EEEN 474
Wireless Communication

Spring 2020

Mobile Radio Propagation:

Large-Scale Path Loss

Free Space Propagation Model

Received power at a distance d from the transmitter:

$$\boxed{P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}}$$

Friis
free space
equation

$P_r(d)$: received power as a function of T-R separation
P_t: transmitted power
G_t: transmitter antenna gain
G_r: receiver antenna gain
λ: wavelength in meters
d: T-R separation distance in meters
L: system loss factor not related to propagation

Note that we divide by d^2 → A slope of -20 dB / decade → 20 dB decrease in received power as distance increases 10 times

Free Space Propagation Model

The gain of an antenna:

$$G = \frac{4\pi A_e}{\lambda^2}$$

A_e: effective aperture (it is related to the physical size of the antenna)
λ: wavelength in meters

$$\lambda = \frac{c}{f} = \frac{2\pi c}{w_c}$$

f: carrier frequency in Hertz
w_c: carrier frequency in radians per second
c: speed of light in meters per second ($\approx 3 \cdot 10^8 m/s$)

Effective isotropic radiated power (EIRP) is defined as: $EIRP = P_t G_t$

Free Space Propagation Model

Path loss:

$$\begin{aligned} PL(d)[dB] &= P_t[dB] - P_r(d)[dB] \\ &= 10 log P_t - 10 log P_r(d) \\ &= 10 log \frac{P_t}{P_r} \\ &= -10 log[\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2}] \end{aligned}$$

(If L = 1)

$$\boxed{PL(d)[dB] = -10 log[\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2}]}$$

(L = 1)

$$PL(d)[dB] = -10 log[\frac{\lambda^2}{(4\pi)^2 d^2}]$$

($L = 1$, $G_t = 1$, $G_r = 1$)

Note that this has a slope of 20 dB / decade → 20 dB increase in path loss as distance increases 10 times

Free Space Propagation Model

Friis free space model is only valid for values of *d* which are in the far-field (Fraunhofer region) of the transmitting antenna, i.e., for $d > d_f$, *where* d_f *is defined as*

$$\boxed{d_f = \frac{2D^2}{\lambda}}$$

D: the largest physical linear dimension of the antenna

$$d_f \gg D$$

$$d_f \gg \lambda$$

Example 3.1

Find the far-field distance for an antenna with maximum dimension of 1 m and operating frequency of 900 MHz.

Solution to Example 3.1

Given:

Largest dimension of antenna, $D = 1$ m

Operating frequency $f = 900$ **MHz**, $\lambda = c/f = \dfrac{3 \times 10^{8} \text{ m/s}}{900 \times 10^{6} \text{ Hz}}$ m

Using equation (3.7.a), far-field distance is obtained as

$$d_f = \frac{2\,(1)^2}{0.33} = 6 \text{ m}$$

Free Space Propagation Model

Friis equation does not hold for $d = 0$
→ Use a close-in distance d_0 as reference

$$\boxed{P_r(d) = P_r(d_0)(\frac{d_0}{d})^2} \qquad d \geq d_0 \geq d_f$$

$$P_r(d)[dB] = 10log[P_r(d_0)] + 20log(\frac{d_0}{d})$$

$$P_r(d)[dBm] = 10log[\frac{P_r(d_0)}{0.001\mathrm{W}}] + 20log(\frac{d_0}{d})$$

$$P_r(d)[dBm] = P_r(d)[dB] + 30$$

Exercise

Show that:

$$\boxed{PL(d)[dB] = PL(d_0)[dB] + 20log(\frac{d}{d_0})}$$

Example 3.2

If a transmitter produces 50 watts of power, express the transmit power in units of (a) dBm, and (b) dBW. If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency, find the received power in dBm at a free space distance of 100 m from the antenna. What is P_r (10 km) ? Assume unity gain for the receiver antenna.

Solution to Example 3.2

Given:

Transmitter power, $P_t = 50\ \text{W}$.

Carrier frequency, $f_c = 900\ \text{MHz}$

Using equation (3.9),

(a) Transmitter power,

$$P_t(\text{dBm}) = 10\log\left[P_t(\text{mW})/(1\ \text{mW})\right]$$
$$= 10\log\left[50 \times 10^3\right] = 47.0\ \text{dBm}.$$

(b) Transmitter power,

$$P_t(\text{dBW}) = 10\log\left[P_t(\text{W})/(1\ \text{W})\right]$$
$$= 10\log[50] = 17.0\ \text{dBW}.$$

The received power can be determined using equation (3.1).

