

Introduction to MATLAB for Control Engineering

The background of the slide features several light gray, wavy, horizontal lines that sweep across the lower right portion of the frame, creating a sense of motion and depth.

Vectors: To create a *column* vector, separate the elements by semicolons. Use square brackets.

```
>> P=[3;7;9]
```

P =

```
3
7
9
```

To create a *row* vector, separate the elements by commas.

```
>> P=[3,7,9]
```

P =

```
3    7    9
```

or by space

```
>> P=[3 7 9]
```

P =

```
3    7    9
```

From a *row* vector

```
>>p = [3, 7, 9]  
p =  
     3     7     9
```

You can create a *column* vector by using the *transpose* notation (').

```
>>p = [3, 7, 9] '  
p =  
     3  
     7  
     9
```

You can also create a column vector by separating the elements by semicolons. For example,

```
>>g = [3;7;9]
```

```
g =
```

```
    3
```

```
    7
```

```
    9
```

You can create vectors by "appending" one vector to another.

For example, to create the row vector u whose first three columns contain the values of

$$r = [2, 4, 20]$$

and whose fourth, fifth, and sixth columns contain the values of

$$w = [9, -6, 3],$$

you type

$$u = [r, w].$$

The result is the vector

$$u = [2, 4, 20, 9, -6, 3].$$

Similarly,

```
>> r=[2;4;20];  
>> w=[9;-6;3];  
>> u=[r;w]
```

u =

```
2  
4  
20  
9  
-6  
3
```

The colon operator (:) easily generates a large vector of regularly spaced elements. Parentheses are not needed but can be used for clarity. Do not use square brackets.

Typing

```
>>x = m:q:n
```

or

```
>>x = (m:q:n)
```

creates a vector x of values with a spacing q .

The first value is m .

The last value is n if $m - n$ is an integer multiple of q .

If not, the last value is less than n .

For example, typing

```
x = 0:2:8
```

creates the vector

```
x = [0, 2, 4, 6, 8],
```

whereas typing

```
x = 0:2:7
```

creates the vector

```
x = [0, 2, 4, 6].
```

To create a row vector z consisting of the values from 5 to 8 in steps of 0.1, type

```
z = 5:0.1:8.
```

If the increment q is omitted, it is presumed to be 1.

Thus typing

```
y = -3:2
```

produces the vector

```
y = [-3, -2, -1, 0, 1, 2].
```


`linspace`

command also creates a linearly spaced row vector, but instead you specify the number of values rather than the increment.

The syntax is

`linspace(x1, x2, n),`

where `x1` and `x2` are the lower and upper limits and `n` is the number of points.

For example, `linspace(1, 11, 100)`

is equivalent to

`1:0.1:11`

If `n` is omitted, the spacing is 1.

The `logspace` command creates an array of *logarithmically* spaced elements.

Its syntax is `logspace(a, b, n)`,
where `n` is the number of points between 10^a and 10^b .

For example,

```
x = logspace(-1, 1, 4)
```

produces the vector

```
x = [0.1000, 0.4642, 2.1544,  
10.000].
```

If `n` is omitted, the number of points defaults to 50.

Norm, Length, and Absolute Value of a Vector

Keep in mind the precise meaning of these terms when using MATLAB.

The `length` command gives the *number of elements* in the vector.

The `norm` command gives the *magnitude* of a vector **x** having elements x_1, x_2, \dots, x_n , computed by $\sqrt{(x_1^2 + x_2^2 + \dots + x_n^2)}$, and shows the vector's geometric length.

The `abs` command gives the *absolute value* of a vector **x**. The result is another vector whose elements are the absolute values of the elements of **x**.

For example,

if $x = [2, -4, 5]$,

- its length is 3; (computed from `length(x)`)
- its magnitude is $\sqrt{2^2 + (-4)^2 + 5^2} = 6.7082$;
(computed from `norm(x)`)
- its absolute value is $[2, 4, 5]$ (computed
from `abs(x)`).

Matrices

A matrix has multiple rows and columns. For example, the matrix

$$\mathbf{M} = \begin{bmatrix} 2 & 4 & 10 \\ 16 & 3 & 7 \\ 8 & 4 & 9 \\ 3 & 12 & 15 \end{bmatrix}$$

has four rows and three columns.

Vectors are special cases of matrices having one row or one column.

