EEEN 322
Communication Engineering

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Spring 2019

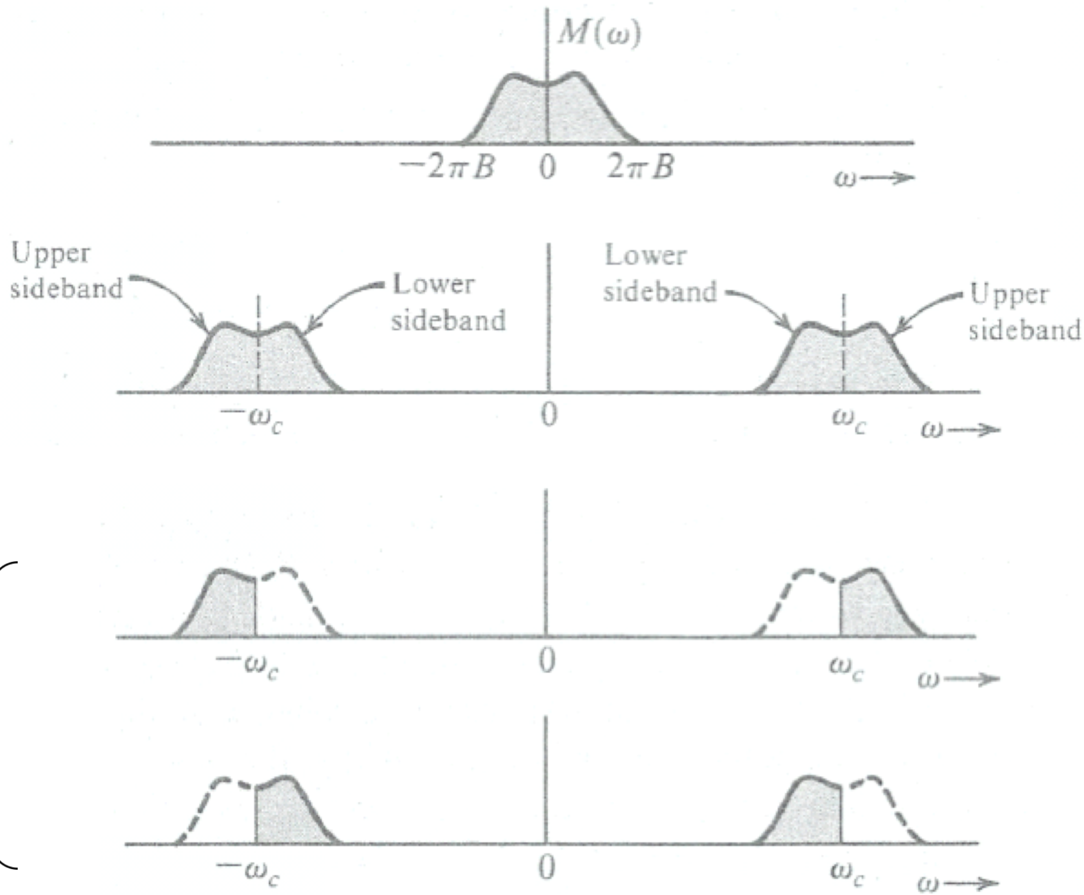
Week 9

Single Sideband (SSB) Modulation

- **DSB spectrum has two sidebands:**
  - **the upper sideband (USB), and**
  - **the lower sideband (LSB)**
- **A scheme in which only one sideband (USB or LSB) is transmitted: Single Sideband (SSB) Modulation**

(Suppressed carrier: SSB-SC. With carrier: SSB+C)

Baseband, DSB and SSB Spectra



Baseband

DSB

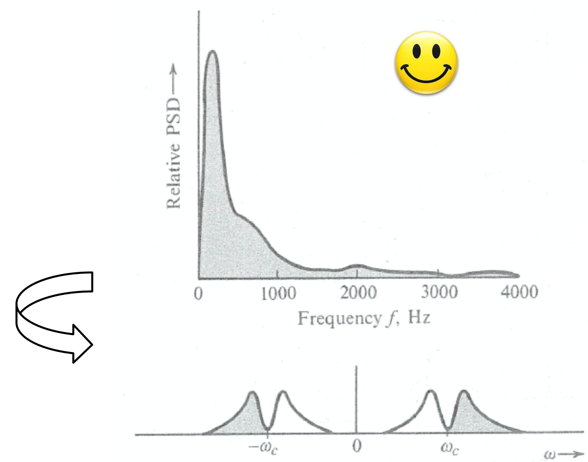
SSB { **USB**, **LSB** }

Illustrations are for the suppressed carrier versions of DSB and SSB

Modulation and Demodulation of SSB-SC Signals

- **Modulation:**
  - **Filtering: First generate DSB-SC, then filter out the undesired bands**

Eg: PSD of speech signal.
Sharp cut-off not needed
for SSB modulation.

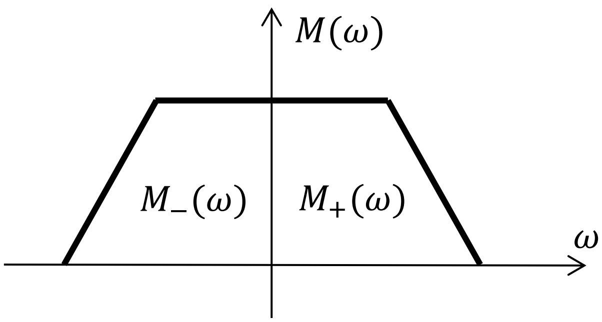


→ **May be difficult to realize if sharp cut-off is needed**

  - **Hilbert Transform**

- **Demodulation:**
  - **Coherent demodulation**

Hilbert Transform



(a) $M_+(\omega) = M(\omega)u(\omega),\ \ M_-(\omega) = M(\omega)u(-\omega)$

(b) $|M_+(\omega)|$ and $|M_-(\omega)|$ are not even functions, hence $m_+(t)$ and $m_-(t)$ must be complex

(c) $m(t)$ is real, therefore $M(-\omega) = M^*(\omega)$ (conjugate symmetry)

$$\Rightarrow M_+(-\omega) = M_-{}^*(\omega)$$

$$\Rightarrow m_+(t) = m_-{}^*(t)$$

(d) $M(\omega) = M_-(\omega) + M_+(\omega) \ \Rightarrow \ m(t) = m_+(t) + m_-(t)$

