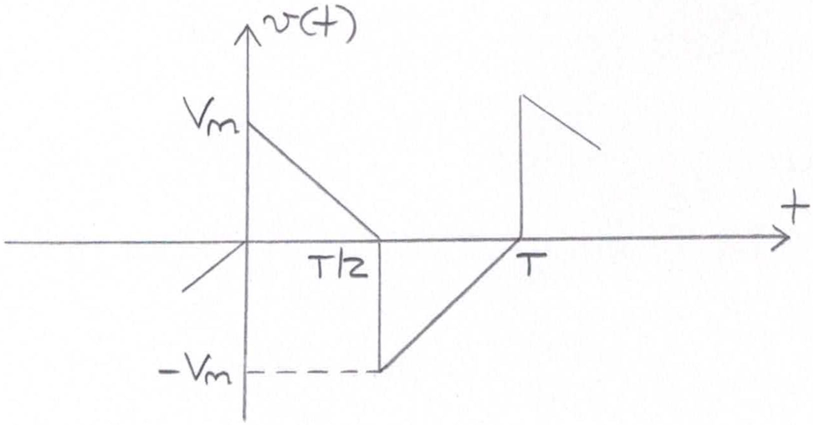
Problem 1) Consider the following periodic function shown as



$V_m = 378\pi$

a. Derive the Fourier series in trigonometric form.
b. Use the first five terms to estimate $v(T/8)$.

Solution.

–we shall express $v(t)$ in closed form as

$$v(t) = \begin{cases} -\frac{2V_m}{T}t + V_m, & 0 \le t < \frac{T}{2} \\ \frac{2V_m}{T}t - 2V_m, & \frac{T}{2} \le t < T \end{cases}$$

–and the trigonometric Fourier series representation is given by

$$v(t) = a_v + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t - \theta_n)$$

where

$$a_n - jb_n = \sqrt{a_n^2 + b_n^2}\,\angle{-\theta_n} = A_n \angle{-\theta_n}$$

–we calculate a_v as follows

$$a_v = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{T}\left[\int_0^{T/2}\left(-\frac{2V_m}{T}t + V_m\right)dt + \int_{T/2}^{T}\left(\frac{2V_m}{T}t - 2V_m\right)dt\right.$$

$$= \frac{1}{T}\left[\left(-\frac{V_m}{T}\right)t^2 + V_m t\,\Big|_0^{T/2} + \frac{V_m}{T}t^2 - 2V_m t\,\Big|_{T/2}^{T}\right]$$

$$= \frac{1}{\cancel{T}}\left[\left(-\frac{V_m}{\cancel{T}}\right)\frac{\cancel{T}^2}{4} + V_m\frac{\cancel{T}}{2} - 0 + \frac{V_m}{\cancel{T}}\cancel{T}^2 - 2V_m\cancel{T} - \frac{V_m}{\cancel{T}}\frac{\cancel{T}^2}{4} + 2V_m\frac{\cancel{T}}{2}\right]$$

$$= -\cancel{\frac{V_m}{4}} + \cancel{\frac{V_m}{2}} + \cancel{V_m} - \cancel{2V_m} - \cancel{\frac{V_m}{4}} + \cancel{V_m}$$

$$= 0$$

Note that ;

- we shall observe that $v(t)$ has half-wave symmetry

Hence ;

$$a_v = 0 \quad \text{(due to half-wave symmetry)}$$

$$a_n = 0 \;,\; b_n = 0 \quad \text{for even } n$$

$$a_n = \frac{4}{T}\int_0^{T/2} v(t)\cos n\omega_0 t\, dt \quad \text{for odd } n$$

$$b_n = \frac{4}{T}\int_0^{T/2} v(t)\sin n\omega_0 t\, dt \quad \text{for odd } n$$

- we calculate

$$a_n = \frac{4}{T}\int_0^{T/2}\left(-\frac{2V_m}{T}t + V_m\right)\cos n\omega_0 t\, dt$$

$$= -\frac{8V_m}{T^2}\int_0^{T/2} \underset{u}{t}\,\underbrace{\cos n\omega_0 t\, dt}_{dv} + \frac{4V_m}{T}\int_0^{T/2}\cos n\omega_0 t\, dt$$

integration by parts

$$= -\frac{8V_m}{T^2}\left[+ \frac{\sin n\omega_0 t}{n\omega_0}\Bigg|_0^{T/2} - \int_0^{T/2}\frac{\sin n\omega_0 t}{n\omega_0}\, dt\right]$$

