Problem 1) Simplify the block diagram shown in Figure P1 and obtain the transfer function $C(s)/R(s)$.

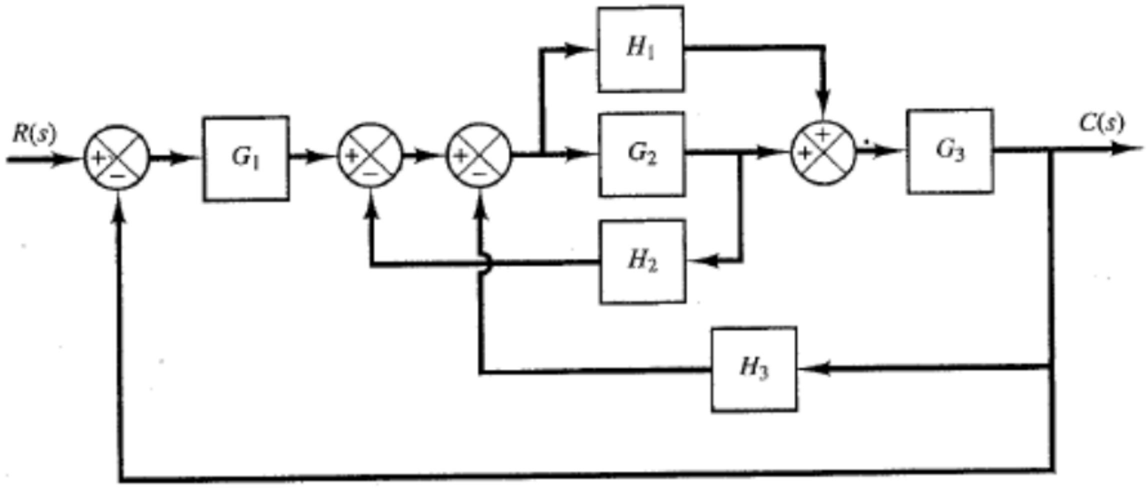


Figure P1

Problem 2) Obtain the transfer functions $C(s)/R(s)$ and $C(s)/D(s)$ of the control system shown in Figure P2.

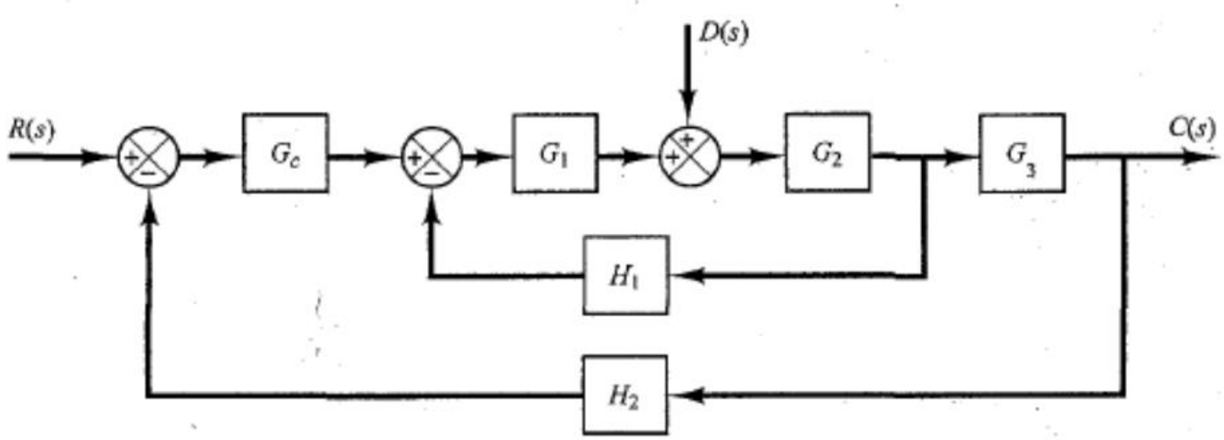


Figure P2

Problem 3) Consider the unit-step response of a unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{1}{(s+1)(s+2)}$$

Obtain the rise time, peak time, maximum percent overshoot and settling time.

Problem 4) Consider the system shown in Figure P4.

