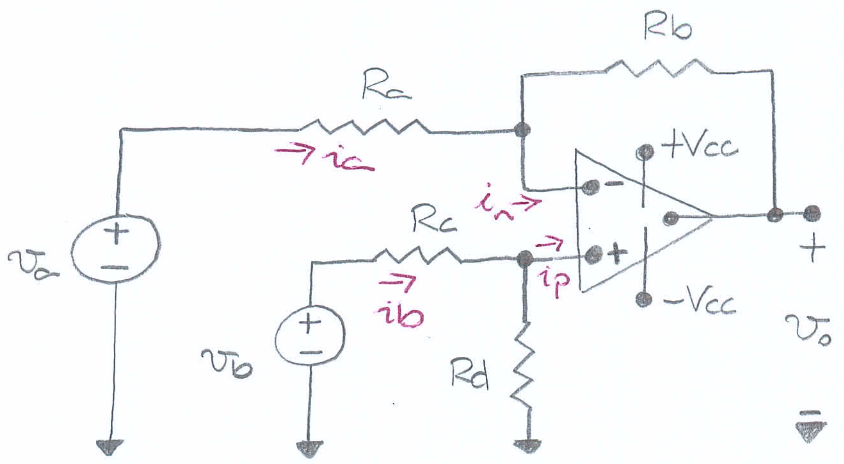
Selected Problems II

Problem 1) The resistors in a difference amplifier shown as



are

$$R_a = 20k\Omega,\ R_b = 80k\Omega,\ R_c = 47k\Omega,\ R_d = 33k\Omega$$

The signal voltages v_a and v_b are

$$v_a = 0.45V,\quad v_b = 0.9V$$

and $V_{cc} = \pm 9V$. Assume that the op amp is ideal.

a. Find v_o.
b. What is the resistance seen by the signal source v_a?
c. What is the resistance seen by the signal source v_b?

Solution.

a. We have $v_p = v_n$ and $i_p = i_n = 0$

KCL at noninverting input :

$$\frac{v_p - 0.9}{47k} + \frac{V_p}{33k} = 0 \Rightarrow 33(v_p - 0.9) + 47v_p = 0$$

$$\Rightarrow 80v_p = 0.9 \cdot 33 \Rightarrow v_p = 0.3712\,V \Rightarrow v_n = 0.3712$$

KCL at inverting input:

$$\underset{(4)}{\frac{0.3712 - 0.45}{20k}} + \underset{(1)}{\frac{0.3712 - v_o}{80k}} = 0 \Rightarrow 5 \cdot 0.3712 - 0.45 = v_o$$

$$\Rightarrow v_o = 1.4062\ V$$

-we then have

$$i_a = \frac{0.45-0.3712}{20k} = 0.0039\ mA$$

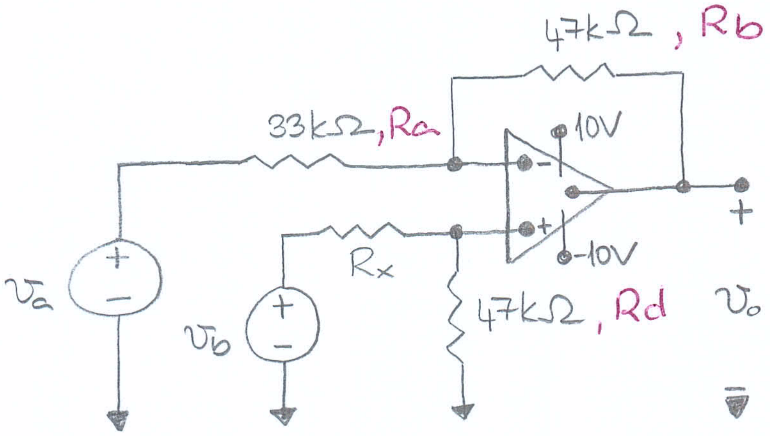
$$i_b = \frac{0.9-0.3712}{47k} = 0.0113\ mA$$

Hence ;

$$R_{\text{seen by } v_a} = \frac{v_a}{i_a} = \frac{0.45}{0.0039 \cdot 10^{-3}} = 115.3846\ k\Omega$$

$$R_{\text{seen by } v_b} = \frac{v_b}{i_b} = \frac{0.9}{0.0113 \cdot 10^{-3}} = 79.6460\ k\Omega$$

Problem 2) In the difference amplifier shown as



What range of values of R_x yields a CMRR ≥ 750 ?