Creating Matrices

If the matrix is small you can type it row by row, separating the *elements* in a given row with *spaces* or *commas* and separating the *rows* with semicolons. For example, typing

```
>>A = [2, 4, 10; 16, 3, 7];
```

creates the following matrix:

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 10 \\ 16 & 3 & 7 \end{bmatrix}$$

Remember, spaces or commas separate elements in different *columns*, whereas semicolons separate elements in different *rows*.

Creating Matrices from Vectors

Suppose

$a = [1, 3, 5]$ and $b = [7, 9, 11]$ (row vectors).

Note the difference between the results given by $[a \ b]$ and $[a;b]$ in the following :

```
>>c = [a b];
```

```
c =
```

```
    1    3    5    7    9   11
```

```
>>D = [a;b]
```

```
D =
```

```
    1    3    5
    7    9   11
```

You need not use symbols to create a new array. For example, you can type

```
>> D = [ [1, 3, 5]; [7, 9, 11] ] ;
```

Array Addressing

The colon operator selects individual elements, rows, columns, or "subarrays" of arrays. Here are some examples:

- $v(:)$ represents all the row or column elements of the vector v .
- $v(2:5)$ represents the second through fifth elements; that is $v(2)$, $v(3)$, $v(4)$, $v(5)$.

Array Addressing, continued

- $A(:,3)$ denotes all the elements in the third column of the matrix A .
- $A(:,2:5)$ denotes all the elements in the second through fifth columns of A .
- $A(2:3,1:3)$ denotes all the elements in the second and third rows that are also in the first through third columns.
-
- $v = A(:)$ creates a vector v consisting of all the columns of A stacked from first to last.
- $A(\text{end},:)$ denotes the last row in A , and $A(:,\text{end})$ denotes the last column.

You can use array indices to extract a smaller array from another array.
For example, if you first create the array **B**

$$\mathbf{B} = \begin{bmatrix} 2 & 4 & 10 & 13 \\ 16 & 3 & 7 & 18 \\ 8 & 4 & 9 & 25 \\ 3 & 12 & 15 & 17 \end{bmatrix}$$

then type `C = B(2:3, 1:3)`,
you can produce the following array:

$$\mathbf{C} = \begin{bmatrix} 16 & 3 & 7 \\ 8 & 4 & 9 \end{bmatrix}$$

Additional Array Functions

`length(A)`

Computes either the number of elements of A if A is a vector

or the largest value of m or n if A is an $m \times n$ matrix.

`size(A)`

Returns a row vector $[m \ n]$ containing the sizes of the $m \times n$ array A .

Additional Array Functions

`sort(A)`

Sorts each column of the array A in ascending order and returns an array the same size as A .

`sum(A)`

Sums the elements in each column of the array A and returns a row vector containing the sums

Additional Array Functions

`max (A)`

Returns the algebraically largest element in A if A is a vector.

Returns a row vector containing the largest elements in each column if A is a matrix.

If any of the elements are complex, `max (A)` returns the elements that have the largest magnitudes.

Additional Array Functions

`[x, k] =
max(A)`

Similar to `max(A)` but stores the maximum values in the row vector `x` and their indices in the row vector `k`.

`min(A)`
and
`[x, k] =
min(A)`

Like `max` but returns minimum values.

The function `size(A)` returns a row vector `[m n]` containing the sizes of the $m \times n$ array **A**.

For example, if

$$\mathbf{A} = \begin{bmatrix} 6 & 2 \\ -10 & -5 \\ 3 & 0 \end{bmatrix}$$

Then `size(A)` returns `[3, 2]`

The `length(A)` function computes either the number of elements of **A** if **A** is a vector or the largest value of *m* or *n* if **A** is an *m* × *n* matrix.

For example, if

$$\mathbf{A} = \begin{bmatrix} 6 & 2 \\ -10 & -5 \\ 3 & 0 \end{bmatrix}$$

Then `length(A)` returns 3.

Similarly,

$$\mathbf{A} = \begin{bmatrix} 6 & 2 \\ -10 & -5 \\ 3 & 0 \end{bmatrix}$$

then `max(A)` returns the vector `[6, 2]`;

`min(A)` returns the vector `[-10, -5]`

Similarly,

$$\mathbf{A} = \begin{bmatrix} 6 & 2 \\ -10 & -5 \\ 3 & 0 \end{bmatrix}$$

then `max(A)` returns the matrix:

$$\mathbf{A} = \begin{bmatrix} -10 & -5 \\ 3 & 0 \\ 6 & 2 \end{bmatrix}$$

Array Addition and Subtraction

For example:

$$\begin{bmatrix} 6 & -2 \\ 10 & 3 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ -12 & 14 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -2 & 17 \end{bmatrix}$$

The addition:

```
>>A = [6,-2;10,3];  
>>B = [9,8;-12,14]  
>>A+B  
ans =  
    15         6  
    -2        17
```

Array subtraction is performed in a similar way.

