Problem 1) a. Using 20nF capacitors, design an active broad-band first-order bandpass filter that has a lower cutoff frequency of 2000 Hz, an upper cutoff frequency of 8000 Hz, and a passband gain of 10dB. Use prototype versions of low-pass and high-pass filters in the design process.
b. Write the transfer function for the scaled filter.
c. Use the transfer function derived in part (b) to find $H(j\omega_0)$, where ω_0 is the center frequency of the filter.

Solution. We first need to calculate the frequency and magnitude scaling factors:

a. High-pass filter:

$$k_f = 2\pi \cdot 2000 = 4000\pi$$

$$20\cdot 10^{-9} = \frac{1}{4000\pi \cdot k_m} \Rightarrow k_m = \frac{10^5}{8\pi} = 3978.9$$

$$R_H = 3978.9 \cdot 1 = 3.9789\ k\Omega \Rightarrow H'_{hp} = -\frac{s}{s+4000\pi}$$

Low-pass filter:

$$k_f = 2\pi \cdot 8000 = 16000\pi$$

$$20\cdot 10^{-9} = \frac{1}{16000\pi \cdot k_m} \Rightarrow k_m = \frac{10^5}{32\pi} = 994.7184$$

$$R_L = 994.7184 \cdot 1 = 9.9472\ k\Omega \Rightarrow H_{lp} = -\frac{16000\pi}{s+16000\pi}$$

Moreover;

$$H(s) = -\frac{R_f}{R_i}\left(-\frac{s}{s+4000\pi}\right)\left(-\frac{16000\pi}{s+16000\pi}\right)$$

$$H_{passband} = |H(j\omega_0)|, \quad \omega_0 = \sqrt{4000\pi \cdot 16000\pi} = 8000\pi$$

$$\left|H(j8000\pi)\right| = \left| -\frac{R_f}{R_i} \left(\frac{-j\,\cancel{8000\pi}}{j\,\underset{2}{\cancel{8000\pi}} + \underset{1}{\cancel{4000\pi}}} \right) \left(\frac{-\cancel{16000\pi}}{j\,\underset{1}{\cancel{8000\pi}} + \underset{2}{\cancel{16000\pi}}} \right) \right|$$

$$= \frac{R_f}{R_i} \frac{4}{\sqrt{5} \cdot \sqrt{5}}$$

$$= 0.8 \frac{R_f}{R_i}$$

$$\Rightarrow \quad 10\,dB = 20 \log_{10} 0.8 \frac{R_f}{R_i} \quad \Rightarrow \quad \frac{R_f}{R_i} = 1.25\sqrt{10}$$

- let us choose $R_i = 10k\Omega$ then $R_f = 39.5285\,k\Omega$

b.

$$H(s) = -\frac{39.5285}{10} \frac{s}{s+4000\pi} \cdot \frac{16000\pi}{s+16000\pi}$$

c.

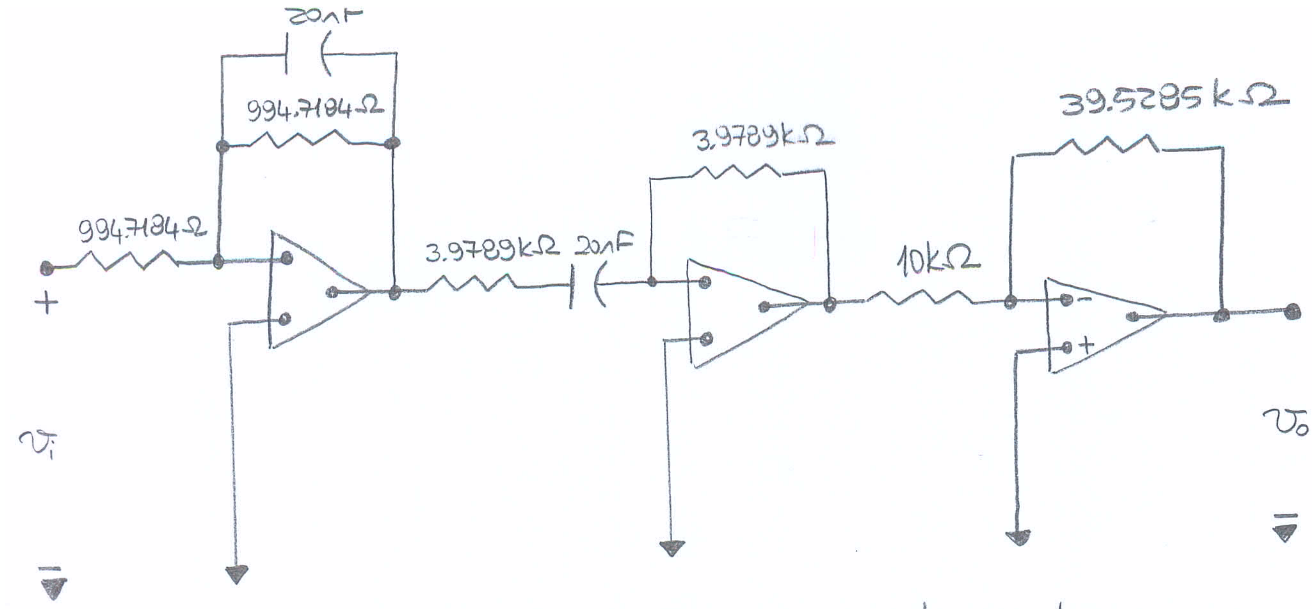
$$H(j\omega_0) = -\frac{39.5285}{10} \frac{j\,\overset{2}{\cancel{8000\pi}}}{j\,\underset{2}{\cancel{8000\pi}} + \cancel{4000\pi}} \cdot \frac{\overset{2}{\cancel{16000\pi}}}{j\,\underset{1}{\cancel{8000\pi}} + \underset{2}{\cancel{16000\pi}}}$$

