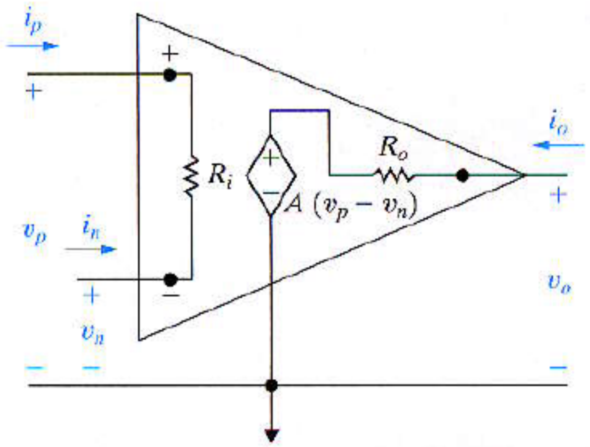
An equivalent circuit for the operational amplifier

- We now consider a more realistic model for the operational amplifier
- We modify the assumptions for an ideal op amp :

  1) a finite input resistance, R_i
  2) a finite open-loop gain, A
  3) a nonzero output resistance, R_0



- We now disregard the assumptions that

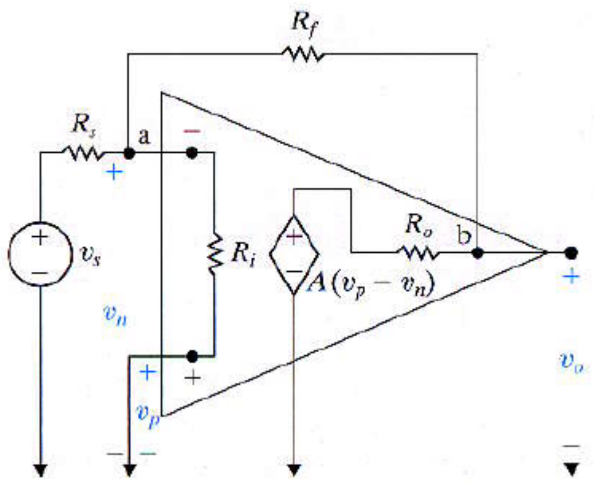
$$\vartheta_p = \vartheta_n \quad \text{and} \quad i_p = i_n = 0$$

- Because of nonzero output resistance, R_0 ,

$$\vartheta_0 = A(\vartheta_p - \vartheta_n) \text{ is NO LONGER valid.}$$

Analysis of the inverting op amp with realistic model

- We consider



- KCL at inverting terminal :

$$\frac{\vartheta_n - \vartheta_s}{R_s} + \frac{\vartheta_n}{R_i} + \frac{\vartheta_n - \vartheta_0}{R_f} = 0$$

- and KCL at output terminal :

$$\frac{\vartheta_0 - \vartheta_n}{R_f} + \frac{\vartheta_0 - A(0 - \vartheta_n)}{R_0} = 0$$

$$\Rightarrow \quad \left(\frac{1}{R_s} + \frac{1}{R_i} + \frac{1}{R_f}\right)\vartheta_n - \frac{1}{R_f}\vartheta_0 = \frac{1}{R_s}\vartheta_s$$

$$\Rightarrow \quad \left(\frac{A}{R_0} - \frac{1}{R_f}\right)\vartheta_n + \left(\frac{1}{R_f} + \frac{1}{R_0}\right)\vartheta_0 = 0$$

- Solving for ϑ_0 yields

$$\vartheta_0 = \frac{-A + (R_0/R_f)}{\frac{R_s}{R_f}\left(1 + A + \frac{R_0}{R_i}\right) + \left(\frac{R_s}{R_i} + 1\right) + \frac{R_0}{R_f}}\vartheta_s$$

