CMPE 352
Signal Processing & Algorithms

Spring 2019

Energy , Power, and dB

Week of March 18, 2019

Signal Energy & Signal Power

Energy

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt$$

Power

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} |x(t)|^2 dt$$

Note: Energy is not used here in the conventional sense: it is rather a measure of signal "size"

A signal is an <u>energy signal</u> if $E < \infty$

A signal is a <u>power signal</u> if $0 < P < \infty$

- Note that power is the time average of the energy
- Since the averaging is over an infinitely long interval, a signal with finite energy has zero power, and a signal with finite power has infinite energy
- Therefore a signal cannot be both an energy and a power signal. If it is one, it cannot be the other
- On the other hand, there are signals that are neither energy nor power signals

Signal Energy / Power Examples (1)

Determine the energy of the following signals:

(a) $x(t) = e^{-t}$

$$\mathrm{E} = \int_{-\infty}^{\infty} e^{-2t} dt \;= \infty$$

Is this an energy signal? → not an energy signal

(b) $x(t) = e^{-t}u(t)$ Note: $u(t)$ is the unit step

$$\mathrm{E} = \int_{0}^{\infty} e^{-2t} dt \;= -\frac{1}{2} e^{-2t}\Big|_{0}^{\infty} = \frac{1}{2}$$

Is this an energy signal? → an energy signal

Signal Energy / Power Examples (2)

Find the energy and the average power of the following signal:

$$x(t) = A\,u(t)$$

$$\mathrm{E} = \int_{-\infty}^{\infty} x^2(t)\,dt \;\; = A^2 \int_0^{\infty} dt = A^2\, t\Big|_0^{\infty} = \infty$$

$$\mathrm{P} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt = \lim_{T\to\infty} \frac{1}{2T}\int_0^{T} A^2\,dt = \lim_{T\to\infty} \frac{A^2}{2T}T = \lim_{T\to\infty} \frac{A^2}{2} = \frac{A^2}{2}$$

Is this an energy or a power signal? Or none?

→ $x(t)$ is a power signal

Signal Energy / Power Examples (3)

Find the energy and average power of the following signal

$$x(t) = Ae^{-\alpha t}u(t) \qquad\qquad (\alpha > 0)$$

$$\mathrm{E} = \int_{-\infty}^{\infty} x^2(t)\,dt = A^2 \int_0^{\infty} e^{-2\alpha t}\,dt = -\frac{A^2}{2\alpha} e^{-2\alpha t}\Big|_0^{\infty} = \frac{A^2}{2\alpha}$$

$$\mathrm{P} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\,dt = \lim_{T\to\infty} \frac{A^2}{2T} \int_0^{T} e^{-2\alpha t}\,dt = \lim_{T\to\infty} \frac{A^2}{2T}\left(-\frac{1}{2\alpha}\right) e^{-2\alpha t}\Big|_0^{T}$$

$$= \lim_{T\to\infty} \frac{A^2}{2T}\left(-\frac{1}{2\alpha}\right)[e^{-2\alpha T} - 1] = 0$$

Is this an energy or a power signal? Or none?

→ $x(t)$ is an energy signal

Signal Energy / Power Examples (4)

Find the average power of the following periodic signal

$$x(t) = A\ \cos(\omega_0 t + \theta)$$

For a periodic signal with period T: $\quad \mathrm{P} = \frac{1}{T}\int_0^T x^2(t)\, dt$

(Note that in this case $\mathrm{E} = \int_{-\infty}^{\infty} x^2(t)\, dt = \infty$)

$$\mathrm{P} = \frac{1}{T}\int_0^T x^2(t)\, dt = \frac{A^2\omega_0}{2\pi}\int_0^{2\pi/\omega_0} cos^2\,(\underbrace{\omega_0 t + \theta}_{\tau})dt = \frac{A^2}{2\pi}\int_\theta^{2\pi+\theta} cos^2\,(\tau)\, d\tau$$

$$= \frac{A^2}{2\pi}\int_0^{2\pi} cos^2\,(\tau)\, d\tau = \frac{A^2}{2\pi}\int_0^{2\pi} \frac{1}{2}(1 + cos2\tau)\ d\tau$$

$$= \frac{A^2}{2\pi}\left[\frac{1}{2}\underbrace{\int_0^{2\pi} d\tau}_{2\pi} + \frac{1}{2}\underbrace{\int_0^{2\pi} cos2\tau\ d\tau}_{0}\right] = \frac{A^2}{2}$$

Decibel (dB)

The **decibel** (**dB**) is a logarithmic unit used to express the ratio of two values of a physical quantity, often power. One of these values is often a standard reference value, in which case the decibel is used to express the level of the other value relative to this reference. The number of decibels is ten times the logarithm to base 10 of the ratio of two power quantities, or of the ratio of the squares of two field amplitude quantities.

Let P_2 denote the reference value

Let P_1 denote the value of the physical quantity of interest

Then $$\boxed{P(dB) = 10\ \log_{10}\frac{P_1}{P_2}}$$

Note: One decibel is one tenth of one **bel**, named in honor of Alexander Graham Bell; however, the bel is seldom used.

