EEEN 474
Wireless Communication

Spring 2020

The Cellular Concept

System Design Fundamentals

-continued-

Trunking and Grade of Service (GOS)

- Cellular radio systems rely on **trunking** to accomodate a large number of users in a limited radio spectrum
- Allows a large number of users to share the relatively small number of chanels in a cell by providing access to each user, on demand, from a pool of available channels
- Each user is allocated a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels

Trunking and Grade of Service (GOS)

- **Trunking theory** is used by telephone company to determine the number of telephone circuits that need to be allocated for office buildings with hundreds of telephones
- This same principle is used in designing cellular radio systems

Trunking and Grade of Service (GOS)

- There is a trade-off between
  - the number of available telephone circuits and
  - the likelihood of a particular user finding that no circuits are available during the peak calling time
- In a trunked mobile radio system, when a particular user requests service and all of the radio channels are already in use, the user is **blocked**, or denied access to the system
- In some systems, a **queue** may be used to hold the users until a channel becomes available

Trunking and Grade of Service (GOS)

- To design trunked radio systems that can handle a specific capacity at a specific GOS, it is essential to understand **trunking** theory and **queuing** theory
- Trunking theory → Erlang (a Danish matematician, late 19th century)
- Today, the measure of traffic intensity bears his name

1 Erlang → the amount of traffic intensiy carried by a channel that is completely occupied (i.e., one call-hour per hour or one call-minute per minute)

e.g. A radio channel that is occupied for thirty minutes during an hour carries 0.5 Erlangs of traffic

Trunking and Grade of Service (GOS)

- Grade of Service (GOS): A measure of the ability of a user to access a trunked system during the **busiest** hour.

  (The busy hours for cellular radio systems typically ocur during rush hours, between 4 p.m. and 6 p.m. on a Thursday or Friday evening.)

- The GOS is a benchmark used to define the desired performance of a particular trunked system by specifying a desired likelihood of a user obtaining channel access given a specific number of channels available in the system.
- Designer's job:
  - To estimate the maximum required capacity
  - To allocate the proper number of channels in order to meet the GOS
- GOS is typically given as the **likelihood**
  - That a call is **blocked**
  - Of a call experiencing a **delay** greater than a certain queuing time

Trunking and Grade of Service (GOS)

$$A_u = \lambda H$$

$$A = UA_u$$

If the traffic is equally distributed among the channels:

$$A_c = A / C = UA_u / C$$

A_u: traffic intensity generated by each user (in Erlangs)
λ: average number of call requests per unit time for each user
H: average duration of a call
A: total offered traffic intensity (in Erlangs)
U: number of users in the system
C: the total number of channels in the trunked system
A_c: traffic intensity per channel

Trunking and Grade of Service (GOS)

- Two types of trunked systems which are commonly used:
  - Offers no queuing for call requests

    → *Blocked Calls Cleared* **(Erlang B)**
  - A queue is provided to hold calls which are blocked

    → *Blocked Calls Delayed* **(Erlang C)**

Erlang B Formula

$$Pr[blocking] = \frac{\frac{A^C}{C!}}{\sum_{k=0}^{C} \frac{A^k}{k!}} = GOS$$

C: number of trunked channels offered by a trunked radio system

A: total offered traffic

Erlang B

A in the formula

Table 2.4 Capacity of an Erlang B System

Number of Channels *C*	Capacity (Erlangs) for GOS = 0.01	= 0.005	= 0.002	= 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