Solution. We have

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| \geq 750$$

-it follows from the difference amplifier formulation :

$$v_o = \frac{R_d(R_a+R_b)}{R_a(R_x+R_d)} v_b - \frac{R_b}{R_a} v_a$$

-defining

$$\left.\begin{aligned} v_{dm} &= v_b - v_a \\ v_{cm} &= (v_a+v_b)/2 \end{aligned}\right\} \Rightarrow \begin{aligned} v_a &= v_{cm} - \frac{1}{2} v_{dm} \\ v_b &= v_{cm} + \frac{1}{2} v_{dm} \end{aligned}$$

Hence;

$$v_o = \frac{R_d(R_a+R_b)}{R_a(R_x+R_d)}\left(v_{cm}+\frac{1}{2}v_{dm}\right) - \frac{R_b}{R_a}\left(v_{cm}-\frac{1}{2}v_{dm}\right)$$

$$= A_{cm}v_{cm} + A_{dm}v_{dm}$$

where

$$A_{cm} = \frac{R_d(R_a+R_b)}{R_a(R_x+R_d)} - \underset{(R_x+R_d)}{\frac{R_b}{R_a}} = \frac{R_aR_d - R_bR_x}{R_a(R_x+R_d)}$$

$$A_{dm} = \frac{R_d(R_a+R_b)}{2R_a(R_x+R_d)} + \frac{R_b}{2R_a} = \frac{R_d(R_a+R_b)+R_b(R_x+R_d)}{2R_a(R_x+R_d)}$$

- we now calculate

$$A_{cm} = \frac{33k\cdot 47k - 47k\cdot R_x}{33k(R_x+47k)} = \frac{47(33-R_x)}{33(R_x+47)}$$

$$A_{dm} = \frac{47k(33k+47k)+47k(R_x+47k)}{2\cdot 33k\cdot(R_x+47k)}$$

$$= \frac{47(R_x+120)}{66(R_x+47)}$$

$$\frac{A_{dm}}{A_{cm}} = \frac{\cancel{47}(R_x+120)}{2\cancel{66}\cancel{(R_x+47)}}\cdot\frac{\cancel{33}\cdot\cancel{(R_x+47)}}{\cancel{47}\cdot(33-R_x)} \geq 750$$

$$\Rightarrow \quad R_x+120 \geq 33\cdot 1500 - 1500R_x$$

$$\Rightarrow \quad 1501R_x \geq 49380$$

$$\Rightarrow \quad R_x \geq 32.8981\ k\Omega$$

PS 2.3

OR

$$\frac{A_{dm}}{A_{cm}} = \frac{R_x+120}{2(33-R_x)} \leq -750$$

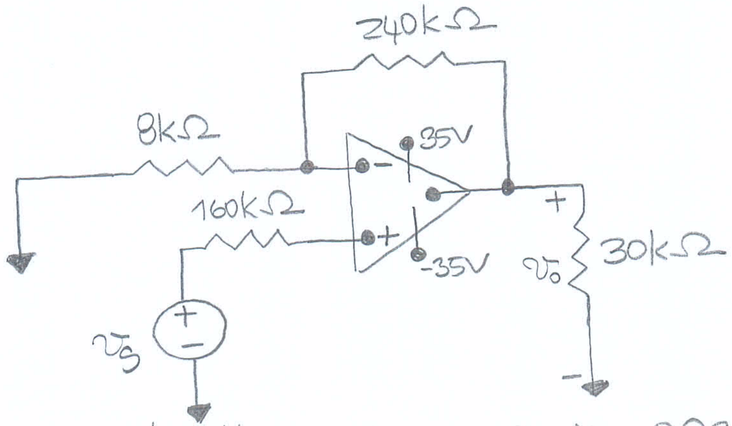
$$\Rightarrow \quad R_x+120 \leq -33.1500+1500R_x$$

$$\Rightarrow \quad R_x \leq 33.1021\ k\Omega$$

Hence ;

$$32.8981\,k\Omega \leq R_x \leq 33.1021\,k\Omega$$

Problem 3) The op amp in the noninverting amplifier circuit has an input resistance of $440\,k\Omega$, an output resistance of $5\,k\Omega$, and an open-loop gain of 10^5.



