Selected Problems IV

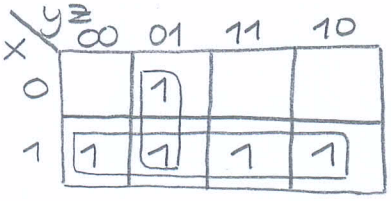
Problem 1) Simplify the following Boolean functions using a three-variable Karnaugh map.

a. $F(x,y,z) = \Sigma m(1,4,5,6,7)$

b. $F(x,y,z) = x'y' + yz + x'yz'$

Solution.

a.



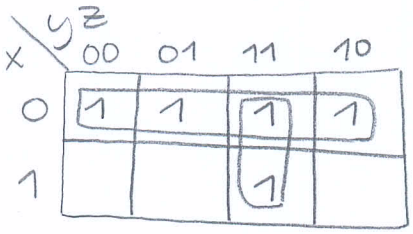
$F(x,y,z) = x + y'z$

→ two adjacent terms

↙ four adjacent terms

b. We first reexpress $F(x,y,z)$ in sum of minterms form

$$F(x,y,z) = x'y'(z+z') + (x+x')yz + x'yz'$$
$$= x'y'z + x'y'z' + xyz + x'yz + x'yz'$$
$$= \Sigma m(0,1,2,3,7)$$



$F(x,y,z) = x' + yz$

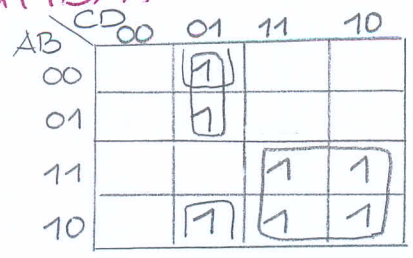
Problem 2) Simplify the following Boolean functions using a four-variable map:

a. $F(A,B,C,D) = \Sigma m(1,5,9,10,11,14,15)$

b. $F(A,B,C,D) = \Sigma m(0,2,4,5,6,7,8,10,13,15)$

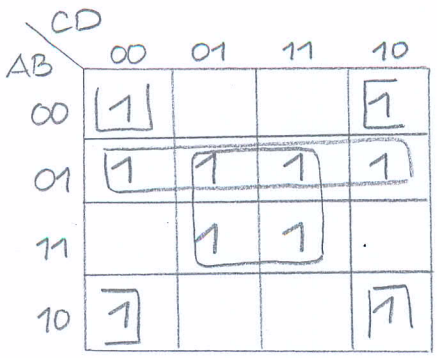
Solution.

a.



$F(A,B,C,D) = AC + B'C'D + A'C'D$

b.



$F(A,B,C,D) = B'D' + A'B + BD$

Problem 3) Simplify the following Boolean functions using a four-variable map:

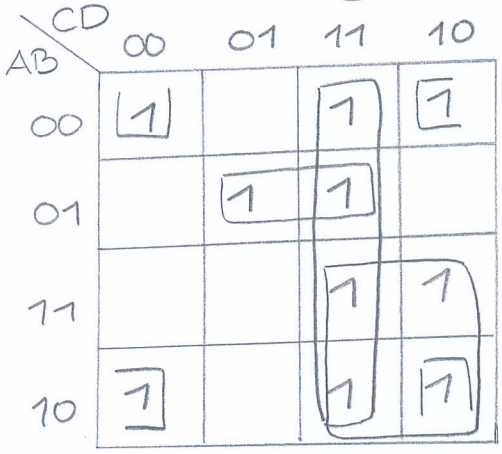
a. $F(A,B,C,D) = AB'C + B'C'D' + BCD + ACD' + A'B'C + A'BC'D$

b. $F(x,y,z,w) = xyz + wy + wxy' + x'y$

Solution.

a. $F(A,B,C,D) = \Sigma m(\underbrace{10,11}_{AB'C}, \underbrace{0,8}_{B'C'D'}, \underbrace{7,15}_{BCD}, \underbrace{10,14}_{ACD'}, \underbrace{2,3}_{A'B'C}, \underbrace{5}_{A'BC'D})$