(c) & (d) $\Rightarrow m_+(t) = \frac{1}{2}[m(t) + jm_h(t)]$ $\rightarrow M_+(\omega) = \frac{1}{2}[M(\omega) + jM_h(\omega)]$

$$m_-(t) = \frac{1}{2}[m(t) - jm_h(t)]$$

$$M_h(\omega) = M(\omega)[-jsgn(\omega)]$$

(a) $M_+(\omega) = M(\omega)u(\omega) = \dfrac{1}{2}M(\omega)[1 + sgn(\omega)] = \dfrac{1}{2}M(\omega) + \dfrac{1}{2}M(\omega)sgn(\omega)$

Remember the Fourier Pair: $sgn(t) \leftrightarrow \dfrac{2}{j\omega}$ From duality principle: $\dfrac{1}{\pi t} \leftrightarrow -jsgn(\omega)$

$$\boxed{m_h(t) = m(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\alpha)}{t-\alpha} d\alpha}$$

Hilbert Transform

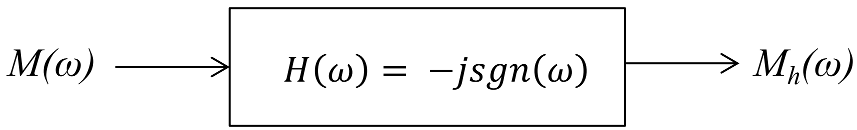
$$\boxed{m(t) \leftrightarrow m_h(t)}$$

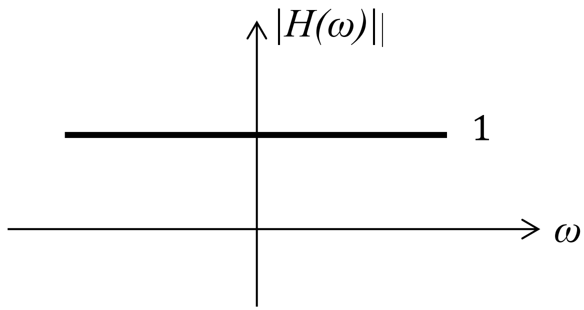
Hilbert Transform Pair

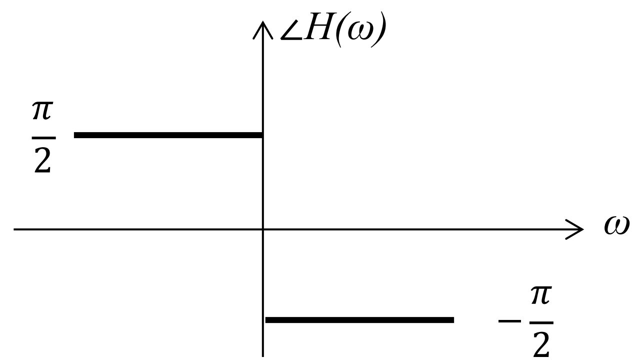
Hilbert Transform

$$\boxed{m_h(t) = m(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\alpha)}{t-\alpha} d\alpha}$$

Hilbert Transformer
(Hilbert Filter)







Note 1: Hilbert transformer is an ideal phase shifter (a phase delay of $\frac{\pi}{2}$); an ideal phase shifter is not realizable

Note 2: Hilbert transform of $\cos(\omega_m t)$ is $\cos\left(\omega_m t - \frac{\pi}{2}\right) = \sin(\omega_m t)$

Note 3: Hilbert transform of $\sin(\omega_m t)$ is $\sin\left(\omega_m t - \frac{\pi}{2}\right) = -\cos(\omega_m t)$

Time-Domain Representation of SSB-SC Signals Using the Hilbert Transform

$$\Phi_{USB}(\omega) = M_+(\omega - \omega_c) + M_-(\omega + \omega_c)$$

$$\begin{aligned}\phi_{USB}(t) &= m_+(t)e^{j\omega_c t} + m_-(t)e^{-j\omega_c t} \\ &= \frac{1}{2}[m(t) + jm_h(t)][\cos(\omega_c t) + j\sin(\omega_c t)] \\ &\quad + \frac{1}{2}[m(t) - jm_h(t)][\cos(\omega_c t) - j\sin(\omega_c t)] \\ &= m(t)\cos(\omega_c t) - m_h(t)\sin(\omega_c t)\end{aligned}$$

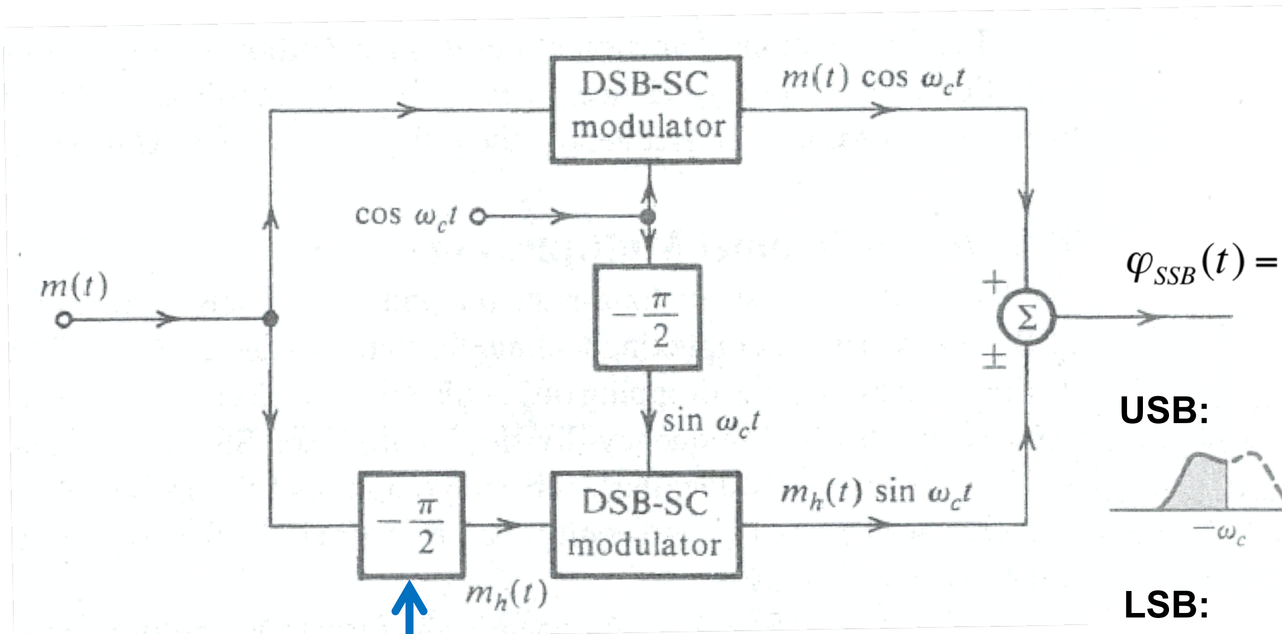
$$\Phi_{LSB}(\omega) = M_+(\omega + \omega_c) + M_-(\omega - \omega_c)$$