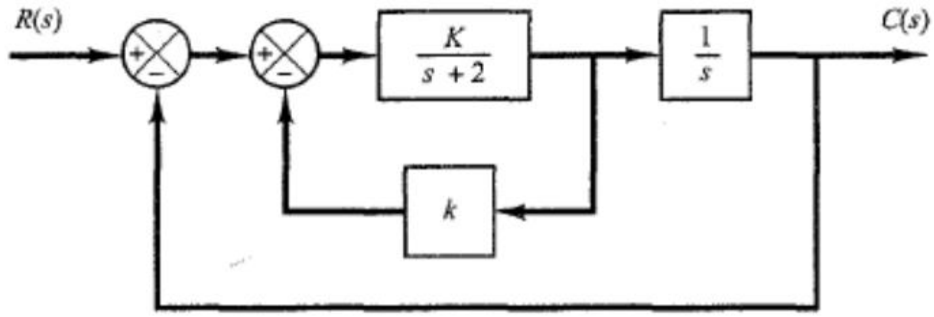
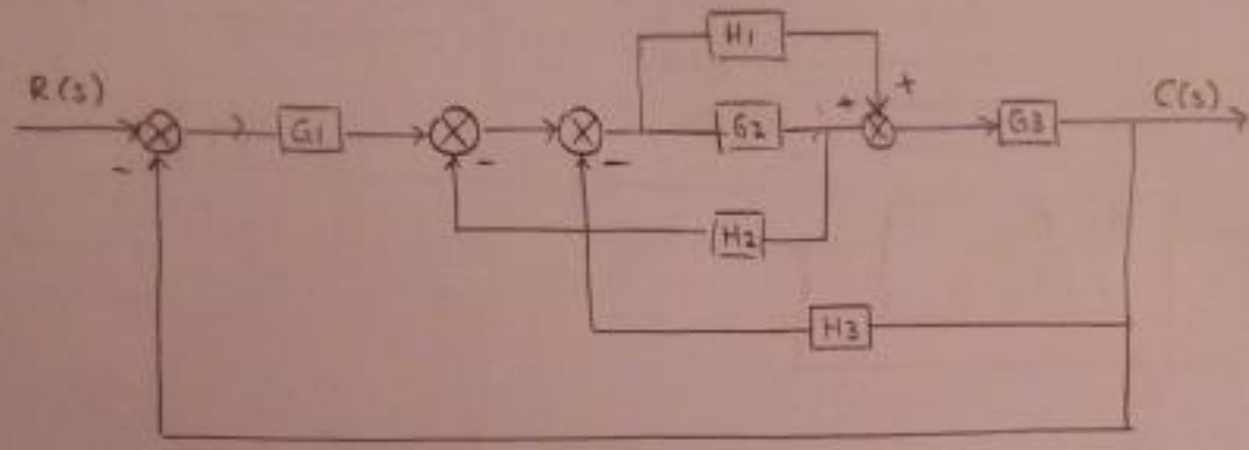
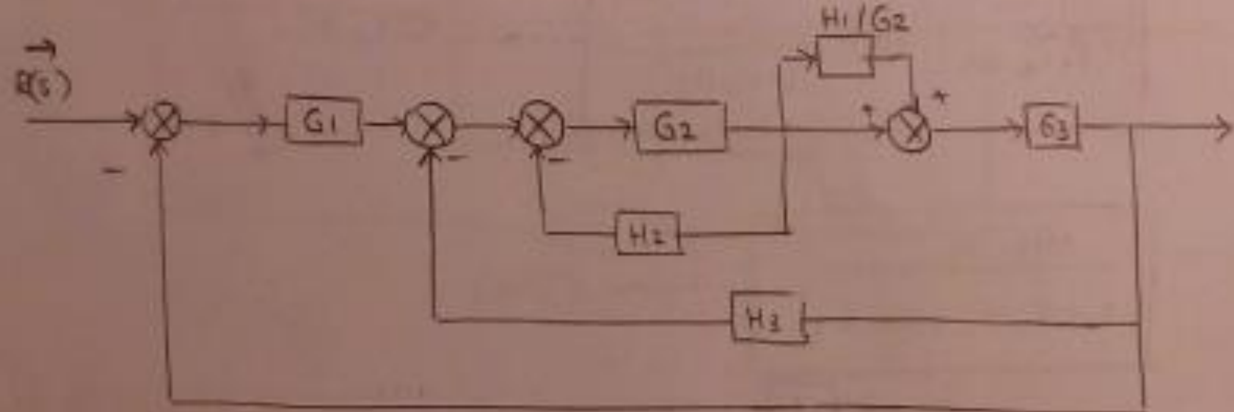


