EEEN 222 / COMP 211
Digital Systems Design

MIDTERM EXAM, Spring 2014-2015 ***Duration: 100 minutes***

Problem 1 (25 points)

Consider a general Boolean algebra which can be defined with more than two elements other than 0 and 1. Given that $A \cdot B = 0$ and $A + B = 1$, use axiom(s) and/or theorems of general Boolean algebra to prove the following identity

$$(A+C)\cdot(A'+B)\cdot(B+C) = B\cdot C$$

(*Hint*: Note that A, B, and C do not necessarily have to be 0 or 1).

Problem 2 (25 points)

Given the following Boolean function:

$$F(A,B,C,D) = \sum m(1,2,9,10,13,14,15) + d(5,7,8)$$

- **a.** Simplify the function $F(A,B,C,D)$ using Quine-McCluskey tabulation method. Show all the tables that you use. **(9 pts.)**
- **b.** Implement the simplified expression using NOR gates. **(8 pts.)**
- **c.** Implement the simplified expression using NAND gates. **(8 pts.)**

Problem 3 (25 points)

Consider a combinational circuit that accepts a 4-bit binary number and generates an output 1 if the number of 1s in the 4-bit binary number is even and yields an output 0 if the number of 1s in the 4-bit binary number is odd.

- **a.** Simplify the output Boolean function f using Karnaugh map approach.
- **b.** Draw the corresponding circuit diagram as a 2-level NOR gate logic circuit.

Problem 4 (25 points)

Given the following Boolean function:

$$F(A,B,C) = A'B + B'C + A'BC'$$

- **a.** Implement $F(A,B,C)$ using a 3×8 decoder and any simple logic gate if necessary. **(7 pts.)**
- **b.** Implement $F(A,B,C)$ using an 8-to-1 multiplexer and any simple logic gate if necessary. **(9 pts.)**
- **c.** Implement $F(A,B,C)$ using a single 4-to-1 multiplexer and any simple logic gate if necessary. **(9 pts.)**

EEEN 222 Midterm Exam
Solution Key

Problem 1) We have

$$A \cdot B = 0$$

$$A + B = 1$$

then we find that

$$\begin{aligned}(A+C)\cdot(A'+B)\cdot(B+C) &= (A+C)\cdot(B+C)\cdot(A'+B) && 05\\ &= (A\cdot B+C)\cdot 1\cdot(A'+B) && 05\\ &= (0+C)\cdot(A+B)\cdot(A'+B) && 05\\ &= C\cdot(A\cdot A'+B) && 05\\ &= C\cdot(0+B) && 05\\ &= CB\\ &= BC\end{aligned}$$

2) $F(A,B,C,D) = \sum m(1,2,9,10,13,14,15)$

$d(A,B,C,D) = \sum m(5,7,8)$

List 1

mi	A	B	C	D	
1	0	0	0	1	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
5	0	1	0	1	✓
9	1	0	0	1	✓
10	1	0	1	0	✓
7	0	1	1	1	✓
13	1	1	0	1	✓
14	1	1	1	0	✓
15	1	1	1	1	✓

List 2

Product terms	A	B	C	D	
(1,5)	0	-	0	1	✓
(1,9)	-	0	0	1	✓
(2,10)	-	0	1	0	PI7
(8,9)	1	0	0	-	PI6
(8,10)	1	0	-	0	PI5
(5,7)	0	1	-	1	✓
(5,13)	-	1	0	1	✓
(9,13)	1	-	0	1	✓
(10,14)	1	-	1	0	PI4
(7,15)	-	1	1	1	✓
(13,15)	1	1	-	1	✓
(14,15)	1	1	1	-	PI3

List 3

Produt terms	A	B	C	D	
(1,5), (9,13)	-	-	0	1	PI1 = C'D
(1,9), (5,13)	-	-	0	1	
(5,7), (13,15)	-	1	-	1	PI2 = BD
(5,13), (7,15)	-	1	-	1	

Problem 3)

A	B	C	D	F
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

2.5

$$F(A,B,C,D) = \Sigma m\,(0,3,5,6,9,10,12,15)$$

a.