$$\begin{aligned}\phi_{LSB}(t) &= m_+(t)e^{-j\omega_c t} + m_-(t)e^{j\omega_c t} \\ &= \frac{1}{2}[m(t) + jm_h(t)][\cos(\omega_c t) - j\sin(\omega_c t)] \\ &\quad + \frac{1}{2}[m(t) - jm_h(t)][\cos(\omega_c t) + j\sin(\omega_c t)] \\ &= m(t)\cos(\omega_c t) + m_h(t)\sin(\omega_c t)\end{aligned}$$

$$\boxed{\phi_{SSB}(t) = m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t)}$$

$-$: USB $+$: LSB

Modulation of SSB-SC Signals Using a Hilbert Transformer



Hilbert Transformer

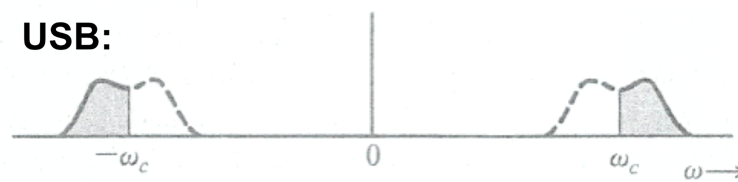
$$m_h(t) = m(t) * \frac{1}{\pi t}$$

$$M_h(\omega) = M(\omega)[-j\,sgn(\omega)]$$

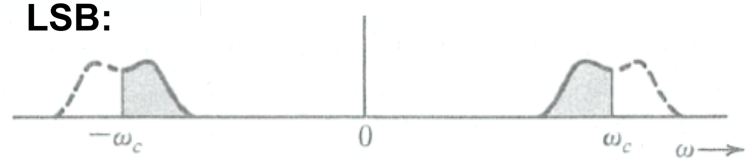
– : USB
+ : LSB

$$\varphi_{SSB}(t) = m(t)\cos\omega_c t \pm m_h(t)\sin\omega_c t$$

USB:

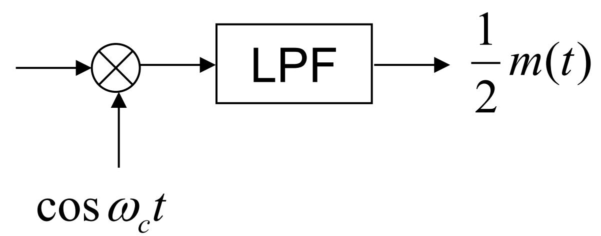


LSB:



Demodulation of SSB-SC Signals

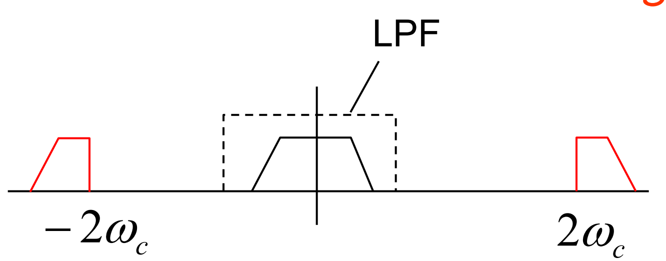
$$\varphi_{SSB}(t) = m(t)\cos\omega_c t \pm m_h(t)\sin\omega_c t$$



Coherent Demodulation

$$\varphi_{SSB}(t)\cos\omega_c t = \left[m(t)\cos\omega_c t \pm m_h(t)\sin\omega_c t\right]\cos\omega_c t \;\; = m(t)\underbrace{\cos^2\omega_c t}_{\frac{1}{2}(1+\cos 2\omega_c t)} \pm m_h(t)\underbrace{\sin\omega_c t\cos\omega_c t}_{\frac{1}{2}\sin 2\omega_c t}$$

$$= \frac{1}{2}m(t) + \underbrace{\frac{1}{2}\left[m(t)\cos 2\omega_c t \pm m_h(t)\sin 2\omega_c t\right]}_{\text{Suppressed by the LPF}}$$



Example: Tone Modulation

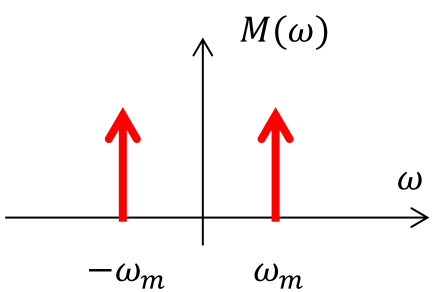
$m(t) = \cos(\omega_m t) \;\Rightarrow m_h(t) = \sin(\omega_m t)$