$$= \frac{-j15.8114}{(1+j2)(2+j)}$$

$$= \frac{-j15.8114}{\cancel{2}+j+j4-\cancel{2}}$$

$$= -3.1623$$

- we shall also draw the opamp circuit diagram of the active broadband bandpass scaled filter as follows:



Problem 2) a. Using 5nF capacitors, design an active broadband first-order bandreject filter with a lower cutoff frequency of 1000 Hz, an upper cutoff frequency of 5000 Hz, and a passband gain of 10dB.

b. Draw the circuit diagram of the filter and label all the components.

c. What is the transfer function of the scaled filter?

d. Evaluate the transfer function derived in (c) at the center frequency of the filter.

e. What is the gain in decibels at the center frequency?

Solution. We consider prototype low-pass and high-pass filters and calculate the frequency and magnitude scaling factors.

Low-pass filter:

$$k_f = 2\pi \cdot 1000 \quad , \quad 5 \cdot 10^{-9} = \frac{1}{2000\pi \cdot k_m} \quad \Rightarrow \quad k_m = 3.1831 \cdot 10^4$$

$$R_L = 3.1831 \cdot 10^4 = 31.831 \text{ k}\Omega$$

High-pass filter:

$$k_f = 2\pi \cdot 5000 , \quad 5 \cdot 10^{-9} = \frac{1}{10000\pi \cdot k_m} \quad \Rightarrow \quad k_m = 6.3662 \cdot 10^3$$

$$R_H = 6.3662 \cdot 10^3 \cdot 1 = 6.3662 \text{ k}\Omega$$

PS 5.3

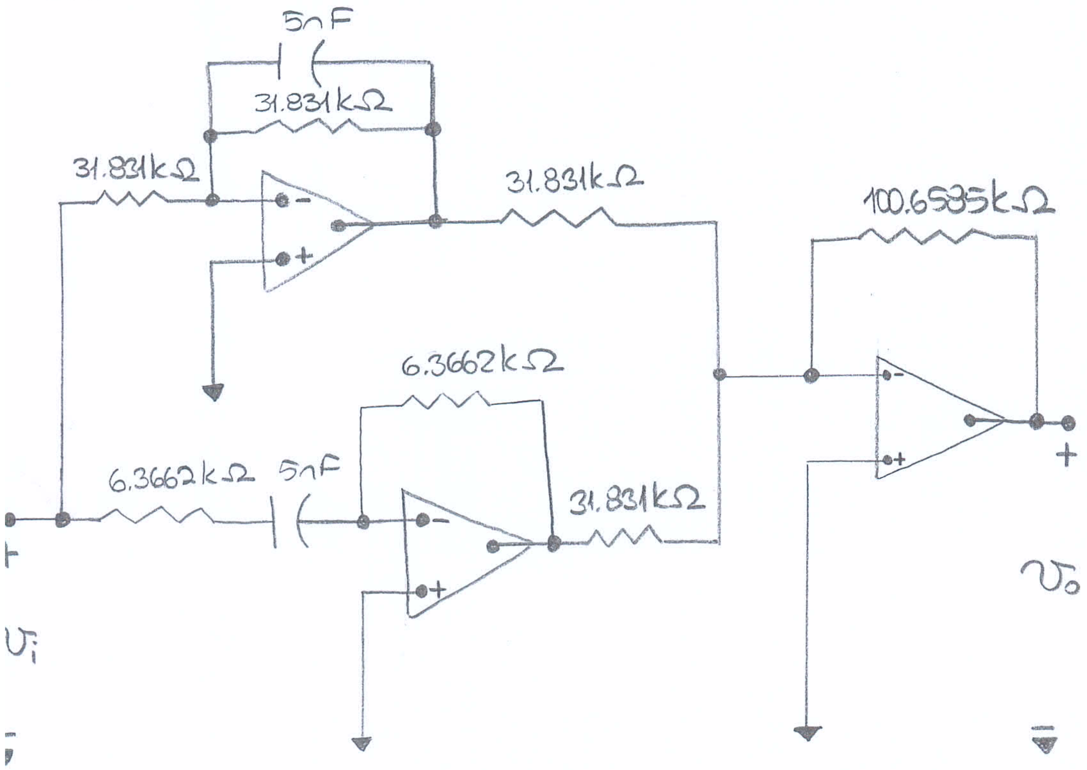
$$H(s) = -\frac{R_f}{R_i}\left(-\frac{2\pi \cdot 1000}{s+2\pi\cdot 1000} - \frac{s}{s+2\pi\cdot 5000}\right)$$