AB \ CD	00	01	11	10
00	$\boxed{1}$		$\boxed{1}$	
01		$\boxed{1}$		$\boxed{1}$
11	$\boxed{1}$		$\boxed{1}$	
10		$\boxed{1}$		$\boxed{1}$

05

$$F(A,B,C,D) = A'B'C'D' + A'B'CD + A'BC'D + A'BCD' + ABC'D' + ABCD + AB'C'D + AB'CD'$$

05

Midterm 3.1

b. We consider sum of minterms representation for $F'(A,B,C,D)$ as follows

$$F'(A,B,C,D) = \sum m(1,2,4,7,8,11,13,14)$$

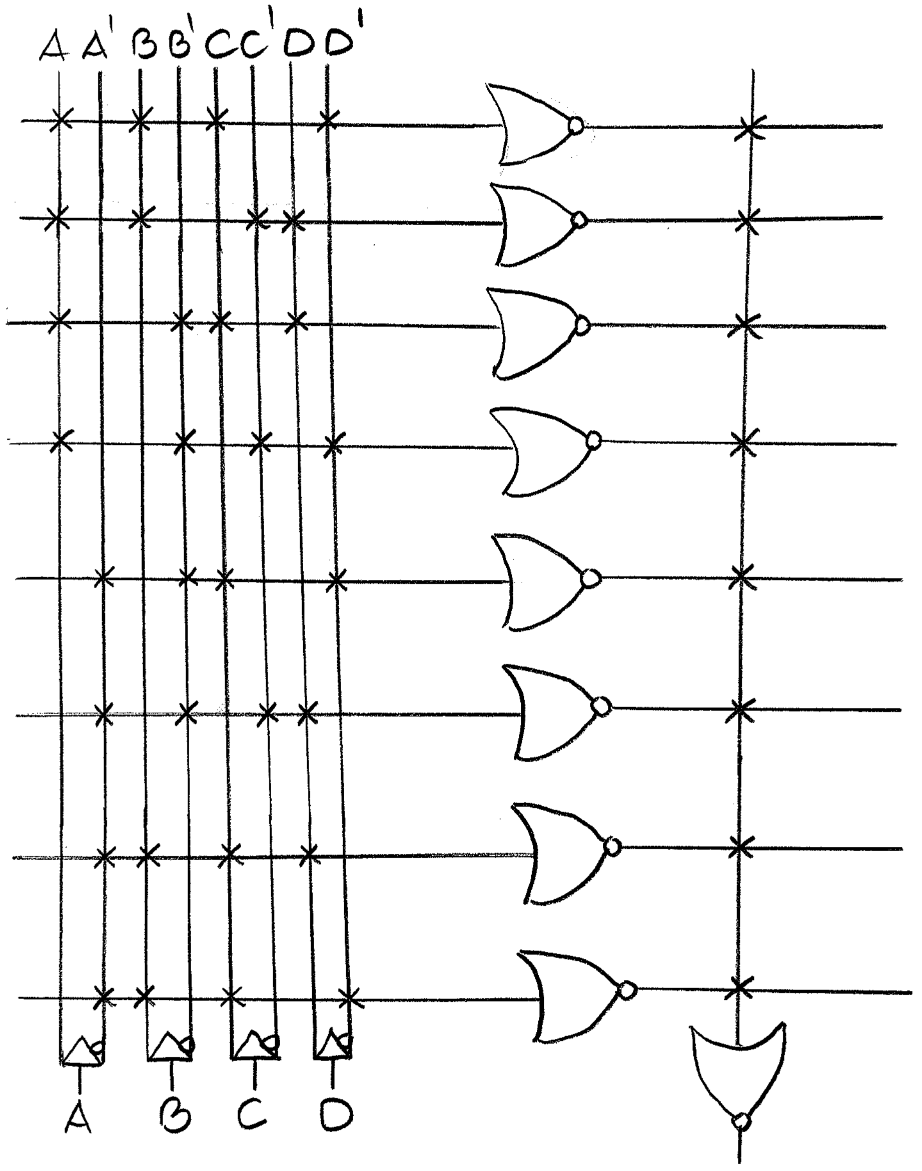
AB \ CD	00	01	11	10
00		1		1
01	1		1	
11		1		1
10	1		1	