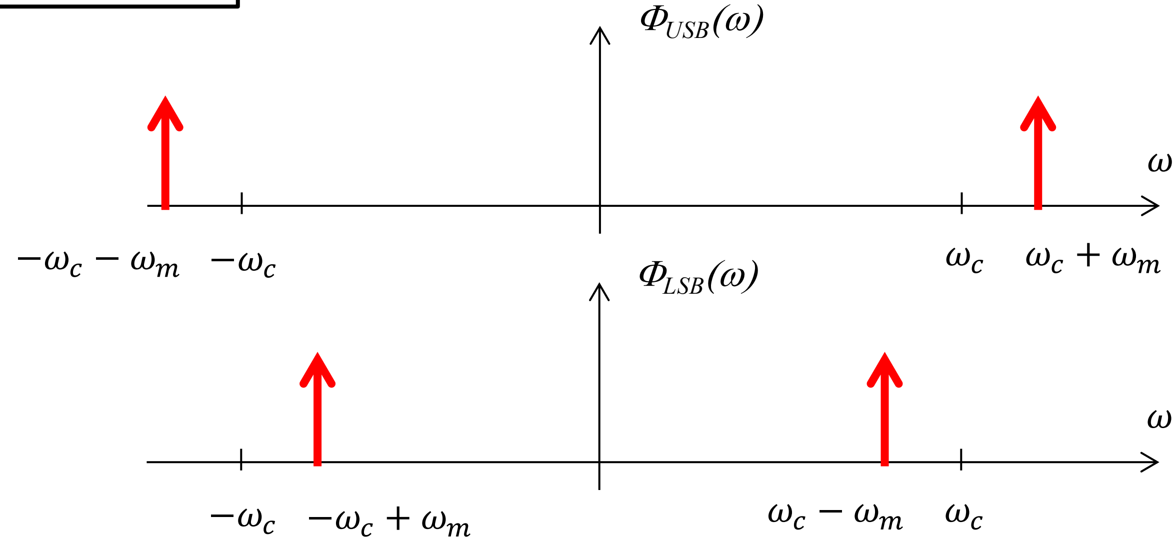
$\phi_{SSB}(t) = m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t)$

Therefore: $\phi_{SSB}(t) = \underbrace{\cos(\omega_m t)\cos(\omega_c t)} \mp \underbrace{\sin(\omega_m t)\sin(\omega_c t)}$

$\frac{1}{2}[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t]$ $\quad\quad$ $\frac{1}{2}[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$

$$\boxed{\phi_{SSB}(t) = \cos(\omega_c \pm \omega_m)t}$$





SSB Modulation with Carrier (SSB+C)

$$\varphi_{SSB+C}(t) = \underbrace{A\cos\omega_c t}_{\text{carrier}} + \underbrace{m(t)\cos\omega_c t \pm m_h(t)\sin\omega_c t}_{\varphi_{SSB-SC}(t)}$$

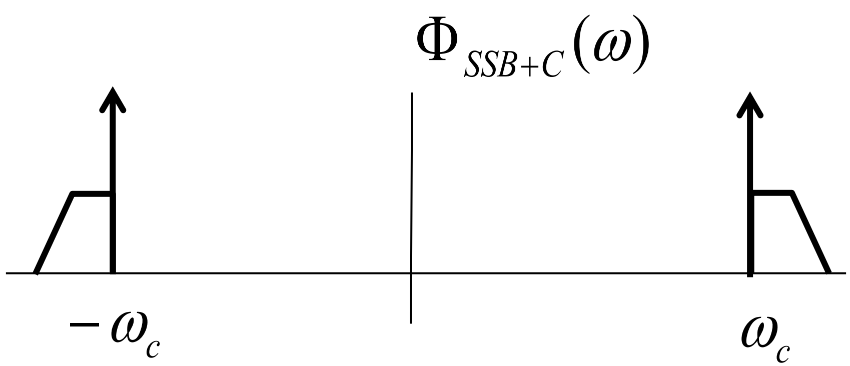


Illustration is for SSB+C where the communicated sideband is the USB

Demodulation of SSB+C Signal

$$\varphi_{SSB+C}(t) = \underbrace{A\cos\omega_c t}_{\text{carrier}} + \underbrace{m(t)\cos\omega_c t \pm m_h(t)\sin\omega_c t}_{\varphi_{SSB-SC}(t)}$$

(1) Coherent demodulation

(2) Envelope (or rectifier) detection if *A* >> |*m(t)*|

$$\varphi_{SSB+C}(t) = [A+m(t)]\cos\omega_c t + m_h(t)\sin\omega_c t = E(t)\cos(\omega_c t+\theta)$$

where $E(t) = \sqrt{[A+m(t)]^2 + m_h^2(t)} = A\sqrt{1+\frac{2m(t)}{A} + \underbrace{\frac{m^2(t)}{A^2} + \frac{m_h^2(t)}{A^2}}_{\approx 0 \text{ if } A >> |m(t)|}}$

then $E(t) \approx A\sqrt{1+\frac{2m(t)}{A}} \approx A\left[1+\frac{m(t)}{A}\right]$

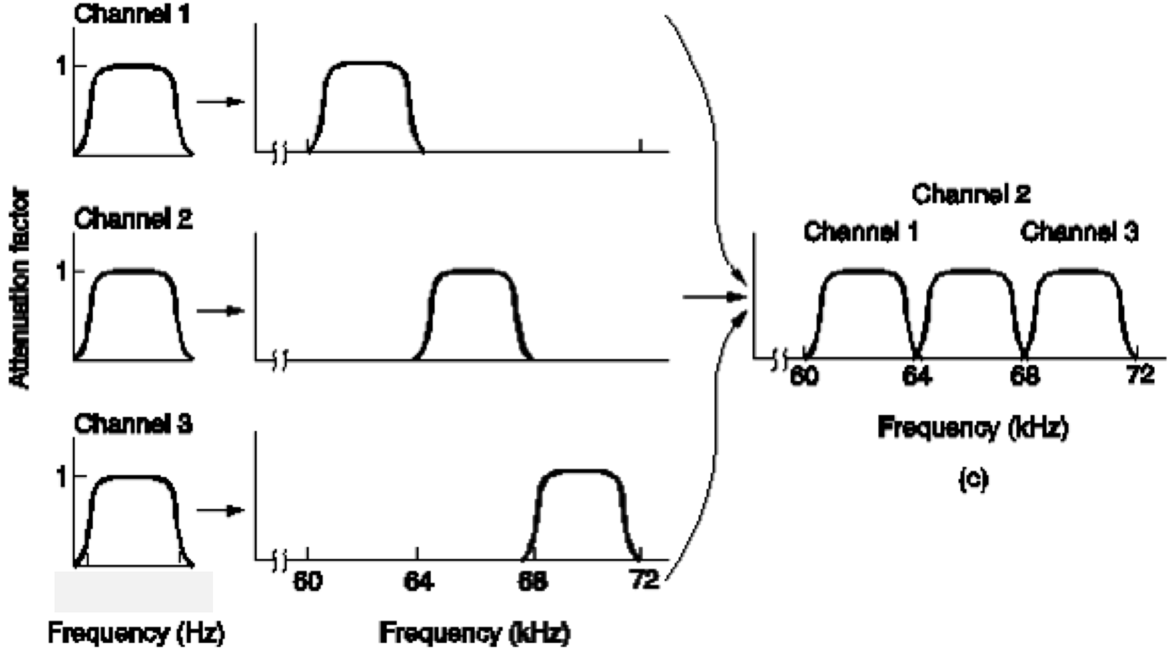
$$\Rightarrow \boxed{E(t) \approx A + m(t)}$$

