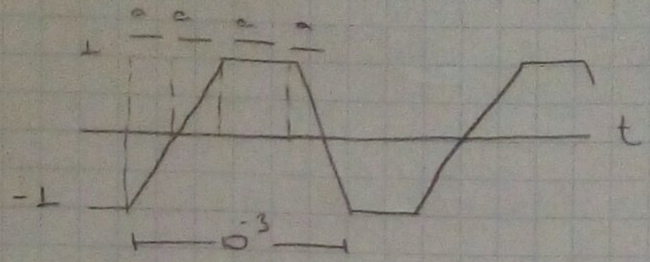
31.04.2018 (PS VII)

Q1) Sketch $\varphi_{FM}(t)$ and $\varphi_{PM}(t)$ for $m(t)$

$w_c = 2\pi \times 10^7 \quad k_p = 25 \times 2\pi \quad k_f = 10^5 \times 2\pi$



$w_c = 10^7 \times 2\pi \quad f_c = 10^7 = 10\ MHz$

Frequency deviation $= \Delta f = k_f . m_p / 2\pi \qquad (m_p = 1)$

$$\dot{m}_p = [\dot{m}(t)]_{max}$$

$$B_{PM} = 2(\Delta f + B)$$

$$\Delta f = \frac{10^5 \times 2\pi \times 1}{2\pi} = 10^5\ Hz.$$

$$f_c + \Delta f = 10.1\ MHz$$

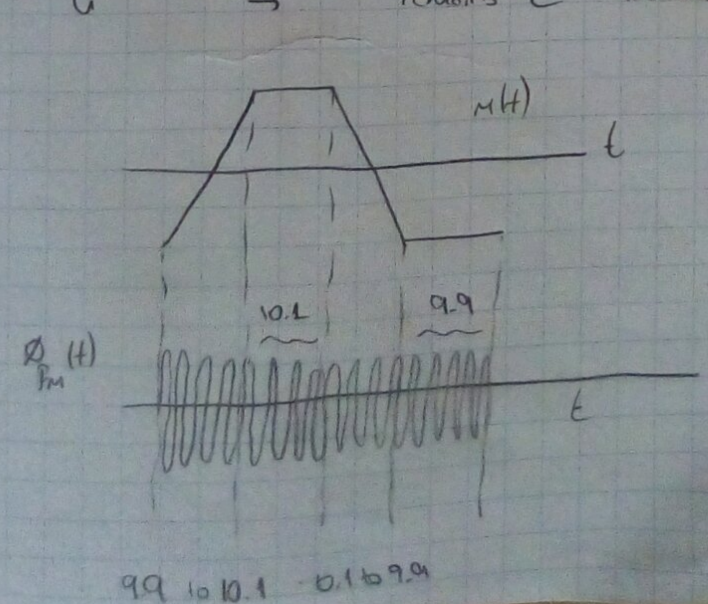
$$f_c - \Delta f = 9.9\ MHz$$

1st quarter → freq. linearly inc. from 9.9 to 10.1 MHz

2nd " $(m(t)=1)$ → " remains @ 10.1 MHz for a sec.

3rd " → freq. dec. lin. from 10.1 to 9.9 MHz.

4th " → " remains @ 9.9 MHz.

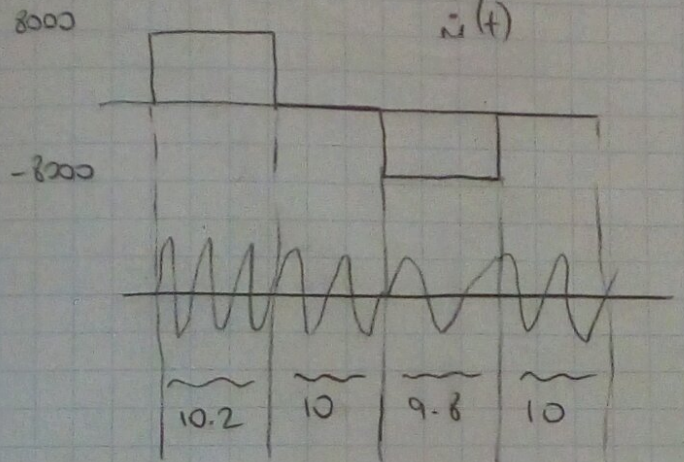


for PM → $\Delta f = \dfrac{k_p \cdot \dot{m}_p}{2\pi}$ $\qquad \dot{m}_p = \dfrac{2}{10^{-3}/4} = 8000$