Multiplication: Multiplying a matrix **A** by a scalar w produces a matrix whose elements are the elements of **A** multiplied by w . For example:

$$3 \begin{bmatrix} 2 & 9 \\ 5 & -7 \end{bmatrix} = \begin{bmatrix} 6 & 27 \\ 15 & -21 \end{bmatrix}$$

This multiplication is performed in MATLAB as follows:

```
>>A = [2, 9; 5, -7];
```

```
>>3*A
```

```
ans =
```

```
     6     27  
    15    -21
```

Multiplication of an array by a scalar is easily defined and easily carried out.

However, multiplication of two *arrays* is not so straightforward.

MATLAB uses two definitions of multiplication:

(1) *array* multiplication (also called *element-by-element* multiplication), and

(2) *matrix* multiplication.

Division and exponentiation must also be carefully defined when you are dealing with operations between two arrays.

MATLAB has two forms of arithmetic operations on arrays. Next we introduce one form, called *array* operations, which are also called *element-by-element* operations. Then we will introduce *matrix* operations. Each form has its own applications.

Division and exponentiation must also be carefully defined when you are dealing with operations between two arrays.

Element-by-element operations:

Symbol	Operation	Form	Examples
+	Scalar-array addition	$A + b$	$[6, 3] + 2 = [8, 5]$
-	Scalar-array subtraction	$A - b$	$[8, 3] - 5 = [3, -2]$
+	Array addition	$A + B$	$[6, 5] + [4, 8] = [10, 13]$
-	Array subtraction	$A - B$	$[6, 5] - [4, 8] = [2, -3]$
.*	Array multiplication	$A .* B$	$[3, 5] .* [4, 8] = [12, 40]$
./	Array right division	$A ./ B$	$[2, 5] ./ [4, 8] = [2/4, 5/8]$
.\	Array left division	$A .\ B$	$[2, 5] .\ [4, 8] = [4\backslash 2, 8\backslash 5]$
.^	Array exponentiation	$A .^ B$	$[3, 5] .^ 2 = [3^2, 5^2]$ $2 .^ [3, 5] = [2^3, 2^5]$ $[3, 5] .^ [2, 4] = [3^2, 5^4]$

Array or *Element-by-element* multiplication is defined only for arrays having the same size. The definition of the product $\mathbf{x} \cdot * \mathbf{y}$, where \mathbf{x} and \mathbf{y} each have n elements, is

$$\mathbf{x} \cdot * \mathbf{y} = [\mathbf{x}(1) \mathbf{y}(1), \mathbf{x}(2) \mathbf{y}(2), \dots, \mathbf{x}(n) \mathbf{y}(n)]$$

if \mathbf{x} and \mathbf{y} are row vectors. For example, if

$$\mathbf{x} = [2, 4, -5], \mathbf{y} = [-7, 3, -8]$$

then $\mathbf{z} = \mathbf{x} \cdot * \mathbf{y}$ gives

$$\mathbf{z} = [2(-7), 4(3), -5(-8)] = [-14, 12, 40]$$

If \mathbf{x} and \mathbf{y} are column vectors, the result of $\mathbf{x} \cdot * \mathbf{y}$ is a column vector. For example $\mathbf{z} = (\mathbf{x}') \cdot * (\mathbf{y}')$ gives

$$\mathbf{z} = \begin{bmatrix} 2(-7) \\ 4(3) \\ -5(-8) \end{bmatrix} = \begin{bmatrix} -14 \\ 12 \\ 40 \end{bmatrix}$$

Note that \mathbf{x}' is a column vector with size 3×1 and thus does not have the same size as \mathbf{y} , whose size is 1×3 .

Thus for the vectors \mathbf{x} and \mathbf{y} the operations $\mathbf{x}' \cdot * \mathbf{y}$ and $\mathbf{y} \cdot * \mathbf{x}'$ are not defined in MATLAB and will generate an error message.

