EEEN 322 PS 3 QUESTIONS

Q1

4.2-1 For each of the following baseband signals: (i) $m(t) = \cos 1000t$; (ii) $m(t) = 2\cos 1000t + \cos 2000t$; (iii) $m(t) = \cos 1000t \cos 3000t$:

(a) Sketch the spectrum of $m(t)$.

(b) Sketch the spectrum of the DSB-SC signal $m(t)\cos 10{,}000t$.

(c) Identify the upper sideband (USB) and the lower sideband (LSB) spectra.

(d) Identify the frequencies in the baseband, and the corresponding frequencies in the DSB-SC, USB, and LSB spectra. Explain the nature of frequency shifting in each case.

* *

Q2

4.2-2 Repeat Prob. 4.2-1 [parts **(a)**, **(b)**, and **(c)** only] if: **(i)** $m(t) = \text{sinc}\,(100t)$; **(ii)** $m(t) = e^{-|t|}$; **(iii)** $m(t) = e^{-|t-1|}$. Observe that $e^{-|t-1|}$ is $e^{-|t|}$ delayed by 1 second. For the last case you need to consider both the amplitude and the phase spectra.

* *

Q3

4.2-3 Repeat Prob. 4.2-1 [parts **(a)**, **(b)**, and **(c)** only] for $m(t) = e^{-|t|}$ if the carrier is $\cos(10{,}000t - \pi/4)$. *Hint*: Use Eq. (3.36).

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Q4

4.2-4 You are asked to design a DSB-SC modulator to generate a modulated signal $km(t)\cos\omega_c t$, where $m(t)$ is a signal band-limited to B Hz. Figure P4.2-4 shows a DSB-SC modulator available in the stock room. The carrier generator available generates not $\cos\omega_c t$, but $\cos^3\omega_c t$. Explain whether you would be able to generate the desired signal using only this equipment. You may use any kind of filter you like.

(a) What kind of filter is required in Fig. P4.2-4?

(b) Determine the signal spectra at points b and c, and indicate the frequency bands occupied by these spectra.

(c) What is the minimum usable value of ω_c?

(d) Would this scheme work if the carrier generator output were $\cos^2\omega_c t$? Explain.

(e) Would this scheme work if the carrier generator output were $\cos^n\omega_c t$ for any integer $n \geq 2$?

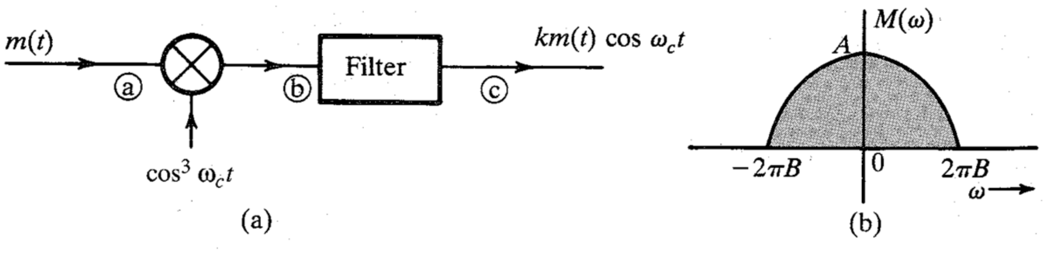

Figure P4.2-4

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Q5

4.2-5 You are asked to design a DSB-SC modulator to generate a modulated signal $km(t)\cos\omega_c t$ with the carrier frequency $f_c = 300$ kHz ($\omega_c = 2\pi \times 300,000$). The following equipment is available in the stock room: **(i)** a signal generator of frequency 100 kHz; **(ii)** a ring modulator; **(iii)** a bandpass filter tuned to 300 kHz.

(a) Show how you can generate the desired signal.

(b) If the output of the modulator is $km(t)\cos\omega_c t$, find k.

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EEEN 322 PS 3 SOLUTIONS

Q1

4.2-1 (i) For $m(t) = \cos 1000t$

$$\varphi_{\text{DSB-SC}}(t) = m(t)\cos 10{,}000t = \cos 1000t \cos 10{,}000t$$
$$= \frac{1}{2}[\underbrace{\cos 9000t}_{\text{LSB}} + \underbrace{\cos 11{,}000t}_{\text{USB}}]$$

(ii) For $m(t) = 2\cos 1000t + \cos 2000t$

$$\varphi_{\text{DSB-SC}}(t) = m(t)\cos 10{,}000t = [2\cos 1000t + \cos 2000t]\cos 10{,}000t$$
$$= \cos 9000t + \cos 11{,}000t + \frac{1}{2}[\cos 8000t + \cos 12{,}000t]$$
$$= \underbrace{[\cos 9000t + \frac{1}{2}\cos 8000t]}_{\text{LSB}} + \underbrace{[\cos 11{,}000t + \frac{1}{2}\cos 12{,}000t]}_{\text{USB}}$$