$$+ \frac{4V_m}{T}\,\frac{\sin n\omega_0 t}{n\omega_0}\Bigg|_0^{T/2}$$

$$= -\frac{8V_m}{T^2}\left[\frac{\cancel{T}}{2}\,\frac{1}{n\omega_0}\sin\left(n\omega_0\frac{T}{2}\right) + \frac{\cos n\omega_0 t}{(n\omega_0)^2}\Big|_0\right]$$

$$+\frac{4V_m}{T}\,\frac{1}{n\omega_0}\sin\left(n\omega_0\frac{T}{2}\right)$$

$$= -\frac{2V_m}{n\pi}\underbrace{\cancel{\sin(n\pi)}}_{0} - \frac{8V_m}{T^2}\,\frac{1}{(n\omega_0)^2}\left(\cos n\pi - 1\right)$$

$$+\frac{2V_m}{n\pi}\underbrace{\cancel{\sin(n\pi)}}_{0} \quad , n \text{ is odd}$$

$$= \frac{\overset{4}{\cancel{16}}V_m}{n^2\cancel{4}\pi^2}$$

$$= \frac{4V_m}{\pi^2 n^2} \quad , n \text{ is odd}$$

Similaryly;

$$b_n = \frac{4}{T}\int_0^{T/2}\left(-\frac{2V_m}{T}t + V_m\right)\sin n\omega_0 t\,dt$$

$$= -\frac{8V_m}{T^2}\int_0^{T/2}\underset{u}{t}\,\underbrace{\sin n\omega_0 t\,dt}_{dv} + \frac{4V_m}{T}\int_0^{T/2}\sin n\omega_0 t\,dt$$

integration by parts

$$= -\frac{8V_m}{T^2}\left[-t\,\frac{\cos n\omega_0 t}{n\omega_0}\Big|_0^{T/2} - \int_0^{T/2} -\frac{\cos n\omega_0 t}{n\omega_0}\,dt\right]$$

$$-\frac{4V_m}{T}\,\frac{\cos n\omega_0 t}{n\omega_0}\Big|_0^{T/2}$$

$$= \frac{8V_m}{\cancel{T^2}}\left[\frac{\cancel{T}}{2}\,\frac{1}{n\omega_0}\cos n\pi - \frac{\sin n\omega_0 t}{(n\omega_0)^2}\Big|_0^{T/2}\right]$$

$$-\frac{4V_m}{n\omega_0 T}\left(\cos n\pi - 0\right) \quad , n \text{ is odd}$$

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$$= -\frac{2V_m}{\pi n} + 0 - 0 + \frac{4V_m}{\pi n}$$

$$= \frac{2V_m}{\pi n} \quad , n \text{ is odd}$$

As a result;

$$A_n \angle -\theta_n = a_n - j b_n$$

$$= \frac{4V_m}{\pi^2 n^2} - j \frac{2V_m}{\pi n} \quad , n \text{ is odd}$$

$$= \frac{2V_m}{\pi n} \left(\frac{2}{\pi n} - j \right)$$

$$= \frac{2.378\cancel{\pi}}{\cancel{\pi} n} \left(\frac{2}{\pi n} - j \right)$$

$$= \frac{756}{n} \left(\frac{2}{\pi n} - j \right)$$

Therefore;

$$v(t) = \sum_{n=1,3,5}^{\infty} A_n \cos(n w_o t - \theta_n)$$

b.

$$A_1 = 756 (0.6366 - j) = 896.1980 \angle -57.5184^\circ$$

$$A_3 = 252 (0.2122 - j) = 257.6112 \angle -78.0195^\circ$$

$$A_5 = 151.2 (0.1273 - j) = 152.4207 \angle -82.7439^\circ$$

$$A_7 = 108 (0.0909 - j) = 108.4457 \angle -84.8035^\circ$$

$$A_9 = 84 (0.0707 - j) = 84.2097 \angle -85.9555^\circ$$

$$v(T/8) = 896.1980 \cos(w_o T/8 - 57.5184)$$
$$+ 257.6112 \cos(3 w_o T/8 - 78.0195)$$
$$+ 152.4207 \cos(5 w_o T/8 - 82.7439)$$