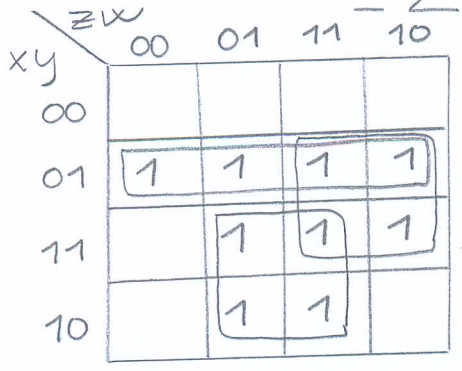
$= \Sigma m(0,2,3,5,7,8,10,11,14,15)$



$F(A,B,C,D) = B'D' + CD + AC + A'BD$

b. $F(x,y,z,w) = \Sigma m(\underbrace{14,15}_{xyz}, \underbrace{5,7,13,15}_{wy}, \underbrace{9,11}_{wxy'}, \underbrace{4,5,6,7}_{x'y})$

$= \Sigma m(4,5,6,7,9,11,13,14,15)$



$F(x,y,z,w) = x'y + yz + xw$

Problem 4) Find the minterms of the following Boolean expressions by first plotting each function in a map:

a. $wyz + w'x' + wxz'$

b. $A'B + A'CD + B'CD + BC'D'$

Solution.

a.

wx \ yz	00	01	11	10
00	1	1	1	1
01				
11	1		1	1
10			1	

$\Sigma m(0,1,2,3,11,12,14,15)$

b.

AB \ CD	00	01	11	10
00			1	
01	1	1	1	1
11	1			
10			1	

$\Sigma m(3,4,5,6,7,11,12)$

Problem 5) Simplify the following Boolean functions to product of sums form:

a. $F(w,x,y,z) = \Sigma m(0,1,2,5,8,10,13)$

b. $F(A,B,C,D) = \Pi M(1,3,5,7,13,15)$

Solution.

a. $F'(w,x,y,z) = \Sigma m(3,4,6,7,9,11,12,14,15)$

wx \ yz	00	01	11	10
00			1	
01	1		1	1
11	1		1	1
10		1	1	

$F'(w,x,y,z) = xz' + yz + wx'z$

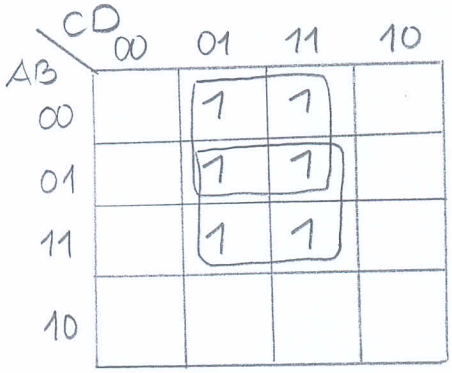
$$\left(F'(w,x,y,z)\right)' = F(w,x,y,z)$$

$$= (xz' + yz + wx'z)'$$

$$= (x'+z)(y'+z')(w'+x+z')$$

$$\therefore F(w,y,z,y) = (x'+z)(y'+z')(w'+x+z')$$

b. $F'(A,B,C,D) = \Sigma m(1,3,5,7,13,15)$



$F'(A,B,C,D) = A'D + BD$

$$\left(F'(A,B,C,D)\right)' = F(A,B,C,D)$$

$$= (A'D + BD)'$$

$$= (A+D')(B'+D')$$

Problem 6) Simplify the following expressions to product-of-sums form:

a. $ACD' + C'D + AB' + ABCD$

b. $(A+C'+D')(A'+B'+D')(A'+B+D')(A'+B+C')$

Solution.

