EEEN-352 (PS-3)

2-7) A system is described by the following dif. eq.

$$\frac{d^3y}{dt^3} + \frac{3d^2y}{dt^2} + \frac{5dy}{dt} + y = \frac{d^3x}{dt^3} + \frac{4d^2x}{dt^2} + 6\frac{dx}{dt} + 8x$$

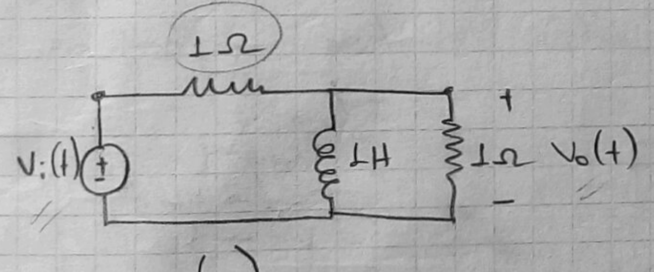
find the exp. for the transfer func. of the system, $Y(s)/X(s)$

Sol. → Laplace t. of the dif. eq., assuming zero initial cond.

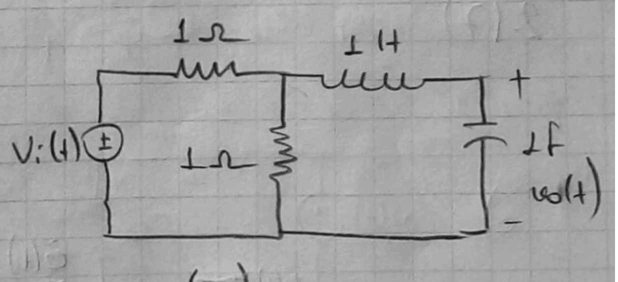
$$(s^3 + 3s^2 + 5s + 1)\,Y(s) = (s^3 + 4s^2 + 6s + 8)\,X(s)$$

$$\frac{Y(s)}{X(s)} = H(s) = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}$$

2-16) Find the transfer func. $G(s) = V_o(s)/V_i(s)$, for each network.



(a)



(b)

Sol. → a) Electrical Netw. Trans. func.

Res → R   Ind. → Ls

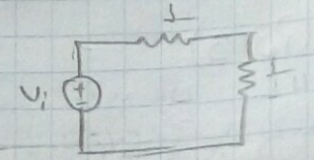
Cap → $\frac{1}{Cs}$

$$\frac{V_o - V_i}{1} + \frac{V_o}{1} + \frac{V_o}{s} = V_i(s)$$

$$V_o(s)s - V_i(s)s + V_o(s)s + V_o = V_i(s)s$$

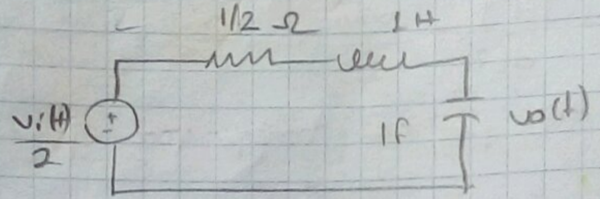
$$V_o(2s+1) = 2V_i(s)s \rightarrow \frac{V_o}{V_i} = \frac{2s}{2s+1} = \frac{s}{s+1/2}$$

b) Thevenin Eq →



$V_{th} = \frac{V_i(t)}{2}$

$R_{eq} = \frac{1 \cdot 1}{1+1} = \frac{1}{2}$

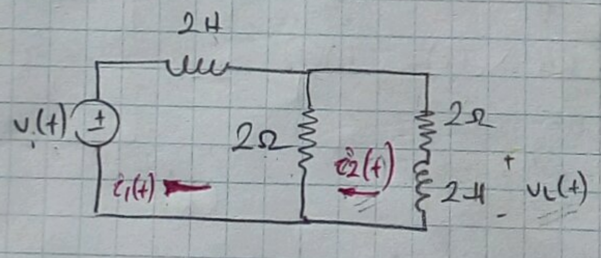


Voltage div. → $\frac{V_i(s)}{2} \cdot \frac{1/s}{\frac{1}{s} + \frac{1}{2} + s} = V_o(s)$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{2s^2+s+2}$$

2-17) Find the transfer f., $G(s) = V_L(s)/V(s)$ for each network.

c)