$$\Delta f = \frac{50\pi \cdot 8000}{2\pi} = 2 \cdot 10^5 \text{ Hz}$$

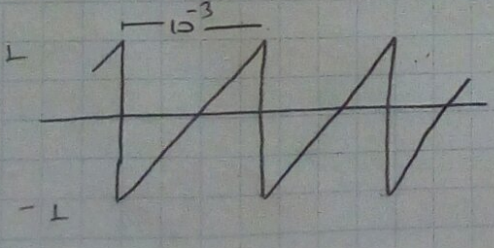
$$(f_c)_{max} = f_c + \Delta f = 10^7 + 2\times10^5 = 10.2 \text{ MHz}$$

$$(f_c)_{min} = f_c - \Delta f = 10^7 - 2\times10^5 = 9.8 \text{ MHz}$$



Q2) $\omega_c = 2\pi\times10^6 \rightarrow f_c = 10^6$ Hz (1 MHz)

$k_f = 20000\pi \qquad k_p = \pi/2 \qquad (k_p < \pi)$



$m_p = 1$

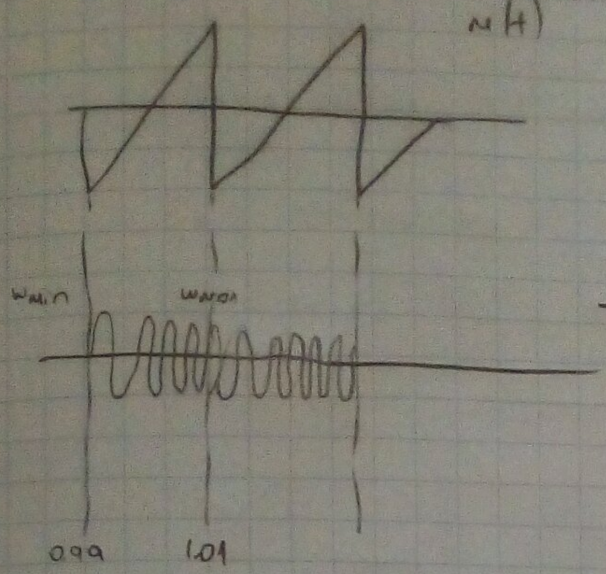
$\dot{m}_p = (1-(-1))/10^{-3} = 2\cdot10^3 = 2000$

FM → $\Delta f = \dfrac{k_f \cdot m_p}{2\pi} = \dfrac{20000\pi\times1}{2\pi} = 10000 = 10^4$ Hz

$f_c = 1$ MHz

$f_{c\,max} = 10^6 + 10^4 = 1.01$ MHz

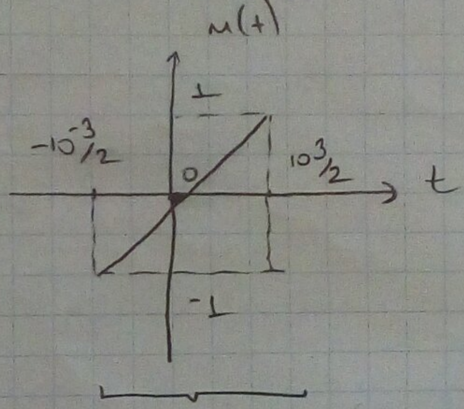
$f_{c\,min} = 10^6 - 10^4 = 0.99$ MHz



→ frequency of the signal increases from 0.99 to 1.01 MHz as m(t) signal linearly increases. At the pt. of jump. disc., freq. drops to its min., 0.99 MHz.

↳ Sudden drop to 0.99 MHz.
Linear increase to 1.01 MHz.