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} = \frac{50\,(1)\,(1)\,(1/3)^2}{(4\pi)^2\,(100)^2\,(1)} = 3.5 \times 10^{-6}\ \text{W} = 3.5 \times 10^{-3}\ \text{mW}$$

$$P_r(\text{dBm}) = 10\log P_r(\text{mW}) = 10\log\left(3.5 \times 10^{-3}\ \text{mW}\right) = -24.5\ \text{dBm}.$$

The received power at 10 km can be expressed in terms of dBm using equation (3.9), where $d_0 = 100$ m and $d = 10$ km

$$P_r(10\ \text{km}) = P_r(100) + 20\log\left[\frac{100}{10000}\right] = -24.5\ \text{dBm} - 40\ \text{dB}$$

$$= -64.5\ \text{dBm}.$$

Ground Reflection (Two-Ray) Model

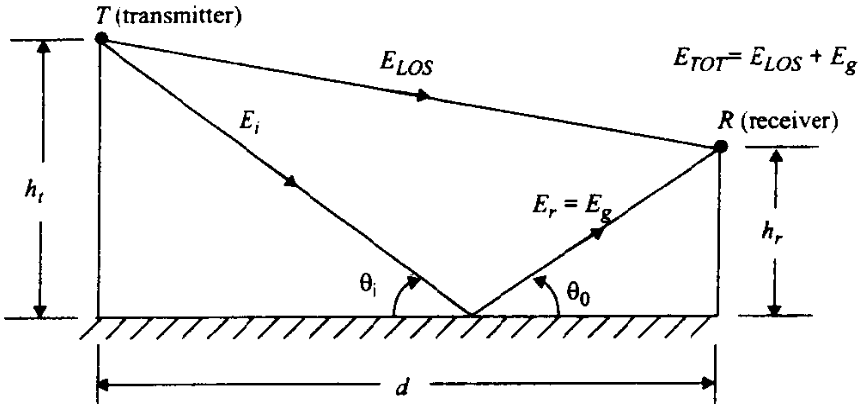


Figure 3.7
Two-ray ground reflection model.

Received power at a distance d from the transmitter:
(h_t: height of the transmitter, h_r: height of the receiver)

$$\boxed{P_r(d) = \frac{P_t G_t G_r h_t^2 h_r^2}{d^4}} \quad \text{whenever} \quad d \gg \frac{20 h_t h_r}{\lambda}$$

Note that we divide by d^4 → A slope of -40 dB / decade → 40 dB decrease in received power as distance increases 10 times

Practical Link Budget Design Using Path Loss Models

- Combination of analytical and empirical methods
- Empirical approach: fitting curves or analytical expressions on measured data
- Takes into account all propagation factors, both known and unknown, through actual field measurements

Log-distance Path Loss Model

$$\overline{PL}(d)[dB] = \overline{PL}(d_0)[dB] + 10nlog(\frac{d}{d_0})$$

Table 3.2 Path Loss Exponents for Different Environments

Environment	Path Loss Exponent, n
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

- Select d_0 properly (large coverage cellular systems 1km, microcellular systems 1m -100m)
- d_0 should always be in the far-field region
- Calculate reference path loss using Friis equation or through field measurements
- Bar denotes ensemble average over all possible values

Log-normal Shadowing

$$PL(d)[dB] = \overline{PL}(d)[dB] + X_\sigma = \overline{PL}(d_0)[dB] + 10nlog(\frac{d}{d_0}) + X_\sigma$$

X_σ: a zero-mean Gaussian distributed random variable (in dB) with standard deviation σ (also in dB)

- In practice, the values of n and σ are estimated from measured data
- $\overline{PL}(d_0)$ is based on either measurement or Friis equation

