

23. Find the transfer function,  $G(s) = X_1(s)/F(s)$ , for the translational mechanical system shown in Figure P2.9. [Section: 2.5]

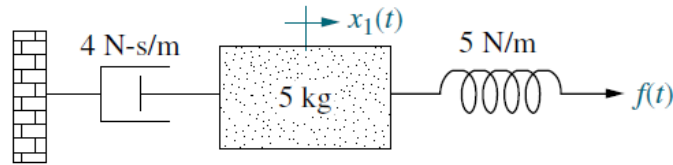


FIGURE P2.9

24. Find the transfer function,  $G(s) = X_2(s)/F(s)$ , for the translational mechanical network shown in Figure P2.10. [Section: 2.5]

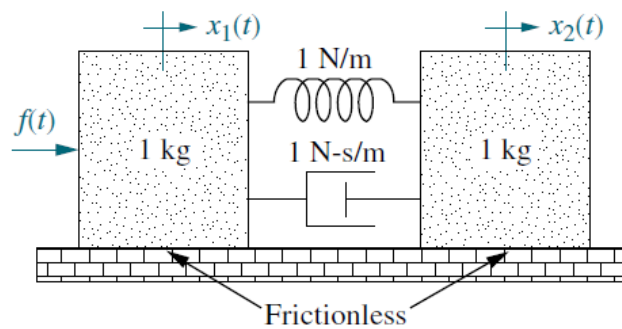


FIGURE P2.10

25. Find the transfer function,  $G(s) = X_2(s)/F(s)$ , for the translational mechanical system shown in Figure P2.11. (Hint: place a zero mass at  $x_2(t)$ .) [Section: 2.5]

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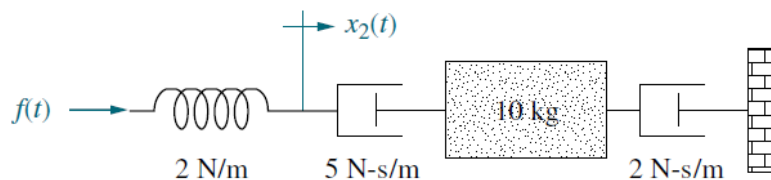


FIGURE P2.11

30. For each of the rotational mechanical systems shown in Figure P2.16, write, but do not solve, the equations of motion. [Section: 2.6]

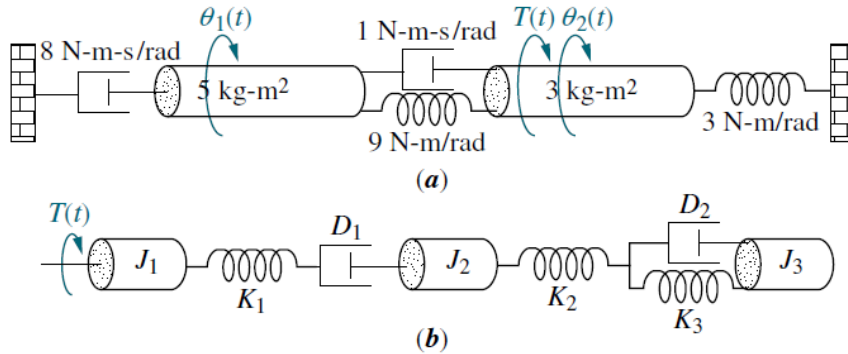


FIGURE P2.16

31. For the rotational mechanical system shown in Figure P2.17, find the transfer function  $G(s) = \theta_2(s)/T(s)$  [Section: 2.6]

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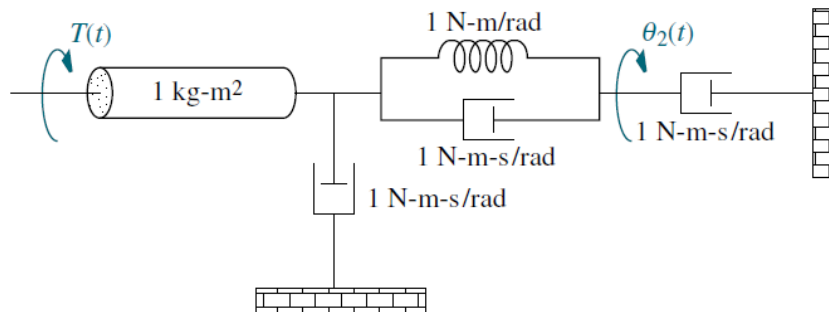


FIGURE P2.17

33. For the rotational system shown in Figure P2.19, find the transfer function,  $G(s) = \theta_2(s)/T(s)$ . [Section: 2.7]

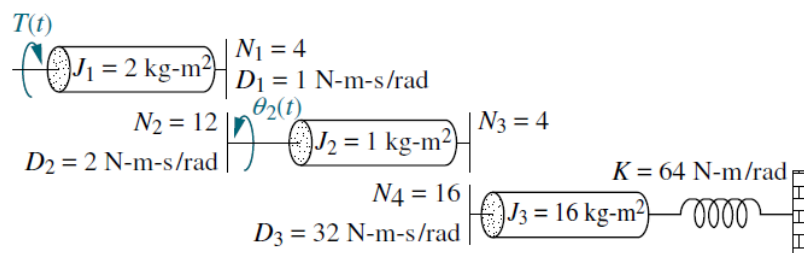


FIGURE P2.19

23.