$$|H(j0)| \cong \left|-\frac{R_f}{R_f}(-1-0)\right|$$

$$= \frac{R_f}{R_i}$$

$$10\,dB = 20\log_{10}\frac{R_f}{R_i} \Rightarrow \frac{R_f}{R_i} = \sqrt{10}$$

– we shall choose $R_i = 31.831\ k\Omega$ then $R_f = 100.6585\ k\Omega$



c. We obtain

$$H(s) = -\sqrt{10}\left(-\frac{2000\pi}{s+2000\pi} - \frac{s}{s+10000\pi}\right)$$

$$= \sqrt{10}\,\frac{s^2+4000\pi s + 2\cdot 10^7\pi^2}{(s+2000\pi)(s+10000\pi)}$$

d. $\omega_0 = \sqrt{2\pi\cdot 1000\cdot 2\pi\cdot 5000} = 2\pi\sqrt{5}\cdot 10^3 = 2000\pi\sqrt{5}$

$$H(j\omega_0) = \sqrt{10}\,\frac{\left(\cancel{-2\cdot 10^7\pi^2} + \overset{2}{\cancel{4000\pi}}\, j\cdot\cancel{2000\pi}\sqrt{5} + \cancel{2\cdot 10^7\pi^2}\right)}{\left(j\,\cancel{2000\pi}\sqrt{5} + \cancel{2000\pi}\right)\left(j\,\cancel{2000\pi}\sqrt{5} + \underset{5}{\cancel{10000\pi}}\right)}$$

PS 5.4

$$= \frac{j2\sqrt{10}\sqrt{5}}{(1+j\sqrt{5})(5+j\sqrt{5})}$$

$$= \frac{\cancel{j}2\sqrt{10}\cancel{\sqrt{5}}}{\cancel{j}6\cancel{\sqrt{5}}}$$

$$= \frac{\sqrt{10}}{3}$$

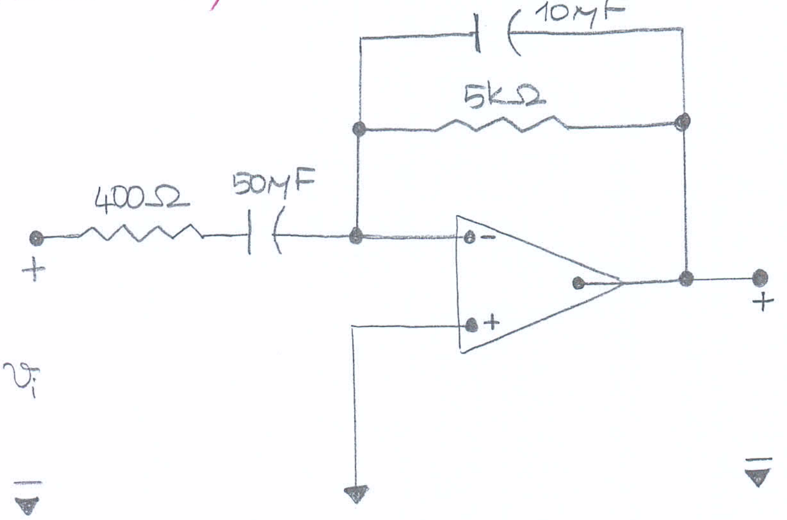
$$= 1.0541$$

e. $$|H(j\omega_0)|_{dB} = 20\log_{10}|H(j\omega_0)|$$

$$= 20\log_{10} 1.0541$$

$$= 0.46\,dB$$

Problem 3) Consider the following circuit.