$+ 108.4457 \cos(7\omega_0 T/8 - 84.8035)$

$+ 84.2097 \cos(9\omega_0 T/8 - 85.9559)$

$= 856.1980 \cos(45 - 57.5184) + 257.6112 \cos(135 - 78.0195)$

$+ 152.4207 \cos(225 - 82.7439) + 108.4457 \cos(315 - 84.8035)$

$+ 84.2097 \cos(405 - 85.9559)$

$= 888.92\ V$

- We also calculate from the defining equation of $v(t)$

$$v(T/8) = -\frac{2V_m}{T}\frac{T}{8} + V_m$$

$$= \frac{3}{4} V_m$$

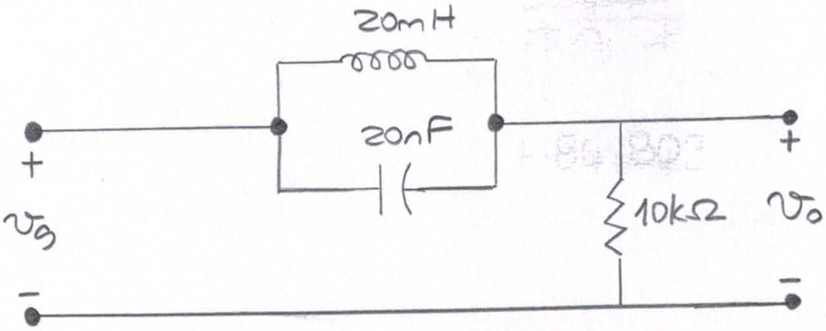
$$= \frac{3}{4} 378\pi$$

$$= 890.64\ V$$

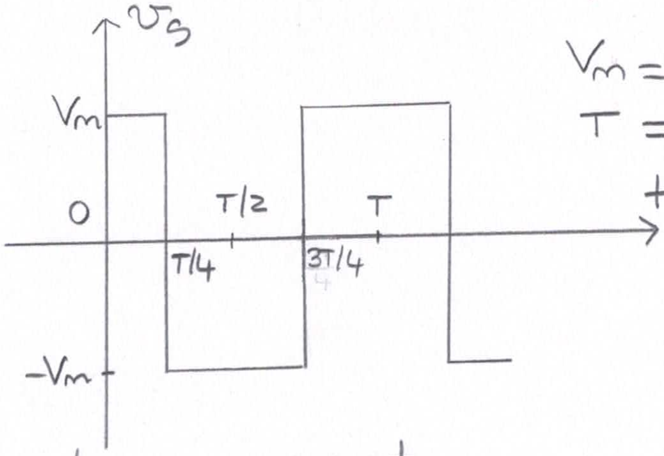
Hence;

$\frac{888.92}{890.64} \times 100 = 99.7957\ \%$ of $v(t)$ is obtained by the first five terms.

Problem 2) Consider the following circuit



The periodic square-wave voltage shown



$V_m = 210\pi$ V
$T = 0.2\pi$ ms

is applied to the above circuit.

a. Derive the first nonzero terms in the Fourier series that represents the steady-state voltage, v_o.

b. Which frequency component in the input voltage is eliminated from the output voltage?

Solution. Note that v_g does not have any symmetry, then

$$a_v = \frac{1}{T}\int_0^T v_g(t)\,dt$$

and

$$a_n = \frac{2}{T}\int_0^{T} v_g(t)\cos n\omega_0 t,$$

$$b_n = \frac{2}{T}\int_0^{T} v_g(t)\sin n\omega_0 t,$$

-we calculate

$$a_v = \frac{1}{T}\left[\int_0^{T/4} V_m\, dt + \int_{T/4}^{3T/4} -V_m\, dt + \int_{3T/4}^{T} V_m\, dt\right]$$

$$= \frac{V_m}{T}\left(t\Big|_0^{T/4} - t\Big|_{T/4}^{3T/4} + t\Big|_{3T/4}^{T} \right)$$

$$= \frac{V_m}{T}\left(\frac{T}{4} - 0 - 3\frac{T}{4} + \frac{T}{4} + T - 3\frac{T}{4} \right)$$

$$= 0$$

and

$$a_n = \frac{2}{T}\left[\int_0^{T/4} V_m \cos n\omega_0 t\, dt + \int_{T/4}^{3T/4} -V_m \cos n\omega_0 t\, dt + \int_{3T/4}^{T} V_m \cos n\omega_0 t\, dt\right]$$

$$= \frac{2V_m}{T}\left(\frac{\sin n\omega_0 t}{n\omega_0}\Big|_0^{T/4} - \frac{\sin n\omega_0 t}{n\omega_0}\Big|_{T/4}^{3T/4} + \frac{\sin n\omega_0 t}{n\omega_0}\Big|_{3T/4}^{T} \right)$$

$$= \frac{2V_m}{n\omega_0 T}\left(\sin\frac{n\pi}{2} - 0 - \sin\frac{3n\pi}{2} + \sin\frac{n\pi}{2} + \underbrace{\cancel{\sin 2\pi n}}_{0} - \sin\frac{3n\pi}{2} \right)$$

$$= \frac{\cancel{2}V_m}{\cancel{2}\pi n}\, 2\left(\sin\frac{n\pi}{2} - \sin\frac{3n\pi}{2} \right)$$

