20.03.18 (WI - PSII)

Q1) $g(t) \rightarrow$ band lim. to B Hz.

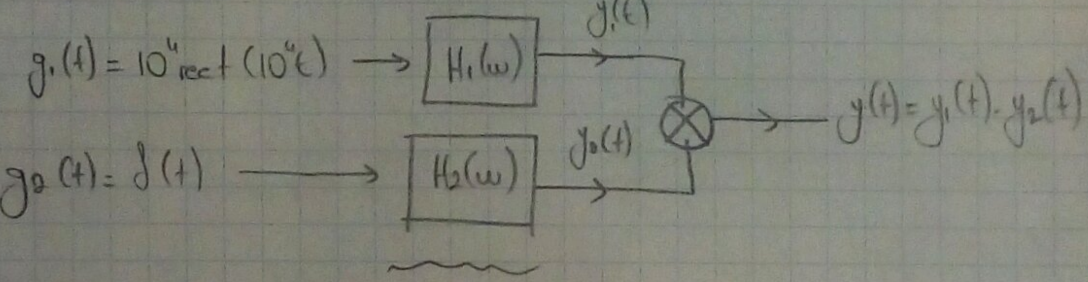
$$g^2(t) \Longleftrightarrow \frac{1}{2\pi}[G(\omega) * G(\omega)] \longrightarrow \text{freq. conv. prop.}$$

Width Prop. $\rightarrow$ If $c_1(x) * c_2(x) = y(x)$

Then the width of $y(x)$ is equal to the sum of the widths of $c_1(x)$ and $c_2(x)$.

$$\underbrace{G(\omega)}_{\text{width}} \longrightarrow \underbrace{G(\omega) * G(\omega)}_{2 * \text{width}}$$

Q2)



$$H_1(\omega) = \text{rect}\left(\frac{\omega}{40000\pi}\right)$$

$$H_2(\omega) = \text{rect}\left(\frac{\omega}{20000\pi}\right)$$

Table 3.1. $\rightarrow$ $\text{rect}\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau \cdot \text{sinc}\left(\frac{\omega\tau}{2}\right)$

$(1/\tau = 10^4)$ $\qquad$ $\underbrace{G_1(\omega) = 10^4 \cdot 10^{-4} \cdot \text{sinc}\left(\frac{\omega}{20000}\right)}$

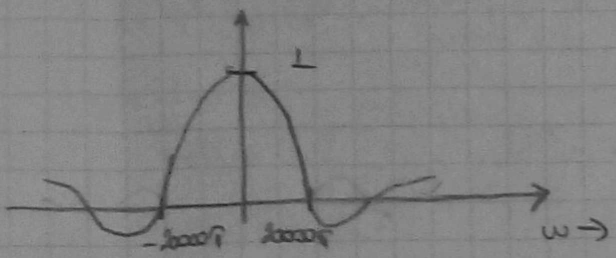
$\tau = 10^{-4}$

$$\delta(t) \Longleftrightarrow \underbrace{1} = G_2(\omega)$$

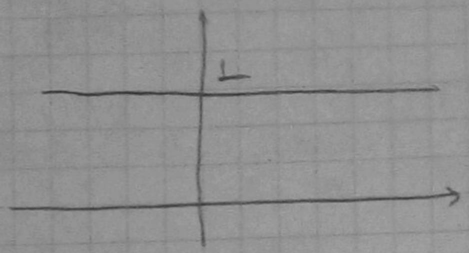
$$Y_1(\omega) = G_1(\omega) \cdot H_1(\omega)$$

$$Y_2(\omega) = G_2(\omega) \cdot H_2(\omega)$$

$$G_1(\omega) = \text{sinc}(\omega/20000)$$



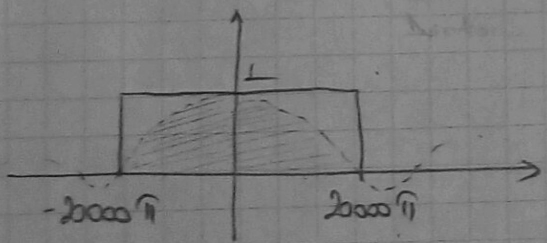
$$G_2(\omega) = 1$$



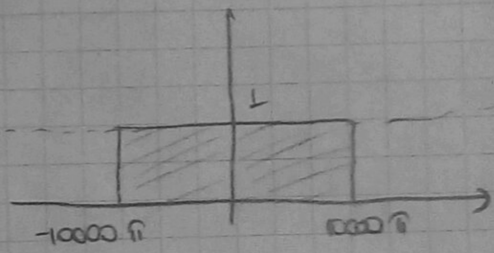
$$\omega/20000 = \pi$$

$$\hookrightarrow \omega = 20000\pi$$

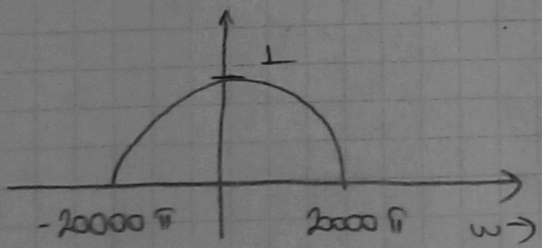
$$H_1(\omega) = \text{rect}\left(\frac{\omega}{40000\pi}\right)$$



