EEEN 352 System Dynamics and Control

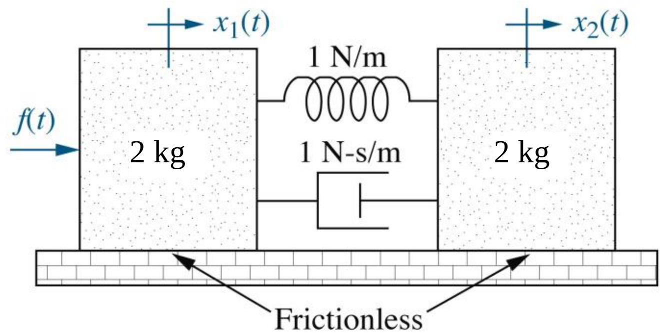
Homework 02

Due: 10-April-2020
Friday 23:59

Problem 1. Find the transfer function,

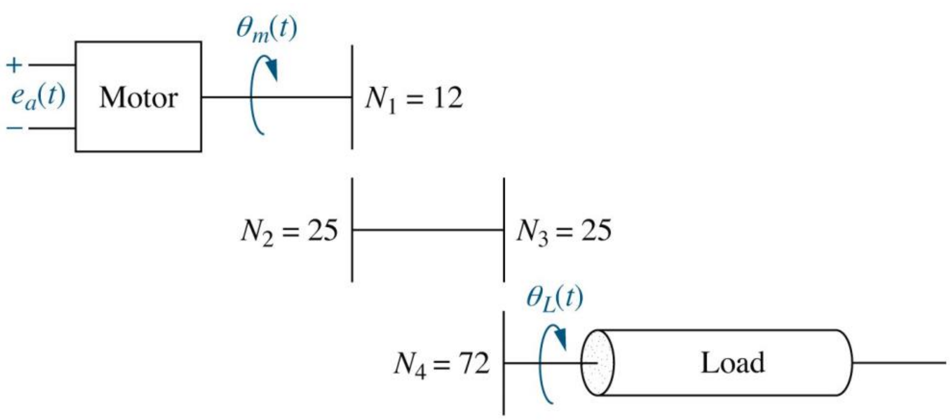
$$G(s) = X_1(s)/F(s)$$

for the translational mechanical system shown in the figure on the right.

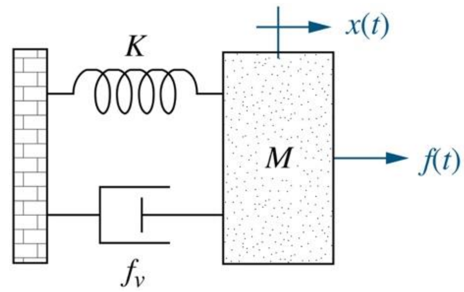


Problem 2. A dc motor develops 55 N-m of torque at a speed of 600 rad/s when 12 Volts are applied. It stalls out at this voltage with 100 N-m of torque. If the inertia and damping of the armature are 7 kg-m2 and 3 N-m-s/rad, respectively,

- **(a)** Find the transfer function, $G_1(s) = \theta_L(s)/E_a(s)$, of this motor if it drives an inertia load of 105 kg-m^2 through a gear train, as shown below.
- **(b)** Find the transfer function for speed, $G_2(s) = \omega_L(s)/E_a(s)$
- **(c)** Plot step responses of $G_1(s)$ and $G_2(s)$ for the range of $t = 0$ to 20 sec. You may use Matlab, Excel, etc.
- **(d)** What is the number of rotations within first 17 secs approximately? (Use Inverse Laplace Transform and make sure that the result validates the step response you plotted in **(c)**.)
- **(e)** What is the steady state value of speed in rad/sec and also in rpm? (You may use Inverse Laplace Transform or Final Value Theorem. Make sure that the result validates the corresponding step response you plotted in **(c)**.)



Problem 3. For the given simple translational mechanical system,



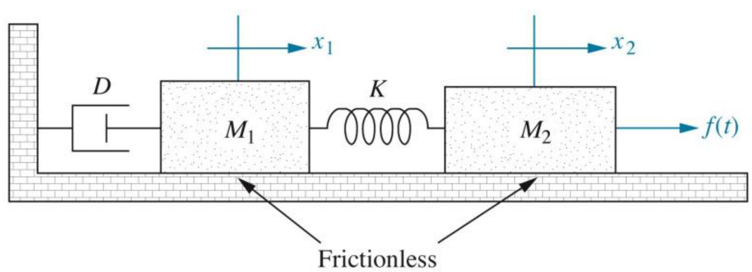
- **(a)** Write the equation of motion using Newton's first law (as a differential equation).
- **(b)** Obtain the transfer function of $G(s) = X(s)/F(s)$.
- **(c)** Represent the same system in state space.
- **(d)** Find the steady-state value of the displacement in meters when a torque of $f(t) = 2\text{N}$ is applied. The values of mass, coefficient of viscous friction and spring constant are M=2kg, f_v=4N-s/m and K= 8N/m, respectively.

Problem 4. Represent the translational mechanical system shown below in state-space (determine the matrices of **A**, **B**, **C**, D).

Hint: Select the state variables as displacements and velocities as follows,

$$\mathbf{x} = \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix}$$

and the output as $y = x_2$



Problem 5. Suppose that we have a satellite system, which can be represented in state-space as,

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t),$$
$$y(t) = [1 \;\; 0]\mathbf{x}(t)$$

Obtain the $Y(s)/U(s)$ transfer function of the system.

Hint: $\frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$