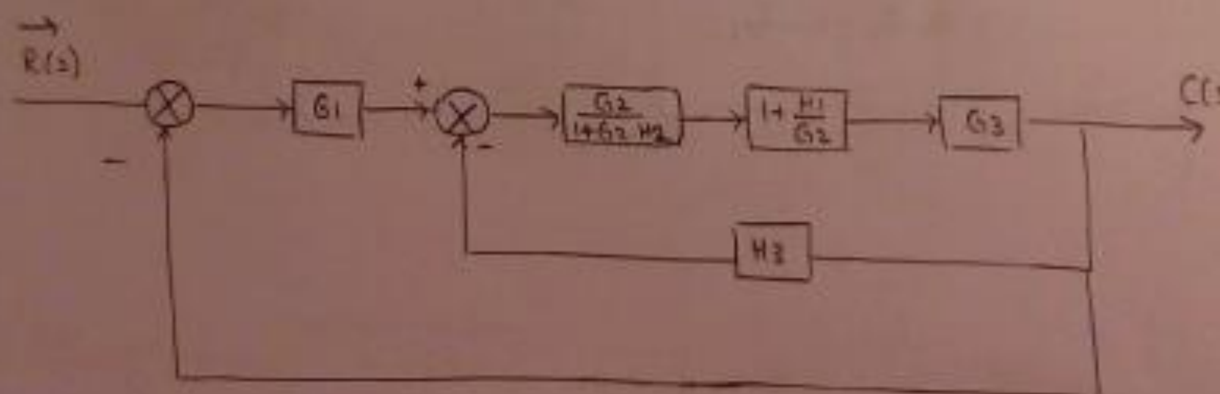
Figure P4

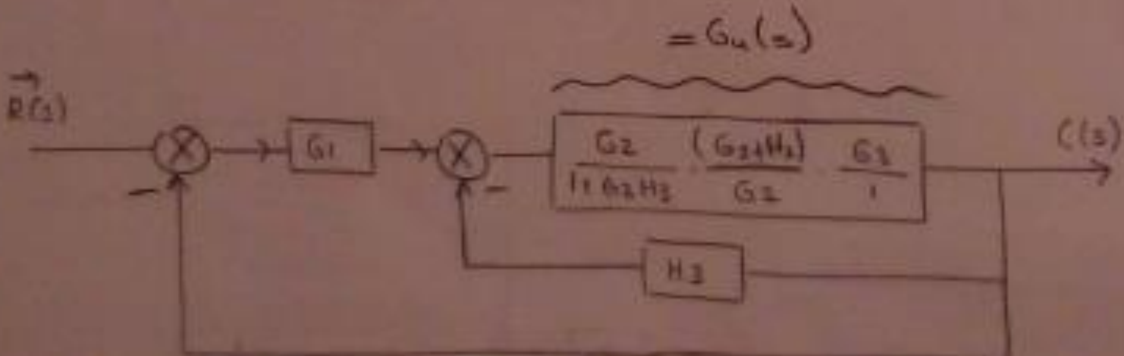
Determine the values of K and k such that the system has a damping ratio ξ of 0.8 and an undamped natural frequency w_n of 5 rad/s.