PM → Jump Discontinuity →



$T = 10^{-3}$

m(t) within this interval,

$$m(t) = 2000t$$

$$(t = 10^{-3}/2 \rightarrow m(t) = 1)$$

$$(t = -10^{-3}/2 \rightarrow m(t) = -1)$$

$$\varphi_{PM}(t) = A.\cos[w_c t + k_p . m(t)] \qquad w_c = 10^6 \times 2\pi$$

$$= \cos[2\pi (10)^6 t + \frac{\pi}{2} . m(t)]$$

$$= \cos[2\pi . (10)^6 t + \frac{\pi}{2} 2000t]$$

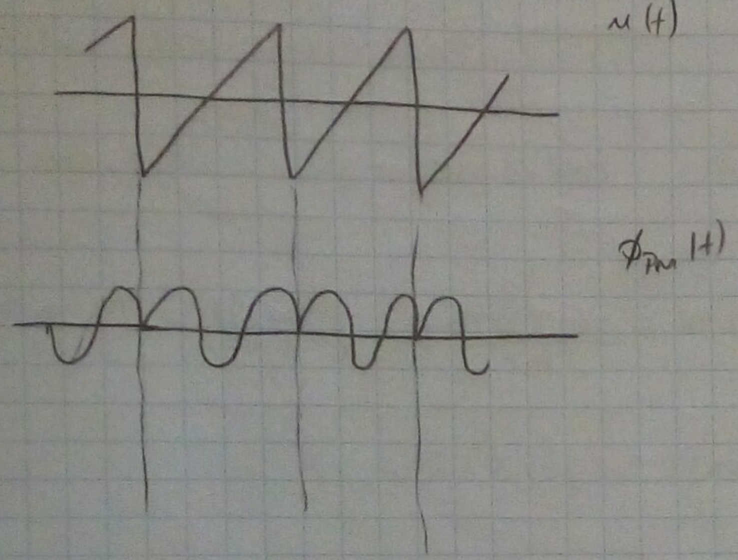
$$= \cos[2\pi (10^6 + 500)t]$$

At the jump disc., amount of jump is $m_d = 2$ (from 1 to -1)

Phase disc. $= k_p \cdot m_d = \frac{\pi}{2} \cdot 2 = \underline{\underline{\pi}}$

$f_c = 10^6 + 500$ Hz

@ disc., there's a phase disc. of π radians.



3) $|t| \leq 1 \rightarrow \varphi_{EM}(t) = 10 \cdot \cos 13000t$

$w_c = 10000$

a) $m(t) = ?$ if PM with $k_p = 1000$

b) $m(t) = ?$ if FM with $k_f = 1000$.

a) $\varphi_{PM}(t) = A \cdot \cos[w_c t + k_p m(t)]$

$$= 10 \cdot \cos[\underbrace{10000t + 1000\, m(t)}_{= 13000t}]$$

$$\underline{\underline{m(t) = 3t}}$$

b) $\varphi_{FM}(t) = A \cdot \cos\left[w_c t + k_f \int^t m(\alpha)\, d\alpha\right]$

$$= 10 \cdot \cos\left[\underbrace{10000t + k_f \int^t m(\alpha)\, d\alpha}_{= 13000t}\right]$$

$\rightarrow k_f \cdot \int^t m(\alpha)\, d\alpha = 3000t$

$\int^t m(\alpha)\, d\alpha = 3t$

$\underline{\underline{m(t) = 3}}$

Q4) $m(t) = 2.\cos 100t + 18.\cos 2000\pi t$

$\varphi_{PM}(t) = ?$ $\varphi_{FM}(t) = ?$

$A = 10$ $w_c = 10^6$ $k_f = 1000\pi$ $k_p = 1.$

$m(t) = 2.\cos 100t + 18.\cos 2000\pi t$

$\dot{m}(t) = -200\sin 100t - 36000\pi \sin 2000\pi t$

$m_p = 20$ $\dot{m}_p = 36000\pi + 200$ $B = \frac{2000\pi}{2\pi} = 1000\ Hz = 1kHz$

FM $\rightarrow$ $\Delta f = \frac{k_f . m_p}{2\pi} = \frac{1000\pi \times 20}{2\pi} = 10000\ Hz$

$B_{FM} = 2(B + \Delta f) = 2(10000 + 1000) = 22kHz$

PM $\rightarrow$ $\Delta f = \frac{k_p . \dot{m}_p}{2\pi} = \frac{1 \times 36000\pi + 200}{2\pi} = 18000 + \frac{100}{\pi}$

$B_{PM} = 2(B + \Delta f) = 2.(18.031 + 1000)$

$= \underline{38.06366}\ kHz$