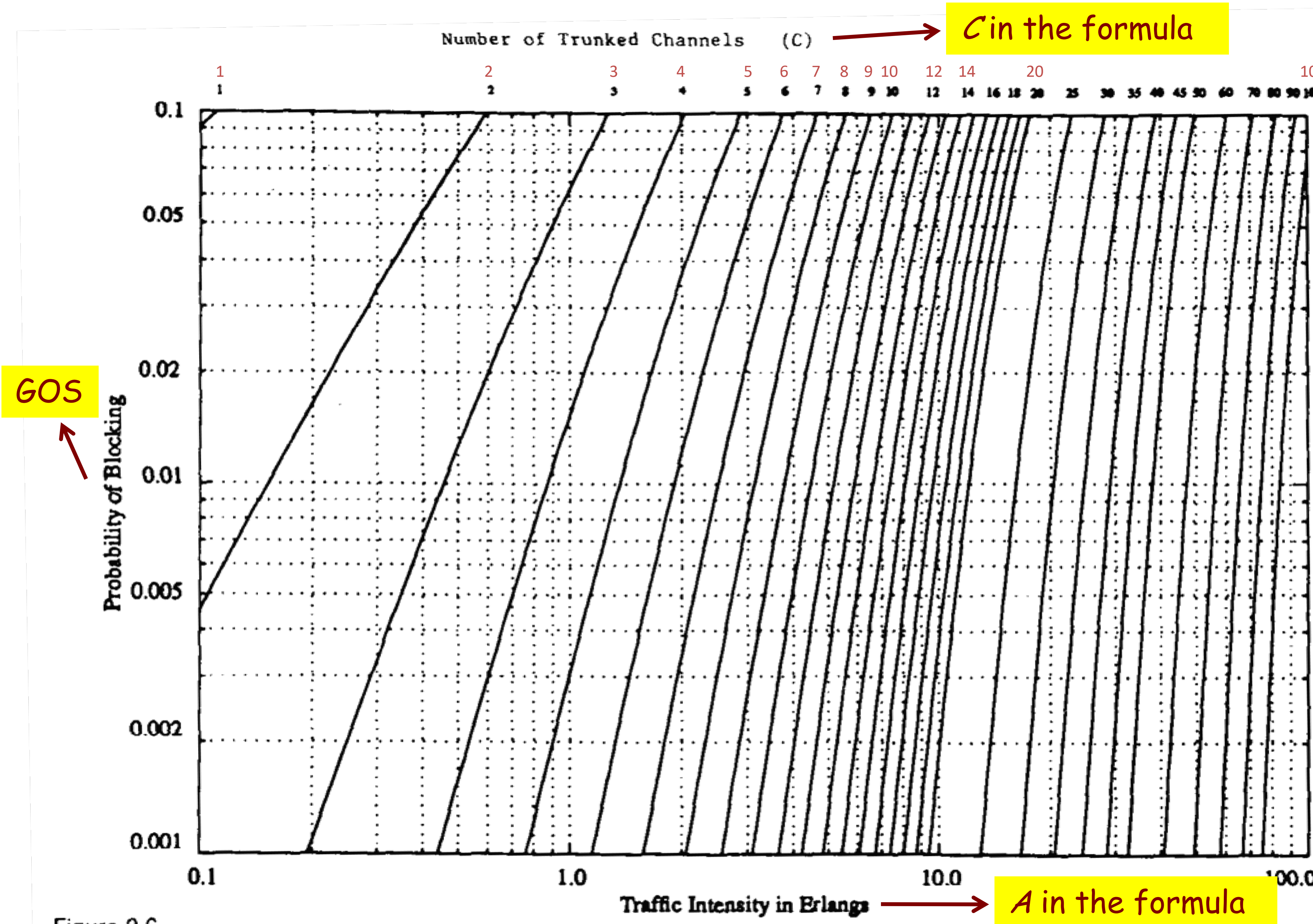

Figure 2.6

The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.

Example 2.4

How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system? ~~(a) 1~~, (b) 5, (c) 10, (d) 20, (e) 100. Assume each user generates 0.1 Erlangs of traffic.

Solution to Example 2.4

From Table 2.4 we can find the total capacity in Erlangs for the 0.5% GOS for different numbers of channels. By using the relation $A = UA_u$, we can obtain the total number of users that can be supported in the system.

~~(a) Given $C = 1$, $A_u = 0.1$, $GOS = 0.005$~~

~~From Figure 2.6, we obtain $A = 0.005$.~~

~~Therefore, total number of users, $U = A/A_u = 0.005/0.1 =$~~

~~But, actually one user could be supported on one channel. S~~

A in the formula

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20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

(b) Given $C = 5$, $A_u = 0.1$, $GOS = 0.005$

From Figure 2.6, we obtain $A = 1.13$.

Therefore, total number of users, $U = A/A_u = 1.13/0.1 \approx 11$ users.

(c) Given $C = 10$, $A_u = 0.1$, $GOS = 0.005$

From Figure 2.6, we obtain $A = 3.96$.

Therefore, total number of users, $U = A/A_u = 3.96/0.1 \approx 39$ users.

(d) Given $C = 20$, $A_u = 0.1$, $GOS = 0.005$

From Figure 2.6, we obtain $A = 11.10$.