Log-normal Shadowing

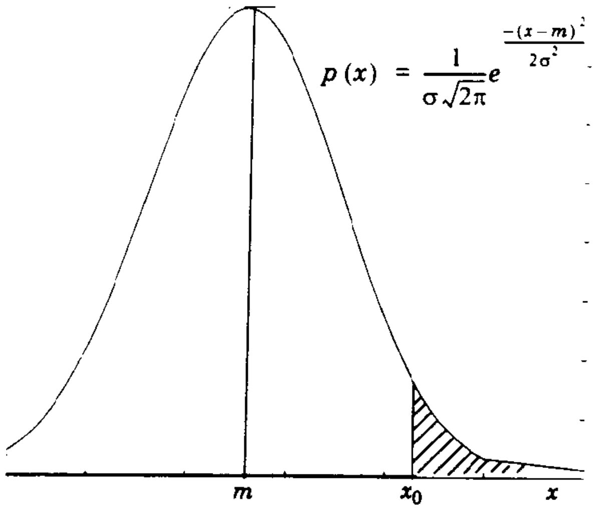
Since $PL(d)$ is a random variable with a normal distribution in dB, so is $P_r(d)$, and the probability that the received signal level (in dB power units) will exceed a certain value γ can be calculated as:

$$Pr[P_r(d) > \gamma] = Q(\frac{\gamma - \overline{P_r(d)}}{\sigma})$$

Similarly, the probability that the received signal level (in dB power units) will be below γ is given by:

$$Pr[P_r(d) < \gamma] = Q(\frac{\overline{P_r(d)} - \gamma}{\sigma})$$

Error function (*erf*)



Gaussian probability density function. Shaded area is $Pr(x \geq x_0)$ for a Gaussian random variable.

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty \exp(-\frac{x^2}{2})dx = \frac{1}{2}[1 - erf(\frac{z}{\sqrt{2}})]$$

$$Q(z) = 1 - Q(-z)$$

Table D.2 Tabulation of the Error Function *erf(z)*

z	$erf(z)$	z	$erf(z)$
0.1	0.11246	1.6	0.97635
0.2	0.22270	1.7	0.98379
0.3	0.32863	1.8	0.98909
0.4	0.42839	1.9	0.99279
0.5	0.52049	2.0	0.99532
0.6	0.60385	2.1	0.99702
0.7	0.67780	2.2	0.99814
0.8	0.74210	2.3	0.99885
0.9	0.79691	2.4	0.99931
1.0	0.84270	2.5	0.99959
1.1	0.88021	2.6	0.99976
1.2	0.91031	2.7	0.99987
1.3	0.93401	2.8	0.99993
1.4	0.95228	2.9	0.99996
1.5	0.96611	3.0	0.99998

Determination of Percentage of Coverage Area

$$\boxed{U(\gamma) = \frac{1}{2}[1 + \exp(\frac{1}{b^2})(1 - erf(\frac{1}{b}))]}$$

$U(\gamma)$: percentage of useful service area (i.e., the percentage of area with a received signal that is equal or greater than γ).

$$b = \frac{10n\log e}{\sigma\sqrt{2}}$$

$$U(\gamma) = \frac{1}{2}[1 + \exp(\frac{1}{b^2})(1 - erf(\frac{1}{b}))]$$

Equation (3.78) may be evaluated for a large number of values of σ and n, as shown in Figure 3.18 [Reu74]. For example, if $n = 4$ and $\sigma = 8$ dB, and if the boundary is to have 75% boundary coverage (75% of the time the signal is to exceed the threshold at the boundary), then the area coverage is equal to 94%. If $n = 2$ and $\sigma = 8$ dB, a 75% boundary coverage provides 91% area coverage. If $n = 3$ and $\sigma = 9$ dB, then 50% boundary coverage provides 71% area coverage.