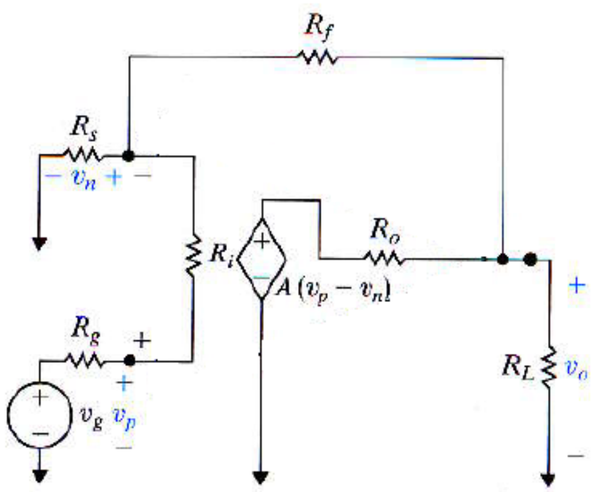
Note that ;

- if $R_0 \to 0\,,\; R_i \to \infty\,,\; A \to \infty$ then we get

$$\vartheta_0 = -\frac{R_f}{R_s}\vartheta_s$$

Analysis of the noninverting amplifier with realistic model

- We now consider a noninverting amplifier with load resistance



- Similarly, we write node-voltage equations at inverting and non-inverting terminals :

$$\frac{\vartheta_n}{R_s} + \frac{\vartheta_n - \vartheta_g}{R_g + R_i} + \frac{\vartheta_n - \vartheta_0}{R_f} = 0$$

$$\frac{\vartheta_0 - \vartheta_n}{R_f} + \frac{\vartheta_0}{R_L} + \frac{\vartheta_0 - A(\vartheta_p - \vartheta_n)}{R_0} = 0$$

- We also have equal currents over R_i and R_g

$$\frac{\vartheta_p - \vartheta_g}{R_g} = \frac{\vartheta_n - \vartheta_g}{R_i + R_g}$$

- Solving for ϑ_0 gives

$$\vartheta_0 = \frac{\left[(R_f + R_s) + (R_s R_0 / A R_i)\right]\vartheta_g}{R_s + \frac{R_0}{A}(1 + Kr) + \frac{R_f R_s + (R_f + R_s)(R_i + R_g)}{A R_i}}$$

where

$$K_r = \frac{R_s + R_g}{R_i} + \frac{R_f + R_s}{R_L} + \frac{R_f R_s + R_f R_g + R_g R_s}{R_i R_L}$$

Note that ;

- if we let $R_0 \to 0$, $A \to \infty$, and $R_i \to \infty$ then for the unloaded $(R_L \to \infty)$ noninverting amplifier we obtain

$$\vartheta_0 = \frac{R_s + R_f}{R_s}\vartheta_g$$

The differential mode

- In differential mode, the op amp produces

  ↳ an amplified replica of $\vartheta_p - \vartheta_n$

- We consider the case when neither ϑ_p nor ϑ_n is set equal to zero.

- The difference between ϑ_p and ϑ_n must be small

  ↳ to avoid saturation of op amp.

Therefore ;

- The op amp is useful as a linear differential amplifier

  when ϑ_p and ϑ_n are approximately equal.

- We consider the following bridge circuit application

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- The purpose of the bridge structure is to generate

  a signal $(\vartheta_n - \vartheta_p)$ when R_x changes from its balance value of R_1R_2/R_4 (i.e. $\vartheta_n = \vartheta_p$)

- When R_x changes because of temperature or strain

  $(\vartheta_n - \vartheta_p)$ is proportional to $\in$

- For an ideal op amp, we shall obtain

$$\vartheta_n - \vartheta_p = \frac{\vartheta_{dc} R_1 R_4}{(R_1 + R_4)^2} \frac{\in}{1 + [R_1/(R_1 + R_4)] \in}$$

- and for small values of $\in$

$$\vartheta_n - \vartheta_p \approx \frac{\vartheta_{dc} R_1 R_4}{(R_1 + R_4)^2} \in$$