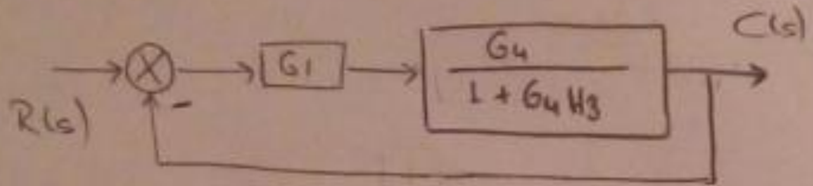
1)


R(s)
G1
H1
G2
G3
C(s)
H2
H3

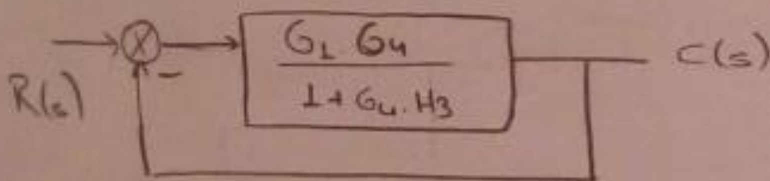

R(s)
G1
H1/G2
G2
G3
H2
H3

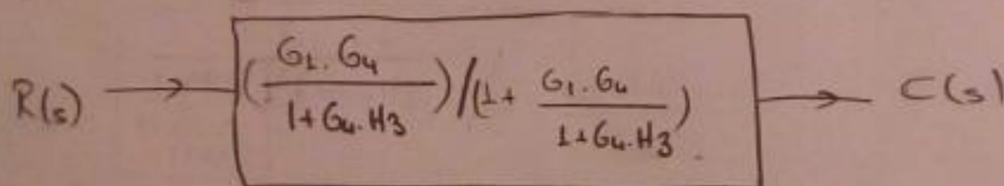

R(s)
G1
G2 / 1+G2 H2
1 + H1/G2
G3
C(s)
H3

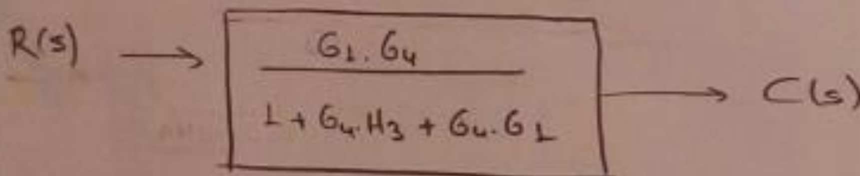

= G4(s)
R(s)
G1
G2 / 1+G2H2 · (G2+H1) / G2 · G3 / 1
C(s)
H3



$$G_4 = \frac{G_2^2 \cdot G_3 + H_1 G_2 G_3}{G_2 + G_2 H_3 G_2}$$

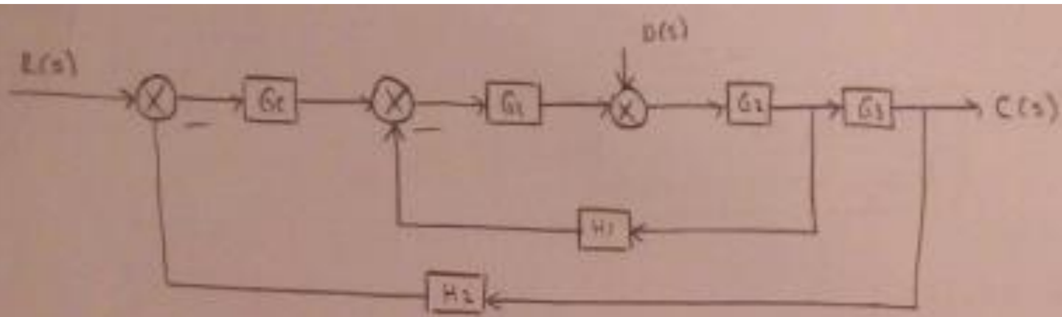


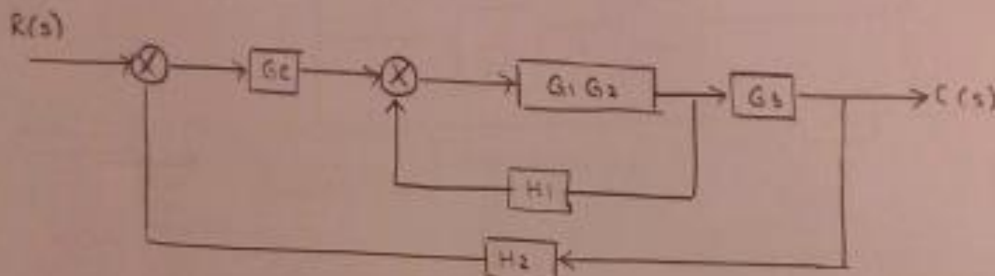


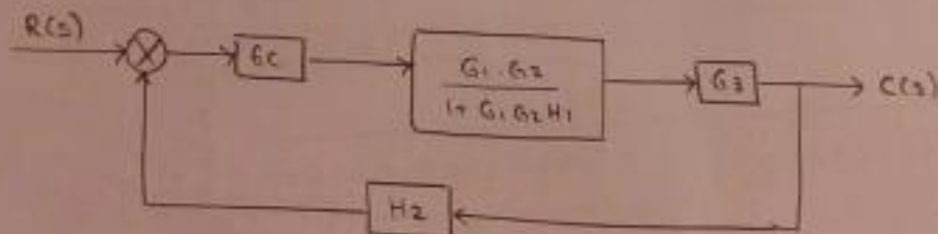


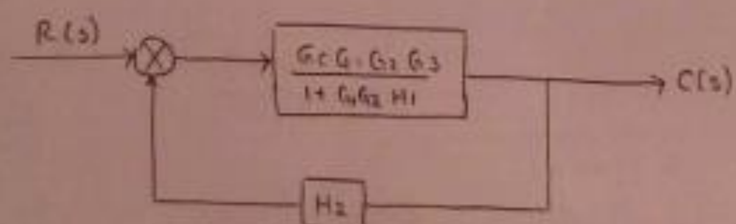