$$= \begin{cases} 0, & \text{for even } n \\ (-1)^{\frac{n-1}{2}} \dfrac{4V_m}{\pi n}, & \text{for odd } n \end{cases}$$

and

$$b_n = \frac{2}{T}\left[\int_0^{T/4} V_m \sin n\omega_0 t\, dt + \int_{T/4}^{3T/4} -V_m \cos n\omega_0 t\, dt + \int_{3T/4}^{T} V_m \cos n\omega_0 t\, dt\right]$$

$$= \frac{2V_m}{T}\left(-\frac{\cos n\omega_0 t}{n\omega_0}\Big|_0^{T/4} + \frac{\cos n\omega_0 t}{n\omega_0}\Big|_{T/4}^{3T/4} - \frac{\cos n\omega_0 t}{n\omega_0}\Big|_{3T/4}^{T} \right)$$

$$= \frac{2V_m}{n\omega_0 T}\left(-\cos\frac{n\pi}{2} + \cancel{1} + \cos\frac{3n\pi}{2} - \cos\frac{n\pi}{2} - \cancel{\cos 2n\pi} + \cos\frac{3n\pi}{2} \right)$$

$$= \frac{2V_m}{2\pi n}\, 2\left(-\cos\frac{n\pi}{2} + \cos\frac{3n\pi}{2}\right)$$

$$= 0 \text{ for all } n$$

Therefore ;

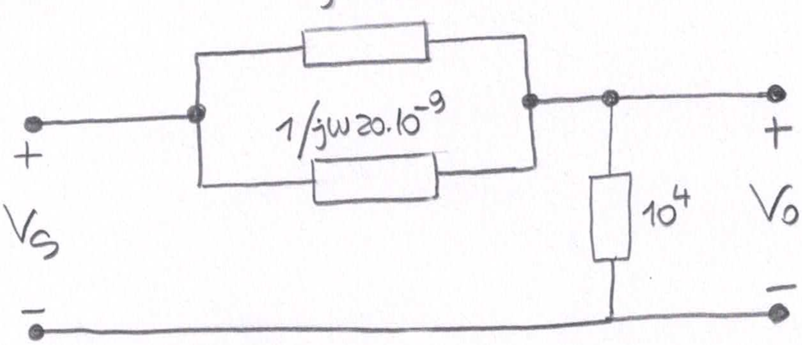
$$a_1 = (-1)^0 \frac{4 \cdot 210 \cancel{\pi}}{\cancel{\pi} \cdot 1} = 840 , \quad b_1 = 0 \quad \Rightarrow \quad V_{g1} = 840 + j \cdot 0$$
$$= 840 \angle 0^\circ$$

$$a_3 = (-1)^1 \frac{4 \cdot \overset{70}{\cancel{210}} \cancel{\pi}}{\cancel{\pi} \cdot \cancel{3}} = -280 , \quad b_3 = 0 \quad \Rightarrow \quad V_{g3} = -280 + j \cdot 0$$
$$= 280 \angle 180^\circ$$

$$a_5 = (-1)^2 \frac{4 \cdot \overset{42}{\cancel{210}} \cancel{\pi}}{\cancel{\pi} \cdot \cancel{5}} = 168 , \quad b_5 = 0 \quad \Rightarrow \quad V_{S5} = 168 + j \cdot 0$$
$$= 168 \angle 0^\circ$$

$$a_7 = (-1)^3 \frac{4 \cdot \overset{30}{\cancel{210}} \cancel{\pi}}{\cancel{\pi} \cdot \cancel{7}} = -120 , \quad b_7 = 0 \quad \Rightarrow \quad V_{S7} = -120 + j \cdot 0$$
$$= 120 \angle 180^\circ$$

-it follows from phasor domain approach that we have



$$j0.02\omega \,//\, \frac{1}{j\omega \cdot 20.10^{-9}} : \quad \frac{j \cdot 0.02\omega \left(1/j\omega . 2.10^{-8}\right)}{j0.02\omega + \dfrac{1}{j\omega \cdot 2.10^{-8}}} = \frac{j \cdot 0.02\omega}{-4\omega^2 . 10^{-10} + 1}$$

$$\Rightarrow \quad \frac{V_o}{V_S} = \frac{10^4}{10^4 + \dfrac{j \cdot 0.02\omega}{-4\omega^2 . 10^{-10} + 1}} = \frac{10^4\left(-4\omega^2 . 10^{-10} + 1\right)}{-4\omega^2 . 10^{-6} + 10^4 + j \cdot 0.02\omega}$$

$$= \frac{\omega^2 - 25.10^8}{\omega^2 - j5000\omega - 25.10^8} \triangleq H(\omega)$$