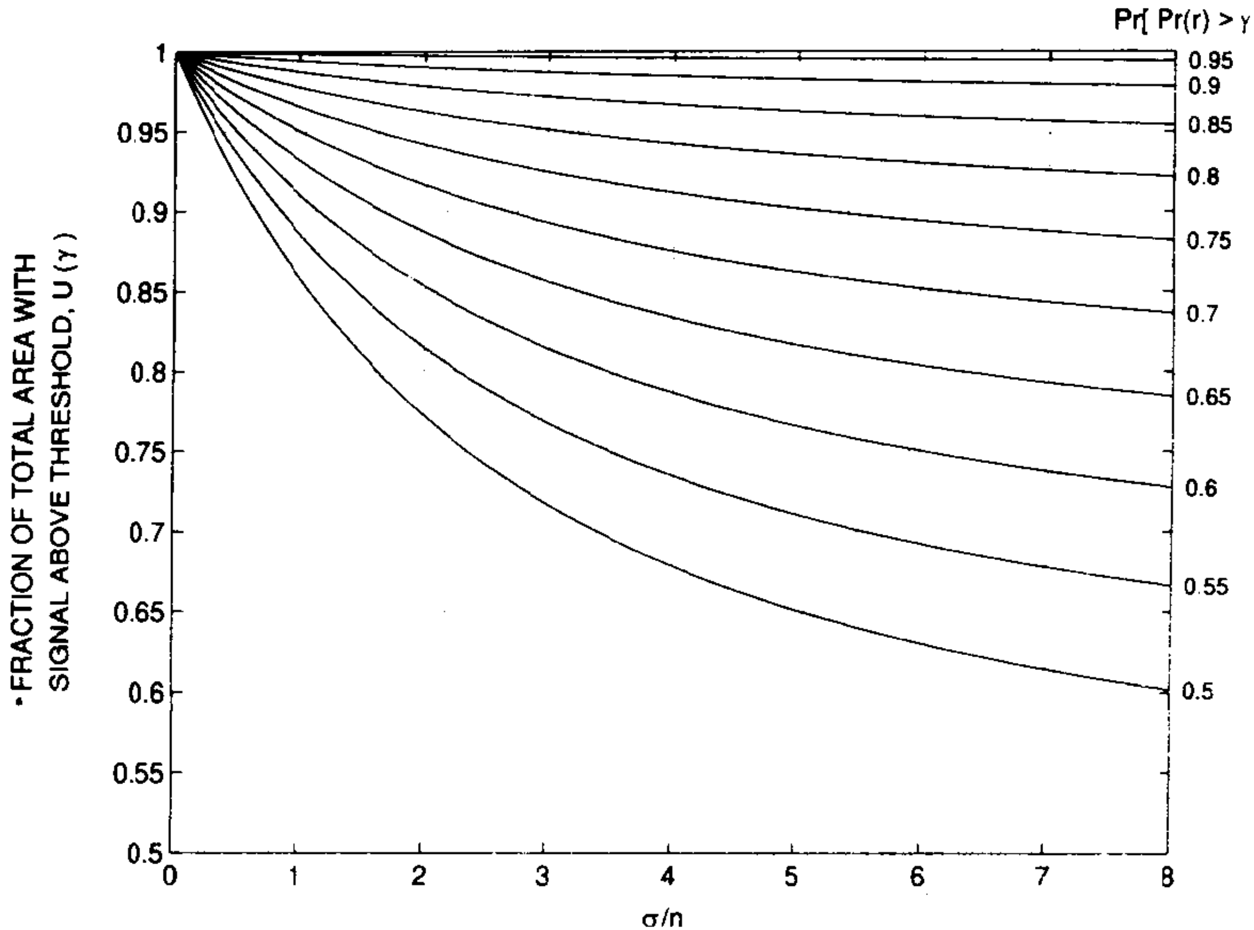


Figure 3.18

Family of curves relating fraction of total area with signal above threshold, $U(\gamma)$ as a function of probability of signal above threshold on the cell boundary.

Link Budget Equation

$$\boxed{P_r[dBm] = P_t[dBm] + \text{Gains}[dB] - \text{Losses}[dB]}$$

Outdoor Propagation Models

- Longley-Rice Model
- Durkin's Model
- **Okumura Model**
- Hata Model
- Walfisch and Bertoni Model
- Wideband PCS Microcell Model

Okumura Model

- A set of curves based on measurements
- Applicable for
  - Frequencies: 150MHz – 1920MHz (extrapolated up to 3000MHz)
  - Distances: 1km – 100km
  - Base station antenna heights: 30m – 1000m
- Simplest and best in terms of accuracy in path loss prediction for mature cellular and land mobile radio systems in cluttered environments
- Fairly good in urban and suburban areas, not as good in rural areas (slow response to rapid changes in terrain)

Okumura Model

$$\boxed{L_{50}[dB] = L_F + A_{mu}(f, d) - G(h_{te}) - G(h_{re}) - G_{AREA}}$$

L_{50}: the 50th percentile (i.e., median) value of propagation path loss
L_F: free space propagation loss
A_{mu}: median attenuation relative to free space
$G(h_{te})$: base station antenna height gain factor
$G(h_{re})$: mobile antenna height gain factor
G_{AREA}: gain due to the type of the environment

Okumura Model

$$\boxed{\begin{aligned} G(h_{te}) &= 20\log\left(\frac{h_{te}}{200}\right) \\ G(h_{re}) &= 10\log\left(\frac{h_{re}}{3}\right) \\ G(h_{re}) &= 20\log\left(\frac{h_{re}}{3}\right) \end{aligned}}$$