Assume that the op amp is operating in its linear region.

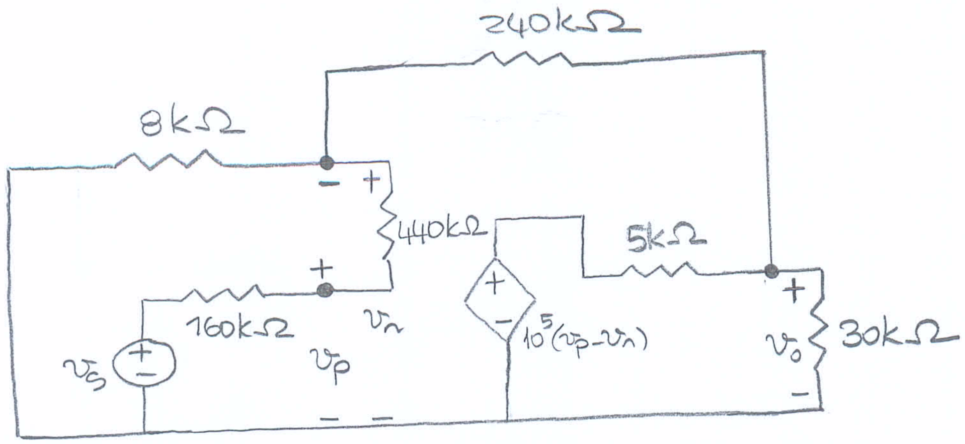
a. Calculate the voltage gain (v_o/v_s).

b. Find the inverting and noninverting input voltages v_n and v_p (in millivolts) if $v_s = 1V$.

c. Calculate the difference ($v_p - v_n$) in microvolts when $v_s = 1V$.

d. Find the current drain in picoamperes on the signal source when $v_s = 1V$

Solution. We first redraw the op amp circuit using the realistic model of the op amp as follows



a.

KCL at noninverting input:

$$\underset{(11)}{\frac{v_p - v_s}{160k}} + \underset{(4)}{\frac{v_p - v_n}{440k}} = 0 \Rightarrow 15v_p - 11v_s - 4v_n = 0$$

KCL at inverting input:

$$\underset{(330)}{\frac{v_n}{8k}} + \underset{(11)}{\frac{v_n - v_o}{240k}} + \underset{(6)}{\frac{v_n - v_p}{440k}} = 0 \Rightarrow 347v_n - 11v_o - 6v_p = 0$$

KCL at output terminal:

$$\frac{v_o - v_n}{240k} + \underset{(48)}{\frac{v_o - 10^5(v_p - v_n)}{5k}} + \underset{(8)}{\frac{v_o}{30k}} = 0$$

$$\Rightarrow 57v_o - 48{\cdot}10^5 v_p + 48{\cdot}10^5 v_n = 0$$

-we have

$$\left.\begin{array}{l} 2/\ 15v_p - 4v_n = 11v_s \\ 5/\ -6v_p + 347v_n = 11v_o \end{array}\right\} \Rightarrow \overset{157}{\cancel{1727}}\, v_n = \overset{2}{\cancel{22}}\, v_s + \overset{5}{\cancel{55}}\, v_o$$

$$\Rightarrow v_n = \frac{2v_s + 5v_o}{157}, \quad v_p = \frac{4(2v_s + 5v_o)}{15.157} + \frac{11}{\underset{157}{15}} v_s$$

$$= \frac{1735v_s + 20v_o}{15.157}$$

PS 2.5

then we have

$$57v_o - 48.10^5\left(\frac{1735v_s+20v_o}{15.157} - \frac{2v_s+5v_o}{\substack{157\\(15)}}\right)$$

$$\Rightarrow 134235v_o - 81840.10^5 v_s + 2640.10^5 v_o = 0$$

$$\Rightarrow 264134235 v_o = 81840.10^5 v_s$$

$$\Rightarrow \frac{v_o}{v_s} = 30.9842$$

b.