Hence;

$$\omega_0 = \frac{2\cancel{\pi}}{0.2\cancel{\pi}.10^{-3}} = 10^4$$

$$H(1.10^4) = \frac{10^8 - 25.10^8}{10^8 - j5.10^7 - 25.10^8} = \frac{-24.\cancel{10^8}}{-24.\cancel{10^8} - j.5.\cancel{10^7}} = \frac{240}{240+j5}$$

$$= 0.9996 - j0.0208$$

$$= 0.9998 \angle -1.1935^\circ$$

$$H(3.10^4) = \frac{9.10^8 - 25.10^8}{9.10^8 - j15.10^7 - 25.10^8} = \frac{-16.10^8}{-16.10^8 - j15.10^7} = \frac{160}{160+j15}$$

$$= 0.9913 - j0.0929$$

$$= 0.9956 \angle -5.3558^\circ$$

$$H(5.10^4) = \frac{25.10^8 - 25.10^8}{25.10^8 - j25.10^7 - 25.10^8} = 0$$

$$H(7.10^4) = \frac{49.10^8 - 25.10^8}{49.10^8 - j35.10^7 - 25.10^8} = \frac{24.10^8}{24.10^8 - j35.10^7} = \frac{240}{240-j35}$$

$$= 0.9792 - j0.1428$$

$$= 0.9895 \angle -8.2971^\circ$$

As a result;

$$V_{oi} = V_{gi}.H(i.10^4), \quad i = 1,3,5,\ldots$$

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$$V_{o1} = 840 \angle 0^\circ \cdot 0.9998 \cdot \angle -1.1935$$

$$= 839.8320 \angle -1.1935$$

$$v_{o1}(t) = 839.8320 \cos(10^4 t - 1.1935) \ V$$

$$V_{o3} = 280 \angle 180^\circ \cdot 0.9956 \cdot \angle -5.3558$$

$$= 278.7680 \angle 174.6442$$

$$v_{o3}(t) = 278.7680 \cos(3 \cdot 10^4 t + 174.6442) \ V$$

$$V_{o5} = 168 \angle 0^\circ \cdot 0 = 0, \quad v_{o5}(t) = 0 \ V$$

$$V_{o7} = 120 \angle 180^\circ \cdot 0.9895 \cdot \angle -8.2971$$

$$= 118.7400 \angle 171.7029$$

$$v_{o7}(t) = 118.74 \cos(7 \cdot 10^4 t + 171.7029) \ V$$

$$\therefore \ v_o(t) = 839.8320 \cos(10^4 t - 1.1935)$$
$$+ 278.7680 \cos(3 \cdot 10^4 t + 174.6442)$$
$$+ 0 + 118.74 \cos(7 \cdot 10^4 t + 171.7029) + \ldots$$

b. The 5^{th} harmonic frequency component in the input voltage is eliminated from the output voltage.

That is ;

$$\omega = 5 \cdot 10^4 = 50 \text{ krad/s} \rightarrow \text{rejected}$$

Because, the circuit is a passive bandreject filter with a centre frequency of 50 krad/s

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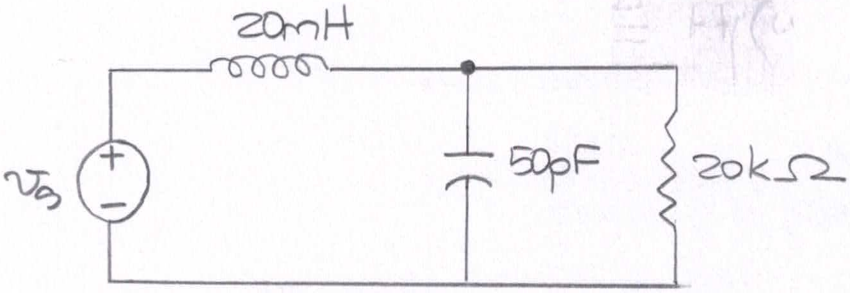
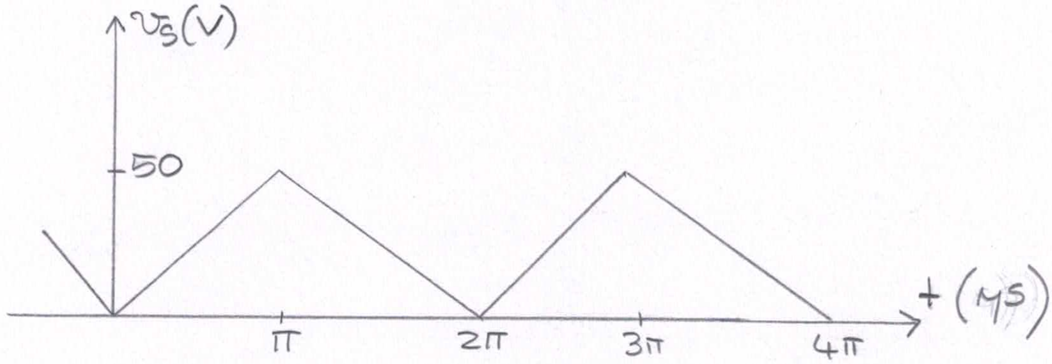
Problem 3) A triangular-wave voltage source is applied to the circuit shown as



The triangular-wave voltage is depicted as follows:



Estimate the average power delivered to the 20kΩ resistor when the circuit is in steady-state operation.

