EEEN 322 PS 8 QUESTIONS

Q1

Example 1 (Example 5.3 in the book)

a) Estimate B_{FM} and B_{PM} for the modulating signal in Fig 5.4(a) for $k_f = 2\pi \times 10^5$ and $k_p = 5\pi$ (Essential bandwidth of $m(t)$ is $B = 15$ kHz)

b) Repeat the problem if the amplitude of $m(t)$ is doubled

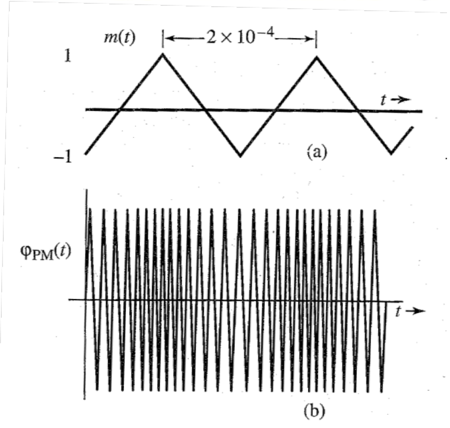

Figure 5.4 FM and PM waveforms.

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Q2

Example 2 (Example 5.4 in the book)

Repeat Example 1 (5.3) if $m(t)$ is time expanded by a factor of 2, that is, the period of $m(t)$ is 4×10^{-4}.

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Q3

Example 3 (5.5 in the book)

An angle-modulated signal with carrier frequency $\omega_c = 2\pi \times 10^5$ is described by the equation

$$\varphi_{EM}(t) = 10\cos(\omega_c t + 5\sin 3000t + 10\sin 2000\pi t)$$

a) Find the power of the modulated signal.

b) Find the frequency deviation Δf.

c) Find the deviation ratio β.

d) Find the phase deviation $\Delta\phi$.

e) Estimate the bandwidth of $\varphi_{EM}(t)$.

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Q4

5.1-3 Over an interval $|t| \leq 1$, an angle modulated signal is given by

$$\varphi_{\text{EM}}(t) = 10 \cos 13,000t$$

It is known that the carrier frequency $\omega_c = 10,000$.

(a) If this were a PM signal with $k_p = 1000$, determine $m(t)$ over the interval $|t| \leq 1$.

(b) If this were an FM signal with $k_f = 1000$, determine $m(t)$ over the interval $|t| \leq 1$.

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Q5

5.2-1 For a modulating signal

$$m(t) = 2 \cos 100t + 18 \cos 2000\pi t$$

(a) Write expressions (do not sketch) for $\varphi_{\text{PM}}(t)$ and $\varphi_{\text{FM}}(t)$ when $A = 10$, $\omega_c = 10^6$, $k_f = 1000\pi$, and $k_p = 1$. For determining $\varphi_{\text{FM}}(t)$, use the indefinite integral of $m(t)$, that is, take the value of the integral at $t = -\infty$ to be 0.

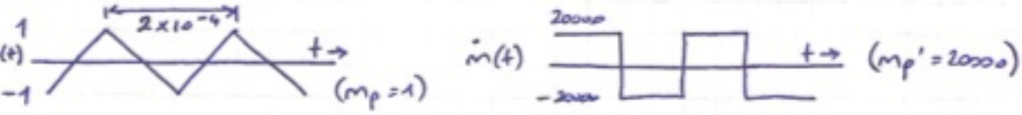
(b) Estimate the bandwidths of $\varphi_{\text{FM}}(t)$ and $\varphi_{\text{PM}}(t)$.

EEEN 322 PS 8 SOLUTIONS

Q1

Sol.

a)



For FM: $\Delta f = \frac{k_f m_p}{2\pi} = \frac{(2\pi \times 10^5)(1)}{2\pi} = 100\text{ kHz}$

$\Rightarrow B_{FM} = 2(\Delta f + B) = 2(100\text{ kHz} + 15\text{ kHz}) = \boxed{230\text{ kHz}}$

For PM: $\Delta f = \frac{k_p m_p'}{2\pi} = \frac{(5\pi)(20000)}{2\pi} = 50\text{ kHz}$

$\Rightarrow B_{PM} = 2(\Delta f + B) = 2(50\text{ kHz} + 15\text{ kHz}) = \boxed{130\text{ kHz}}$

b) Doubling $m(t)$ doubles its peak value. Hence, $m_p = 2$,

$m_p' = 40000$, but B is unchanged and $B = 15\text{ kHz}$

For FM: $\Delta f = \frac{k_f m_p}{2\pi} = \frac{(2\pi \times 10^5)(2)}{2\pi} = 200\text{ kHz}$

$\Rightarrow B_{FM} = 2(\Delta f + B) = 2(200\text{ kHz} + 15\text{ kHz}) = \boxed{430\text{ kHz}}$

For PM: $\Delta f = \frac{k_p m_p'}{2\pi} = \frac{(5\pi)(40000)}{2\pi} = 100\text{ kHz}$

$\Rightarrow B_{PM} = 2(\Delta f + B) = 2(100\text{ kHz} + 15\text{ kHz}) = \boxed{230\text{ kHz}}$

* Observe that doubling the signal amplitude roughly doubles the bandwidth of both FM and PM waveforms

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Q2

Recall that time expansion of a signal by a factor of 2 reduces the signal spectral width (bandwidth) by a factor of 2. Hence, $B = 7.5\text{ kHz}$, which is half the previous bandwidth. Time expansion does not affect the peak amplitude, so that $m_p = 1$. However, m_p' is halved, that is, $m_p' = 10000$.