$$v_o = 30.9842 \cdot 1 = 30.9842 \text{ V}$$

$$v_p = \frac{1735.1 + 20.30.9842}{2355} = 999.8658 \text{ mV}$$

$$v_n = \frac{2.1 + 5.30.9842}{157} = 999.4968 \text{mV}$$

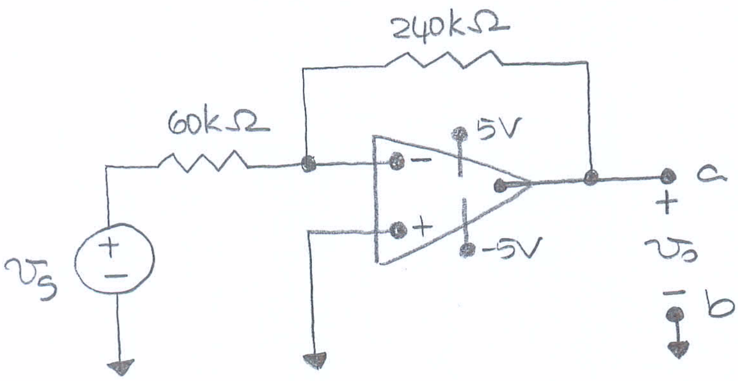
c. $v_p - v_n = 999.8658 - 999.4968$

$$= 369.0021 \ \mu\text{V}$$

d. $$i_s = \frac{(1000 - 999.8658).10^{-3}}{160.10^3}$$

$$= 838.6412 \text{ pA}$$

Problem 4) Consider the following inverting amplifier circuit



a. Find the Thévenin equivalent circuit with respect to the output terminals a, b. The dc signal source

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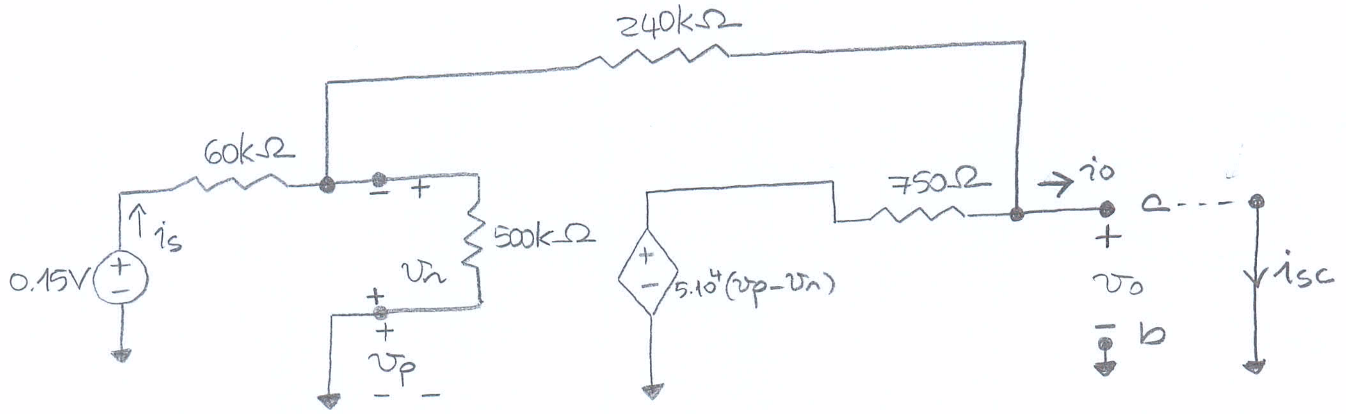
has a value of 150 mV. The op amp has an input resistance of 750 Ω and an open-loop gain of $5 \cdot 10^4$.

b. What is the output resistance of the inverting amplifier?

c. What is the resistance (in ohms) seen by the signal source v_s when the load at the terminals a,b is 150 Ω?

