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Suppose you perform DSB+C (AM) modulation and the modulated signal is
$\varphi_{AM}(t) = [2 + 0.5\cos 500\pi t]\cos(20000\pi t)$.

a) Give the mathematical expression of the message signal $m(t)$. (20p)

b) Give the value of the carrier frequency in Hz. (10p)

c) Calculate the percentage value of the modulation index μ. (20p)

d) Write the mathematical expression of the spectrum $\Phi_{AM}(\omega)$. (30p)

e) Sketch the spectrum $\Phi_{AM}(\omega)$. (20p)

(*You may find the attached reference sheets useful*)

a) $m(t) = 0.5\cos 500\pi t$

b) $\omega_c = 20000\pi$ rad/s $\Rightarrow f_c = \dfrac{\omega_c}{2\pi} = \dfrac{20000\pi}{2\pi} = 10000\,Hz$
$= 10\,kHz$

c) $A = 2$, $m_p = 0.5 \Rightarrow \mu = \dfrac{m_p}{A} = \dfrac{0.5}{2} = 0.25 = 25\%$

d) $\varphi_{AM}(t) = [A + m(t)]\cos\omega_c t$

$= A\cos\omega_c t + m(t)\cos\omega_c t$

$\Rightarrow \Phi_{AM}(\omega) = A\pi\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right] + \dfrac{1}{2}\left[M(\omega-\omega_c) + M(\omega+\omega_c)\right]$

Note that $M(\omega) = 0.5\pi\left[\delta(\omega-500\pi) + \delta(\omega+500\pi)\right]$

$\Rightarrow \Phi_{AM}(\omega) = 2\pi\left[\delta(\omega-20000\pi) + \delta(\omega+20000\pi)\right]$

$+ \dfrac{1}{2}\Big\{ 0.5\pi\left[\delta(\omega-500\pi-20000\pi) + \delta(\omega+500\pi-20000\pi)\right]$

$+ 0.5\pi\left[\delta(\omega-500\pi+20000\pi) + \delta(\omega+500\pi+20000\pi)\right]\Big\}$

$= 2\pi\,\delta(\omega-20000\pi) + 2\pi\,\delta(\omega+20000\pi)$

$+ 0.25\pi\,\delta(\omega-20500\pi) + 0.25\pi\,\delta(\omega-19500\pi)$

$+ 0.25\pi\,\delta(\omega+19500\pi) + 0.25\pi\,\delta(\omega+20500\pi)$

e)