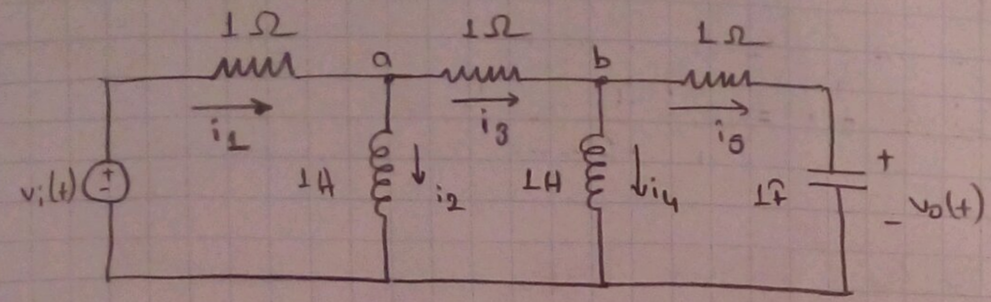
EEEN-352 PS-6/2



Differential equation for each energy storage element.

$$L.\ \frac{di_2}{dt} = v_1 \sim \frac{di_2}{dt} = v_1$$

$$L.\ \frac{di_4}{dt} = v_2 \sim \frac{di_4}{dt} = v_2$$

$$C.\ \frac{dv_o}{dt} = i_5 \sim \frac{dv_o}{dt} = i_5$$

Therefore, the state vector is $x = \begin{bmatrix} i_2 \\ i_4 \\ v_o \end{bmatrix}$

Now obtain $v_1, v_2,$ and i_5 in terms of the state variables.

① KVL for the outer loop:

$$-v_i + i_1 (1) + i_3(1) + i_5(1) + v_o = 0$$

② KCL for the nodes a and b yields,

ⓐ $i_3 = i_1 - i_2$

ⓑ $i_5 = i_3 - i_4 = i_1 - i_2 - i_4$

• Substituting ⓐ and ⓑ into the 1st equation,

$$-v_i + i_1 + (i_1 - i_2) + (i_1 - i_2 - i_4) + v_o = 0$$

Solving for i_1,

$$i_1 = \frac{2}{3} i_2 + \frac{1}{3} i_4 - \frac{1}{3} v_o + \frac{1}{3} v_i$$

(2) KVL for the leftmost loop,

$$v_i = i_1 \cdot (1) + v_1$$

$$v_1 = v_i - i_1$$

(A) $$v_1 = -\frac{2}{3} i_2 - \frac{1}{3} i_4 + \frac{1}{3} v_o + \frac{2}{3} v_i$$

Also,

$$i_3 = i_1 - i_2$$

$$= -\frac{1}{3} i_2 + \frac{1}{3} i_4 - \frac{1}{3} v_o + \frac{1}{3} v_i$$

and

$$i_5 = i_3 - i_4$$

(B) $$i_5 = -\frac{1}{3} i_2 - \frac{2}{3} i_4 - \frac{1}{3} v_o + \frac{1}{3} v_i$$

Finally, KVL for the rightmost loop,

$$v_2 = v_o + i_5 (1)$$

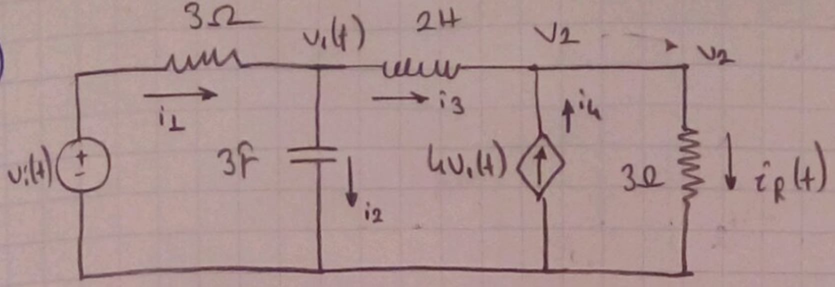
$$= v_o + i_5$$

(C) $$v_2 = -\frac{1}{3} i_2 - \frac{2}{3} i_4 + \frac{2}{3} v_o + \frac{1}{3} v_i$$