$\Rightarrow$ If $A >> |m(t)|$ **, SSB+C can be demodulated by an envelope detector**

Notes: SSB+C has very low power efficiency

Remember: AM requires $A > |m(t)|$ **(a smaller carrier amplitude A!)**

Frequency-Division Multiplexing of Telephone Channels using SSB Modulation



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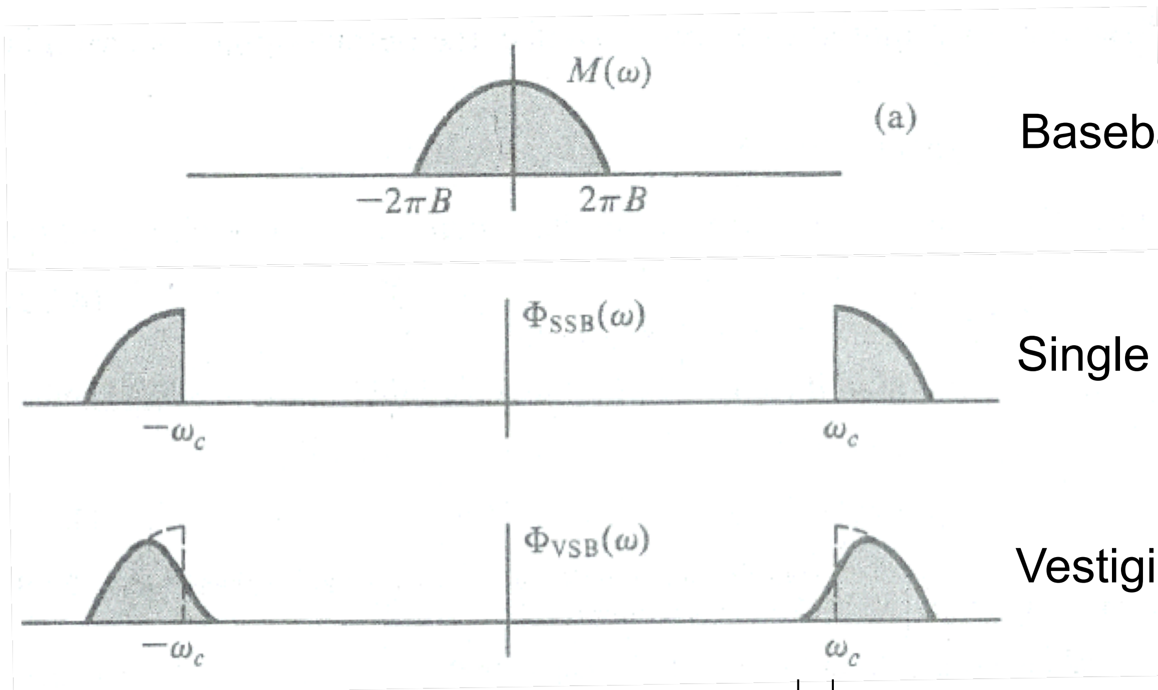
USB Modulated Spectra

Vestigial Sideband (VSB) Modulation

- Generation of SSB signals may be difficult (costly)
- Generation of DSB signals much easier but requires twice the bandwidth
- Vestigial Sideband Modulation (VSB) idea:



Baseband

Single Sideband (SSB)

Vestigial Sideband (VSB)

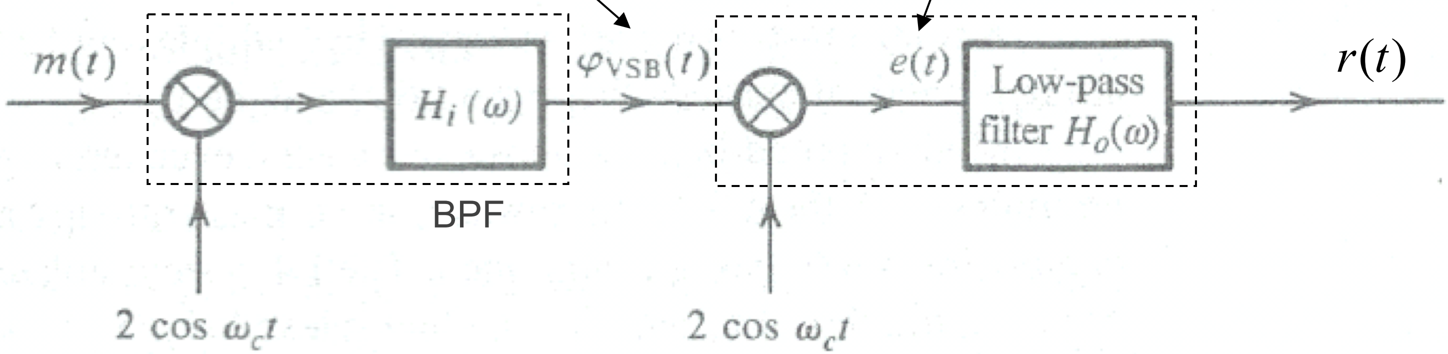
About 25% to 30% increase in bandwidth

VSB is a compromise between DSB and SSB

VSB Modulation and Demodulation

$$\Phi_{VSB}(\omega) = \left[M(\omega+\omega_c) + M(\omega-\omega_c)\right]H_i(\omega)$$