Show that the circuit behaves as a bandpass filter.

a. Find the center frequency, bandwidth and gain for this bandpass filter.

b. Find the cutoff frequencies and the quality factor for this bandpass filter.

Solution. Let us find the transfer function for this circuit, we know that

$$\frac{V_o}{V_i} \triangleq H(s) = -\frac{Z_f}{Z_i}$$

where

$$Z_f \equiv 5k\Omega \,//\, 10\mu F$$

$$Z_i \equiv 400\Omega \sim 50\mu F$$

Hence;

$$Z_f = \frac{5000 \cdot (1/s \cdot 10 \cdot 10^{-6})}{5000 + \frac{1}{s \cdot 10 \cdot 10^{-6}}} = \frac{5000}{0.05s+1}$$

$$Z_i = 400 + \frac{1}{s \cdot 50 \cdot 10^{-6}} = \frac{0.02s+1}{5s \cdot 10^{-5}}$$

$$\Rightarrow \quad H(s) = -\frac{5000}{0.05s+1} \cdot \frac{5s \cdot 10^{-5}}{0.02s+1}$$

$$= -\frac{250s}{(5s+100)(0.2s+10)}$$

$$= -\frac{250s}{s^2+70s+1000}$$

$$= -\frac{25}{7} \frac{70s}{s^2+70s+1000}$$

$$= -K \frac{\beta s}{s^2+\beta s+\omega_0^2}$$

$\therefore$ the structure of the transfer function, $H(s)$ indicates that the circuit behaves as a bandpass filter.

2. $\omega_0 = \sqrt{1000} = 10\sqrt{10} = 31.6228$ rad/s

$\beta = 70$ rad/sec

$K = \frac{25}{7} = 3.5714$

b.

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2}$$

$$= -\frac{70}{2} + \sqrt{\left(\frac{70}{2}\right)^2 + 1000}$$

$$= -35 + 47.1655$$

$$= 12.1655 \text{ rad/s}$$

$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2}$$

$$= \frac{70}{2} + \sqrt{\left(\frac{70}{2}\right)^2 + 1000}$$

$$= 35 + 47.1655$$

$$= 82.1655 \text{ rad/sec}$$

$$Q = \frac{\omega_0}{\beta} = \frac{31.6228}{70} = 0.4518$$

Problem 4) Design a unity-gain bandpass filter, using a cascade connection, to give a center frequency of 50 krad/s and a bandwidth of 300 krad/s. Use 150 nF capacitors. Specify f_{c1}, f_{c2}, R_L and R_H.

Solution. We have

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = 50 \cdot 10^3 \quad \Rightarrow \quad \omega_{c2}\omega_{c1} = 2500 \cdot 10^6$$

$$\beta = \omega_{c2} - \omega_{c1} = 300 \cdot 10^3$$

$$\Rightarrow (\omega_{c2} - \omega_{c1})^2 + 4\omega_{c2}\omega_{c1} = \omega_{c2}^2 - 2\omega_{c2}\omega_{c1} + \omega_{c1}^2 + 4\omega_{c2}\omega_{c1}$$

$$= \omega_{c2}^2 + 2\omega_{c2}\omega_{c1} + \omega_{c1}^2$$

$$= (\omega_{c2} + \omega_{c1})^2$$

$$\Rightarrow \omega_{c2} + \omega_{c1} = \sqrt{(\omega_{c2} - \omega_{c1})^2 + 4\omega_{c2}\omega_{c1}}$$

$$= \sqrt{\beta^2 + 4\omega_0^2}$$

$\Rightarrow \quad \omega_{c2}+\omega_{c1} = \sqrt{5\cdot10^{10}+10^{10}} = \sqrt{10^{11}} = 3.1623\cdot10^{5}$

- we shall now solve for ω_{c2} and ω_{c1} as

$$\begin{array}{r} \omega_{c2}-\omega_{c1} = 3\cdot10^{5} \\ + \quad \omega_{c2}+\omega_{c1} = 3.1623\cdot10^{5} \\ \hline 2\omega_{c2} = 0.1623\cdot10^{5} \end{array} \Rightarrow \quad \begin{array}{l} \omega_{c2} = 3.0811\cdot10^{5} \text{ rad/s} \\ \omega_{c1} = 8.1139\cdot10^{3} \text{ rad/s} \end{array}$$