Writing the equations of motion, where  $x_2(t)$  is the displacement of the right member of spring,

$$(5s^2+4s+5)X_1(s) - 5X_2(s) = 0$$

$$-5X_1(s) + 5X_2(s) = F(s)$$

Adding the equations,

$$(5s^2+4s)X_1(s) = F(s)$$

From which,  $\frac{X_1(s)}{F(s)} = \frac{1}{s(5s+4)} = \frac{1/5}{s(s+4/5)}$ .

24.

Writing the equations of motion,

$$(s^2 + s + 1)X_1(s) - (s + 1)X_2(s) = F(s)$$

$$-(s + 1)X_1(s) + (s^2 + s + 1)X_2(s) = 0$$

Solving for  $X_2(s)$ ,

$$X_2(s) = \frac{\begin{vmatrix} (s^2 + s + 1) & F(s) \\ -(s + 1) & 0 \end{vmatrix}}{\begin{vmatrix} (s^2 + s + 1) & -(s + 1) \\ -(s + 1) & (s^2 + s + 1) \end{vmatrix}} = \frac{(s + 1)F(s)}{s^2(s^2 + 2s + 2)}$$

From which,

$$\frac{X_2(s)}{F(s)} = \frac{(s + 1)}{s^2(s^2 + 2s + 2)}.$$

25.

Let  $X_1(s)$  be the displacement of the left member of the spring and  $X_3(s)$  be the displacement of the mass.

Writing the equations of motion

$$2x_1(s) - 2x_2(s) = F(s)$$

$$-2X_1(s) + (5s + 2)X_2(s) - 5sX_3(s) = 0$$

$$-5sX_2(s) + (10s^2 + 7s)X_3(s) = 0$$

Solving for  $X_2(s)$ ,

$$X_2(s) = \frac{\begin{vmatrix} 5s^2+10 & F(s) \\ -10 & 0 \end{vmatrix}}{\begin{vmatrix} 5s^2+10 & -10 \\ -10 & \frac{1}{5}s+10 \end{vmatrix}} = \frac{10F(s)}{s(s^2+50s+2)}$$

$$\text{Thus, } \frac{X_2(s)}{F(s)} = \frac{1}{10} \frac{(10s + 7)}{s(5s + 1)}$$

30.

a.

Writing the equations of motion,

$$\begin{aligned}(5s^2 + 9s + 9)\theta_1(s) - (s + 9)\theta_2(s) &= 0 \\ -(s + 9)\theta_1(s) + (3s^2 + s + 12)\theta_2(s) &= T(s)\end{aligned}$$

b.

Defining

$\theta_1(s)$  = rotation of  $J_1$

$\theta_2(s)$  = rotation between  $K_1$  and  $D_1$

$\theta_3(s)$  = rotation of  $J_3$

$\theta_4(s)$  = rotation of right - hand side of  $K_2$

the equations of motion are

$$\begin{aligned}(J_1s^2 + K_1)\theta_1(s) - K_1\theta_2(s) &= T(s) \\ -K_1\theta_1(s) + (D_1s + K_1)\theta_2(s) - D_1s\theta_3(s) &= 0 \\ -D_1s\theta_2(s) + (J_2s^2 + D_1s + K_2)\theta_3(s) - K_2\theta_4(s) &= 0 \\ -K_2\theta_3(s) + (D_2s + (K_2 + K_3))\theta_4(s) &= 0\end{aligned}$$

31.

Writing the equations of motion,

$$(s^2 + 2s + 1)\theta_1(s) - (s + 1)\theta_2(s) = T(s)$$

$$-(s + 1)\theta_1(s) + (2s + 1)\theta_2(s) = 0$$

Solving for  $\theta_2(s)$

$$\theta_2(s) = \frac{\begin{vmatrix} (s^2 + 2s + 1) & T(s) \\ -(s + 1) & 0 \end{vmatrix}}{\begin{vmatrix} (s^2 + 2s + 1) & -(s + 1) \\ -(s + 1) & (2s + 1) \end{vmatrix}} = \frac{T(s)}{2s(s + 1)}$$

Hence,

$$\frac{\theta_2(s)}{T(s)} = \frac{1}{2s(s + 1)}$$

33.

Reflecting all impedances to  $\theta_2(s)$ ,

$$\left\{ \left[ J_2 + J_1 \left( \frac{N_2}{N_1} \right)^2 + J_3 \left( \frac{N_3}{N_4} \right)^2 \right] s^2 + \left[ f_2 + f_1 \left( \frac{N_2}{N_1} \right)^2 + f_3 \left( \frac{N_3}{N_4} \right)^2 \right] s + \left[ K \left( \frac{N_3}{N_4} \right)^2 \right] \right\} \theta_2(s) = T(s) \frac{N_2}{N_1}$$

Substituting values,

$$\left\{ \left[ 1 + 2(3)^2 + 16 \left( \frac{1}{4} \right)^2 \right] s^2 + \left[ 2 + 1(3)^2 + 32 \left( \frac{1}{4} \right)^2 \right] s + 64 \left( \frac{1}{4} \right)^2 \right\} \theta_2(s) = T(s)(3)$$

Thus,

$$\frac{\theta_2(s)}{T(s)} = \frac{3}{20s^2 + 13s + 4}$$