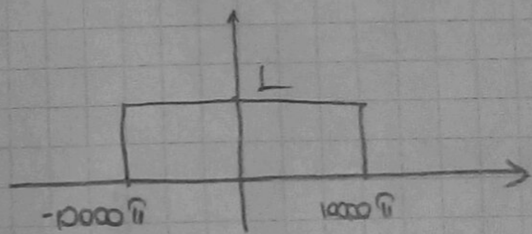
$$H_2(\omega) = \text{rect}\left(\frac{\omega}{20000\pi}\right)$$



$$Y_1(\omega) = G_1(\omega) \cdot H_1(\omega)$$



$$Y_2(\omega) = G_2(\omega) \cdot H_2(\omega)$$



Q3) $E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt \overset{?}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(\omega)|^2 d\omega \quad \left(g(t) = e^{-at}u(t), a>0 \right)$

$$E_g = \int_0^{\infty} e^{-2at} dt = \left(\frac{e^{-2at}}{-2a} \right)_0^{\infty} = 0 - \left(\frac{1}{-2a} \right) = \frac{1}{2a}$$

Table 3.1. $\rightarrow \quad g(t) = e^{-at} \Longleftrightarrow G(\omega) = \frac{1}{j\omega + a}$

Parseval's $\rightarrow \quad E_g = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{\omega^2+a^2} d\omega = \left(\frac{1}{2\pi} \cdot \frac{1}{a} \cdot \tan^{-1}\frac{\omega}{a} \right)_{-\infty}^{\infty}$

$$= \frac{1}{2\pi a}\left[\underbrace{\tan^{-1}(\infty)}_{\pi/2} - \underbrace{\tan^{-1}(-\infty)}_{-\pi/2} \right]$$

$(\tan(-x) = -\tan(x))$

Q4) $g(t) = e^{-at} \cdot u(t)$

Ess. bw. $\rightarrow$ Contains 95% of the signal eng.

$G(\omega) = \frac{1}{j\omega + a}$

ESD $\rightarrow |G(\omega)^2| = \frac{1}{\omega^2 + a^2}$

$E_g = \frac{1}{2a}$

95% of the sig. eg. $\rightarrow \frac{1}{2a}\cdot(0.95)$

$$\frac{0.95}{2a} = \frac{1}{2\pi}\int_{-w}^{w}\frac{1}{w^2+a^2}\,dw$$

$$= \frac{1}{2\pi a}\cdot\tan^{-1}\left(\frac{w}{a}\right)\Big|_{-w}^{w}$$

$$\tan^{-1}\left(\frac{w}{a}\right) - \underbrace{\tan^{-1}\left(\frac{-w}{a}\right)}_{=-\tan^{-1}\left(\frac{w}{a}\right)}$$

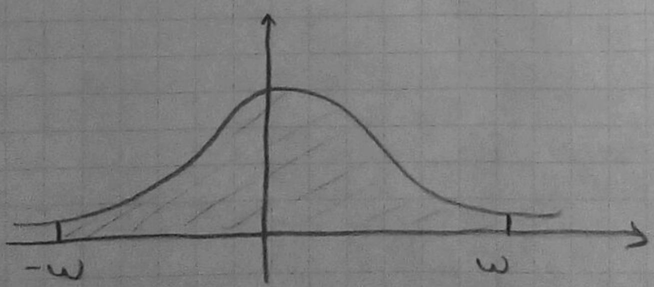
$$= 2\cdot\tan^{-1}\left(\frac{w}{a}\right)$$

$$= \frac{1}{\pi\cdot a}\cdot\tan^{-1}\left(\frac{w}{a}\right)$$

$$\rightarrow \frac{0.95}{2a} = \frac{1}{\pi a}\cdot\tan^{-1}\left(\frac{w}{a}\right)$$

$$\frac{0.95\pi}{2} = \tan^{-1}\left(\frac{w}{a}\right)$$

$$\tan\left(\frac{0.95\pi}{2}\right)\cdot a = w \rightarrow w = 12.706a \text{ rad/sec.}$$



0-w is the ess. bw.

Q5) $n_i(t) \longrightarrow \boxed{d/dt} \longrightarrow \underbrace{n_o(t)}_{\text{Output noise signal}}$

(PSD) $\rightarrow S_{n_i}(\omega) = K$

① PSD of output $\rightarrow S_{n_o}(\omega)$

② Power of output $\rightarrow N_o$

Transfer func. of an ideal diff. is $H(\omega) = j\omega$

$$S_{n_o}(\omega) = \cdots$$

① $\rightarrow$ $Y(\omega) = H(\omega) . G(\omega)$

Therefore,

$$\underbrace{|Y(\omega)|^2}_{\text{Output signal ESD}} = |H(\omega)|^2 . \underbrace{|G(\omega)|^2}_{\text{Input signal ESD}}$$

①' ... $\underbrace{S_{n_o}(\omega)}_{\text{Output signal PSD}} = |H(\omega)|^2 . \underbrace{S_{n_i}(\omega)}_{\text{Input signal PSD}}$

$$S_{n_o}(\omega) = |j\omega|^2 . K$$

② Power (N_o) is $\frac{1}{2\pi}$ times the area under the output PSD

$$N_o = \frac{1}{2\pi} . \int_{-2\pi B}^{2\pi B} \underbrace{K . \omega^2}_{S_{n_o}(\omega)} d\omega = K \int_{-2\pi B}^{2\pi B} \omega^2 d\omega$$

$$= \frac{8\pi^2 B^3 K}{3}$$

⑤