Solution. Note v_g is an even function which can be defined in one period of interval as follows:

$$v_g = \begin{cases} V_m t, & 0 \leq t < \pi\cdot10^{-6} \\ V_m(-t+2\pi\cdot10^{-6}), & \pi\cdot10^{-6} \leq t \leq 2\pi\cdot10^{-6} \end{cases} \qquad V_m = \frac{50}{\pi}10^{6}, \quad \omega_0 = \frac{2\pi}{2\pi\cdot10^{-6}} = 10^{6}\ \text{rad/s}$$

$$a_v = \frac{V_m}{2\pi}\left[\int_0^{\pi\cdot10^{-6}} t\,dt + \int_{\pi\cdot10^{-6}}^{2\pi\cdot10^{-6}} (-t+2\pi)\,dt\right]$$

$$= \frac{V_m}{2\pi\cdot10^{-6}}\left(\frac{1}{2}t^2\Big|_0^{\pi\cdot10^{-6}} - \frac{1}{2}t^2\Big|_{\pi\cdot10^{-6}}^{2\pi\cdot10^{-6}} + 2\pi t\Big|_{\pi\cdot10^{-6}}^{2\pi\cdot10^{-6}}\right)$$

$$= \frac{V_m}{2\pi\cdot10^{-6}}\left(\frac{1}{2}\pi^2 - 0 - \frac{1}{2}4\pi^2 + \frac{1}{2}\pi^2 + 4\pi^2 - 2\pi^2\right)10^{-12}$$

$$= \frac{V_m}{2\pi}\pi^2\cdot10^{-6}$$

$$= 25V$$

$$a_n = \frac{2}{T}\left[\int_0^{T/2} V_m \underset{\downarrow\, u}{t}\,\underbrace{\cos(n\omega_0 t)\,dt}_{dv} + \int_{T/2}^{T} V_m(-t+2\pi)\cos(n\omega_0 t)\,dt\right]$$

-using integration by parts gives

$$a_n = \frac{2V_m}{T}\left(\frac{t\sin n\omega_0 t}{n\omega_0}\Bigg|_0^{T/2} - \int_0^{T/2}\frac{\sin n\omega_0 t}{n\omega_0}\,dt\right.$$

$$-\frac{t\sin n\omega_0 t}{n\omega_0}\Bigg|_{T/2}^{T} + \int_{T/2}^{T}\frac{\sin n\omega_0 t}{n\omega_0}\,dt$$

$$\left.+\,2\pi\,\frac{\sin n\omega_0 t}{n\omega_0}\Bigg|_{T/2}^{T}\right)$$

$$= \frac{2V_m}{T}\left[0-0+\frac{\cos n\omega_0 t}{(n\omega_0)^2}\Bigg|_0^{T/2} - 0 + 0\right.$$

$$\left.-\frac{\cos n\omega_0 t}{(n\omega_0)^2}\Bigg|_{T/2}^{T} + 0 - 0\right]$$

$$= \frac{2V_m}{T(n\omega_0)^2}\left(\cos n\pi - 1 - \cos n2\pi + \cos n\pi\right)$$

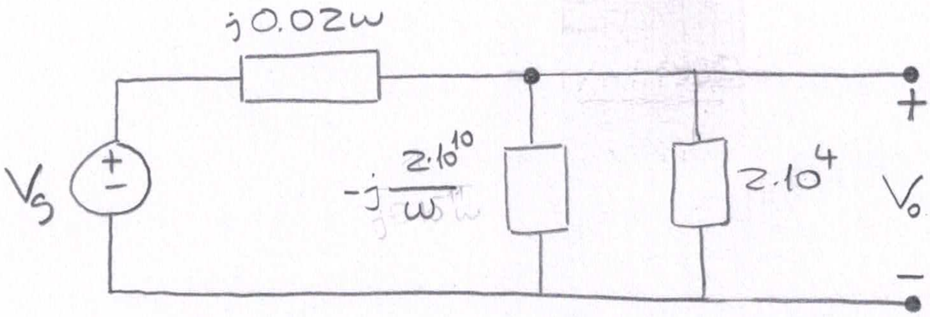
$$= \frac{2V_m}{T(n\omega_0)^2}\,2\left(\cos n\pi - 1\right)$$

