İstanbul Bilgi University Faculty of Engineering and Natural Sciences Department of Elecetrical and Electronics Engineering	Course: EEEN 202-Electrical and Electronic Circuits II Instructor: Prof. Dr. Mehmet Nur Alpaslan Parlakçı, Dr. Revna Acar Vural, Dr. Mustafa Berke Yelten Exam/Date: Midterm Exam/02.04.2019, 15:00 Duration: 100 min.

Problem	1	2	3	4	Total
Maximum score	25	25	25	25	100
Course learning outcome	2	1	2	2	1, 2

Problem 1)

Consider the circuit shown in Figure P1. Assume that there is no energy stored in the circuit at t = 0. Write your answers in the table shown below.

a. Find the transfer function $H(s) = V_L(s) / V_G(s)$. **(15 points)**
b. Find $v_L(t)$ if $v_G(t) = 2u(t)$, where $u(t)$ is the unit-step signal. **(10 points)**

	Answer
$H(s)$	
$v_L(t)$	

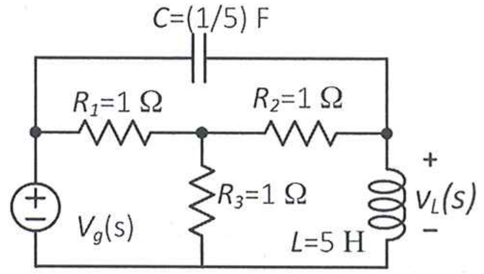


Figure P1

Problem 2)

Assume that the op amp shown in Figure P2 is ideal and signal voltages are provided in terms of differential mode and common mode voltages, v_{dm} and v_{cm}, respectively.

$$v_A = v_{cm} - \frac{1}{2} v_{dm}$$

$$v_B = v_{cm} + \frac{1}{2} v_{dm}$$

$R1$= 1 kΩ, $R2$= 10 kΩ

$R3$= 1 kΩ, $R4$= 1 kΩ

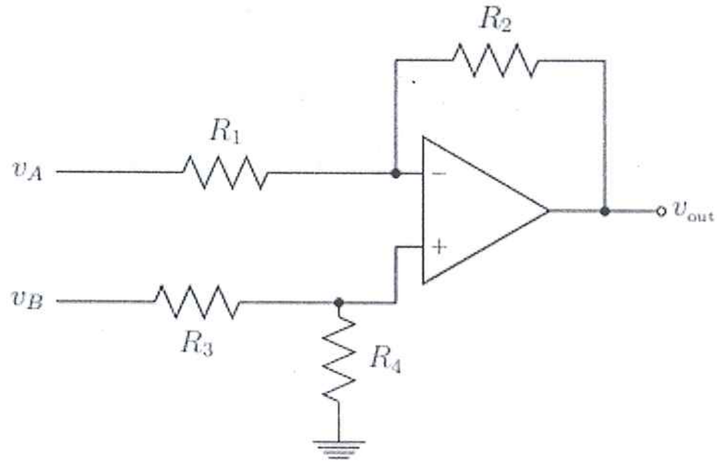


Figure P2

a. Derive an expression for the output voltage in terms of v_{dm} and v_{cm}. **(15 points)**
b. Calculate the common-mode gain. (Specify your units in either V/V or dB). **(3 points)**
c. Calculate the differential-mode gain. (Specify your units in either V/V or dB). **(3 points)**
d. Calculate the CMRR. **(4 points)**

Problem 3)

Consider the circuit shown in Figure P3.

a. Derive the transfer function, $H(s) = V_o(s)/V_s(s)$, of the circuit shown in Figure P3. **(10 points)**
b. Determine the type of the filter shown in Figure P3. **(5 points)**
c. Calculate the cutoff frequency(ies) of the filter shown in Figure P3. **(5 points)**
d. Find the output response of the filter shown in Figure P3 when $v_s(t) = cos(2t + 30^0)$. **(5 points)**

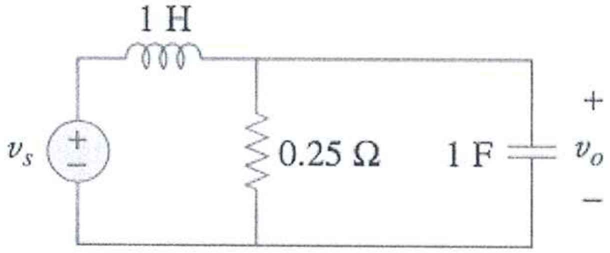


Figure P3

Problem 4)