$$E(\omega) = \Phi_{VSB}(\omega+\omega_c) + \Phi_{VSB}(\omega-\omega_c)$$



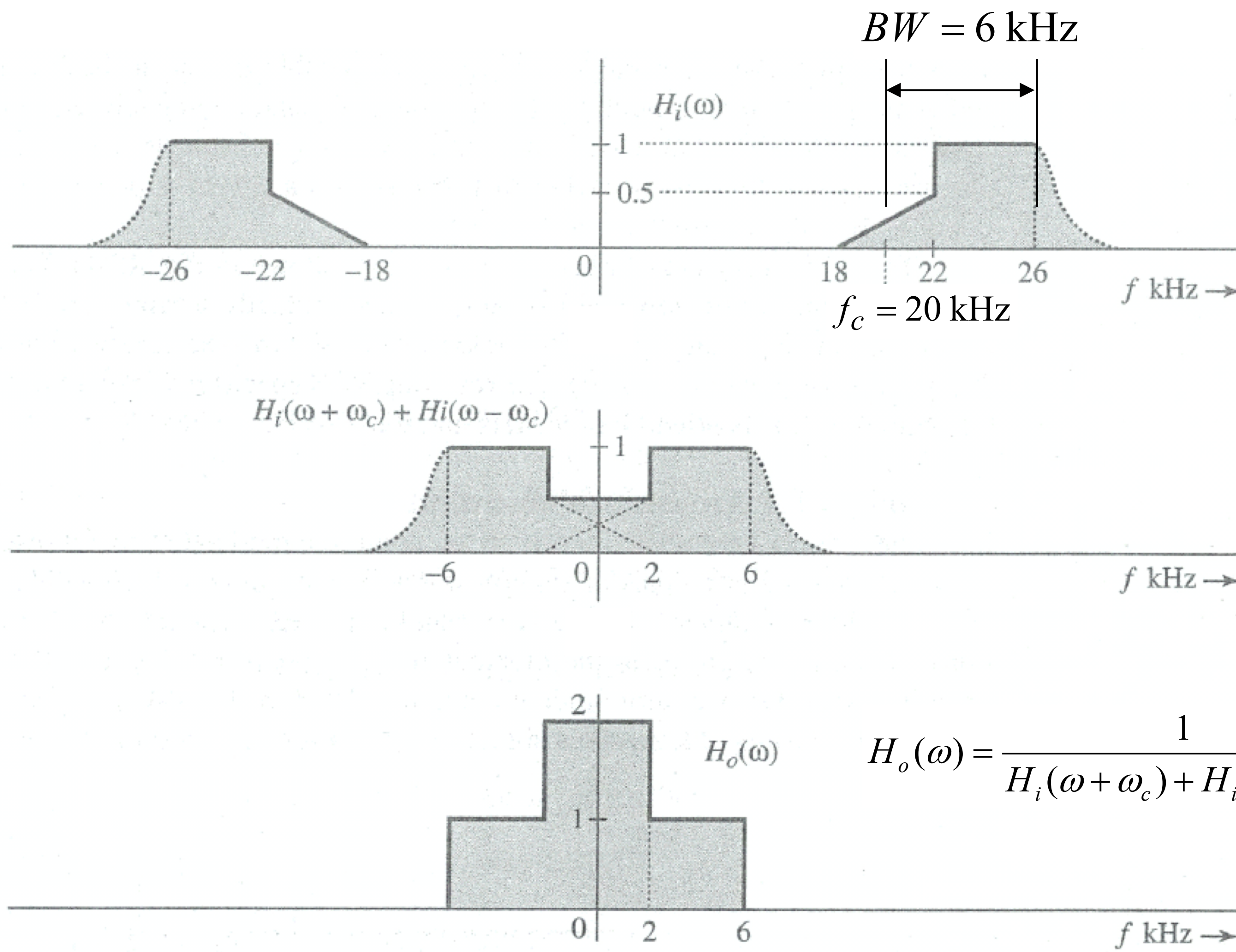
We want $R(\omega) = M(\omega) \Rightarrow \left[\underbrace{\Phi_{VSB}(\omega+\omega_c)}_{\left[\cancel{M(\omega+2\omega_c)}+M(\omega)\right]H_i(\omega+\omega_c)} + \underbrace{\Phi_{VSB}(\omega-\omega_c)}_{\left[M(\omega)+\cancel{M(\omega-2\omega_c)}\right]H_i(\omega-\omega_c)}\right]H_o(\omega) = M(\omega)$

Suppressed by H_0

$$\Rightarrow M(\omega)\left[H_i(\omega+\omega_c) + H_i(\omega-\omega_c)\right]H_o(\omega) = M(\omega)$$

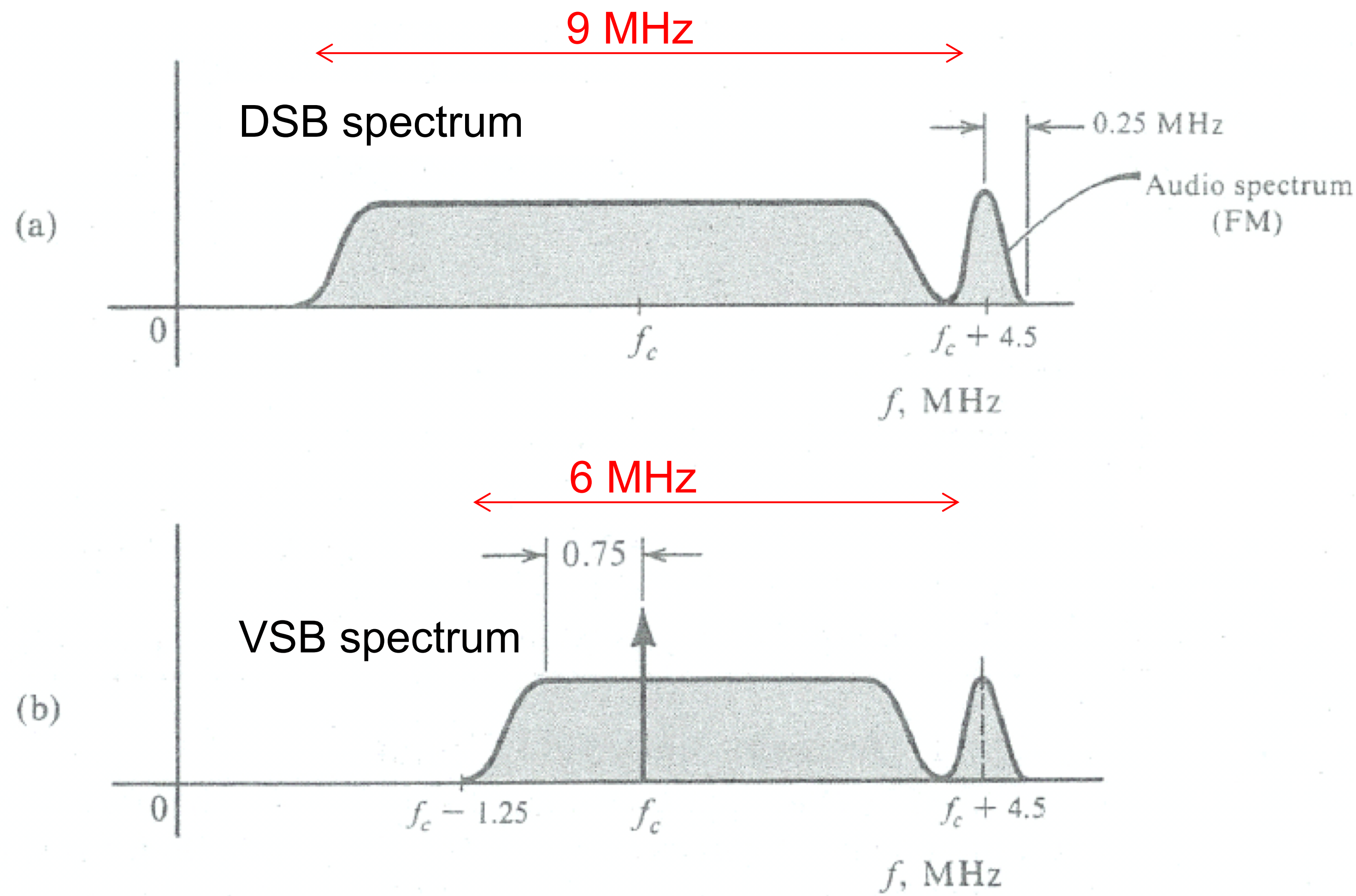
$$\boxed{\Rightarrow H_o(\omega) = \frac{1}{H_i(\omega+\omega_c) + H_i(\omega-\omega_c)}} \qquad -2\pi B \le \omega \le 2\pi B$$