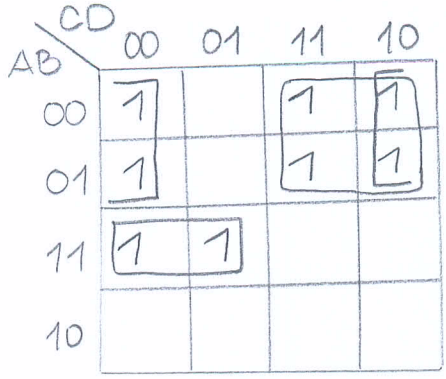
a. The expression can be reexpressed as

$$F(A,B,C,D) = \Sigma m(\underbrace{10,14}_{ACD'}, \underbrace{1,5,5,14}_{C'D}, \underbrace{8,9,10,11}_{AB'}, \underbrace{15}_{ABCD})$$

$$= \Sigma m(1,5,8,9,10,11,14,15)$$

Hence;

$$F'(A,B,C,D) = \Sigma m(0,2,3,4,6,7,12,13)$$



$$F'(A,B,C,D) = A'D' + A'C + ABC'$$

$$F(A,B,C,D) = (F'(A,B,C,D))'$$

$$= (A'D' + A'C + ABC')'$$

$$= (A+D)(A+C')(A'+B'+C)$$

b. Let us rewrite the expression as

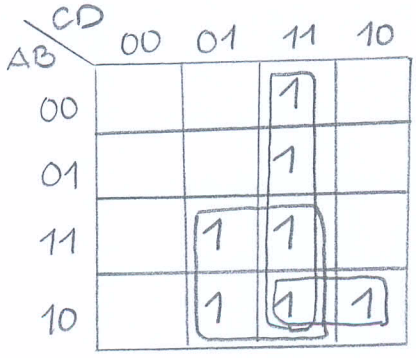
$$F(A,B,C,D) = (A+C'+D')(A'+B'+D')(A'+B+D')(A'+B+C')$$

then we consider

$$F'(A,B,C,D) = A'CD + ABD + AB'D + AB'C$$

$$= \Sigma m(\underbrace{3,7}_{A'CD}, \underbrace{13,15}_{ABD}, \underbrace{9,13}_{AB'D}, \underbrace{10,11}_{AB'C})$$

$$= \Sigma m(3,7,9,10,11,13,15)$$



$$F'(A,B,C,D) = AD + CD + AB'C$$

$$F(A,B,C,D) = (F'(A,B,C,D))'$$

$$= (AD + CD + AB'C)'$$

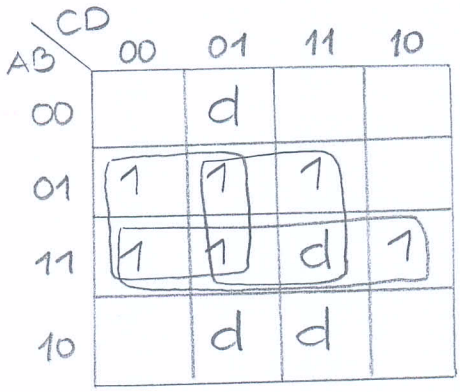
$$= (A'+D')(C'+D')(A'+B+C')$$

Problem 7) Simplify the following Boolean function together with don't care conditions, d:

$$F(A,B,C,D) = \Sigma m(4,5,7,12,13,14)$$

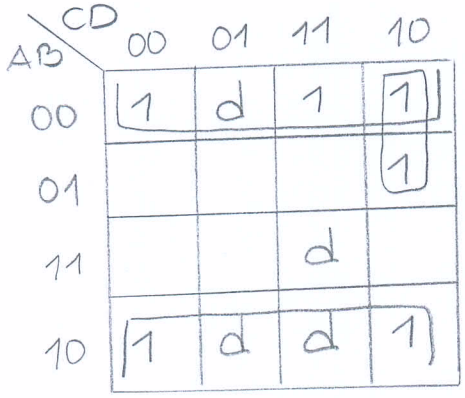
$$d(A,B,C,D) = \Sigma m(1,5,11,15)$$

Solution.