For FM: $\Delta f = \frac{k_f m_p}{2\pi} = \frac{(2\pi \times 10^5)(1)}{2\pi} = 100\text{ kHz}$

$\Rightarrow B_{FM} = 2(\Delta f + B) = 2(100\text{ kHz} + 7.5\text{ kHz}) = \boxed{215\text{ kHz}}$

For PM: $\Delta f = \frac{k_p m_p'}{2\pi} = \frac{(5\pi)(10000)}{2\pi} = 25\text{ kHz}$

$\Rightarrow B_{PM} = 2(\Delta f + B) = 2(25\text{ kHz} + 7.5\text{ kHz}) = \boxed{65\text{ kHz}}$

* Note that time expansion of $m(t)$ has very little effect on the FM bandwidth, but it halves the PM bandwidth. This verifies our observation that the PM spectrum is strongly dependent on the spectrum of $m(t)$.
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Q3

The signal bandwidth is the highest frequency in $m(t)$ (or its derivative).

In this case $B = \frac{2000\pi}{2\pi} = 1000\,Hz$

a) The carrier amplitude A is 10 and the power is $P = \frac{A^2}{2} = \frac{10^2}{2} = 50$

b) We need to find the frequency deviation Δf

$$\omega_i = \frac{d\theta(t)}{dt} = \frac{d}{dt}\{\omega_c t + 5\sin 3000t + 10\sin 2000\pi t\}$$

$$= \omega_c + 15000\cos 3000t + 20000\pi\cos 2000\pi t$$

The two sinusoids $15000\cos 3000t$ and $20000\pi\cos 2000\pi t$ will add in phase at some point in time, and the maximum value of $15000\cos 3000t + 20000\pi\cos 2000\pi t$ is $15000 + 20000\pi$.

$\Rightarrow \Delta\omega = 15000 + 20000\pi \Rightarrow \Delta f = \frac{\Delta\omega}{2\pi} = \frac{15000 + 20000\pi}{2\pi}$

$= 12,387.32\ Hz$

c) $\beta = \frac{\Delta f}{B} = \frac{12,387.32}{1000} = 12.387$

d) $\theta(t) = \omega_c t + 5\sin 3000t + 10\sin 2000\pi t$. The two sinusoids $5\sin 3000t$ and $10\sin 2000\pi t$ will add in phase at some point in time, therefore $\Delta\phi = 5 + 10 = 15$ rad

e) $B_{FM} = 2(\Delta f + B) = 2(12,387.32 + 1000)$

$= 26,774.65\ Hz$

* Observe the generality of this method of estimating the bandwidth of an angle-modulated waveform. We need not know whether it is FM or PM, or some other kind of angle modulation. It is applicable to any angle-modulated signal.
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Q4

5.1-3

$$\text{(a)} \qquad \varphi_{PM}(t) = A\cos\left[\omega_c t + k_p m(t)\right] = 10\cos[10,000t + k_p m(t)]$$

We are given that $\varphi_{PM}(t) = 10\cos(13,000t)$ with $k_p = 1000$. Clearly, $m(t) = 3t$ over the interval $|t| \leq 1$.

$$\text{(b)} \qquad \varphi_{FM}(t) = A\cos\left[\omega_c t + k_f \int^t m(\alpha)\,d\alpha\right] = 10\cos\left[10,000t + k_f \int^t m(\alpha)\,d\alpha\right]$$

$$\text{Therefore} \qquad k_f \int^t m(\alpha)\,d\alpha = 1000 \int^t m(\alpha)\,d\alpha = 3000t$$

$$\text{Hence} \qquad 3t = \int^t m(\alpha)\,d\alpha \qquad \Longrightarrow m(t) = 3$$

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Q5

5.2-1 In this case $k_f = 1000\pi$ and $k_p = 1$. For

$$m(t) = 2\cos 100t + 18\cos 2000\pi t \qquad \text{and} \qquad \dot{m}(t) = -200\sin 100t - 36,000\pi\sin 2000\pi t$$

Therefore $m_p = 20$ and $m'_p = 36,000\pi + 200$. Also the baseband signal bandwidth $B = 2000\pi/2\pi = 1$ kHz.

For FM : : $\Delta f = k_f m_p/2\pi = 10,000$, and $B_{FM} = 2(\Delta f + B) = 2(20,000 + 1000) = 42$ kHz.

For PM : : $\Delta f = k_p m'_p/2\pi = 18,000 + \frac{100}{\pi}$ Hz, and $B_{PM} = 2(\Delta f + B) = 2(18,031.83 + 1000) = 38.06366$ kHz.