$1000 \text{ m} > h_{te} > 30 \text{ m}$

$h_{re} \leq 3 \text{ m}$

$10 \text{ m} > h_{re} > 3 \text{ m}$

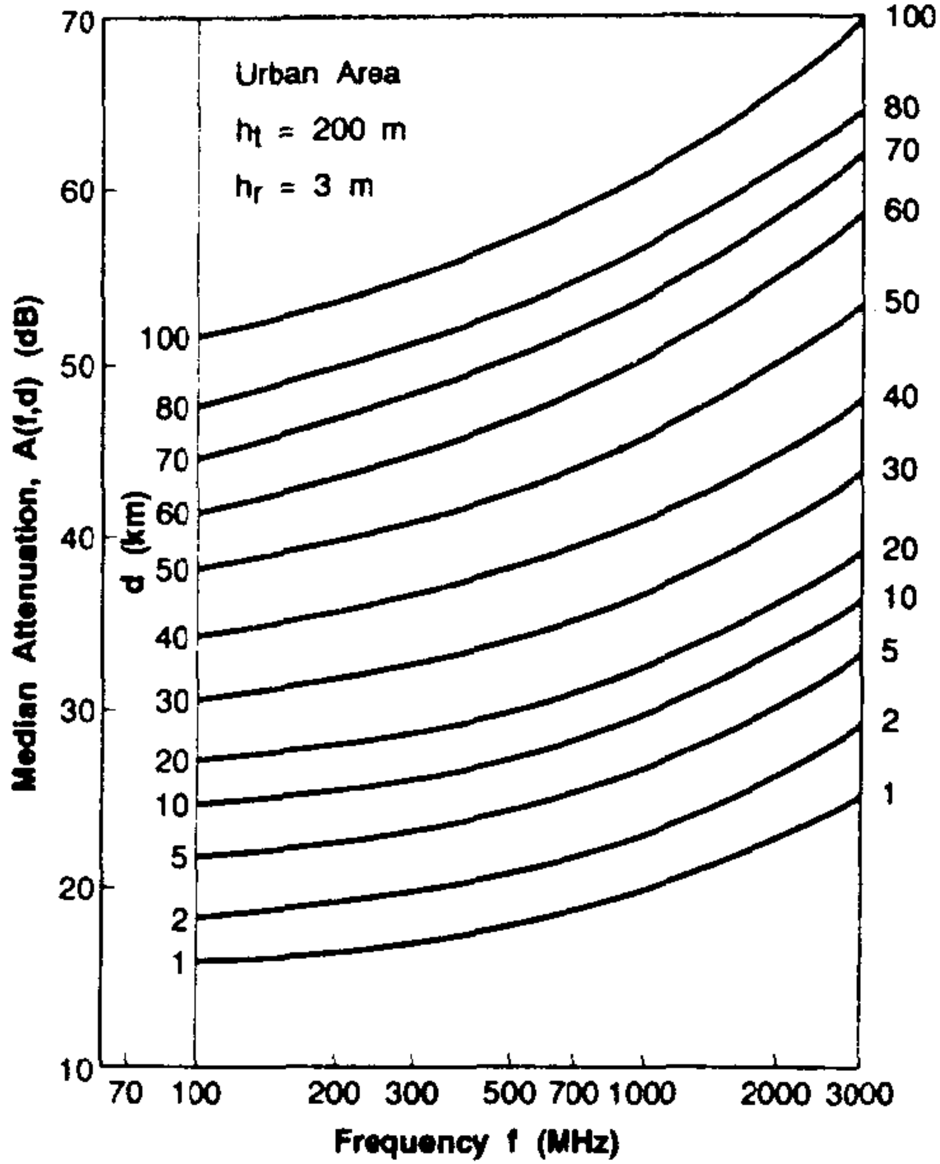


Figure 3.23
Median attenuation relative to free space $(A_{mu}(f,d))$, over a quasi-smooth terrain [From [Oku68] © IEEE].