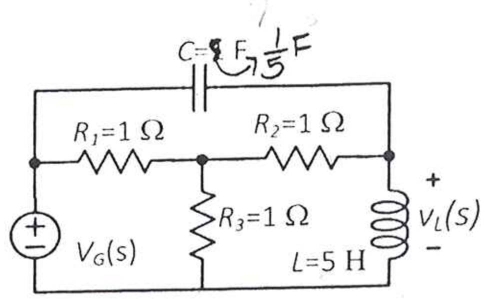
A series RLC bandpass passive filter has cutoff frequencies at 1 kHz and 10 kHz. The input impedance at resonance is 6 Ω.

a. Calculate the bandwidth, the center frequency and the quality factor of the filter. **(5 points)**
b. Calculate the values of circuit components L, C and R. Draw the circuit indicating component values and input and output configuration. **(10 points)**
c. Derive the transfer function of the bandpass filter. **(10 points)**

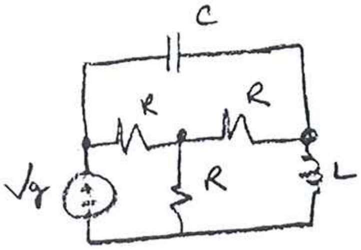
EEEN 202
Midterm Exam Solution Key

Problem 1)

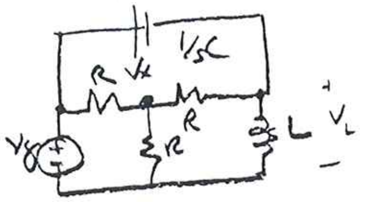
1) Consider the circuit below. Assume there is no energy stored in the circuit at $t = 0$. Write your answers in the table below:
   a. Find the transfer function $H(s) = V_L(s) / V_G(s)$.
   b. Find $v_L(t)$ if $v_G(t) = 2u(t)$.



	Answer
H(s)	$= \frac{s(5+3s)}{3s^2+10s+3}$
$v_L(t)$	$= (e^{-3t} + e^{-\frac{1}{3}t})u(t)$



$\Rightarrow$



I $\quad \frac{V_x}{R} + \frac{V_x - V_g}{R} + \frac{V_x - V_L}{R} = 0$

II $\quad (V_L - V_g)sC + \frac{V_L}{sL} + \frac{V_L - V_x}{R} = 0$

From I $\quad \frac{3V_x}{R} = \frac{V_g}{R} + \frac{V_L}{R} \Rightarrow V_x = \frac{V_g + V_L}{3}$

In II $\quad V_L\left(sC + \frac{1}{sL} + \frac{1}{R}\right) = V_g sC + \frac{V_x}{R} = V_g sC + \frac{V_g + V_L}{3R}$

$$V_L\left(sC + \frac{1}{sL} + \frac{1}{R} - \frac{1}{3R}\right) = V_g\left(sC + \frac{1}{3R}\right)$$

$$V_L\left(\frac{s^2LC(3R) + 3R + 2sL}{\cancel{3}sRL}\right) = V_g \frac{1+3sRC}{\cancel{3R}}$$

$\Rightarrow \frac{V_L}{V_g} = \frac{s(1+3RCs)L}{3RLCs^2 + 2sL + 3R} \Rightarrow$ with the values $\Rightarrow \frac{V_L}{V_g} = \frac{5s(1+\frac{3}{5}s)}{3s^2 + 10s + 3} = \frac{s(5+3s)}{3s^2+10s+}$

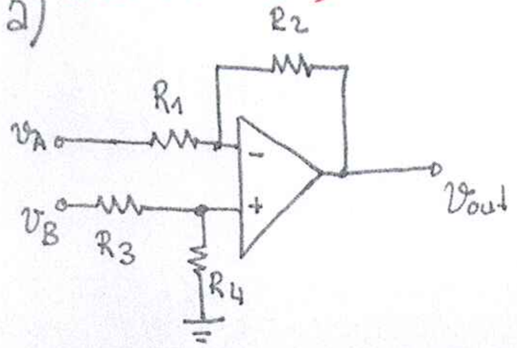
$V_L(s) = \frac{2}{\cancel{s}} \frac{\cancel{s}(5+3s)}{3s^2+10s+3} = \frac{2(5+3s)}{(s+3)(3s+1)} = \frac{A}{s+3} + \frac{B}{3s+1}$

