Question 1 (30 points)

Use the axiom(s) and/or theorems of general Boolean algabra to prove the following identities:

a. $(X + Y)'(X' + Y') = X'Y'$ ***(10 points)***
b. $X'Z' + XYZ + XZ' = Z' + XY$ ***(10 points)***
c. $Y + X'Z + XY' = X + Y + Z$ ***(10 points)***

P1)

a) $(X+Y)'(X'+Y') = X'Y'$

$$X'Y'(X'+Y') = X'Y'$$

$$X'Y'X' + X'Y'Y' = X'Y'$$

$$X'Y' + X'Y' = X'Y'$$

$$X'Y' = X'Y' \quad ✓$$

b) $X'Z' + XYZ + XZ' = Z' + XY$

$$Z'\underbrace{(X'+X)}_{1} + XYZ = Z' + XY$$

$$Z' + XYZ = Z' + XY$$

$$(Z' + XY)\underbrace{(Z'+Z)}_{1} = Z' + XY$$

$$Z' + XY = Z' + XY \quad ✓$$

c) $Y + X'Z + XY' = X + Y + Z$

$$(Y+X)\underbrace{(Y+Y')}_{1} + X'Z = X + Y + Z$$

$$Y + X + X'Z = X + Y + Z$$

$$Y + \underbrace{(X+X')}_{1}(X+Z) = X + Y + Z$$

$$X + Y + Z = X + Y + Z \quad ✓$$

Question 2 (35 points)

Design a combinational logic circuit to detect following prime numbers (2,3,5,7,11,13) in BCD. The combinational circuit must have 4 binary inputs (x_1, x_2, x_3, x_4) and a single output (y). The output value will be "1" when the binary input is a prime number, otherwise the output will be "0".

a. Derive the truth table of the design. ***(10 points)***
b. Simplfy the boolean function in sum of products form using Karnaugh Map method. ***(15 points)***
c. Draw the gate implementation of the corresponding logic circuit. ***(10 points)***

Answer Q2

a-)

inputs				output
x_1	x_2	x_3	x_4	y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

b-)

x_1x_2 \ x_3x_4	00	01	11	10
00	0	0	1	1
01	0	1	1	0
11	0	1	0	0
10	0	0	1	0

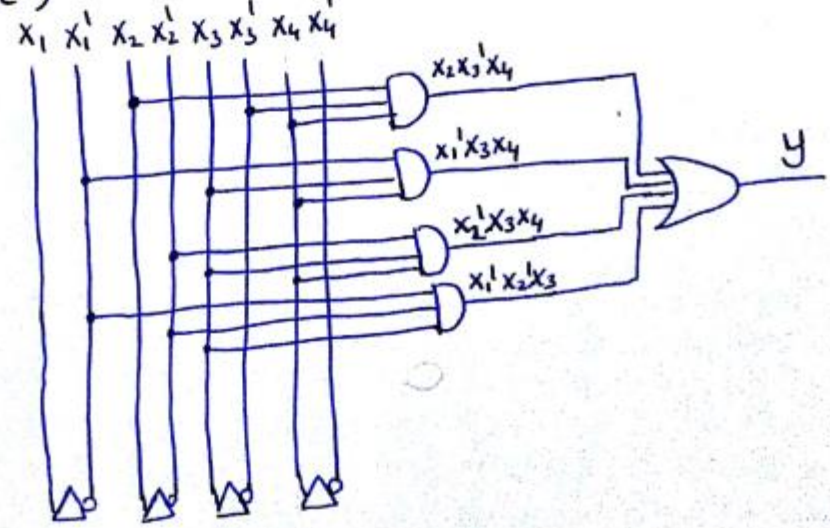
x_3, x_2, x_1, x_4

$$y = x_2x_3'x_4 + x_1'x_3x_4 + x_2'x_3x_4 + x_1'x_2'x_3$$

OR

$$y = x_2x_3'x_4 + x_1'x_2x_4 + x_2'x_3x_4 + x_1'x_2'x_3$$

c-)



Question 3 (35 points)

For the boolean function $F(A,B,C,D) = \sum m(0,2,5,7,11,14)$

a. Simplify the function using Karnaugh-Map method. **(10 points)**

b. Implement the function using 8-to-1 line multiplexer and external AND-OR logic gates if necassary. **(25 points)**

a.

AB \ CD	00	01	11	10
00	1			1
01		1	1	
11				1
10			1	

$$F = A'B'D' + A'BD + ABCD' + AB'CD$$

-we shall select A, B, D as control inputs of the 3-to-8 line decoder

That is ;

$$F = (A'B'D')\cdot 1 + (A'BD)\cdot 1 + (ABD')\cdot C + (AB'D)\cdot C$$

$$= m_0\cdot 1 + m_3\cdot 1 + m_6\cdot C + m_5\cdot C$$

$$+ m_1\cdot 0 + m_2\cdot 0 + m_4\cdot 0 + m_7\cdot 0$$

$$= \sum_{i=0}^{7} m_i I_i$$

where $I_0 = I_3 = 1$, $I_1 = I_2 = I_4 = I_7 = 0$ and $I_5 = I_6 = C$