Therefore, total number of users, $U = A/A_u = 11.1/0.1 = 110$ users.

(e) Given $C = 100$, $A_u = 0.1$, $GOS = 0.005$

From Figure 2.6, we obtain $A = 80.9$.

Therefore, total number of users, $U = A/A_u = 80.9/0.1 = 809$ users.

Example 2.5

An urban area has a population of 2 million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages 2 calls per hour at an average call duration of 3 minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

Solution to Example 2.5

System A

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 19$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 19$, from the Erlang B chart, the total carried traffic, A, is obtained as 12 Erlangs.

Therefore, the number of users that can be supported per cell is

$$U = A/A_u = 12/0.1 = 120.$$

Since there are 394 cells, the total number of subscribers that can be supported by System A is equal to $120 \times 394 = 47280$.

System B

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 57$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 57$, from the Erlang B chart, the total carried traffic, A, is obtained as 45 Erlangs.

Therefore, the number of users that can be supported per cell is

$$U = A/A_u = 45/0.1 = 450.$$

Since there are 98 cells, the total number of subscribers that can be supported by System B is equal to $450 \times 98 = 44100$.

System C

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 100$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 100$, from the Erlang B chart, the total carried traffic, A, is obtained as 88 Erlangs.

Therefore, the number of users that can be supported per cell is

$$U = A/A_u = 88/0.1 = 880.$$

Since there are 49 cells, the total number of subscribers that can be supported by System C is equal to $880 \times 49 = 43120$

Therefore, total number of cellular subscribers that can be supported by these three systems are $47280 + 44100 + 43120 = 134500$ users.

Since there are 2 million residents in the given urban area and the total number of cellular subscribers in System A is equal to 47280, the percentage market penetration is equal to

$$47280/2000000 = 2.36\ \%$$

Similarly, market penetration of System B is equal to

$$44100/2000000 = 2.205\ \%$$

and the market penetration of System C is equal to

$$43120/2000000 = 2.156\ \%$$

The market penetration of the three systems combined is equal to

$$134500/2000000 = 6.725\ \%$$

Example 2.6

A certain city has an area of 1,300 square miles and is covered by a cellular system using a 7-cell reuse pattern. Each cell has a radius of 4 miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute (a) the number of cells in the service area, (b) the number of channels per cell, (c) traffic intensity of each cell, (d) the maximum carried traffic, (e) the total number of users that can be served for 2% GOS

The area of a hexagon is 2.5981R^2 (R being its radius)

Solution to Example 2.6

(a) Given:

Total coverage area = 1300 miles

Cell radius = 4 miles

The area of a cell (hexagon) can be shown to be $2.5981R^2$, thus each cell covers

$$2.5981 \times (4)^2 = 41.57 \text{ sq mi.}$$

Hence, the total number of cells are $N_c = 1300/41.57 = 31$ cells.

(b) The total number of channels per cell (C)

$$= \text{allocated spectrum} / (\text{channel width} \times \text{frequency reuse factor})$$

$$= 40,000,000/(60,000 \times 7) = 95 \text{ channels/cell}$$

(c) Given:

$C = 95$, and $GOS = 0.02$

From the Erlang B chart, we have

traffic intensity per cell $A = 84$ Erlangs/cell

(d) Maximum carried traffic = number of cells × traffic intensity per cell

$$= 31 \times 84 = 2604 \text{ Erlangs.}$$

(e) Given traffic per user = 0.03 Erlangs

Total number of users = Total traffic / traffic per user

= 2604 / 0.03 = 86,800 users.

Erlang C Formula

$$Pr\,[delay > 0] = \frac{A^C}{A^C + C!\left(1 - \frac{A}{C}\right)\sum_{k=0}^{C-1}\frac{A^k}{k!}}$$