Solution. We consider the realistic model of the op amp



$v_{Th} \equiv v_o \quad , \quad v_p = 0$

KCL at inverting input:

$$\underset{(100)}{\frac{v_n - 0.15}{60k}} + \underset{(25)}{\frac{v_n - v_{Th}}{240k}} + \underset{(12)}{\frac{v_n}{500k}} = 0$$

$$\Rightarrow \quad 137 v_n - 15 - 25 v_{Th} = 0 \quad \Rightarrow \quad v_n = \frac{25 v_{Th} + 15}{137}$$

KCL at output terminal:

$$\underset{(25)}{\frac{v_{Th} - v_n}{240k}} + \underset{(8k)}{\frac{v_{Th} - 5 \cdot 10^4 (\overset{0}{\cancel{v_p}} - v_n)}{750}} = 0$$

$$\Rightarrow \quad 8025 v_{Th} + 4 \cdot 10^8 v_n = 0$$

$$\Rightarrow 8025\,v_{Th} + 4\cdot 10^{8}\,\frac{25 v_{Th} + 15}{137} = 0$$

$$\Rightarrow v_{Th} \cong -\frac{15}{25} = -0.6\ \text{V}$$

- then we find the short-circuit current, i_{sc}

KCL at inverting input:

$$\frac{v_n - 0.15}{60k} + \frac{v_n}{240k} + \frac{v_n}{500k} = 0$$

$$(100) \qquad (25) \qquad (12)$$

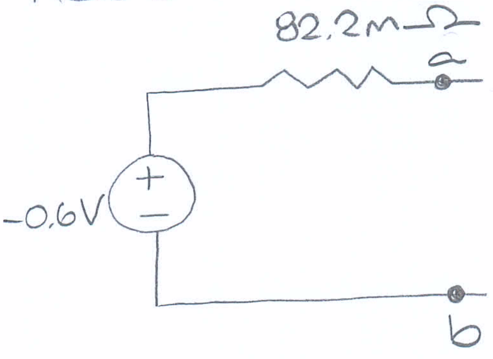
$$\Rightarrow 137 v_n = 15 \quad \Rightarrow \quad v_n = 0.1095\ \text{V}$$

$$\Rightarrow i_{sc} = \frac{-5\cdot 10^{4}\cdot 0.1095}{750} + \frac{0.1095}{240k} \cong -7.3\ \text{A}$$

Hence;

$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{-0.6}{-7.3} = 82.2\ \text{m}\Omega$$

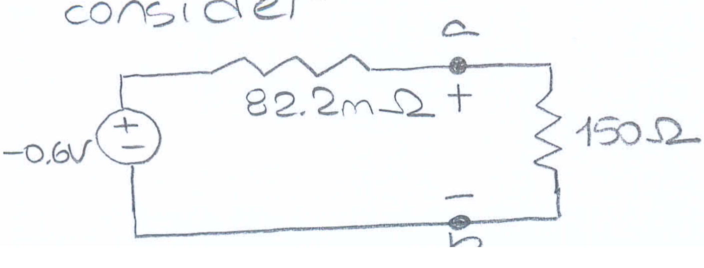
- we have



b. The output resistance is the same as the Thévenin resistance, i.e.

$$R_o = R_{Th} = 82.2\ \text{m}\Omega$$

c. We consider



$$v_{ab} = \frac{150}{150 + 0.0822}(-0.6)$$

$$= -0.5997\ \text{V}$$

PS 2.8

-then we apply KCL at inverting input:

$$\frac{v_n - 0.15}{60k} + \frac{v_n + 0.5997}{240k} + \frac{v_n}{500k} = 0$$

$$(100) \qquad (25) \qquad (12)$$

$$\Rightarrow 137 v_n - 15 + 14.9918 = 0 \quad \Rightarrow v_n = 54.745 \mu A$$

$$i_s = \frac{0.15 - 54.745.10^{-6}}{60\,k} = 2.4991 \mu A$$

Thus;

$$R_s = \frac{0.15}{2.4991.10^{-6}} = 60.022\ k\Omega$$