Using the equations (A), (B), and (C)

$$\dot{x} = \begin{bmatrix} -2/3 & -1/3 & 1/3 \\ -1/3 & -2/3 & 2/3 \\ -1/3 & -2/3 & -1/3 \end{bmatrix} x + \begin{bmatrix} 2/3 \\ 1/3 \\ 1/3 \end{bmatrix} v_i \qquad y = [0 \ 0 \ 1]\, x$$

2)



Differential equation for each energy storage element,

$$C.\frac{dv_1}{dt} = i_2 \sim \frac{dv_1}{dt} = \frac{i_2}{3}$$

$$L.\frac{di_3}{dt} = V_L \sim \frac{di_3}{dt} = \frac{v_L}{2}$$

Therefore the state vector is $x = \begin{bmatrix} v_1 \\ i_3 \end{bmatrix}$

Now obtain v_L and i_2 in terms of the state variables.

1) $v_L = v_1 - v_2$

$v_2 = i_R.(3)$

$v_L = v_1 - 3i_R$ where $i_R = i_3 + i_4$

$= i_3 + 4v_1$

$v_L = v_1 - 3(i_3 + 4v_1)$

A) $v_L = -11v_1 - 3i_3$

2) $i_2 = i_1 - i_3$ where $i_1 = \frac{v_i - v_1}{3}$

B) $i_2 = \frac{v_i}{3} - \frac{v_1}{3} - i_3$

Also the output is

$y = i_R = 4v_1 + i_3$

Hence,

$$\dot{x} = \begin{bmatrix} -1/9 & -1/3 \\ -11/2 & -3/2 \end{bmatrix} + \begin{bmatrix} 1/9 \\ 0 \end{bmatrix} v_i$$

$$y = [4 \quad 1] \, x$$

4) Equations of motion in Laplace;

① $(2s^2 + 3s + 2)X_1(s) - (s+2)X_2(s) - sX_3(s) = 0$

② $-(s+2)X_1(s) + (s^2 + 2s + 2)X_2(s) - sX_3(s) = F(s)$

③ $-sX_1(s) - sX_2(s) + (s^2 + 3s)X_3(s) = 0$

Equations of motion in time domain,

① $2\dfrac{d^2x_1}{dt^2} + 3\dfrac{dx_1}{dt} + 2x_1 - \dfrac{dx_2}{dt} - 2x_2 - \dfrac{dx_3}{dt} = 0$

② $\dfrac{d^2x_2}{dt^2} + 2\dfrac{dx_2}{dt} + 2x_2 - \dfrac{dx_1}{dt} - 2x_1 - \dfrac{dx_3}{dt} = F(t)$

③ $\dfrac{d^2x_3}{dt^2} + 3\dfrac{dx_3}{dt} - \dfrac{dx_1}{dt} - \dfrac{dx_2}{dt} = 0$

Define state variables.

$z_1 = x_1$

$z_2 = \dfrac{dx_1}{dt}$

$z_3 = x_2$

$z_4 = \dfrac{dx_2}{dt}$

$z_5 = x_3$

$z_6 = \dfrac{dx_3}{dt}$

Substituting the state variables into the equations of motion,

$$\frac{dz_1}{dt} = z_2, \quad \frac{dz_3}{dt} = z_4, \quad \frac{dz_5}{dt} = z_6$$

$$\frac{dz_1}{dt} = z_2$$

(1) $$\frac{dz_2}{dt} = \left(-2z_1 - 3z_2 + 2z_3 + z_4 + z_6\right) \cdot \frac{1}{2}$$

$$\frac{dz_3}{dt} = z_4$$

(2) $$\frac{dz_4}{dt} = 2z_1 + z_2 - 2z_3 - 2z_4 + z_6 + f(t)$$

$$\frac{dz_5}{dt} = z_6$$

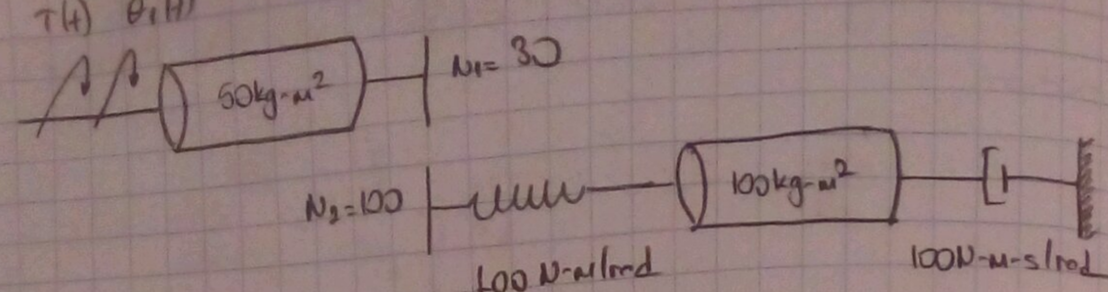
(3) $$\frac{dz_6}{dt} = z_2 + z_4 - 3z_6$$

The output is $x_3 = z_5$.