The array operations are performed between the elements in corresponding locations in the arrays. For example, the array multiplication operation $\mathbf{A} \cdot * \mathbf{B}$ results in a matrix \mathbf{C} that has the same size as \mathbf{A} and \mathbf{B} and has the elements $c_{ij} = a_{ij}b_{ij}$. For example, if

$$\mathbf{A} = \begin{bmatrix} 11 & 5 \\ -9 & 4 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} -7 & 8 \\ 6 & 2 \end{bmatrix}$$

then $\mathbf{C} = \mathbf{A} \cdot * \mathbf{B}$ gives this result:

$$\mathbf{C} = \begin{bmatrix} 11(-7) & 5(8) \\ -9(6) & 4(2) \end{bmatrix} = \begin{bmatrix} -77 & 40 \\ -54 & 8 \end{bmatrix}$$

The built-in MATLAB functions such as `sqrt(x)` and `exp(x)` automatically operate on array arguments to produce an array result the same size as the array argument `x`.

Thus these functions are said to be *vectorized* functions.

For example, in the following session the result `y` has the same size as the argument `x`.

```
>>x = [4, 16, 25];  
>>y = sqrt(x)  
y =  
    2    4    5
```

However, when multiplying or dividing these functions, or when raising them to a power, you must use element-by-element operations if the arguments are arrays.

For example, to compute $z = (e^y \sin x) \cos^2 x$, you must type

```
z = exp(y) .* sin(x) .* (cos(x) ) .^2.
```

You will get an error message if the size of x is not the same as the size of y . The result z will have the same size as x and y .

Array Division

The definition of array division is similar to the definition of array multiplication except that the elements of one array are divided by the elements of the other array. Both arrays must have the same size. The symbol for array right division is `./`. For example, if

$$x = [8, 12, 15] \qquad y = [-2, 6, 5]$$

then $z = x ./ y$ gives

$$z = [8/(-2), 12/6, 15/5] = [-4, 2, 3]$$

Also, if

$$\mathbf{A} = \begin{bmatrix} 24 & 20 \\ -9 & 4 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} -4 & 5 \\ 3 & 2 \end{bmatrix}$$

then $\mathbf{C} = \mathbf{A} \cdot / \mathbf{B}$ gives

$$\mathbf{C} = \begin{bmatrix} 24/(-4) & 20/5 \\ -9/3 & 4/2 \end{bmatrix} = \begin{bmatrix} -6 & 4 \\ -3 & 2 \end{bmatrix}$$

Array Exponentiation

MATLAB enables us not only to raise arrays to powers but also to raise scalars and arrays to *array* powers.

To perform exponentiation on an element-by-element basis, we must use the `.^` symbol.

For example, if

```
x = [3, 5, 8],
```

then typing

```
x.^3
```

produces the array

```
[33, 53, 83] = [27, 125, 512].
```

We can raise a scalar to an array power. For example, if

$p = [2, 4, 5]$,

then typing

$3.^p$

produces the array

$[3^2, 3^4, 3^5] = [9, 81, 243]$.

Remember that $.^$ is a *single* symbol. The dot in $3.^p$ is *not* a *decimal point* associated with the number 3. The following operations, with the value of p given here, are equivalent and give the correct answer:

$3.^p$

$3.0.^p$

$3.^.^p$

$(3).^p$

$3.^{[2, 4, 5]}$

Matrix-Matrix Multiplication

In the product of two matrices **AB**, the number of *columns* in **A** must equal the number of *rows* in **B**. The row-column multiplications form column vectors, and these column vectors form the matrix result. The product **AB** has the same number of *rows* as **A** and the same number of *columns* as **B**. For example,

$$\begin{bmatrix} 6 & -2 \\ 10 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 9 & 8 \\ -5 & 12 \end{bmatrix} = \begin{bmatrix} (6)(9) + (-2)(-5) & (6)(8) + (-2)(12) \\ (10)(9) + (3)(-5) & (10)(8) + (3)(12) \\ (4)(9) + (7)(-5) & (4)(8) + (7)(12) \end{bmatrix}$$
$$= \begin{bmatrix} 64 & 24 \\ 75 & 116 \\ 1 & 116 \end{bmatrix}$$

Use the operator `*` to perform matrix multiplication in MATLAB. The following MATLAB session shows how to perform the matrix multiplication shown in

```
>>A = [6,-2;10,3;4,7];
```

```
>>B = [9,8;-5,12];
```

```
>>A*B
```

```
ans =
```

```
    64    24
```

```
    75   116
```

```
     1   116
```

Matrix multiplication does not have the commutative property; that is, in general,

$$\mathbf{AB} \neq \mathbf{BA}.$$

A simple example will demonstrate this fact:

$$\mathbf{AB} = \begin{bmatrix} 6 & -2 \\ 10 & 3 \end{bmatrix} \begin{bmatrix} 9 & 8 \\ -12 & 14 \end{bmatrix} = \begin{bmatrix} 78 & 20 \\ 54 & 122 \end{bmatrix}$$

whereas

$$\mathbf{BA} = \begin{bmatrix} 9 & 8 \\ -12 & 14 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ 10 & 3 \end{bmatrix} = \begin{bmatrix} 134 & 6 \\ 68 & 66 \end{bmatrix}$$

Reversing the order of matrix multiplication is a common and easily made mistake.

