

Q1012-01

Answer:

$$(i) G_1(s) = \frac{18}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5}$$

(a)

$$A = \frac{18}{s+5} \Big|_{s=-2} = \frac{18}{3} = 6$$

$$B = \frac{18}{s+2} \Big|_{s=-5} = \frac{18}{-3} = -6$$

$$G_1(s) = \frac{6}{s+2} - \frac{6}{s+5}$$

$$\mathcal{L}^{-1}\{G_1(s)\} = 6e^{-2t} - 6e^{-5t}$$

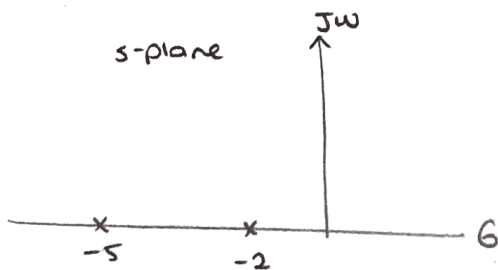
$$g_1(t) = 6e^{-2t} - 6e^{-5t}$$

$$\rightarrow G(s) = \frac{18}{s^2 + 7s + 10} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \Rightarrow \omega_n^2 = 10$$

$$2\zeta\omega_n = 7$$

$$\omega_n = 3.16 \text{ (natural frequency)}$$

$$\zeta = 1.108 \text{ (damping ratio)}$$



$$(b) G_1(s) = \frac{18}{(s+2)(s+5)} \rightarrow C(s) = \frac{18}{s(s+2)(s+5)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5}$$

$$A = \frac{18}{(s+2)(s+5)} \Big|_{s=0} = 1.8$$

$$C_1(s) = \frac{1.8}{s} + \frac{3}{s+2} + \frac{1.2}{s+5}$$

$$B = \frac{18}{s(s+5)} \Big|_{s=-2} = -3$$

$$c_1(t) = 1.8 - 3e^{-2t} + 1.2e^{-5t}$$

(OVERDAMPED)

$$C = \frac{18}{s(s+2)} \Big|_{s=-5} = 1.2$$

(c)

At steady state,

i.e.  $t \rightarrow \infty$

$$c(t) = 1 - \frac{e^{-3.500} \cdot \cos(\omega_0 t + \phi_1)}{\sqrt{1 - \delta^2}^2}$$

$$\boxed{c(t) = 1} \text{ at steady state}$$

$$\therefore \xi > 1$$

(d)

$$\therefore \text{peak value} = 1$$

$$\text{Settling time } t_s = \frac{4}{\xi \cdot \omega_n}$$

$$= \frac{4}{3.5}$$

$$= 1.142 \text{ sec}$$

peak time cannot be calculated. As it does not form any peak, but get to its final value is system is overdamped.