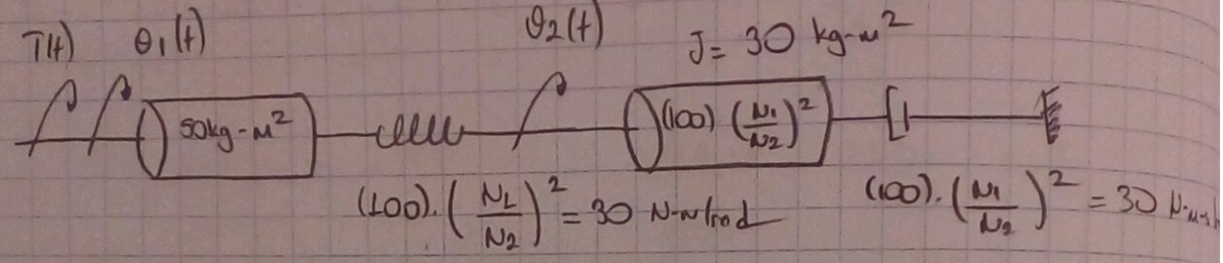
In vector-matrix form:

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -2 & -3 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & -2 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & -3 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} f(t)$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} . z$$

6) 

Drawing the equivalent network



Writing the equations of motion,

① $(50s^2 + 30)\theta_1(s) - 30.\theta_2(s) = T(s)$

② $-30.\theta_1(s) + (30s^2 + 30s + 30)\theta_2(s) = 0$

Taking the inverse Laplace transform and simplifying,

① $5\ddot{\theta}_1 + 3\theta_1 - 3\theta_2 = \frac{1}{10}T$

② $-\theta_1 + \ddot{\theta}_2 + \dot{\theta}_2 + \theta_2 = 0$

Defining the state variables,

$x_1 = \theta_1$ $\quad$ $x_3 = \theta_2$

$x_2 = \dot{\theta}_1$ $\quad$ $x_4 = \dot{\theta}_2$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{3}{5}x_1 + \frac{3}{5}x_3 + \frac{1}{50}T$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = x_1 - x_3 - x_4$$

$\theta_1(t)$ is the output,

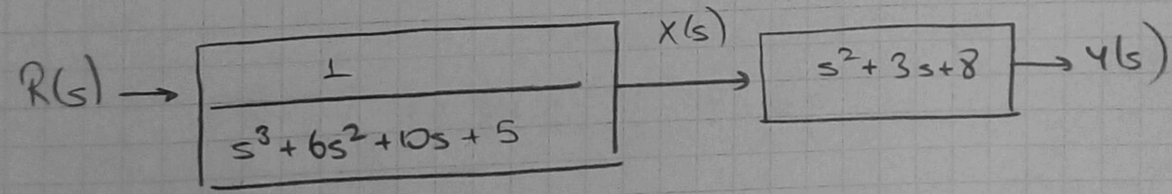
$$y = \theta_1 = x_1$$

In vector-matrix form,

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -3/5 & 0 & 3/5 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/50 \\ 0 \\ 0 \end{bmatrix} T$$

$$y = [1 \ 0 \ 0 \ 0]\, x$$

13



Differential equation for the 1^{st} box:

$$\dddot{x} + 6\ddot{x} + 10\dot{x} + 5x = r(t)$$

Defining the state variables,

$$x_1 = x$$

$$x_2 = \dot{x}$$

$$x_3 = \ddot{x}$$

Thus,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -5x - 10\dot{x} - 6\ddot{x} + r(t)$$

$$\dot{x}_3 = -5x_1 - 10x_2 - 6x_3 + r(t)$$

From the 2nd box,

$$y = \ddot{x} + 3\dot{x} + 8x$$

$$y = 8x_1 + 3x_2 + x_3$$

In vector-matrix form:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -10 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$y = \begin{bmatrix} 8 & 3 & 1 \end{bmatrix} x$$