$$= \begin{cases} 0, & \text{for even } n \\ -\dfrac{200}{\pi^2 n^2}, & \text{for odd } n \end{cases}$$

and

$$b_n = 0, \text{ for all } n$$

-using phasor domain approach allows to get



$$-j\frac{2\cdot10^{10}}{\omega} \,//\, 2\cdot10^{4} \Rightarrow \frac{-j\frac{4\cdot10^{14}}{\omega}}{-j\frac{2\cdot10^{10}}{\omega}+2\cdot10^{4}} = \frac{-j4\cdot10^{14}}{2\cdot10^{4}\omega-j2\cdot10^{10}}$$

$$\Rightarrow \frac{V_o}{V_s} = \frac{-j\cdot4\cdot10^{14}\,/\,(2\cdot10^{4}\omega-j\cdot2\cdot10^{10})}{j\cdot0.02\omega-\frac{j4\cdot10^{14}}{2\cdot10^{4}\omega-j\cdot2\cdot10^{10}}}$$

$$= \frac{-j4\cdot10^{14}}{j\cdot0.02\omega\left(2\cdot10^{4}\omega-j\cdot2\cdot10^{10}\right)-j4\cdot10^{14}}$$

$$= \frac{-j\cdot4\cdot10^{14}}{j4\cdot10^{2}\omega^{2}+4\cdot10^{8}\omega+j4\cdot10^{14}}$$

$$= \frac{-10^{12}}{\omega^{2}-j10^{6}\omega-10^{12}}$$

$$\triangleq H(\omega)$$

Hence;

$$H(1\cdot10^{6}) = \frac{-10^{12}}{\cancel{10^{12}}-j\cdot10^{12}-\cancel{10^{12}}} = -j = 1\angle-90^{\circ}$$

$$H(3\cdot10^6) = \frac{-10^{12}}{9\cdot10^{12} - j3\cdot10^{12} - 10^{12}} = \frac{-1}{8-j3}$$

$$= -0.1096 - j0.0411$$

$$= 0.1170 \angle -159.4440^\circ$$

$$H(5\cdot10^6) = \frac{-10^{12}}{25\cdot10^{12} - j5\cdot10^{12} - 10^{12}} = \frac{-1}{24-j5}$$

$$= -0.0399 - j0.0083$$

$$= 0.0408 \angle -168.2317^\circ$$

$\vdots$

and

$$H(0) = \frac{-10^{12}}{0-0-10^{12}} = 1$$

$$a_1 = -\frac{200}{\pi^2} = -20.2642 \Rightarrow V_{S1} = -20.2642 + j\cdot 0 = 20.2642 \angle 180^\circ$$

$$a_3 = -\frac{200}{9\pi^2} = -2.2516 \Rightarrow V_{S3} = -2.2516 + j\cdot 0 = 2.2576 \angle 180^\circ$$

$$a_5 = -\frac{200}{25\pi^2} = -0.8106 \Rightarrow V_{S5} = -0.8106 + j\cdot 0 = 0.8106 \angle 180^\circ$$

$$a_7 = -\frac{200}{49\pi^2} = -0.4136 \Rightarrow V_{S7} = -0.4136 + j\cdot 0 = 0.4136 \angle 180^\circ$$

Hence;

$$v_o(t) = 25 + 20.2642\cos(10^6 t - 90^\circ) - 0.2634\cos(3\cdot10^6 t - 159.4440)$$
$$-0.0331\cos(5\cdot10^6 t - 168.2317) - \ldots$$

- it appears that only the first four terms are effective in the rms value of $v_o(t)$

That is ;

$$V_{orms} = \sqrt{25^2 + \frac{1}{2}\left(20.2642^2 + 0.2634^2 + 0.0331^2\right)}$$

$$= 28.8159 \text{ V (rms)}$$

As a result ;

$$P_{av} = \frac{28.8159^2}{2.10^4} = 0.0415 \text{ W}$$

Problem 4) The voltage and current at the terminals of a network are

$$v = 80 + 200\cos(500t + 45) + 60\sin 1500t \text{ V}$$

$$i = 10 + 6\sin(500t + 75) + 3\cos(1500t + 30) \text{ A}$$

The current is in the direction of the voltage drop across the terminals.

a. What is the average power at the terminals?

b. What is the rms value of the voltage?

c. What is the rms value of the current?