C: number of trunked channels offered by a trunked radio system

A: total offered traffic

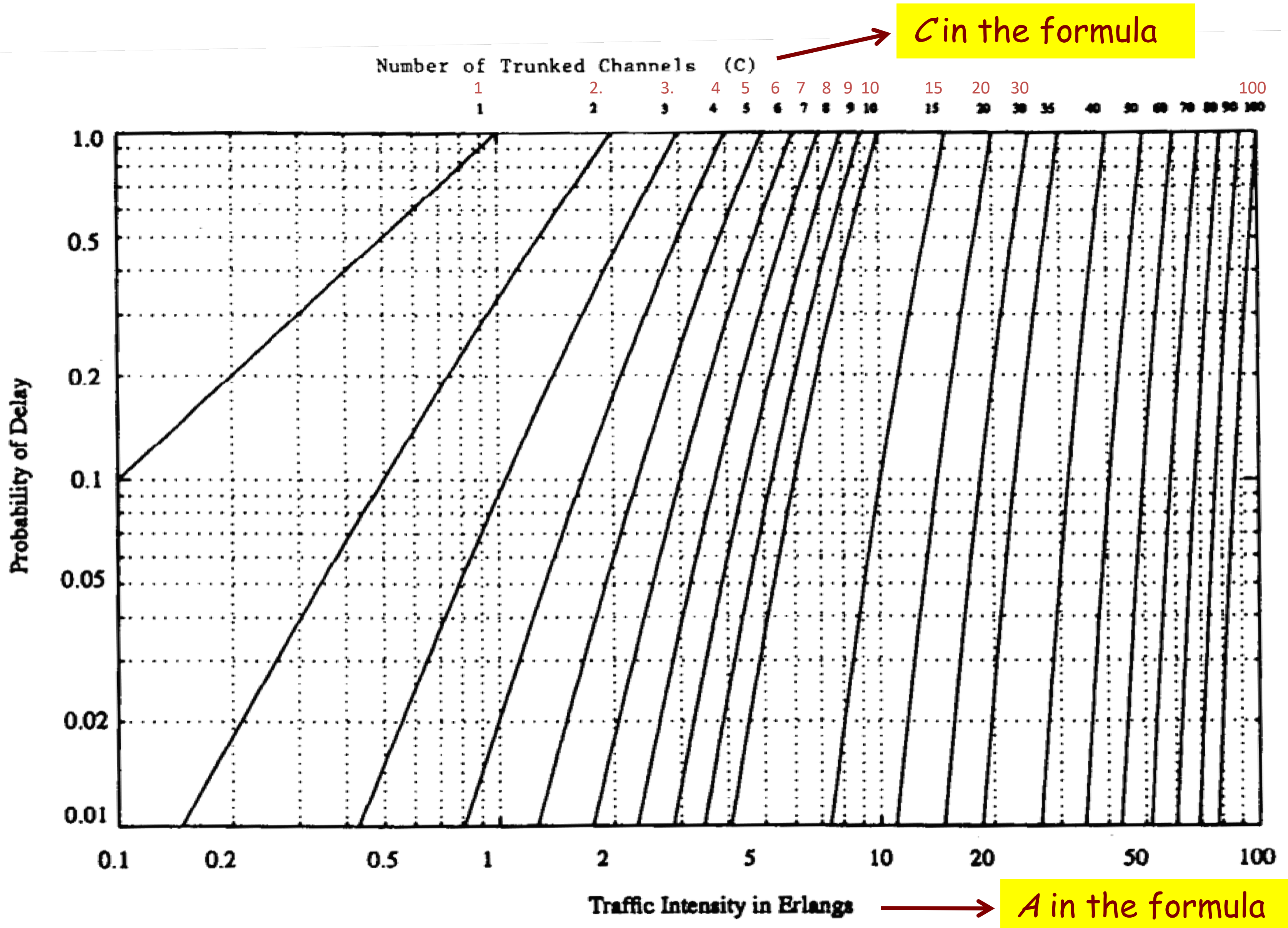


Figure 2.7
The Erlang C chart showing the probability of a call being delayed as a function of the number of channels and traffic intensity in Erlangs.

$$
\begin{aligned}
Pr[delay > t] &= Pr[delay > 0]\, Pr[delay > t \mid delay > 0] \\
&= Pr[delay > 0] \exp(-(C-A)t/H)
\end{aligned}
$$

$$
D = Pr[delay > 0] \frac{H}{C-A}
$$

C: number of trunked channels offered by a trunked radio system

A: total offered traffic

H: average duration of a call

D: average delay for all calls in a queued system

Example 2.7

A hexagonal cell within a 4-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and $\lambda = 1$ call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call:

(a) How many users per square kilometer will this system support?

(a) What is the probability that a delayed call will have to wait for more than 10 s?

(c) What is the probability that a call will be delayed for more than 10 seconds?

