Selected Problems - I

Problem 1) What is the exact number of bits in a system that contains

a. 32 K bytes

b. 64 M bytes

c. 6.4 G bytes

Solution. We know that

$1 \text{ byte} \equiv 8 \text{ bits}$

$1K \equiv 2^{10} = 1024$, $1M \equiv 2^{20} = 1024^2 = 1048576$

$1G \equiv 2^{30} = 1024^3 = 1073741824$

c. $32 \text{ K bytes} \equiv 2^5 \cdot 2^{10} \cdot 8$

$= 32 \cdot 1024 \cdot 8$

$= 262144 \text{ bits}$

b. $64M \text{ bytes} \equiv 2^8 \cdot 2^{20} \cdot 8$

$= 64 \cdot 1048576 \cdot 8$

$= 536870912 \text{ bits}$

c. $6.4G \text{ bytes} \equiv 6.4 \cdot 1073741824 \cdot 8$

$\cong 54975581388 \text{ bits}$

Problem 2) Convert the decimal 8723 to both BCD and ASCII codes. For ASCII, an even parity bit is to be appended at the left.

Solution. We represent

8	7	2	3
BCD: 1000	0111	0010	0011

Four Different Binary Codes for the Decimal Digits

Decimal Digit	BCD 8421	2421	Excess-3	8, 4, −2, −1
0	0000	0000	0011	0000
1	0001	0001	0100	0111
2	0010	0010	0101	0110
3	0011	0011	0110	0101
4	0100	0100	0111	0100
5	0101	1011	1000	1011
6	0110	1100	1001	1010
7	0111	1101	1010	1001
8	1000	1110	1011	1000
9	1001	1111	1100	1111
Unused bit combi-nations	1010	0101	0000	0001
	1011	0110	0001	0010
	1100	0111	0010	0011
	1101	1000	1101	1100
	1110	1001	1110	1101
	1111	1010	1111	1110

American Standard Code for Information Interchange (ASCII)

	$b_7b_6b_5$							
$b_4b_3b_2b_1$	000	001	010	011	100	101	110	111
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	"	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	'	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	|
1101	CR	GS	−	=	M	]	m	}
1110	SO	RS	.	>	N	^	n	~
1111	SI	US	/	?	O	−	o	DEL

Control characters

NUL	Null	DLE	Data-link escape
SOH	Start of heading	DC1	Device control 1
STX	Start of text	DC2	Device control 2
ETX	End of text	DC3	Device control 3
EOT	End of transmission	DC4	Device control 4
ENQ	Enquiry	NAK	Negative acknowledge
ACK	Acknowledge	SYN	Synchronous idle
BEL	Bell	ETB	End-of-transmission block
BS	Backspace	CAN	Cancel
HT	Horizontal tab	EM	End of medium
LF	Line feed	SUB	Substitute
VT	Vertical tab	ESC	Escape
FF	Form feed	FS	File separator
CR	Carriage return	GS	Group separator
SO	Shift out	RS	Record separator
SI	Shift in	US	Unit separator
SP	Space	DEL	Delete

PS 1.2

ASCII:

parity bit	8	7	2	3
0001_011	011_1000	011_0111	011_0010	011_0011

Problem 3) Write the expression "G. Boole" in ASCII using an eight-bit code. Include the period and the space. Treat the leftmost bit of each character as a parity bit. Each eight-bit code should have even parity.

Solution.

G	(dot)	(space)	B
01000111	00101110	10100000	01000010

o	o	l
01101111	01101111	01101100

e
01100101

Problem 4) Decode the following ASCII code:

