MATH 233 **FALL 2018** **FINAL EXAM**

Remarks: Calculate everything using pen and pencil only. Show your thinking steps; do not just write a number or an expression as a result. Write neatly; if your writing is not readable it is not my problem. Do not write multiple solutions; none of them will be considered in this case.

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1. **(15 pts)** Using **induction**, prove that the sum of the first n powers of 2 is $2^n - 1$. (That is to say, show that $2^0 + 2^1 + 2^2 \ldots + 2^{n-1} = 2^n - 1$ using induction)

2. **(15 pts)** You have **n** 1 TL coins. How can you distribute these coins to **k** children, if each child will get at least 2 TL ? Assume that $n > 2k$.

3. **(20 pts)** In experiment 1, **two dice** are rolled and in experiment 2, **three dice** are rolled. Is it more **probable** to get a total of **8** in the second experiment or the first experiment?

a) Write the sample space of experiment 1.
b) Write the sample space for experiment 2.
c) Write the **size** of the sample space of experiment 1.
d) Write the **size** of the sample space of experiment 2.
e) Write the **outcomes** in $E_{1,8}$, the event that a total of 8 results in the 1st experiment.
f) Write the **outcomes** in $E_{2,8}$, the event that a total of 8 results in the 2nd experiment.
g) What is $|E_{1,8}|$? (size of the event $E_{1,8}$)
h) What is $|E_{2,8}|$? (size of the event $E_{2,8}$)
i) What is $P(E_{1,8})$?
j) What is $P(E_{2,8})$?

4. **(15 pts)** Prove that if n, m are integers:

$$\mathbf{n.m = gcd(n, m).lcm(n, m)}$$

Remember that **gcd(n, m)** is the **greatest common divisor** of n and m;
lcm(n, m) is the **least common multiple** of n and m.

5. **(17 pts)** Draw all trees that are **subgraphs** of a **complete** graph on 4 vertices, G_4. Remember that a **complete** graph is a graph G=(V,E) where any two vertex pairs is connected by an edge.

a) **(5 pts)** Draw the **complete graph G_4** on 4 vertices.

b) **(12 pts)** Write down all trees that are a subgraphs of G_4.Remember that a **tree** is a graph which is **connected** and has **no cycle**.

6. **(18 pts)** Ayşe and Bekir communicate using substitution cipher. They use the following substitution table (which is their **secret key**):

A	**B**	**C**	**Ç**	**D**	**E**	**F**	**G**	**Ğ**	**H**	**I**	**İ**	**J**	**K**	**L**	**M**	**N**	**O**	**Ö**	**P**	**R**	**S**	**Ş**	**T**	**U**	**Ü**	**V**	**Y**	**Z**
D	F	Z	R	Y	Ü	M	A	V	C	Ö	U	B	Ç	K	Ğ	I	Ş	J	G	N	İ	L	T	S	P	O	H	E

a) If Ayşe wants to send the following message:

İYİYILLAR

What is the **ciphertext** she is going to send over the insecure channel?

b) Assume Bekir replies back to Ayşe using the same secret key and Ayşe gets the following ciphertext:

TÜLÜÇÇPNKÜN

When Ayşe decrypts the **ciphertext** above, what is the **plaintext** that she gets?

c) What is problematic with this particular **secret key**?