(iii) For $m(t) = \cos 1000t \cos 3000t$

$$\varphi_{\text{DSB-SC}}(t) = m(t)\cos 10{,}000t = \frac{1}{2}[\cos 2000t + \cos 4000t]\cos 10{,}000t$$
$$= \frac{1}{2}[\cos 8000t + \cos 12{,}000t] + \frac{1}{2}[\cos 6000t + \cos 14{,}000t]$$
$$= \frac{1}{2}\underbrace{[\cos 8000t + \cos 6000t]}_{\text{LSB}} + \frac{1}{2}\underbrace{[\cos 12{,}000t + \cos 14{,}000t]}_{\text{USB}}$$

This information is summarized in a table below. Figure S4.2-1 shows various spectra.

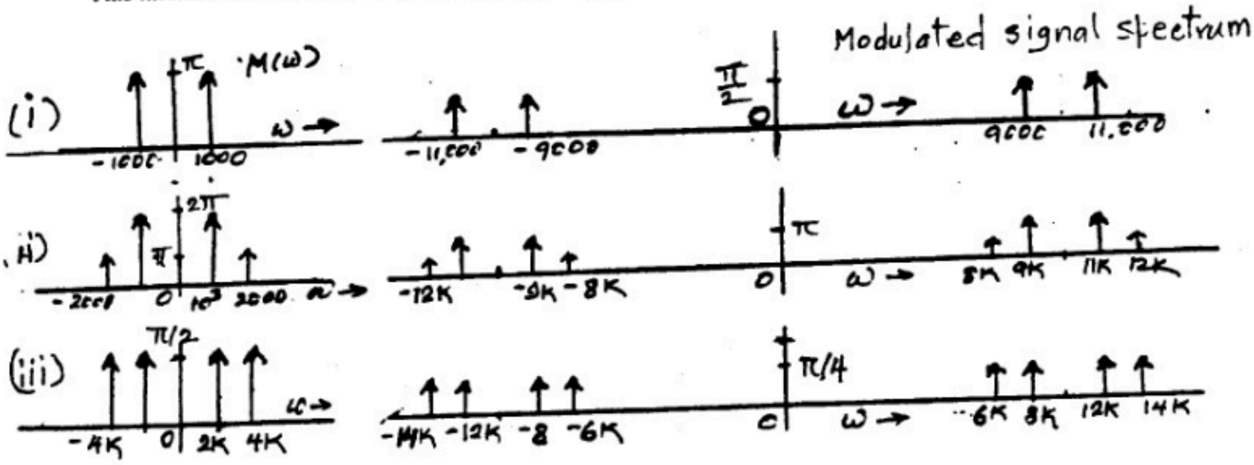

Fig. S4.2-1

case	Baseband frequency	DSB frequency	LSB frequency	USB frequency
i	1000	9000 and 11,000	9000	11,000
ii	1000	9000 and 11,000	9000	11,000
	2000	8000 and 12,000	8000	12,000
iii	2000	8000 and 12,000	8000	12,000
	4000	6000 and 14,000	6000	14,000