$F(A,B,C,D) = BC' + BD + AB$ SOP form

-and we shall minimize in POS form as



$F'(A,B,C,D) = \Sigma m(0,2,3,6,8,10)$

$d(A,B,C,D) = \Sigma m(1,5,11,15)$

$F'(A,B,C,D) = B' + A'CD'$

Hence;

$$F(A,B,C,D) = (F'(A,B,C,D))'$$
$$= (B' + A'CD')'$$
$$= B(A + C' + D)$$ POS form

Problem 8) Simplify the following functions and implement them with two-level NOR gate circuits:

a. $F = wx' + y'z' + w'yz'$

b. $F(w,x,y,z) = \Sigma m(1,2,13,14)$

Solution.

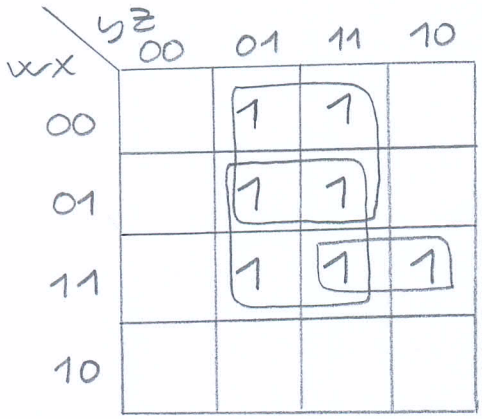
a. We need to simplify F in product-of-sums form:

$$F = \Sigma m(\underbrace{8,9,10,11}_{wx'}, \underbrace{0,4,8,12}_{y'z'}, \underbrace{2,6}_{w'yz'})$$

$$= \Sigma m(0,2,4,6,8,9,10,11,12)$$

Hence;