Example Shaping and Receiving Filters in a VSB System



$$H_o(\omega) = \frac{1}{H_i(\omega + \omega_c) + H_i(\omega - \omega_c)}$$

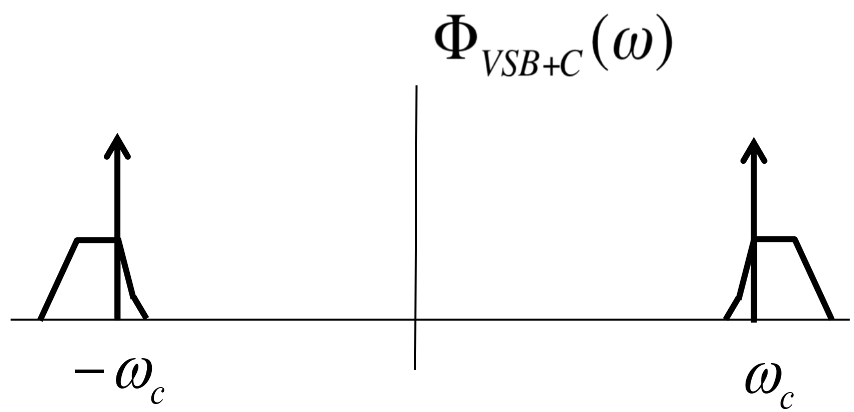
DSB and VSB Spectra of Television Signals



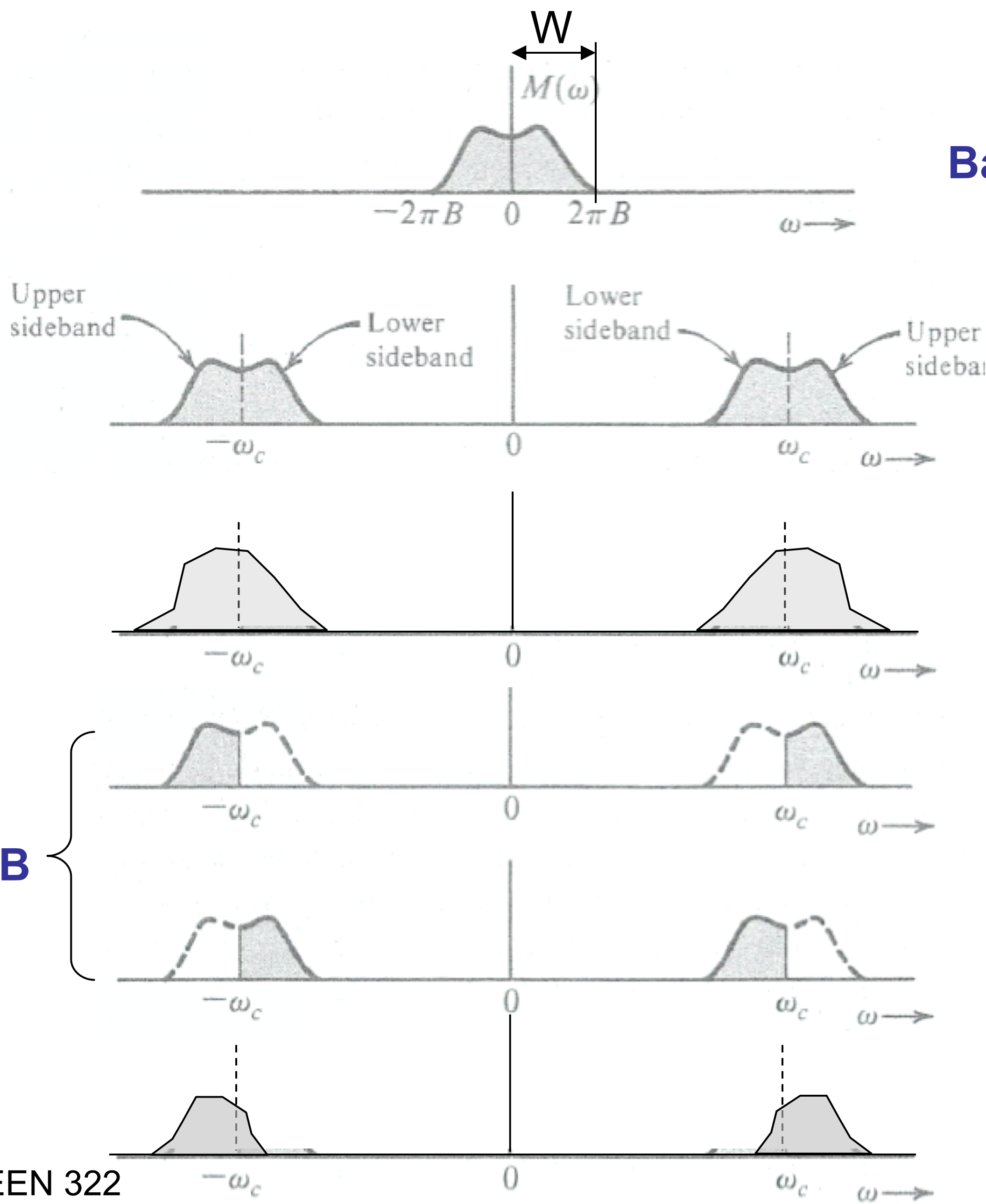
Note: SSB would require 4.5 MHz of bandwidth