* *

Q2

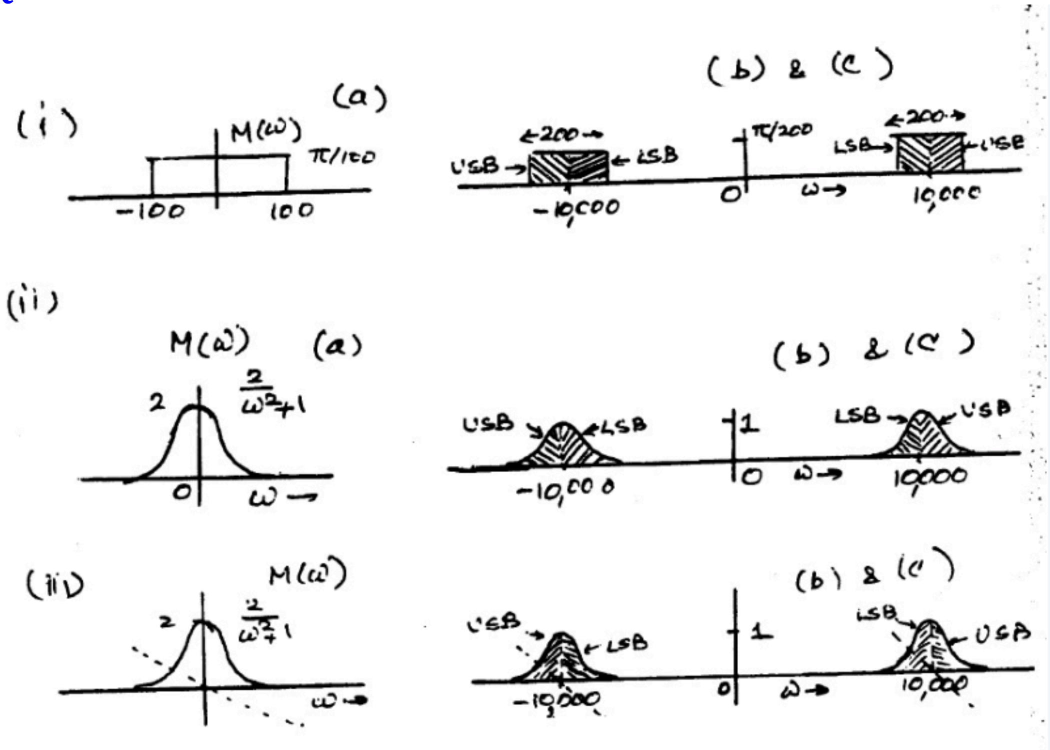

Fig. S4.2-2

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Q3

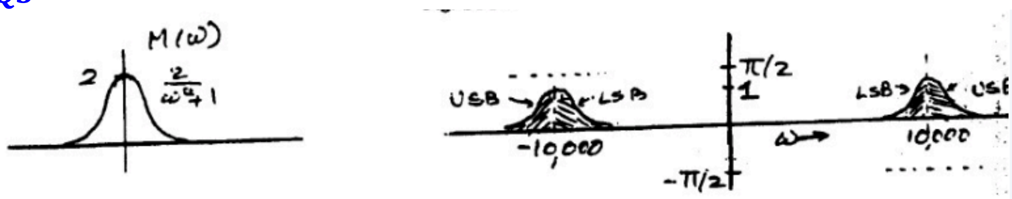

Fig. S4.2-3

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Q4

4.2-4 (a) The signal at point b is

$$
\begin{aligned}
g_a(t) &= m(t)\cos^3 \omega_c t \\
&= m(t)\left[\frac{3}{4}\cos\omega_c t + \frac{1}{4}\cos 3\omega_c t\right]
\end{aligned}
$$

The term $\frac{3}{4}m(t)\cos\omega_c t$ is the desired modulated signal, whose spectrum is centered at $\pm\omega_c$. The remaining term $\frac{1}{4}m(t)\cos 3\omega_c t$ is the unwanted term, which represents the modulated signal with carrier frequency $3\omega_c$ with spectrum centered at $\pm 3\omega_c$ as shown in Fig. S4.2-4. The bandpass filter centered at $\pm\omega_c$ allows to pass the desired term $\frac{3}{4}m(t)\cos\omega_c t$, but suppresses the unwanted term $\frac{1}{4}m(t)\cos 3\omega_c t$. Hence, this system works as desired with the output $\frac{3}{4}m(t)\cos\omega_c t$.

(b) Figure S4.2-4 shows the spectra at points b and c.

(c) The minimum usable value of ω_c is $2\pi B$ in order to avoid spectral folding at dc.

(d)

$$
\begin{aligned}
m(t)\cos^2\omega_c t &= \frac{m(t)}{2}\left[1 + \cos 2\omega_c t\right] \\
&= \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2\omega_c t
\end{aligned}
$$

The signal at point b consists of the baseband signal $\frac{1}{2}m(t)$ and a modulated signal $\frac{1}{2}m(t)\cos 2\omega_c t$, which has a carrier frequency $2\omega_c$ not the desired value ω_c. Both the components will be suppressed by the filter, whose center center frequency is ω_c. Hence, this system will not do the desired job.

(e) The reader may verify that the identity for $\cos n\omega_c t$ contains a term $\cos\omega_c t$ when n is odd. This is not true when n is even. Hence, the system works for a carrier $\cos^n \omega_c t$ only when n is odd.

