

Last lecture's unresolved problem:

May 26, 2018

Question:

Consider the following grammar with the productions

$$\begin{aligned} S &\rightarrow aBCDe \\ B &\rightarrow \varepsilon \mid b \mid CD \\ C &\rightarrow \varepsilon \mid c \mid DB \\ D &\rightarrow \varepsilon \mid BCS \end{aligned}$$

Construct an equivalent grammar which has no ε -productions. You should describe the process by which you obtain the new grammar.

Solution:

Eliminate $D \rightarrow \varepsilon$ using our ε elimination procedure:

$$\begin{aligned} S &\rightarrow aBCDe \mid aBCe \\ B &\rightarrow \varepsilon \mid b \mid CD \mid C \\ C &\rightarrow \varepsilon \mid c \mid DB \mid B \\ D &\rightarrow BCS \end{aligned}$$

Now eliminate $B \rightarrow C$ and $C \rightarrow B$ (which in fact means they correspond to the same variable):

$$\begin{aligned} S &\rightarrow aBCDe \mid aBCe \\ B &\rightarrow \varepsilon \mid b \mid CD \mid c \mid DB \\ C &\rightarrow \varepsilon \mid c \mid DB \mid b \mid CD \\ D &\rightarrow BCS \end{aligned}$$

Now, we see that in the above grammar B and C generate exactly the same right-hand side. Thus, they are the *same* variable (generating the same sequence of terminals). Then replace both B and C with the new variable X .

$$\begin{aligned} S &\rightarrow aXXDe \mid aXXe \\ X &\rightarrow \varepsilon \mid b \mid XD \mid c \mid DX \\ D &\rightarrow XXS \end{aligned}$$

Now eliminate $X \rightarrow \varepsilon$:

$$\begin{aligned} S &\rightarrow aXXDe \mid aXXe \mid aXDe \mid aDe \mid aXe \mid ae \\ X &\rightarrow b \mid XD \mid c \mid DX \mid D \\ D &\rightarrow XXS \mid XS \mid S \end{aligned}$$

We are done.