

MATH 233

Fall 2018

Quiz #1 A Solutions.

Duration: 50 minutes.

Remark: Show your thinking/work. Do not just write a number as a result.

1. A fair coin is flipped 10 times where each flip comes up either **head** or **tails**.

- a) How many possible outcomes are there total?
- b) How many of these outcomes contain a single tail or a single head?

a) Outcomes are:

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....

....

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There are $2^{10} = 1024$ difference outcomes.

b) Outcomes containing a single head or a single tail are:

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...

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and

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...

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Thus, there are $10 + 10 = 20$ such outcomes.

2. Prove the following identity using induction:

$$1.2 + 2.3 + 3.4 + \dots + (n-1).n = ((n-1).n.(n+1)) / 3$$

a) Base case: Does the assertion hold for $n=2$?

left hand side of the equation is $1.2 = 2$ for $n=2$.

right hand side of the equation is $(1.2.3)/3 = 2$

Therefore the equation holds for $n=2$.

(See that it also holds for $n=1$)

b) Inductive step: Assuming that the assertion holds for k , show that it also holds for $k+1$.

$1.2 + 2.3 + 3.4 + \dots + (k-1).k = ((k-1).k.(k+1)) / 3$ is given.

Show that : $1.2 + 2.3 + 3.4 + \dots + (k-1).k + k.(k+1) = ((k).(k+1).(k+2)) / 3$.

$$\begin{aligned} 1.2 + 2.3 + 3.4 + \dots + (k-1).k + k.(k+1) &= ((k-1).k.(k+1)) / 3 + k.(k+1) \\ &= (k+1) ((k^2-k)/3 + k) \\ &= (k+1) (k^2-k + 3k)/3 \\ &= (k+1) (k^2+2k)/3 \\ &= (k+1). k(k+2)/3 \end{aligned}$$

Base case and inductive steps show that the assertion holds for all n larger than or equal to 0.