1000010 1101001 1101100 1101100 1000111

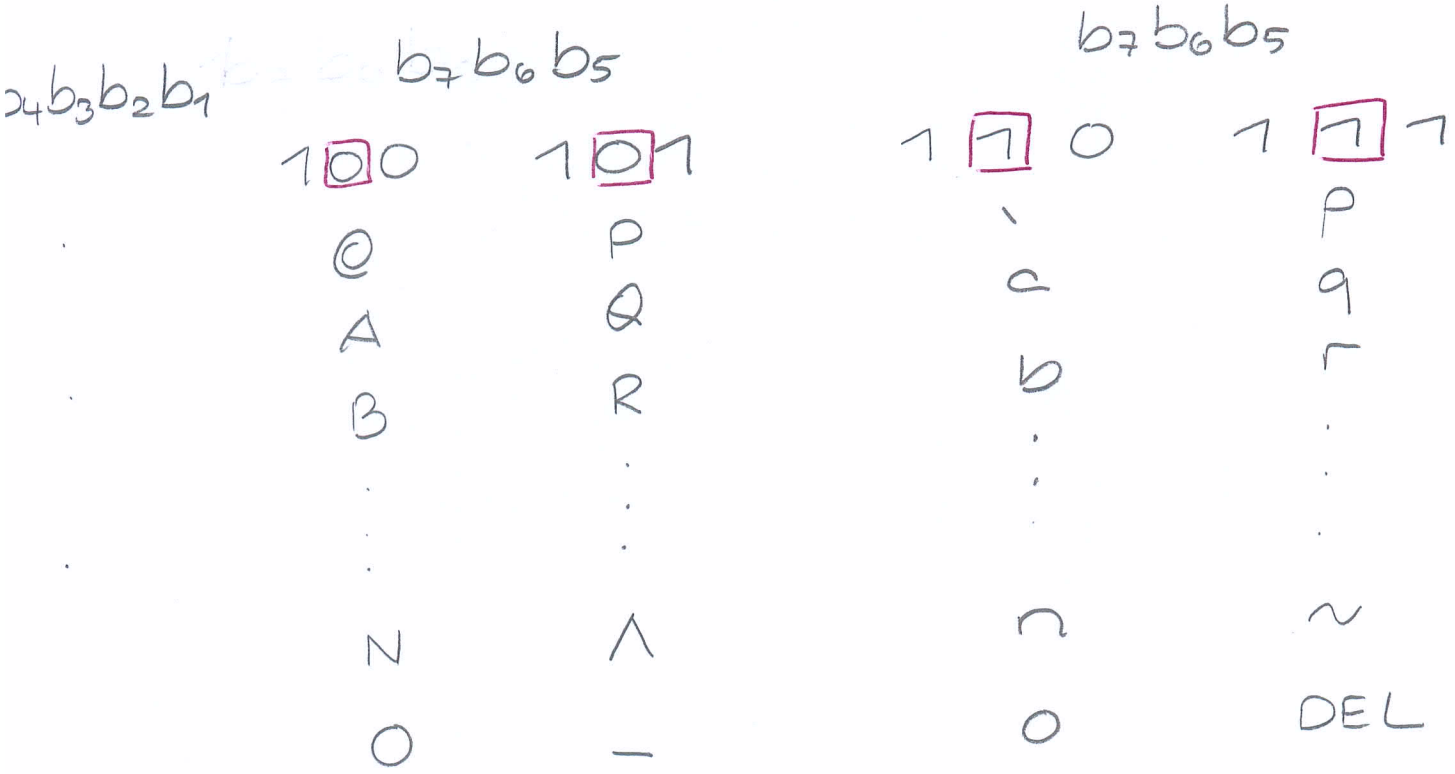
1100001 1110100 1100101 1110011

Solution. We find

Bill Gates

Problem 5) What bit must be complemented to change an ASCII letter from capital to lowercase and vice versa?

Solution. We have



Hence;

- the bit 6 from the right should be complemented

Problem 6) Assign a binary code in some orderly manner to the 52 playing cards. Use the minimum number of bits.

PS 1.4

Solution. We calculate the length of the code as follows:

$$s = \lceil \log_2 52 \rceil$$

$$= \lceil 5.7004 \rceil$$

$$= 6 \quad \Rightarrow \text{ we need } 6 \text{ bits}$$

Indeed;

$$2^{6-1} < 52 < 2^6$$

That is

$$32 < 52 < 64$$

- Let the msb's (most significant bit) select the suit, i.e.

diamonds, hearts, clubs, spades

which can be ENCODED with 2 bits

00 → diamonds

01 → hearts

10 → clubs

11 → spades

- The remcining 4 bits can be used to ENCODE the "number" of the cards

0000 → ace (A)

0001 → 1

1010 → 10

1011 → jack

1100 → queen

1101 → king

e.g.

- 11_10111 → jack of spades

Note that ;

- using 6 bits, we can ENCODE at most 64 patterns

However ;

- we have used only 52 out of 64 codes