Hence;

Low-pass filter;

$k_f = 3.0811\cdot10^{5}$, $\quad 150\cdot10^{-9} = \dfrac{1}{3.0811\cdot10^{5}\cdot k_m}$

$\Rightarrow \quad k_m = 21.6373$

$\Rightarrow \quad R_L = 21.6373\cdot1 = 21.6373\,\Omega$

High-pass filter :

$k_f = 8.1139\cdot10^{3}$, $\quad 150\cdot10^{-9} = \dfrac{1}{8.1139\cdot10^{3}\cdot k_m}$

$\Rightarrow \quad k_m = 821.6353$

$\Rightarrow \quad R_H = 821.6353\cdot1 = 821.6353\,\Omega$

$$f_{c1} = \frac{8.1139\cdot10^{3}}{2\pi} = 1291.4 \text{ Hz}$$

$$f_{c2} = \frac{3.0811\cdot10^{5}}{2\pi} = 49.037 \text{ kHz}$$

Problem 5) Design a parallel bandreject filter with a center frequency of 5kHz, a bandwidth of 30kHz, and a passband gain of 4. Use 250nF capacitors and specify all resistor values.

Solution. We have

$$\omega_0 = 2\pi\cdot5000 = 10000\pi = \sqrt{\omega_{c1}\omega_{c2}}$$

$$\omega_{c2} - \omega_{c1} = 2\pi \cdot 30\,000 = 60000\pi$$

$$\omega_{c2} + \omega_{c1} = \sqrt{(\omega_{c2} - \omega_{c1})^2 + 4\omega_{c2}\omega_{c1}}$$

$$= \sqrt{(60000\pi)^2 + 4 \cdot (10000\pi)^2}$$

$$= \sqrt{40} \cdot 10^4 \pi$$

$$= 6.3246 \cdot 10^4 \pi$$

Hence;

$$\omega_{c2} + \omega_{c1} = 6.3246 \cdot 10^4 \pi$$

$$- \quad \omega_{c2} - \omega_{c1} = 6 \cdot 10^4 \pi$$

$$2\omega_{c1} = 1.0196 \cdot 10^4 \quad \Rightarrow \quad \omega_{c1} = 5098.1 \text{ rad/s}$$

$$\omega_{c2} = 193593.7 \text{ rad/s}$$

Low-pass filter:

$$k_f = 5098.1 \quad , \quad 250 \cdot 10^{-9} = \frac{1}{k_m \cdot 5098.1} \quad \Rightarrow k_m = 784.6060$$

$$\Rightarrow R_L = 784.6060 \cdot 1 = 784.6060\ \Omega$$

High-pass filter:

$$k_f = 193593.7 \quad , \quad 250 \cdot 10^{-9} = \frac{1}{k_m \cdot 193593.7} \quad \Rightarrow k_m = 20.6618$$

$$\Rightarrow R_H = 20.6618 \cdot 1 = 20.6618\ \Omega$$

- we also have

$$4 = \frac{R_f}{R_i}$$

- choosing $R_i = 1\text{k}\Omega$ yields $R_f = 4\text{k}\Omega$

Problem 6) a. Determine the order of a low-pass Butterworth filter that has a cutoff frequency of 1000 Hz and a gain of at least -40dB at 4000 Hz.

b. What is the actual gain, in decibels at 4000 Hz?

PS 5.9

Solution. The Butterworth filter has the transfer fn. having the magnitude:

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_c)^{2n}}}$$

a.

$$-\cancel{4}^{2}0\,dB = \cancel{2}0 \log_{10} |H(j\,2\pi\,4000)|$$

$$\Rightarrow |H(j8000\pi)| = \frac{1}{100}$$

Hence;

$$\frac{1}{\sqrt{1+(4000/1000)^{2n}}} = \frac{1}{100}$$

$$\Rightarrow \quad 1+16^n = 10000 \quad \Rightarrow \quad 16^n = 9999$$

$$\Rightarrow \quad n.\ln 16 = \ln 9999 \quad \Rightarrow \quad n = \left\lceil \frac{\ln 9999}{\ln 16} \right\rceil = 4$$

∴ the order of the Butterworth low-pass filter is 4

b.

$$|H(j8000\pi)| = \frac{1}{\sqrt{1+(4000/1000)^{24}}} = 0.0039$$

and in decibels

$$20\log_{10} 0.0039 = -48.1649\ dB$$