$\left.\begin{array}{l} 3A + B = 6 \\ A + 3B = 10 \end{array}\right\} \begin{array}{l} A = 1 \\ B = 3 \end{array}$

$V_L(s) = \frac{1}{s+3} + \frac{3}{3s+1} \Rightarrow v_L(t) = (e^{-3t} + e^{-\frac{1}{3}t})u(t)$

Problem 2)

a)



Assume opamp is ideal $v_p = v_n$

$$v_p = \frac{v_B . R_4}{(R_3+R_4)}$$

$$\frac{v_A - v_n}{R_1} = \frac{v_n - v_{out}}{R_2}$$

$$\frac{v_A - v_B . \frac{R_4}{(R_3+R_4)}}{\left(\frac{R_1}{R_2}\right)} = \frac{v_B . \frac{R_4}{(R_3+R_4)} - v_{out}}{\frac{\cancel{R_2}}{\cancel{R_2}}} \Rightarrow \boxed{v_{out} = v_B \frac{R_4}{(R_3+R_4)}\left(1+\frac{R_2}{R_1}\right) - v_A . \frac{R_2}{R_1}} \quad (1)$$

$v_A, v_B = f(v_{cm}, v_{dm})$, rewrite (1)

$$\boxed{v_{out} = v_{cm}\left[\left(\frac{R_4}{(R_3+R_4)} . \left(1+\frac{R_2}{R_1}\right)\right) - \frac{R_2}{R_1}\right] + \frac{1}{2} v_{dm}\left[\left(\frac{R_4}{R_3+R_4} . \left(1+\frac{R_2}{R_1}\right)\right) + \frac{R_2}{R_1}\right]}$$

$R_4 = 1k\Omega$, $R_2 = 10k\Omega$, $R_3 = 1k\Omega$, $R_1 = 1k\Omega$

$$v_{out} = v_{cm}\left[\left(\frac{1}{2} \times 11\right) - 10\right] + v_{dm} \times \frac{1}{2}\left[\frac{11}{2} + 10\right]$$

$$v_{out} = v_{cm} \times (-4.5) + v_{dm} . (7.75)$$

b) $A_{cm} = -4.5$ V/V

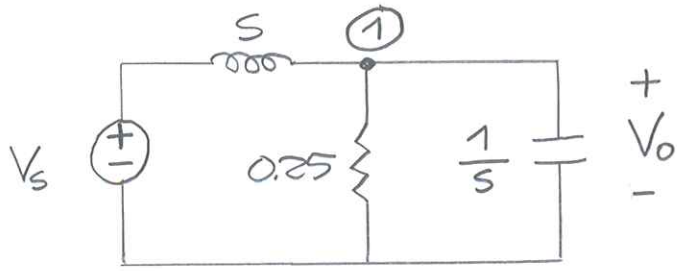
c) $A_{dm} = 7.75$ V/V

d) $CMRR = 20 \log_{10}\left|\left(\frac{A_{dm}}{A_{cm}}\right)\right|$

$$CMRR = 20 \log_{10} 1.72 = 4.72 \text{ dB}$$

Problem 3)

a. We consider the s-domain equivalent circuit



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KCL at node 1 :

$$\frac{V_o - V_s}{s} + \frac{V_o}{0.25} + \frac{V_o}{1/s} = 0$$

$$\Rightarrow \left(\frac{1}{s} + 4 + s\right) V_o = \frac{V_s}{s} \Rightarrow V_s = (s^2 + 4s + 1) V_o$$ 05

$$\Rightarrow H(s) = \frac{V_o}{V_s} = \frac{1}{s^2 + 4s + 1}$$

b. $s = j\omega \Rightarrow H(j\omega) = \dfrac{1}{(j\omega)^2 + j4\omega + 1}$

$$|H(j\omega)| = \frac{1}{\sqrt{(1-\omega^2)^2 + 16\omega^2}} \quad , \quad \angle H(j\omega) = 0 - \tan^{-1}\frac{4\omega}{1-\omega^2} = -\tan^{-1}\frac{4\omega}{1-\omega^2}$$

$\omega = 0$:

$$|H(j0)| = \frac{1}{\sqrt{(1-0)^2 + 0^2}} = 1 \quad , \quad \angle H(j0) = 0°$$ 05