Ind → Ls
Cap → $1/Cs$
Res → R

Mesh eqs: ① $(2s+2)I_1(s) - 2 \cdot I_2(s) = V_i(s)$

② $(2s+4) \cdot I_2(s) - 2I_1(s) = 0$

$I_1(s) = (s+2)I_2(s)$

$(2s+2) \cdot (s+2)I_2(s) - 2I_2(s) = V_i(s)$

$I_2(s)/V_i(s) = 1/(2s^2+6s+2)$

$I_2(s) \rightarrow V_L(s)$

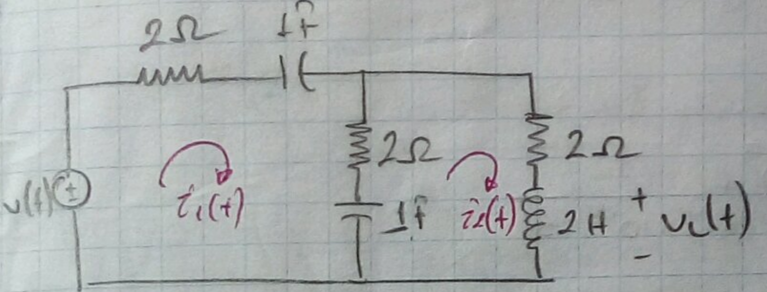
$v_L(t) = L \cdot \frac{di_2(t)}{dt}$

$V_L(s) = L \cdot s \cdot I_2(s) = 2 \cdot s \cdot I_2(s)$

$I_2(s) = \frac{V_L(s)}{2s}$

$V_L(s) / V_i(s) = 2s / (2s^2 + 6s + 2)$

b)



① $(4 + \frac{2}{s}) \cdot I_1(s) - (2 + \frac{1}{s}) \cdot I_2(s) = V(s)$

② $(-2 - \frac{1}{s}) \cdot I_1(s) + (4 + \frac{1}{s} + 2s) \cdot I_2(s) = 0$

$I_1(s) = \frac{(4 + 1/s + 2s)\, I_2(s)}{(2 + \frac{1}{s})}$

$(4 + \frac{2}{s}) \cdot \left(\frac{4 + 1/s + 2s}{(2 + 1/s)} \right) I_2(s) - (2 + \frac{1}{s}) I_2(s) = V(s)$

$I_2(s) \cdot \left\{ \left[8 + 2/s + 4s \right] - (2 + \frac{1}{s}) \right] = V(s) \quad / s$

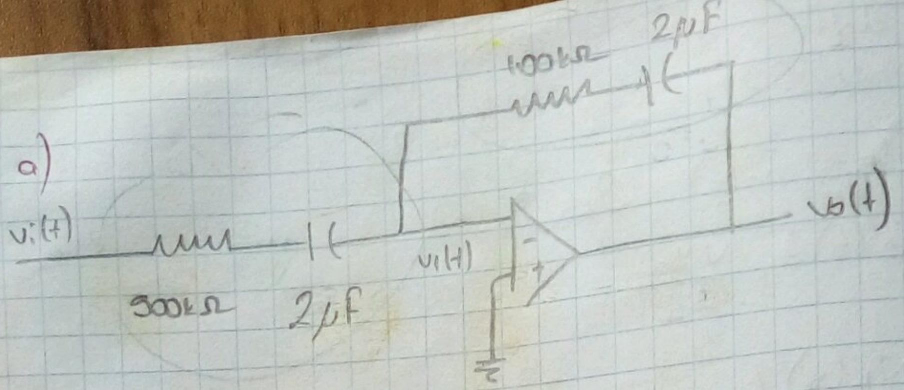
$I_2(s) \left\{ \left[(4s^2 + 8s + 2) - (2s + 1) \right] = sV(s)$

$I_2(s) = \frac{sV(s)}{4s^2 + 6s + 1}$

$V_L(s) = 2 \cdot s\, I_2(s) \rightarrow I_2(s) = V_L(s) / 2s$

$\frac{V_L(s)}{V(s)} = \frac{2s^2}{4s^2 + 6s + 1}$

2-21) a)



Find the transfer func., $G(s) = V_o(s)/V_i(s)$, for each operational amplifier circuit.