VSB+C: VSB Modulation with Carrier

- Similarly to SSB+C, it is possible to generate VSB signals with carrier and detect them with envelope detection
- SSB+C requires a much larger carrier power than DSB+C for envelope detection
- Because VSB+C is intermediate, the added carrier in VSB+C is larger than that in AM but smaller than that in SSB+C



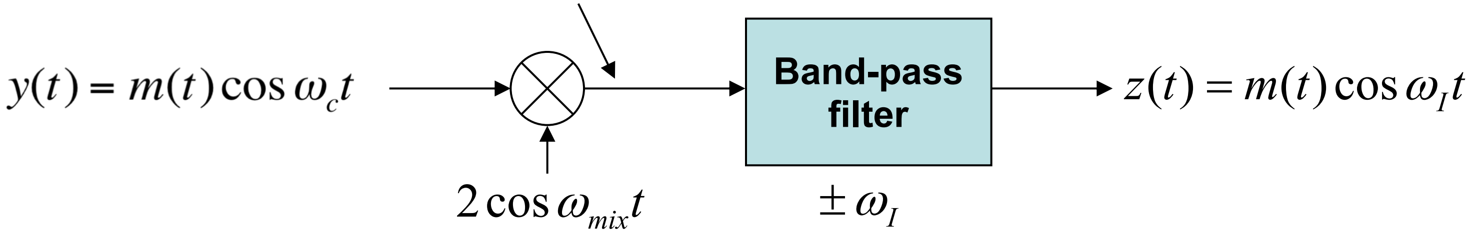
Modulation Schemes and Their Efficiencies



		Efficiency
	Baseband	1msg per W Hz
	DSB	1msg per 2W Hz
	QAM	2msg per 2W Hz
SSB	USB	1msg per W Hz
SSB	LSB	1msg per W Hz
	VSB	1msg per ~1.25W Hz

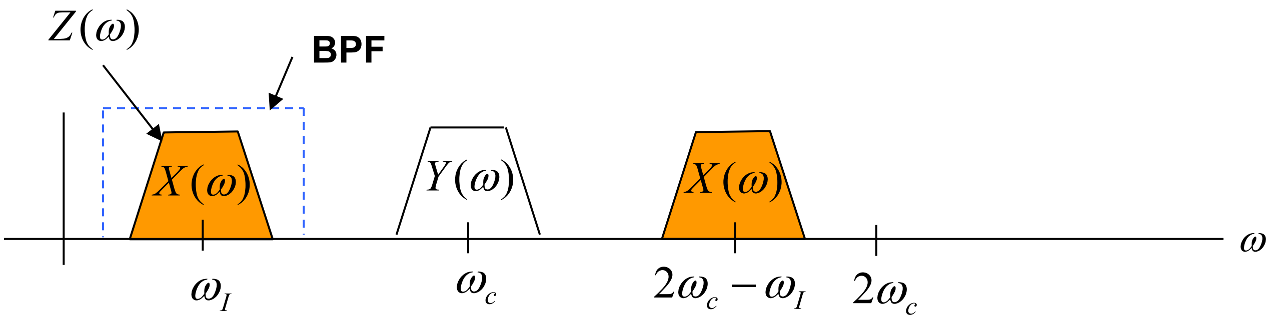
Frequency Mixer: Changing the Carrier Frequency

$$x(t) = m(t)\left[\cos(\omega_c - \omega_{mix})t + \cos(\omega_c + \omega_{mix})t\right]$$



Down conversion: $\boxed{\omega_{mix} = \omega_c - \omega_I}$

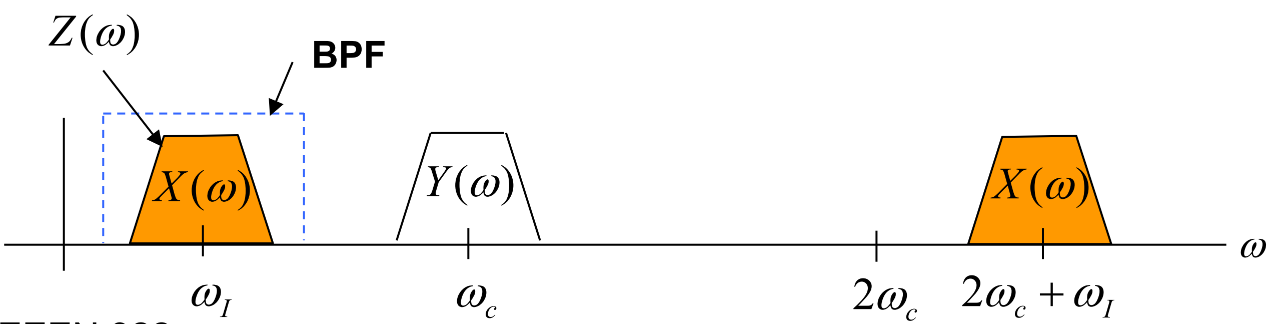
Heterodyning = frequency mixing



Up conversion: $\boxed{\omega_{mix} = \omega_c + \omega_I}$

Necessary conditions:

$$\omega_c - \omega_I \geq 2\pi B$$
$$\omega_I \geq 2\pi B$$



Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j \sin x$$

$$\cos x = \frac{1}{2}(e^{jx} + e^{-jx})$$

$$\sin x = \frac{1}{2j}(e^{jx} - e^{-jx})$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin x$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos x$$

$$2 \sin x \cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \frac{1}{4}(3 \cos x + \cos 3x)$$

$$\sin^3 x = \frac{1}{4}(3 \sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2}[\sin(x - y) + \sin(x + y)]$$

$$a \cos x + b \sin x = C \cos(x + \theta)$$

in which $C = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$