Decibel (dB) -- Examples

$$\boxed{P\ (dB) = 10\ \log_{10}\frac{P_1}{P_2}}$$

Particular case:

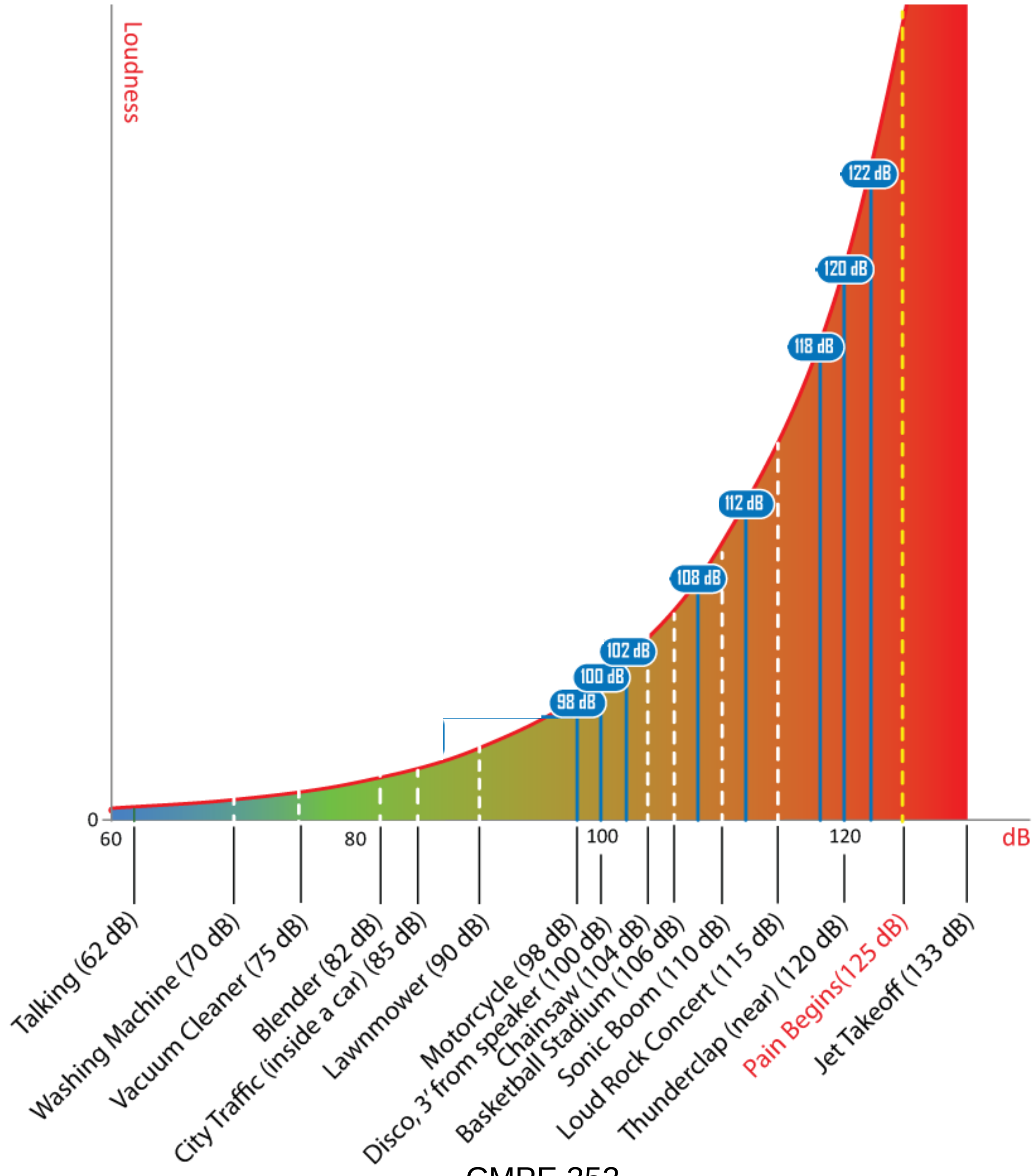
If $P_2 = 1\ mW$, then we use dBm instead of dB

Note also that:

$$\log a^b = b \log a$$

Power ratio	dB
1000	30
100	20
10	10
2	~3
1	0
0.5	~(-3)
0.1	-10
0.01	-20
0.001	-30

Decibel (dB) -- Examples



Signal Energy / Power in dB -- Example

Compute the power ratio of the following two signals in Db:

$$x_1(t) = A\ \cos(\omega_0 t + \theta) \qquad x_2(t) = 4A\ \cos(3\omega_0 t + 5\theta)$$

$$P_1 = \frac{A^2}{2} \qquad P_2 = \frac{16A^2}{2}$$

$$10\log_{10}\frac{P_1}{P_2} = 10\log_{10}\frac{1}{16}$$

$$= -10\log_{10} 16$$

$$= -10\log_{10} 2^4$$

$$= -4 \times 10\log_{10} 2 \ = -4 \times 3$$

$$= -12\ \text{dB}$$