Special Matrices

Two exceptions to the noncommutative property are the *null* or *zero* matrix, denoted by **0** and the *identity*, or *unity*, matrix, denoted by **I**.

The null matrix contains all zeros and is not the same as the *empty* matrix $[]$, which has no elements.

These matrices have the following properties:

$$\mathbf{0A} = \mathbf{A0} = \mathbf{0}$$

$$\mathbf{IA} = \mathbf{AI} = \mathbf{A}$$

The identity matrix is a square matrix whose diagonal elements are all equal to one, with the remaining elements equal to zero.

For example, the 2×2 identity matrix is

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The functions `eye(n)` and `eye(size(A))` create an $n \times n$ identity matrix and an identity matrix the same size as the matrix `A`.

For example

```
>> eye(3)
```

```
ans =
```

```
1  0  0
0  1  0
0  0  1
```

```
>> A=[2 3 4;5 6 7; 8 9 10]
```

```
A =
```

```
2  3  4
5  6  7
8  9 10
```

```
>> eye(size(A))
```

```
ans =
```

```
1  0  0
0  1  0
0  0  1
```

Sometimes we want to initialize a matrix to have all zero elements. The `zeros` command creates a matrix of all zeros.

Typing `zeros(n)` creates an $n \times n$ matrix of zeros, whereas typing `zeros(m,n)` creates an $m \times n$ matrix of zeros.

Typing `zeros(size(A))` creates a matrix of all zeros having the same dimension as the matrix **A**. This type of matrix can be useful for applications in which we do not know the required dimension ahead of time.

The syntax of the `ones` command is the same, except that it creates arrays filled with ones.

Matrix Left Division and Linear Algebraic Equations

$$6x + 12y + 4z = 70$$

$$7x - 2y + 3z = 5$$

$$2x + 8y - 9z = 64$$

```
>>A = [6,12,4;7,-2,3;2,8,-9];
```

```
>>B = [70;5;64];
```

```
>>Solution = A\B
```

```
Solution =
```

```
3
```

```
5
```

```
-2
```

The solution is $x = 3$, $y = 5$, and $z = -2$.

Polynomial Coefficients

The function `poly(r)` computes the coefficients of the polynomial whose roots are specified by the vector `r`. The result is a *row vector* that contains the polynomial's coefficients arranged in descending order of power.

For example,

```
>>c = poly([-2, -5])  
c =  
    1     7    10
```

Polynomial Roots

The function `roots(a)` computes the roots of a polynomial specified by the coefficient array `a`. The result is a *column vector* that contains the polynomial's roots.

For example,

```
>>r = roots([2, 14, 20])  
r =  
    -2  
    -5
```

Polynomial Multiplication and Division

The function

`conv(a, b)`

computes the product of the two polynomials described by the coefficient arrays `a` and `b`. The two polynomials need not be the same degree. The result is the coefficient array of the product polynomial.

The function

`[q, r] = deconv(num, den)`

computes the result of dividing a numerator polynomial, whose coefficient array is `num`, by a denominator polynomial represented by the coefficient array `den`. The quotient polynomial is given by the coefficient array `q`, and the remainder polynomial is given by the coefficient array `r`.

Polynomial Multiplication and Division: Examples

```
>>a = [9,-5,3,7];  
>>b = [6,-1,2];  
>>product = conv(a,b)  
product =  
    54    -39    41    29    -1    14  
>>[quotient, remainder] = deconv(a,b)  
quotient =  
    1.5    -0.5833  
remainder =  
    0     0    -0.5833    8.1667
```

Plotting Polynomials

The function `polyval(a, x)` evaluates a polynomial at specified values of its independent variable `x`, which can be a matrix or a vector. The polynomial's coefficients of descending powers are stored in the array `a`. The result is the same size as `x`.

Example of Plotting a Polynomial

To plot the polynomial $f(x) = 9x^3 - 5x^2 + 3x + 7$

for $-2 \leq x \leq 5$, you type

```
>>a = [9,-5,3,7];  
>>x = -2:0.01:5;  
>>f = polyval(a,x);  
>>plot(x,f),xlabel('x'),ylabel('f(x)')
```

Figure