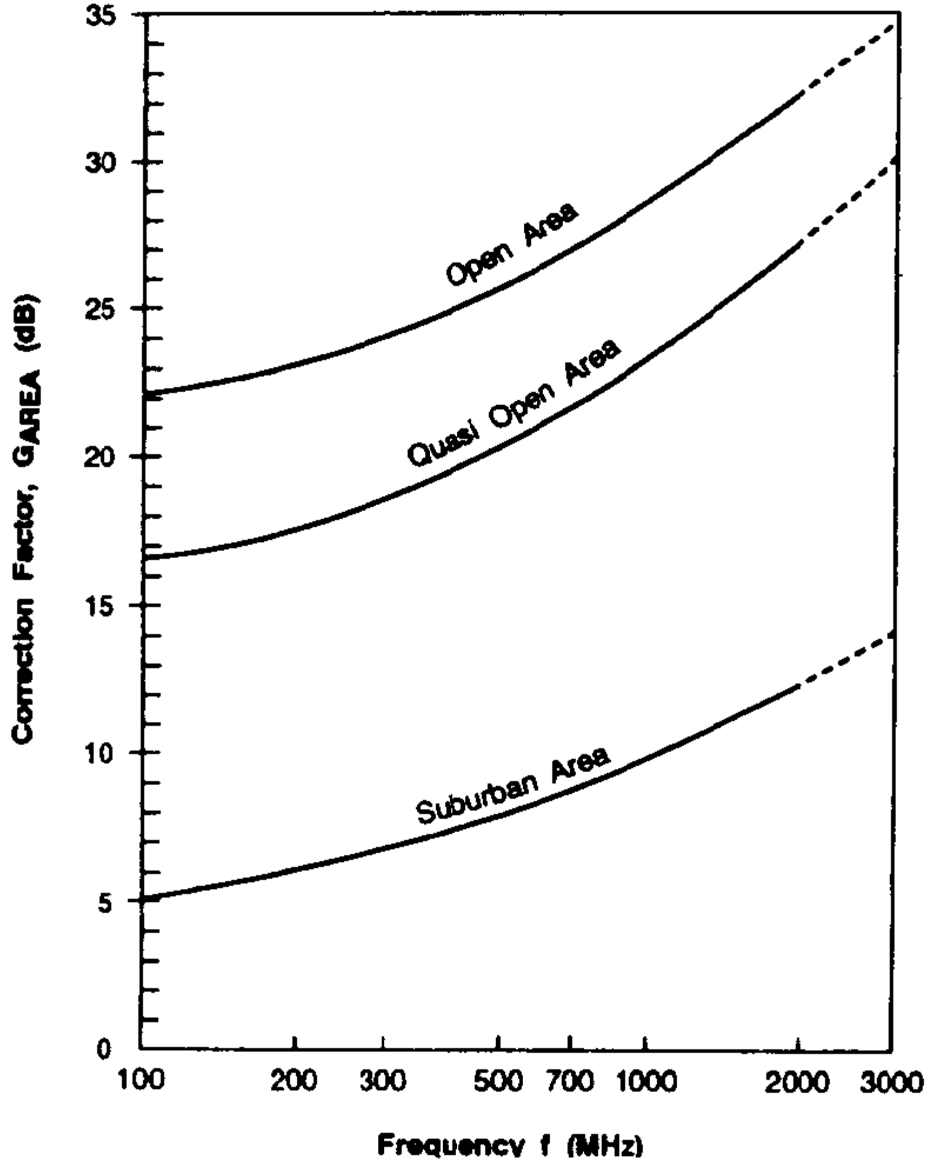


Figure 3.24
Correction factor, G_{AREA}, for different types of terrain [From [Oku68] © IEEE].

Example 3.10

Find the median path loss using Okumura's model for $d = 50$ km, $h_{te} = 100$ m, $h_{re} = 10$ m in a suburban environment. If the base station transmitter radiates an EIRP of 1 kW at a carrier frequency of 900 MHz, find the power at the receiver (assume a unity gain receiving antenna).

Solution to Example 3.10

The free space path loss L_F can be calculated using equation (3.6) as

$$L_F = 10\log\left[\frac{\lambda^2}{(4\pi)^2 d^2}\right] = 10\log\left[\frac{(3\times 10^8/900\times 10^6)^2}{(4\pi)^2\times(50\times 10^3)^2}\right] = 125.5 \text{ dB}.$$

From the Okumura curves

$$A_{mu}(900 \text{ MHz}(50 \text{ km})) = 43 \text{ dB}$$

and

$$G_{AREA} = 9 \text{ dB}.$$

Using equation (3.81.a) and (3.81.c) we have

$$G(h_{te}) = 20\log\left(\frac{h_{te}}{200}\right) = 20\log\left(\frac{100}{200}\right) = -6 \text{ dB}.$$

$$G(h_{re}) = 20\log\left(\frac{h_{re}}{3}\right) = 20\log\left(\frac{10}{3}\right) = 10.46 \text{ dB}.$$

Using equation (3.80) the total mean path loss is

$$\begin{aligned} L_{50}(\text{dB}) &= L_F + A_{mu}(f,d) - G(h_{te}) - G(h_{re}) - G_{AREA} \\ &= 125.5 \text{ dB} + 43 \text{ dB} - (-6) \text{ dB} - 10.46 \text{ dB} - 9 \text{ dB} \\ &= 155.04 \text{ dB}. \end{aligned}$$

Therefore, the median received power is

$$\begin{aligned} P_r(d) &= EIRP(\text{dBm}) - L_{50}(\text{dB}) + G_r(\text{dB}) \\ &= 60 \text{ dBm} - 155.04 \text{ dB} + 0 \text{ dB} = -95.04 \text{ dBm}. \end{aligned}$$

Indoor Propagation

- Partition losses (same floor) (table 3.3)
  - Due to partitions (hard or soft) and obstacles within the floor, depends on structure and material type
- Partition losses between floors (table 3.4 and 3.5)
  - Due to external dimensions and materials of the building, and, type of the construction used to create the floors and the external surroundings
- Indoor path loss obeys the distance power law (identical to log-normal shadowing model) (table 3.6)

Table 3.3 Average Signal Loss Measurements Reported by Various Researchers for Radio Paths Obstructed by Common Building Material.

Material Type	Loss (dB)	Frequency	Reference
All metal	26	815 MHz	[Cox83b]
Aluminium siding	20.4	815 MHz	[Cox83b]
Foil insulation	3.9	815 MHz	[Cox83b]
Concrete block wall	13	1300 MHz	[Rap91c]
Loss from one floor	20-30	1300 MHz	[Rap91c]
Loss from one floor and one wall	40-50	1300 MHz	[Rap91c]
Fade observed when transmitter turned a right angle corner in a corridor	10-15	1300 MHz	[Rap91c]
Light textile inventory	3-5	1300 MHz	[Rap91c]
Chain-like fenced in area 20 ft high containing tools, inventory, and people	5-12	1300 MHz	[Rap91c]
Metal blanket — 12 sq ft	4-7	1300 MHz	[Rap91c]
Metallic hoppers which hold scrap metal for recycling - 10 sq ft	3-6	1300 MHz	[Rap91c]
Small metal pole — 6" diameter	3	1300 MHz	[Rap91c]
Metal pulley system used to hoist metal inventory — 4 sq ft	6	1300 MHz	[Rap91c]
Light machinery < 10 sq ft	1-4	1300 MHz	[Rap91c]
General machinery - 10 - 20 sq ft	5-10	1300 MHz	[Rap91c]
Heavy machinery > 20 sq ft	10-12	1300 MHz	[Rap91c]
Metal catwalk/stairs	5	1300 MHz	[Rap91c]
Light textile	3-5	1300 MHz	[Rap91c]
Heavy textile inventory	8-11	1300 MHz	[Rap91c]
Area where workers inspect metal finished products for defects	3-12	1300 MHz	[Rap91c]
Metallic inventory	4-7	1300 MHz	[Rap91c]
Large 1-beam — 16 - 20"	8-10	1300 MHz	[Rap91c]
Metallic inventory racks — 8 sq ft	4-9	1300 MHz	[Rap91c]
Empty cardboard inventory boxes	3-6	1300 MHz	[Rap91c]
Concrete block wall	13-20	1300 MHz	[Rap91c]
Ceiling duct	1-8	1300 MHz	[Rap91c]
2.5 m storage rack with small metal parts (loosely packed)	4-6	1300 MHz	[Rap91c]
4 m metal box storage	10-12	1300 MHz	[Rap91c]
5 m storage rack with paper products (loosely packed)	2-4	1300 MHz	[Rap91c]

Table 3.3 Average Signal Loss Measurements Reported by Various Researchers for Radio Paths Obstructed by Common Building Material.

Material Type	Loss (dB)	Frequency	Reference
5 m storage rack with large paper products (tightly packed)	6	1300 MHz	[Rap91c]
5 m storage rack with large metal parts (tightly packed)	20	1300 MHz	[Rap91c]
Typical N/C machine	8-10	1300 MHz	[Rap91c]
Semi-automated assembly line	5-7	1300 MHz	[Rap91c]
0.6 m square reinforced concrete pillar	12-14	1300 MHz	[Rap91c]
Stainless steel piping for cook-cool process	15	1300 MHz	[Rap91c]
Concrete wall	8-15	1300 MHz	[Rap91c]
Concrete floor	10	1300 MHz	[Rap91c]
Commercial absorber	38	9.6 GHz	[Vio88]
Commercial absorber	51	28.8 GHz	[Vio88]
Commercial absorber	59	57.6 GHz	[Vio88]
Sheetrock (3/8 in) — 2 sheets	2	9.6 GHz	[Vio88]
Sheetrock (3/8 in) — 2 sheets	2	28.8 GHz	[Vio88]
Sheetrock (3/8 in) — 2 sheets	5	57.6 GHz	[Vio88]
Dry plywood (3/4 in) — 1 sheet	1	9.6 GHz	[Vio88]
Dry plywood (3/4 in) — 1 sheet	4	28.8 GHz	[Vio88]
Dry plywood (3/4 in) — 1 sheet	8	57.6 GHz	[Vio88]
Dry plywood (3/4 in) — 2 sheets	4	9.6 GHz	[Vio88]
Dry plywood (3/4 in) — 2 sheets	6	28.8 GHz	[Vio88]
Dry plywood (3/4 in) — 2 sheets	14	57.6 GHz	[Vio88]
Wet plywood (3/4 in) — 1 sheet	19	9.6 GHz	[Vio88]
Wet plywood (3/4 in) — 1 sheet	32	28.8 GHz	[Vio88]
Wet plywood (3/4 in) — 1 sheet	59	57.6 GHz	[Vio88]
Wet plywood (3/4 in) — 2 sheets	39	9.6 GHz	[Vio88]
Wet plywood (3/4 in) — 2 sheets	46	28.8 GHz	[Vio88]
Wet plywood (3/4 in) — 2 sheets	57	57.6 GHz	[Vio88]
Aluminium (1/8 in) — 1 sheet	47	9.6 GHz	[Vio88]
Aluminium (1/8 in) — 1 sheet	46	28.8 GHz	[Vio88]
Aluminium (1/8 in) — 1 sheet	53	57.6 GHz	[Vio88]

Table 3.4 Total Floor Attenuation Factor and Standard Deviation σ (dB) for Three Buildings. **Each point represents the average path loss over a 20λ measurement track [Sei92a].**

Building	915 MHz FAF (dB)	σ (dB)	Number of locations	1900 MHz FAF (dB)	σ (dB)	Number of locations
Walnut Creek						
One Floor	33.6	3.2	25	31.3	4.6	110
Two Floors	44.0	4.8	39	38.5	4.0	29
SF PacBell						
One Floor	13.2	9.2	16	26.2	10.5	21
Two Floors	18.1	8.0	10	33.4	9.9	21
Three Floors	24.0	5.6	10	35.2	5.9	20
Four Floors	27.0	6.8	10	38.4	3.4	20
Five Floors	27.1	6.3	10	46.4	3.9	17
San Ramon						
One Floor	29.1	5.8	93	35.4	6.4	74
Two Floors	36.6	6.0	81	35.6	5.9	41
Three Floors	39.6	6.0	70	35.2	3.9	27

Table 3.5 Average Floor Attenuation Factor in dB for One, Two, Three, and Four Floors in Two Office Buildings [Sei92b].

Building	FAF (dB)	σ (dB)	Number of locations
Office Building 1:			
Through One Floor	12.9	7.0	52
Through Two Floors	18.7	2.8	9
Through Three Floors	24.4	1.7	9
Through Four Floors	27.0	1.5	9
Office Building 2:			
Through One Floor	16.2	2.9	21
Through Two Floors	27.5	5.4	21
Through Three Floors	31.6	7.2	21

Table 3.6 Path loss exponent and standard deviation measured in different buildings [And94]

Building	Frequency (MHz)	n	σ (dB)
Retail Stores	914	2.2	8.7
Grocery Store	914	1.8	5.2
Office, hard partition	1500	3.0	7.0
Office, soft partition	900	2.4	9.6
Office, soft partition	1900	2.6	14.1
Factory LOS			
Textile/Chemical	1300	2.0	3.0
Textile/Chemical	4000	2.1	7.0
Paper/Cereals	1300	1.8	6.0
Metalworking	1300	1.6	5.8
Suburban Home			
Indoor Street	900	3.0	7.0
Factory OBS			
Textile/Chemical	4000	2.1	9.7
Metalworking	1300	3.3	6.8