 2500 Hz, the bandwidth of $x(t) = m^2(t)$ is $2 \times 2500 = 5000$ Hz

since $X(\omega) = \frac{1}{2\pi} M(\omega) * M(\omega)$

Hence, for FM

$$\Delta f = \frac{k_f x_p}{2\pi} = \frac{20000\pi \times 4}{2\pi} = 40 \text{ kHz}$$

$B = 5000$ Hz $= 5$ kHz

$\Rightarrow B_{FM} = 2(40+5) = 90$ kHz

and for PM

$$\Delta f = \frac{k_p x_p'}{2\pi} = \frac{10\pi \times 16000}{2\pi} = 80 \text{ kHz}$$

$B = 5000$ Hz $= 5$ kHz

$\Rightarrow B_{PM} = 2(80+5) = 170$ kHz

c) Squaring the message signal has effects both on the amplitude and spectral properties, therefore, both B_{FM} and B_{PM} have been increased from (a) to (b) due to the increases in both the frequency deviations and the bandwidth of the message. The increase in B_{PM} is greater due to the fact that

$$\frac{x_p'}{m_p'} > \frac{x_p}{m_p}.$$