a) $$Z_1(s) = 5\times10^5 + \frac{1}{2\times10^{-6}\cdot s}$$

INVERTING

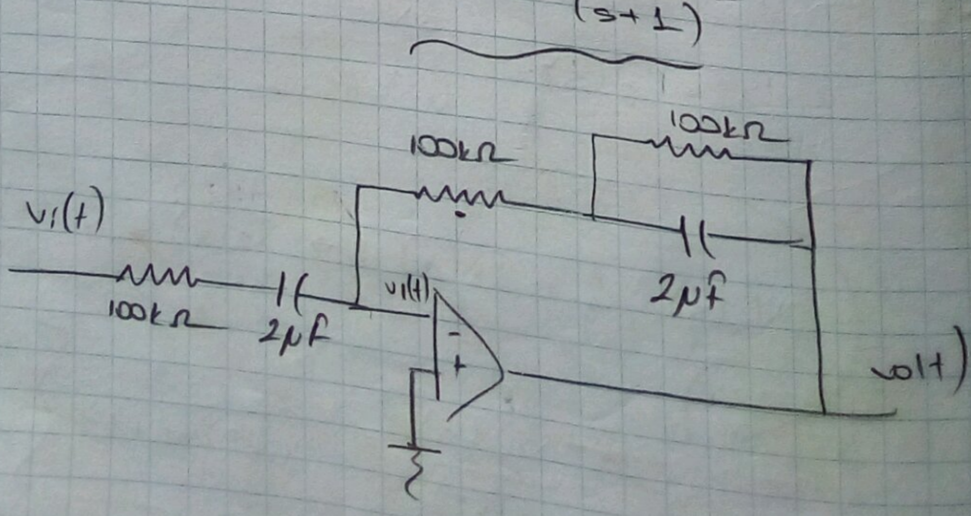
$$Z_2(s) = 10^5 + \frac{1}{2\times10^{-6}s}$$

$$-\frac{Z_2(s)}{Z_1(s)} = -\frac{\left[\dfrac{10^5\times2\times10^{-6}\times s+1}{\cancel{2\times10^{-6}s}}\right]}{\left[\dfrac{5\times10^5\times2\times10^{-6}\times s+1}{\cancel{2\times10^{-6}s}}\right]}$$

$$= -\frac{(2.10^{-1}s+1)}{(s+1)} =$$

$$= -\frac{1/5\,(s+5)}{(s+1)}$$

b)



b) $Z_2(s) = 10^5 + \dfrac{10^5 \cdot \left(\dfrac{1}{2\times10^{-6}s}\right)}{10^5 + \left(\dfrac{1}{2\times10^{-6}s}\right)} = 10^5 + \dfrac{0.5\times10^{11}}{\left(\dfrac{0.2s+1}{2\times10^{-6}s}\right)}$

$= 10^5 + \dfrac{\overbrace{0.5\times2\times10^{11}\times10^{-6}}^{10^5}s}{0.2s+1}$

~~2-22)~~

$= 10^5 + \dfrac{10^5 s}{0.2s+1 \to 0.2(s+5)}$

$Z_1(s) = 10^5 + \dfrac{1}{2\cdot10^{-6}s}$

$\boxed{Z_2(s) = 10^5\left[1 + \dfrac{5s}{s+5}\right]}$

$= \dfrac{10^5\times2\times10^{-6}s+1}{2\times10^{-6}s}$

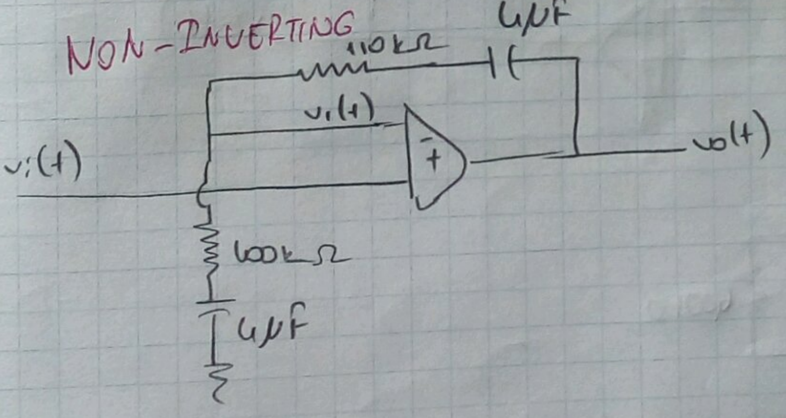
$= \dfrac{2\times10^{-1}s+1}{2\times10^{-6}s}$