The area of a hexagon is 2.5981R^2 (R being its radius)

Solution to Example 2.7

Given,

Cell radius, $R = 1.387$ km

Area covered per cell is $2.598 \times (1.387)^2 = 5$ sq km

Number of cells per cluster = 4

Total number of channels = 60

Therefore, number of channels per cell = 60 / 4 = 15 channels.

(a) From Erlang C chart, for 5% probability of delay with $C = 15$, traffic intensity = 9.0 Erlangs.

Therefore, number of users = total traffic intensity / traffic per user

$= 9.0/0.029 = 310$ users

$= 310$ users$/5$ sq km $= 62$ users/sq km

(b) Given $\lambda = 1$, holding time

$H = A_u/\lambda = 0.029$ hour $= 104.4$ seconds.

The probability that a delayed call will have to wait for more than 10 s is

$$Pr[delay > t|delay] = \exp(-(C-A)t/H)$$

$$= \exp(-(15-9.0)10/104.4) = 56.29\ \%$$

(c) Given $Pr[delay > 0] = 5\% = 0.05$

Probability that a call is delayed more than 10 seconds,

$$Pr[delay > 10] = Pr[delay > 0]Pr[delay > t|delay]$$

$$= 0.05 \times 0.5629 = 2.81\ \%$$

Improving Coverage and Capacity in Cellular Systems

- As the demand increases, number of channels assigned to a cell becomes insufficient
- Cellular design techniques are needed to provide more channels per unit coverage area
  - Cell splitting

    → Increases the number of base stations in order to increase capacity
  - Sectoring

    → Uses directional antennas to further control the interference and frequency reuse of channels.
  - Coverage zone approaches

Cell Splitting

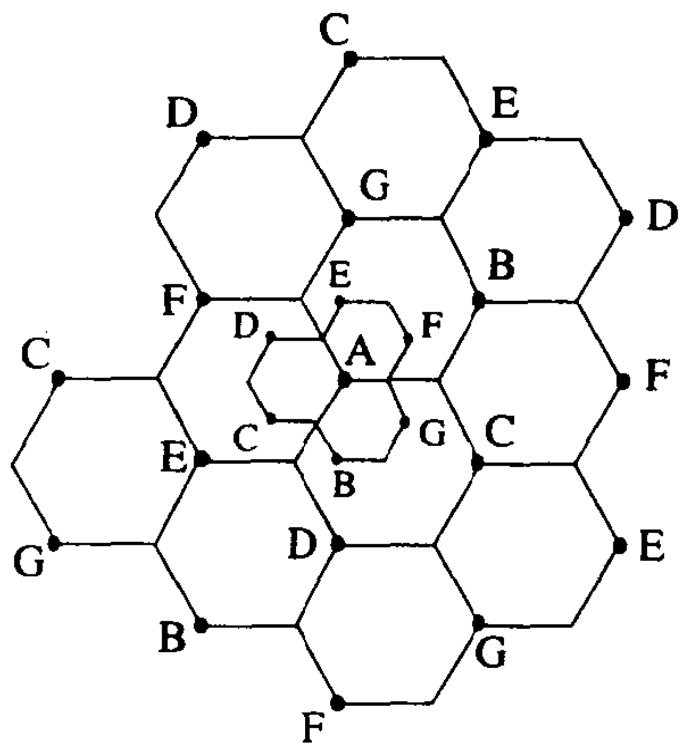


Illustration of cell splitting.

Cell Splitting

- Is the process of subdividing a congested cell into smaller cells
  - Each with its own base station
  - And a corresponding reduction in
    - Antenna height
    - Transmitter power
- Increases the capacity of a cellular system since it increases the number of times that channels are reused

Cell Splitting

$$P_r \text{ [at old cell boundary] } \propto P_{t1}R^{-n}$$

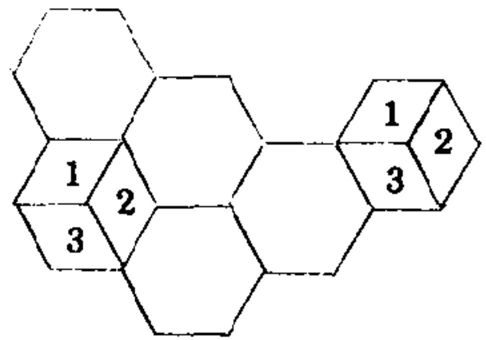
$$P_r \text{ [at new cell boundary] } \propto P_{t2}(R/2)^{-n}$$