$$F' = \Sigma m(1,3,5,7,13,14,15)$$

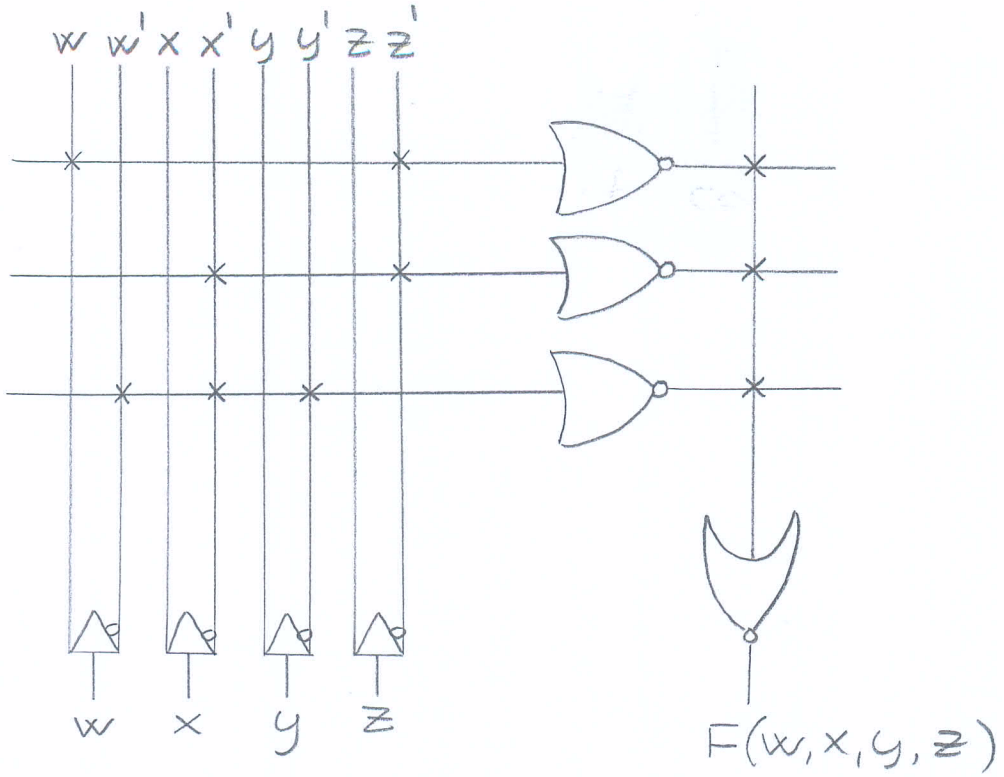


$$F'(w,x,y,z) = w'z + xz + wxy$$

$$F(w,x,y,z) = \left(F'(w,x,y,z)\right)'$$

$$= (w'z + xz + wxy)'$$

$$= (w+z')(x'+z')(w'+x'+y')$$



b. We consider

$$F'(w,x,y,z) = \Sigma m\,(0,3,4-12,15)$$

wx \ yz	00	01	11	10
00	1		1	
01	1	1	1	1
11	1		1	
10	1	1	1	1

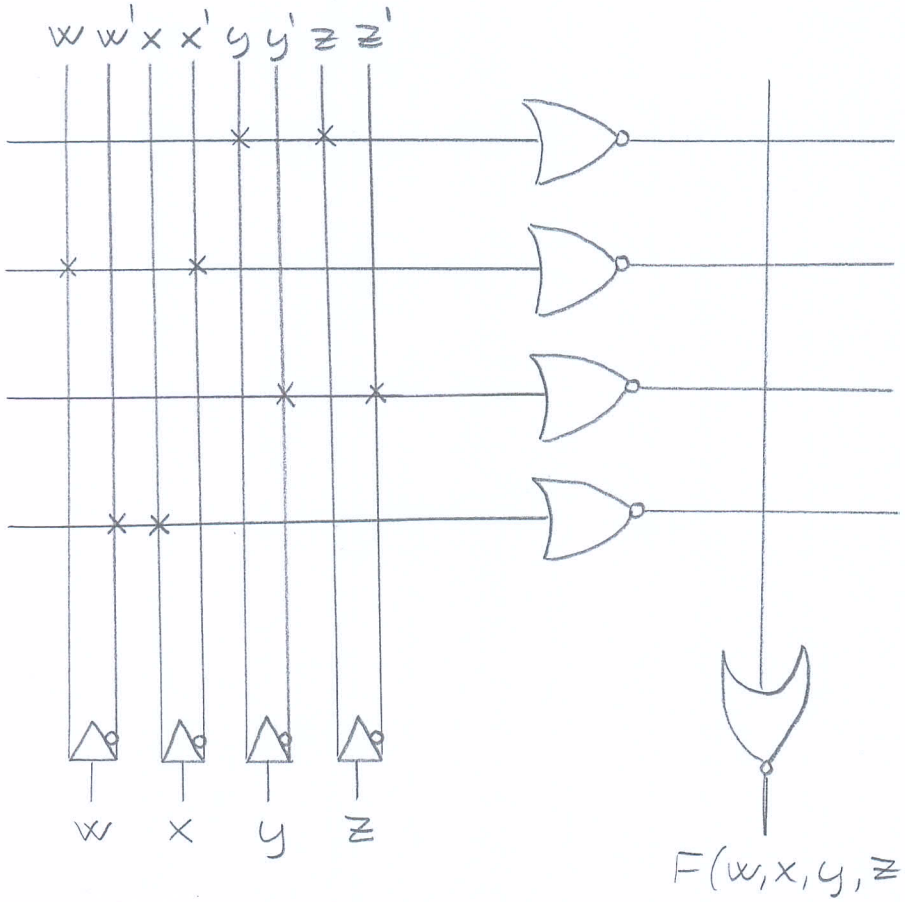
$$F'(w,x,y,z) = y'z' + w'x + yz + wx'$$

$$F(w,x,y,z) = \left(F'(w,x,y,z)\right)'$$

$$= (y'z' + w'x + yz + wx')'$$

$$= (y+z)(w+x')(y'+z')(w'+x)$$

-we shall draw the two-level NOR logic circuit diagram as follows:



Problem 9) Implement the following Boolean function F, together with the don't care conditions d as a two-level NAND gate circuit:

$$F(A,B,C,D) = \Sigma m(2,4,6,10,12)$$

$$d(A,B,C,D) = \Sigma m(0,8,9,13)$$

Solution.

AB \ CD	00	01	11	10
00	d			1
01	1			1
11	1	d		
10	d	d		1

$$F(A,B,C,D) = B'D' + AC' + A'D'$$

-then we shall drew the two-level NAND gate logic circuit diagram as follows: