MATH 233
Fall 2018
Class (Week 5)

1. A batch of 100 manufactured items is checked by an inspector. The inspector examines 10 items selected at random. If none of the 10 items is defective, he accepts the whole batch. Otherwise the batch is subjected to further inspection. How many ways can the inspector select these 10 items?

A: The problem is selection 10-combination from a set of 100 elements.
C (100, 10) = 100! / (10! 90!) = 17,310,309,456,440

2. A kindergarted in Canada has 30 students. 5 students are Asian, 10 students are European, and the rest of the students are African. Assume the teacher wants to select a football team of size 11. In how many ways can the selection be made, so that there are **at least 5 European students** in the team and **at least 5 African students?**

A. List all possible team arrangements of 11 (F : African, E : European, A : Asian)
1. (5F, 5E, A)
2. (5F, 6E)
3. (6F, 5E)

# of ways for selecting (5F, 5E, A) = C(15, 5).C(10, 5). C(5, 1)
= 15! / (10! 5!) . 10! / (5! 5!). 5
= 5. 15! / (5! 5! 5!)
= 3783780

# of ways for selecting (5F, 6E) = C(15, 5). C(10, 6)
= 15! / (10! 5!) . 10! / (6! 4!)
= 630630

# of ways for selecting (6F, 5E) = C(15, 6). C(10, 5)
= 15! / (9! 6!) . 10! / (5! 5!)
= 1261260

Total # of ways of selecting the 11-student team = 3783780 + 630630 + 1261260 = 5,675,670

3 How many ways are there to distribute six objects to five boxes if

a) both the objects and boxes are labeled?
b) the objects are unlabeled, but the boxes are labeled?

a) O1, O2, O3, O4, O5, O6 are objects. B1, B2, B3, B4, B5 are the boxes. Think about the possible placements :

B1(O1, O2), B2(O3), B3(O4), B4(O5,O6), B5()
B1(O2, O3), B2(O1), B3(O4), B4(O5,O6), B5()
B1(O1, O2, O3, O4, O5, O6), B2(), B3(), B4(), B5()

A box can be filled in 64 (2^6) different ways.

Set of objects **Set of boxes**

Think of a six-letter word where each letter can be one of B1, … B5.

B1 B2 B1 B1 B1 B1 is the placement where O2 is placed in B2 and all others are in B1.

B1 B2 B3 B4 B5 B5 is the placement where O1 is placed in B1, O2 in B2, O3 in B3, O4 in B3 and O5 and O6 in B5

Thus each letter can take 6 different values (each object can go to 6 different boxes). The result is 5^6 = 15625

This also corresponds to the number of functions from a set of 6 elements to a set of 5 elements.

b) O, O, O, O, O, O are objects. B1, B2, B3, B4, B5 are the boxes. Here are two placements:

B1(O, O), B2(O), B3(O), B4(O,O), B5()
B1(), B2(O, O, O, O, O, O), B3(), B4(), B5()

Encode the two placements as:

OO I O I O I OO I
I OOOOOO III

Thus, the problem reduces to placing six Os and 4 Is in 10-letter word (made of the letters O and I). Thus number of bitstrings of size 10 with 4 ones in it. C(10, 4)

4. What is the probability that the sum of the numbers on two dice is even when they are rolled? What is the sample space?

Sample Space = { (1,1), (1,2), … (1,6),
(2,1), (2,2), …(2,6),
….
(6,1), (6,2), …. (6,6) } (list of all possible outcomes)
Thus, size of the sample space is = 36

Event that sum of two dice is even is the following subset of the sample space:
{ (1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6.4), (6,6) }

Size of the outcomes in this event is 18.

P(E) = lEl / lS.Spacel = 18 / 36 = 1/2

5. Which is more likely: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled?

Experiment 1 : Rolling two dice
Experiment 2 : Rolling three dice

Sample Space 1: { (1,1), (1,2), … (1,6),
(2,1), (2,2), …(2,6),
….
(6,1), (6,2), …. (6,6) }

Sample Space 2 : { (1,1,1), (1,1,2), … (1,1,6),
(1,2,1), (1,2,2), …(1,2,6),
…
…
….
(6,6,1), (6,6,2), …. (6,6,6) }

Event 1 : A total of 8 when two dice are rolled = {(2,6), (3,5), (4,4), (5,3), (6,2) }
Event 2 : A total of 8 when three dice are rolled = {(1,1,6), (1,2,5), (1,3,4), (1,4,3), (1,5,2), (1,6,1), (2,1,5), (2,2,3), (2,3,3), (2,4,2), (2,5,1), (3,1,4), (3,2,3), (3,3,2), (3,4,1), (4,1,3), (4,2,2), (4,3,1), (5,1,2), (5,2,1), (6,1,1)}

l Event 1 l = 5
l Event 2 l = 21

P (Event 1) = 5 / 36
P (Event 2)= 21 / 216 = 7 / 72

Event 1 is more likely since 5 / 36 is greater than 7 / 72