If the path loss exponent $n=4$

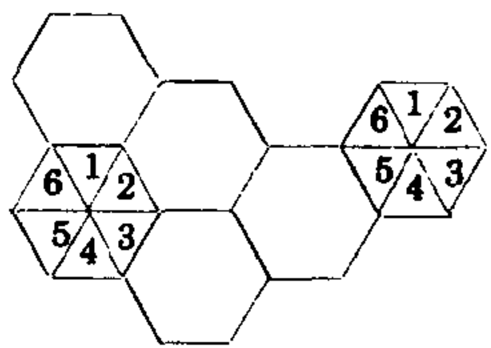
$$P_{t2} = P_{t1} / 16$$

The transmit power must be reduced by 12 dB in order to fill in the original coverage area by microcells

Sectoring



(a)



(b)

(a) 120° sectoring.
(b) 60° sectoring.

Sectoring

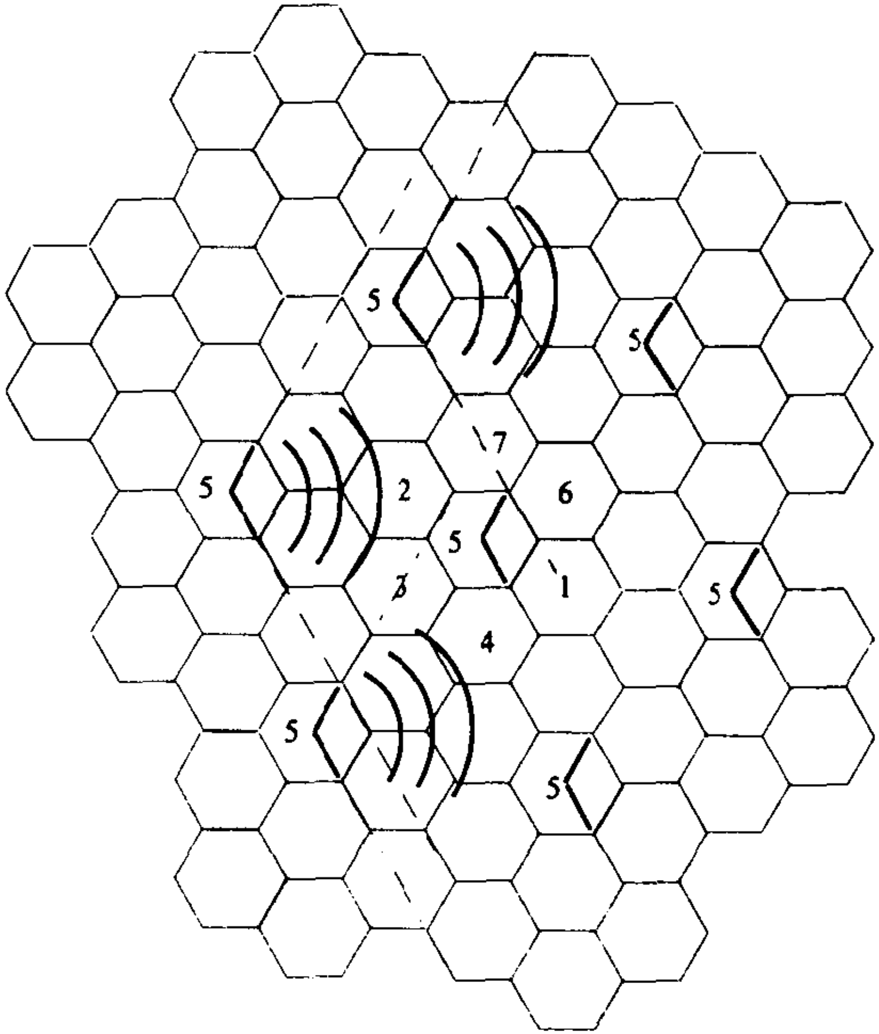


Illustration of how 120° sectoring reduces interference from co-channel cells. Out of the 6 co-channel cells in the first tier, only 2 of them interfere with the center cell. If omni-directional antennas were used at each base station, all 6 co-channel cells would interfere with the center cell.

S/I=24.2 dB (improved)

(Worst case S/I=17 dB in the omnidirectional case)

→ This improvement allows the designer to decrease *N*, which will increase the capacity

→ Penalty: A decrease in trunking efficiency