$\omega \to \infty$:

$$\lim_{\omega \to \infty} |H(j\omega)| = \lim_{\omega \to \infty} \frac{1}{\sqrt{(1-\omega^2)^2 + 16\omega^2}} = \lim_{\omega \to \infty} \frac{1}{\sqrt{\omega^4 + 16\omega^2}} = 0°$$

$$\lim_{\omega \to \infty} \angle H(j\omega) = \lim_{\omega \to \infty} -\tan^{-1}\frac{4\omega}{1-\omega^2} = \lim_{\omega \to \infty} -\tan^{-1}\frac{4\omega}{-\omega^2} = 0°$$

$\therefore$ H is a low pass filter !

c.

$$|H(j\omega)| = \frac{1}{\sqrt{2}} \cdot \underbrace{H_{max}}_{=1} \Rightarrow \omega \equiv \omega_c$$

$$\Rightarrow \frac{1}{\sqrt{(1-\omega_c^2)^2+16\omega_c^2}} = \frac{1}{\sqrt{2}} \cdot 1 \Rightarrow (1-\omega_c^2)^2+16\omega_c^2 = 2$$

$$\Rightarrow \omega_c^4+14\omega_c^2+1 = 2 \Rightarrow \omega_c^4+14\omega_c^2-1=0$$

05

$$\Rightarrow \omega_c^2 = \frac{-14+\sqrt{14^2+4}}{2} = -7+5\sqrt{2}$$

$$\Rightarrow \omega_c = \sqrt{5\sqrt{2}-7} = 0.2666 \text{ rad/sec}$$

d. The steady-state output response is calculated as

$$v_{o,ss}(t) = |H(j2)| \cos\left(2t+30^\circ + \angle H(j2)\right)$$

where

$$|H(j2)| = \frac{1}{\sqrt{(1-4)^2+16.4}} = \frac{1}{\sqrt{73}} = 0.117$$

$$\angle H(j2) = -\tan^{-1}\frac{4 \cdot 2}{1-4} = \tan^{-1}\frac{8}{3} = 69.444^\circ$$

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Hence;

$$v_{o,ss}(t) = 0.117 \cos\left(2t+30^\circ+65.444^\circ\right)$$

$$= 0.117 \cos\left(2t+95.444^\circ\right) \text{ V}$$

Problem 4)

a) At resonance input impedance of series RLC circuit is equal to R. Hence $R = 6\,\Omega$

$$\beta = 2\pi(f_{c2} - f_{c1}) = 2\cdot\pi\cdot 9\times10^{3} = \overset{\beta}{56.52\ \text{krad/s}}$$

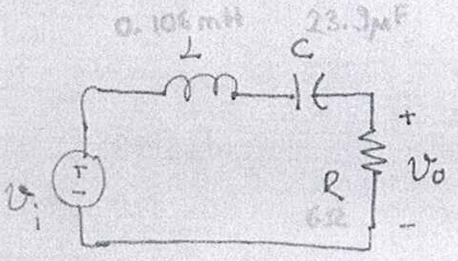
$$\omega_0^2 = \omega_{c1}\cdot\omega_{c2} \Rightarrow \omega_0 = \sqrt{4\pi^2\times10\times10^6} = \overset{\omega_0}{19.85\ \text{krad/s}}$$

$$Q = \frac{\omega_0}{\beta} = \frac{19.85\ \text{krad/s}}{56.52\ \text{krad/s}} = \overset{Q}{0.35}$$

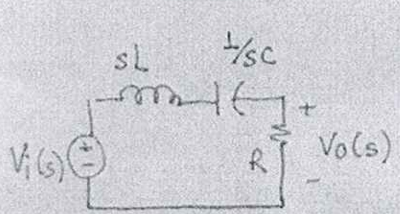
b) $R = 6\,\Omega$

$$\beta = \frac{R}{L} \Rightarrow L = \frac{6}{56.52\times10^3} = \overset{L}{0.106\ \text{mH}}$$

$$\omega_0^2 = \frac{1}{LC} \Rightarrow C = \frac{1}{0.106\times10^{-3}\times(19.85\times10^3)^2} = \overset{C}{23.9\ \mu F}$$



c) s_domain



$$\frac{V_o(s)}{V_i(s)} = \frac{R}{sL + \frac{1}{sC} + R} = \frac{sRC \times \frac{1}{LC}}{(s^2LC + sRC + 1)\times\frac{1}{LC}}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{s\,R/L}{s^2 + s\,R/L + \frac{1}{LC}}}$$

where $\beta = R/L$

$\omega_0^2 = 1/LC$