$$T(s) = \frac{G_1 \cdot G_4}{1 + G_4 \cdot H_3 + G_4 \cdot G_1}$$ where $$G_4 = \frac{G_2^2 \cdot G_3 + H_1 \cdot G_2 \cdot G_3}{G_2 + G_2^2 \cdot H_3}$$

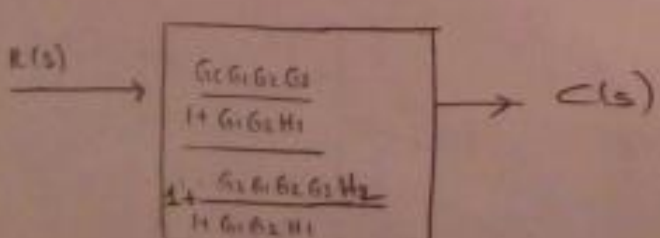
2)


R(s)
D(s)
Gc
G1
G2
G3
C(s)
H1
H2


R(s)
Gc
G1 G2
G3
C(s)
H1
H2

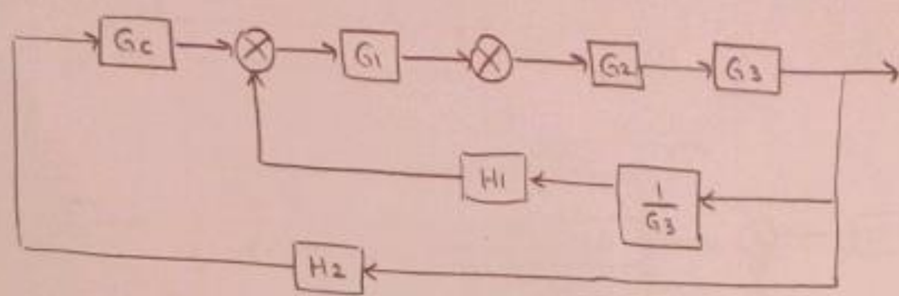

R(s)
Gc
G1 . G2 / 1+ G1 G2 H1
G3
C(s)
H2

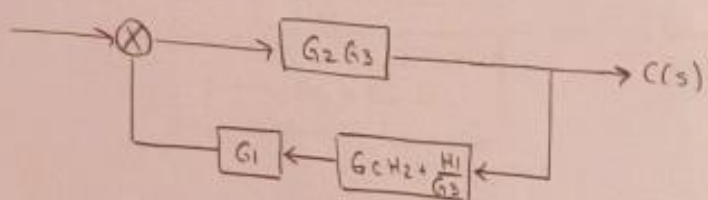

R(s)
Gc G1 G2 G3 / 1+ G1G2 H1
C(s)
H2


R(s)
Gc G1 G2 G3 / 1+ G1G2 H1
1+ Gc G1 G2 G3 H2 / 1+ G1 G2 H1
C(s)

$$R(s) \longrightarrow \boxed{\dfrac{G_c G_1 G_2 G_3}{1 + G_1 G_2 H_1 + G_c G_1 G_2 G_3 H_2}} \longrightarrow C(s)$$

