

Problem 5) Consider the unity-feedback control system with the following open-loop transfer function:

$$G(s) = \frac{10}{s(s+1)(3s+2)}$$

Determine if this system is stable or not.

Problem 6) Determine the range of gain K for stability of a unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}$$

1. For the unity feedback system shown in Figure P7.1, where

$$G(s) = \frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}$$

find the steady-state errors for the following test inputs: $5u(t)$, $37tu(t)$, $47t^2u(t)$. [Section: 7.2]

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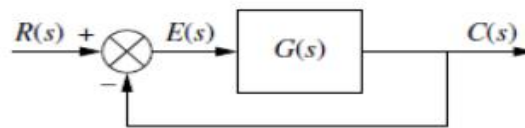


FIGURE P7.1

4. For the system shown in Figure P7.3, what steady-state error can be expected for the following test inputs: $15u(t)$, $15tu(t)$, $15t^2u(t)$. [Section: 7.2]

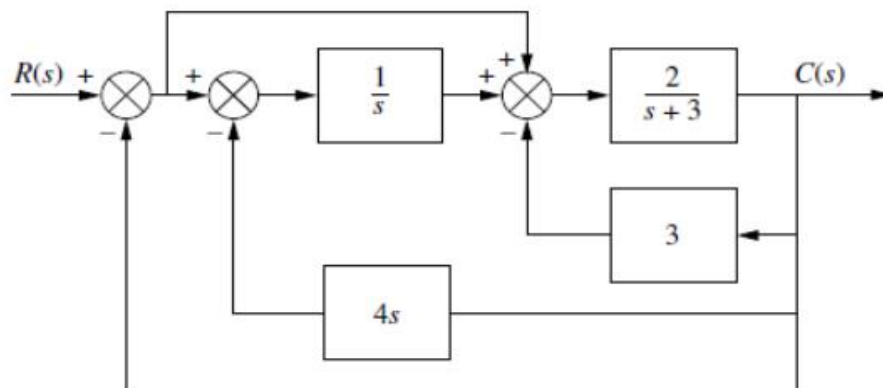


FIGURE P7.3

$$5) G(s) = \frac{10}{s(s+1)(3s+2)}$$

$$\text{unity feedback} \\ H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)} = \frac{10}{\frac{s(s+1)(3s+2)}{1 + \frac{10}{s(s+1)(3s+2)}}} = \frac{\frac{10}{(s^2+s)(3s+2)}}{\frac{(s^2+s)(3s+2)+10}{(s^2+s)(3s+2)}}$$

$$\frac{C(s)}{R(s)} = \frac{10}{(s^2+s)(3s+2)+10}$$

$$\Rightarrow 3s^3 + 2s^2 + 3s^2 + 2s + 10$$

$$3s^3 + 5s^2 + 2s + 10 = 0$$

$$\begin{array}{rcl} +s^3 & 3 & 2 \\ +s^2 & 5 & 10 \\ -s^1 & -4 & 0 \\ +s^0 & +10 & 0 \end{array} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{LHP} \\ \text{RHP} \\ \text{RHP} \end{array}$$

$\therefore 1 \text{ LHP} / 2 \text{ RHP} \Rightarrow \text{system is unstable}$

$$6) G(s) = \frac{K}{(s+1)(s+2)(s+3)}$$

$$H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)} = \frac{K}{\frac{(s+1)(s+2)(s+3)}{1 + \frac{K}{(s+1)(s+2)(s+3)}}}$$

$$\frac{K}{(s+1)(s+2)(s+3)+K} = \frac{K}{s^3 + 6s^2 + 11s + 6 + K}$$

$$\Rightarrow s^3 + 6s^2 + 11s + 6 + K = 0$$

$$\begin{array}{rcl} s^3 & 6 & 11 & 0 \end{array}$$

$$\begin{array}{rcl} s^2 & 6 & 6+K & 0 \end{array}$$

$$\begin{array}{rcl} s^1 & \frac{60-K}{6} & 0 & 0 \end{array}$$

$$\begin{array}{rcl} s^0 & 6+K & \end{array}$$

$$\frac{60-K}{6} > 0 \rightarrow \underline{K < 60}$$

$$6+K > 0 \rightarrow \underline{K > -6}$$

$$\boxed{-6 < K < 60}$$

1.

$$e(\infty) = \lim_{s \rightarrow 0} s E(s) = \lim_{s \rightarrow 0} \frac{s R(s)}{1+G(s)}$$

where

$$G(s) = \frac{450(s+12)(s+8)(s+15)}{s(s+38)(s^2+2s+28)}$$

For step, $e(\infty) = 0$. For $37tu(t)$, $R(s) = \frac{37}{s^2}$. Thus, $e(\infty) = 6.075 \times 10^{-2}$. For parabolic input, $e(\infty) = \infty$.

4.

Reduce the system to an equivalent unity feedback system by first moving $1/s$ to the left past the summing junction. This move creates a forward path consisting of a parallel pair, $\left(\frac{1}{s} + 1\right)$ in cascade with a feedback loop consisting of $G(s) = \frac{2}{s+3}$ and $H(s) = 7$. Thus,

$$G_e(s) = \left(\frac{s+1}{s}\right) \left(\frac{2/(s+3)}{1+14/(s+3)}\right) = \frac{2(s+1)}{s(s+17)}$$

Hence, the system is Type 1 and the steady-state errors are as follows:

Steady-state error for $15u(t) = 0$.

Steady-state error for $15tu(t) = \frac{15}{K_v} = \frac{15}{2/17} = 127.5$.

Steady-state error for $15t^2u(t) = \infty$

Input	Steady-state error formula	Type 0		Type 1		Type 2	
		Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p = \text{Constant}$	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v = \text{Constant}$	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a = \text{Constant}$	$\frac{1}{K_a}$