$= \dfrac{\cancel{0.2}^{\,10^{-1}}(s+5)}{\cancel{2}\times10^{-6}(s)} = \boxed{\dfrac{10^5(s+5)}{s}}$

$\to G(s) = -\dfrac{Z_2(s)}{Z_1(s)} = \dfrac{s(s+10)}{(s+5)^2}$

2-22)

NON-INVERTING



$Z_1(s) = 4\times10^5 + \dfrac{1}{4\times10^{-6}s}$

$Z_2(s) = 1.1\times10^5 + \dfrac{1}{4\times10^{-6}s}$

$G(s) = \dfrac{Z_1(s) + Z_2(s)}{Z_1(s)}$

$$Z_1(s)+Z_2(s) = 5.1\times10^5 + \frac{2}{4\times10^{-6}s}$$

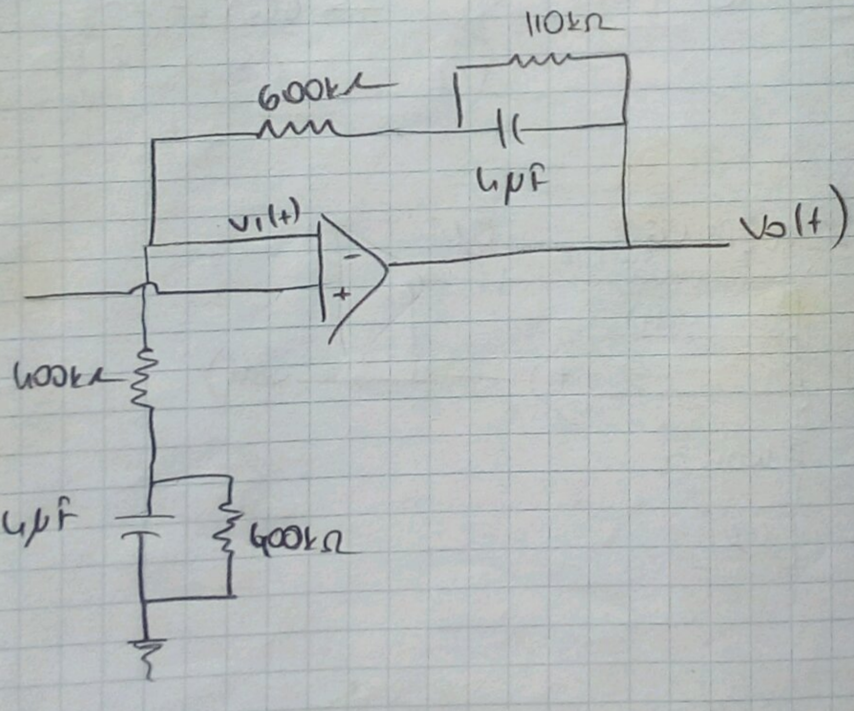
$$Z_1(s) = 4\times10^5 + \frac{1}{4\times10^{-6}s}$$

$$= \frac{4\times5.1\times10^{-1}s + 2}{4\times4\times10^{-1}s + 1}$$

$$= \frac{2.04\,(s+ 2/2.04)}{1.6\,(s+1/1.6)}$$

$$\boxed{G(s) = 1.275\left(\frac{s+0.98}{s+0.625}\right)}$$

b)



$$Z_1(s) = 4\times10^5 + \frac{R\cdot(1/cs)}{R+(1/cs)}$$

$$= 4\times10^5 + \frac{4\times10^5\left(\frac{1}{4\times10^{-6}s}\right)}{4\times10^5 + \frac{1}{4\times10^{-6}s}}$$

$$Z_1(s) = \frac{10^{11}/s}{4\times10^5 + \frac{0.25\times10^6}{s}}$$

$$Z_2(s) = 6\times10^5 + \frac{110\times10^3 \cdot \left(\frac{1}{4\times10^{-6}s}\right)}{110\times10^3 + \left(\frac{0.25\times10^6}{s}\right)} = \frac{27.5\times\frac{10^9}{s}}{\text{" " "}}$$

$$\boxed{G(s) = \frac{Z_1(s) + Z_2(s)}{Z_1(s)} = \frac{2640s^2 + 8400s + 4275}{1056s^2 + 3500s + 2500}}$$