$$F'(A,B,C,D) = A'B'C'D + A'B'CD' + A'BC'D' + A'BCD + ABC'D + ABCD' + AB'C'D' + AB'CD$$

2.5

$$\Rightarrow \left[F'(A,B,C,D)\right]' = F(A,B,C,D) = (A+B+C+D')\cdot(A+B+C'+D)\cdot(A+B'+C+D)\cdot(A+B'+C'+D')\cdot(A'+B'+C+D')\cdot(A'+B'+C'+D)\cdot(A'+B+C+D)\cdot(A'+B+C'+D')$$

05



05

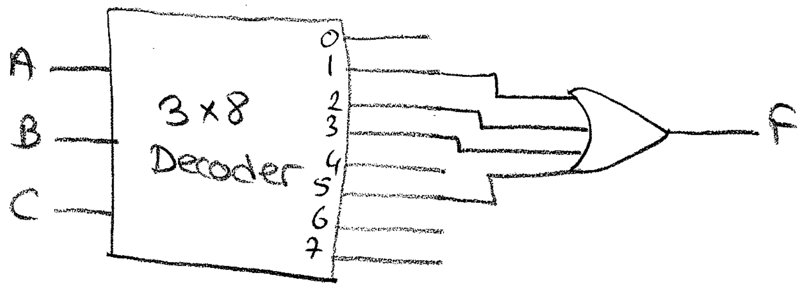
4)

$$F = A'B + B'C + A'BC'$$

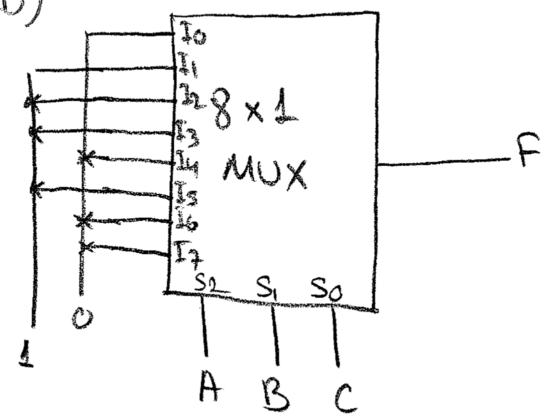
A \ BC	00	01	11	10
0		1	1	1
1		1		

$$F = \Sigma m(1,2,3,5)$$

a)



b)



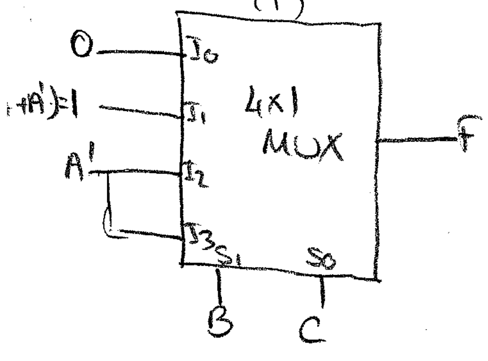
c) $F = A'B'C + A'BC' + A'BC + AB'C$

$= A(B'C) + A'(B'C + BC' + BC)$ (i)

$= B(A'C' + A'C) + B'(A'C + AC)$ (ii)

$= C(A'B' + A'B + AB') + C'(A'B)$ (iii)

(i)



(ii)