Therefore ;

$$\vartheta_0 = -A(\vartheta_n - \vartheta_p)$$

$$= \frac{-\vartheta_{dc} R_1 R_4 A \in}{(R_1 + R_4)^2} \qquad \text{for } |\in| \ll 1$$

- In order to guarantee linear region of operation

$$\frac{\vartheta_{dc} R_1 R_4 A \in}{(R_1 + R_4)^2} \leq V_{cc}$$

OR

$$|\in| \leq \frac{V_{cc}(R_1 + R_4)^2}{\vartheta_{dc} R_1 R_4 A}$$

Note that ;

- when $\in = 0$, ϑ_0 should be zero because $\vartheta_n = \vartheta_p$

However ;

- with $\vartheta_n = \vartheta_p$, there will be some output from the amplifier

  because balance situation can NOT occur perfectly.

- Applying the same signal to the input terminals of a differential amplifier is

  known as "common-mode".

Hence ;

- How well an operational amplifier can reject a common-mode signal is

  referred to as "common-mode rejection" property.

- The ability of a differential amplifier to reject a signal common to both input terminals is

  expressed quantitatively by the "common-mode rejection ratio(CMRR)"

The common-mode rejection ratio

- In practical op amps, $\vartheta_0 \neq 0$ when $\vartheta_n = \vartheta_p$

  caused by unavoidable imperfections

- Thus impossible to make identical amplifying channels.

- We consider

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- Let $\vartheta_p = 0$ and $\vartheta_n = \vartheta_s$ then

$$\vartheta_0 = A_1 \vartheta_s \qquad , \ A_1 : gain\ via\ inverting\ channel$$

- We now reverse the process, **i.e.** $\vartheta_n = 0$, $\vartheta_p = \vartheta_s$

$$\vartheta_0 = A_2 \vartheta_s \qquad , \ A_2 : gain\ via\ noninverting\ channel$$

- For an ideal op amp, we would expect

$$A_1 \vartheta_s = -A_2 \vartheta_s$$

$$\Rightarrow \qquad A = -A_1 = A_2$$

- To quantitatively discuss the difference between channels, we shall write ϑ_0 as

$$\vartheta_0 = A_1\vartheta_n + A_2\vartheta_p$$

which reduces to

$$\vartheta_0 = A(\vartheta_p - \vartheta_n)$$

Common-mode and differential-mode components

- The common-mode component of the two signals is defined as

$$\vartheta_c = \frac{1}{2}(\vartheta_p + \vartheta_n)$$

- and the differential-mode component is defined as

$$\vartheta_d = \vartheta_n - \vartheta_p$$

Output in terms of ϑ_c and ϑ_d

- We can now describe quantitatively

  the op amp's ability to reject the common-mode component.

- We first solve for ϑ_p and ϑ_n in terms of ϑ_c and ϑ_d as

$$\left.\begin{aligned} \vartheta_p + \vartheta_n &= 2\vartheta_c \\ \vartheta_n - \vartheta_p &= \vartheta_d \end{aligned}\right\} \quad \begin{aligned} \vartheta_n &= \vartheta_c + \tfrac{1}{2}\vartheta_d \\ \vartheta_p &= \vartheta_c - \tfrac{1}{2}\vartheta_d \end{aligned}$$

- Then the output becomes

$$\begin{aligned} \vartheta_0 &= A_1\left(\vartheta_c + \frac{1}{2}\vartheta_d\right) + A_2\left(\vartheta_c - \frac{1}{2}\vartheta_d\right) \\ &= (A_1 + A_2)\vartheta_c + \frac{1}{2}(A_1 - A_2)\vartheta_d \\ &\triangleq A_c\vartheta_c + A_d\vartheta_d \end{aligned}$$

where

$$A_c = A_1 + A_2 \quad , \quad A_d = \frac{1}{2}(A_1 - A_2)$$