* when $R(s) = 0$





$$\frac{C(s)}{D(s)} = \frac{G_2 G_3}{1 + G_2 G_3 G_1\left(G_c H_2 + \frac{H_1}{G_3}\right)} = \boxed{\frac{G_2 G_3}{1 + G_1 G_2 G_3 G_c H_2 + G_1 G_2 H_1}}$$

3) $G(s) = \dfrac{1}{(s+1)(s+2)}$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{C(s)}{R(s)} = \frac{\frac{1}{(s+1)(s+2)}}{1 + \frac{1}{(s+1)(s+2)}} \quad \Rightarrow \underline{\frac{C(s)}{R(s)} = \frac{1}{s^2+3s+3}}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

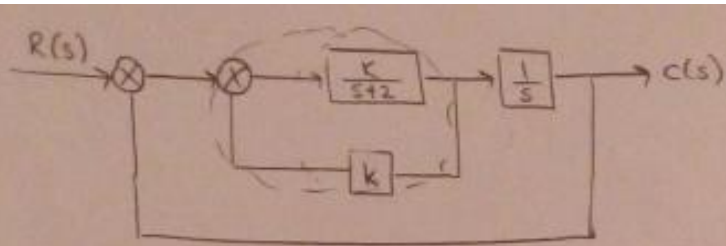
$$\boxed{\begin{array}{l} 2\zeta\omega_n = 3 \\ \omega_n^2 = 3 \;\therefore\; \omega_n = \sqrt{3} \\ \zeta = 0.866 \end{array}}$$

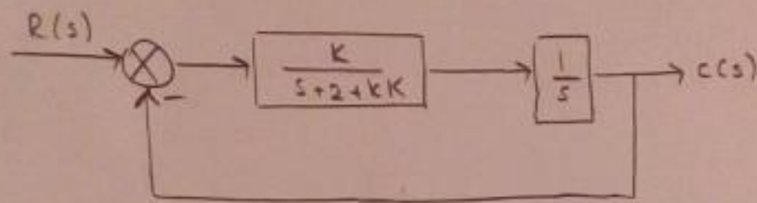
* $T_P = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \boxed{3.627 \text{ sec}}$

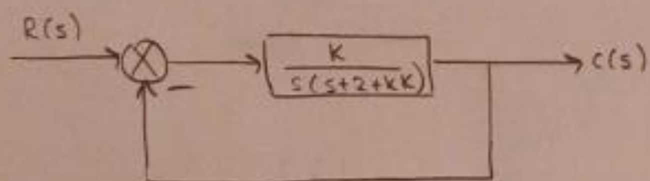
* $T_S = \dfrac{4}{\zeta\omega_n} = \boxed{2.667 \text{ sec}}$

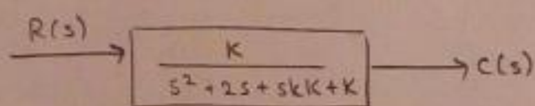
* $\% OS = e^{-\left(\zeta\pi / \sqrt{1-\zeta^2}\right)} \times 100 = \boxed{\% 0.43}$ * $T_r = \dfrac{2.16\,3\zeta + 0.6}{\omega_n} = \boxed{1.426}$

4)









$\therefore \boxed{\frac{K}{s^2+(2+kK)s+K}}$

$$\Rightarrow \frac{K}{s^2+(2+kK)s+K}$$

$\omega_n = 5 \text{rad/s}$ $\quad \zeta = 0.8$ $\qquad G(s) = \frac{1}{s^2+2\zeta\omega_n s+\omega_n^2}$

$\Rightarrow \omega_n^2 = K \quad \Rightarrow \boxed{K = 25}$

$2+25k = 8 = 2\zeta\omega_n$

$25k = 6 \quad \boxed{k = 0.24}$