Solution.a. We shall rewrite v and i in Fourier series representation of trigonometric form as

$$v = 80 + 200\cos(500t + 45) + 60\cos(1500t - 90)$$

$$i = 10 + 6\cos(500t - 15) + 3\cos(1500t + 30)$$

-we know that

$$P = V_{dc} I_{dc} + \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n \cos(\theta_{vn} - \theta_{in})$$

$$= 80 \cdot 10 + \frac{1}{2}\left(200 \cdot 6 \cos(60) + 60 \cdot 3 \cos(-120)\right)$$

$$= 800 + \frac{1}{2}(600 - 90)$$

$$= 1055 \text{ W}$$

b.

$$v_{rms} = \sqrt{80^2 + \left(\frac{200}{\sqrt{2}}\right)^2 + \left(\frac{60}{\sqrt{2}}\right)^2}$$

$$= 167.9286 \text{ V}$$

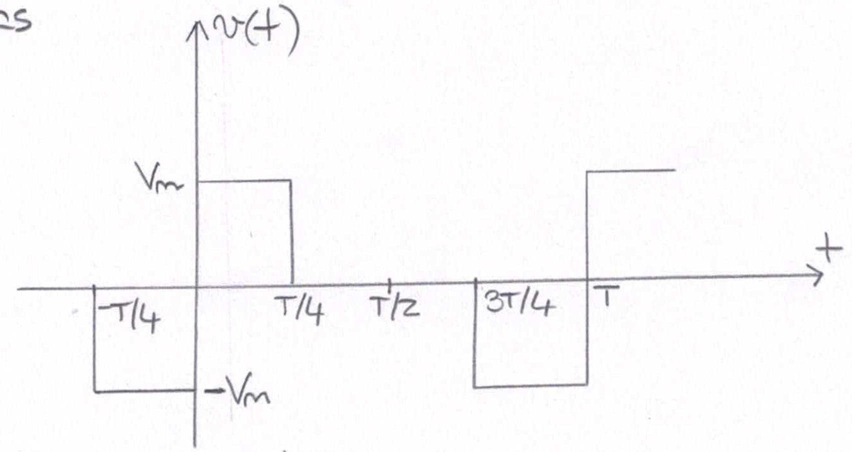
c.

$$i_{rms} = \sqrt{10^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2}$$

$$= \sqrt{100 + 22.5}$$

$$= 11.0680 \text{ A}$$

Problem 5) Use the exponential form of the Fourier series to write an expression for the voltage shown as



Solution. We shall write

$$v(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T}$$

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with

$$C_n = \frac{1}{T}\int_{-T/4}^{3T/4} v(t)e^{-jn\omega_0 t}\,dt \quad , \quad n \neq 0$$

$$= \frac{1}{T}\left[\int_0^{T/4} V_m e^{-jn\omega_0 t}\,dt + \int_{-T/4}^{0}(-V_m)e^{-jn\omega_0 t}\,dt\right]$$

$$= \frac{V_m}{T}\left(\left.\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right|_0^{T/4} - \left.\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right|_{-T/4}^{0}\right)$$

$$= \frac{V_m}{-jn2\pi}\left(e^{-jn\frac{\pi}{2}} - 1 - 1 + e^{jn\frac{\pi}{2}}\right)$$

$$= -\frac{V_m}{jn2\pi}\left(2\cos\frac{n\pi}{2} - 2\right)$$

$$= \frac{V_m}{jn\pi}\left(1 - \cos\frac{n\pi}{2}\right)$$

and

$$C_0 = \frac{1}{T}\int_{-T/4}^{3T/4} v(t)\,dt$$

$$= \frac{1}{T}\left(\int_0^{T/4} V_m\,dt + \int_{-T/4}^{0} -V_m\,dt\right)$$

$$= \frac{V_m}{T}\left(\left. t\right|_0^{T/4} - \left. t\right|_{-T/4}^{0}\right)$$

$$= \frac{V_m}{T}\left(\frac{T}{4} - 0 - 0 + \frac{T}{4}\right)$$

$$= 0$$

Hence;

$$C_n = \begin{cases} 0, & \text{if } n \text{ is odd or zero} \\ \dfrac{V_m}{jn\pi}\left[1-(-1)^{n/2}\right], & \text{if } n \text{ is even} \end{cases}$$

OR

$$C_n = \begin{cases} \dfrac{2V_m}{jn\pi} & \text{if } n \text{ is even but } \frac{n}{2} \text{ is odd} \\ 0 & \text{else} \end{cases}$$

As a result;

$$v(t) = \sum_{\substack{n \text{ is even} \\ \frac{n}{2} \text{ is odd}}} \frac{2V_m}{jn\pi} e^{jn\omega_0 t}$$

$$= \sum_{k=-1,-3,-5,\ldots}^{1,3,5,\ldots} \frac{V_m}{jk\pi} e^{j2k\omega_0 t}$$