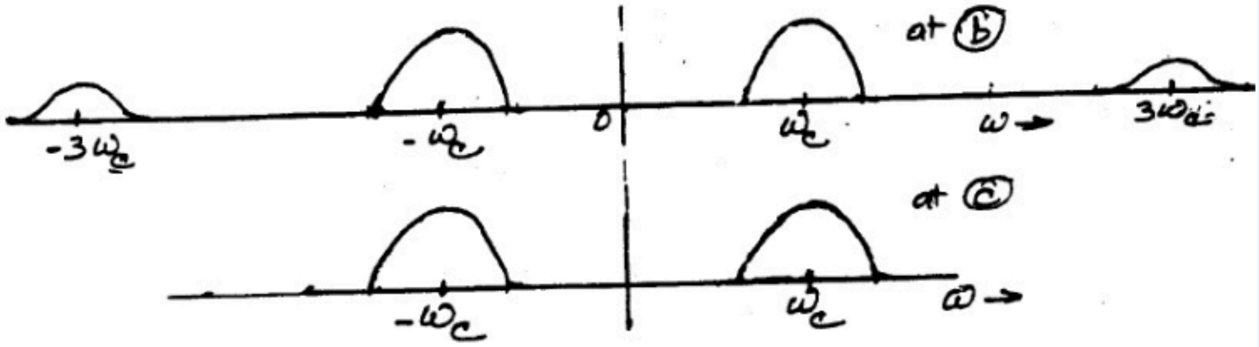

Fig. S4.2-4

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Q5

4.2-5 We use the ring modulator shown in Fig. 4.6 with the carrier frequency $f_c = 100$ kHz ($\omega_c = 200\pi \times 10^3$), and the output bandpass filter centered at $f_c = 300$ kHz. The output $v_i(t)$ is found in Eq. (4.7b) as

$$
v_i(t) = \frac{4}{\pi}\left[m(t)\cos\omega_c t - \frac{1}{3}m(t)\cos 3\omega_c t + \frac{1}{5}m(t)\cos 5\omega_c t + \cdots\right]
$$

The output bandpass filter suppresses all the terms except the one centered at 300 kHz (corresponding to the carrier $3\omega_c t$). Hence, the filter output is

$$
y(t) = \frac{-4}{3\pi}m(t)\cos 3\omega_c t
$$

This is the desired output $km(t)\cos\omega_c t$ with $k = -4/3\pi$.

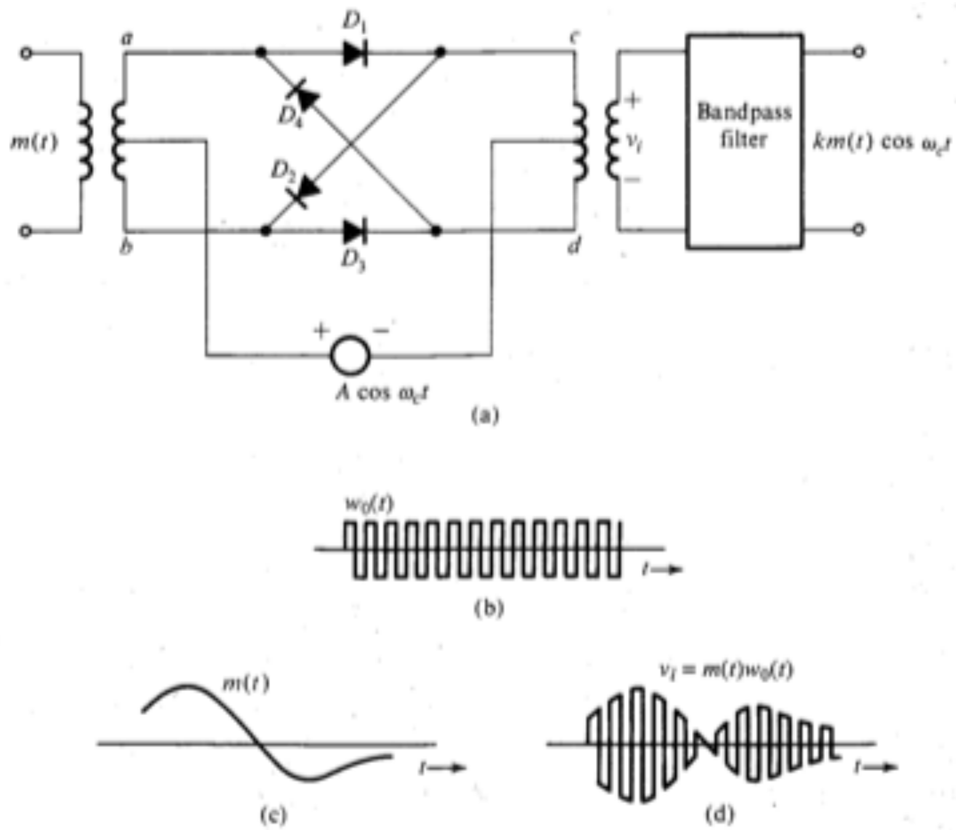

Figure 4.6 Ring modulator.

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