13.03.18 (wⅡ - PSI)

Q1) Signal Energy → $E_g = \int_{-\infty}^{\infty} |g(t)|^2 \, dt$

↓

(if not finite) Power → $P_g = \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 \, dt$

(Mean-square → Avg. of the square of the signal)

Rms → Square root of the avg. power (P_g)

$$g_{rms} = \sqrt{P_g} = \sqrt{\underbrace{\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2}}_{\text{Mean}} \underbrace{|g(t)|^2}_{\text{sq.}} \, dt}$$

Root

a) $g(t) = \underbrace{10}_{\substack{\text{Amp.} \\ (8)}} . \cos(\underbrace{100}_{\substack{\text{F. freq.} \\ (w_0)}} t + \underbrace{\frac{\pi}{3}}_{\substack{\text{Phase} \\ (\theta)}})$

Power regardless of these

Ex: $g(t) = A . \cos . (w_0 t + \theta)$

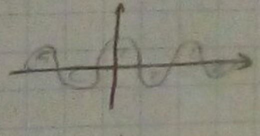
$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |A . \cos(w_0 t + \theta)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 . \cos^2(w_0 t + \theta) \, dt$$

$(\cos^2 \emptyset) = \frac{1}{2}(1 + \cos 2\emptyset)$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 . [1 + \cos(2w_0 t + 2\theta)] \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T/2}^{T/2} A^2 \, dt + \underbrace{\lim_{T\to\infty} \frac{1}{2T} \int_{-T/2}^{T/2} \cos(2w_0 t + 2\theta) \, dt}_{=0}$$



1 $= A^2 . \frac{1}{2T} \left(\frac{T}{2} - \left(-\frac{T}{2}\right)\right)$

$= \frac{A^2}{2}$

a) $g(t) = 10 \cdot \cos(100t + \frac{\pi}{3}) \rightarrow P_g = \frac{A^2}{2} = \frac{100^2}{2} = 50$

b) $\frac{C_1^2}{2} + \frac{C_2^2}{2} = \frac{10^2}{2} + \frac{16^2}{2} = P_g$

$g(t) = 10 \cdot \cos(100t + \frac{\pi}{3}) + 16\sin(150t + \frac{\pi}{6})$

d) $g(t) = 10 \cdot \cos 5t \cdot \cos 10t$

$= 10 \cdot \frac{1}{2} [\cos(15t) + \cos(5t)]$

$= 5 \cdot \cos(15t) + 5 \cdot \cos(5t)$

$P_g = \frac{5^2}{2} + \frac{5^2}{2} = 25$

f) $e^{j\alpha t} \cdot \cos\omega_0 t$

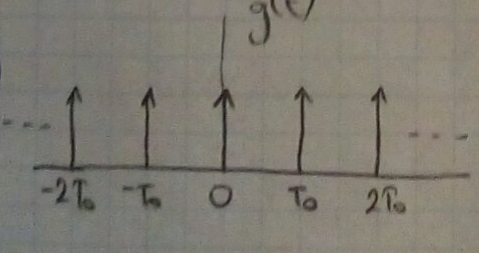
$= e^{j\alpha t} \cdot \frac{1}{2} (e^{j\omega_0 t} + e^{-j\omega_0 t})$

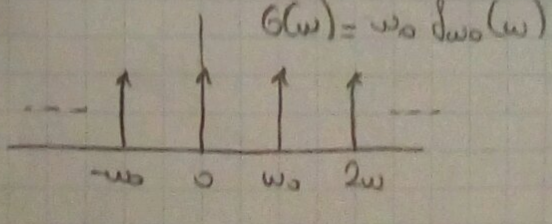
$= \frac{1}{2} (e^{j(\alpha+\omega_0)t} + e^{j(\alpha-\omega_0)t})$

$\frac{1}{2} \cdot [(\cos --- + j\sin ---) + (\cos --- - j\sin ---)]$

$= \frac{(1/2)^2}{2} + --- + --- + --- = 4 \cdot \frac{1}{8} = 1/2$

2)





$= \delta_{T_0}(t)$

Periodic sig. $g(t) \Rightarrow$ Exp. Four. Ser.

① $$g(t) = \sum_{n=-\infty}^{\infty} D_n . e^{jn\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T}$$

② $$\mathcal{F}\{g(t)\} = \sum_{n=-\infty}^{\infty} \mathcal{F}\{D_n . e^{jn\omega_0 t}\}$$

Spectrum of an everlasting exp. $\rightarrow$ $e^{j\omega_0 t} \Longleftrightarrow 2\pi . \delta(\omega - \omega_0)$ ③

$\hookrightarrow$ Ext. $\rightarrow$ $$\mathcal{F}^{-1}[\delta(\omega-\omega_0)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0) . e^{j\omega t} \, d\omega$$

$$= \frac{1}{2\pi} . e^{j\omega_0 t} \Longleftrightarrow \delta(\omega-\omega_0)$$

$$e^{j\omega_0 t} \Longleftrightarrow 2\pi \delta(\omega-\omega_0)$$

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② + ③ $\rightarrow$ ④ $\rightarrow$ $$2\pi \sum_{n=-\infty}^{\infty} D_n . \delta(\omega - n\omega_0) \Leftrightarrow g(t)$$

⑤ $$\delta_{T_0}(t) = \sum_{n=-\infty}^{\infty} D_n . e^{jn\omega_0 t} \qquad \omega_0 = 2\pi / T_0$$

$$D_n = \frac{1}{T_0} \underbrace{\int_{\langle T_0 \rangle} \delta_{T_0}(t) . e^{-jn\omega_0 t} \, dt}$$

$$= e^{-jn\omega_0 t}_{(t=0)} = 1$$

⑥ $D_n = 1/T_0$

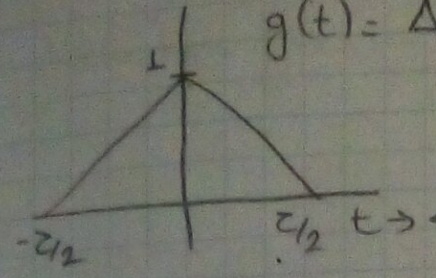
⑦ $\rightarrow$ ⑤ + ⑥ $\rightarrow$ $$\frac{1}{T_0} \sum_{n=-\infty}^{\infty} e^{jn\omega_0 t} = \delta_{T_0}(t)$$

④ + ⑦ $\rightarrow$ $$\delta_{T_0}(t) \Leftrightarrow \underbrace{\frac{2\pi}{T_0}}_{\omega_0} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

③

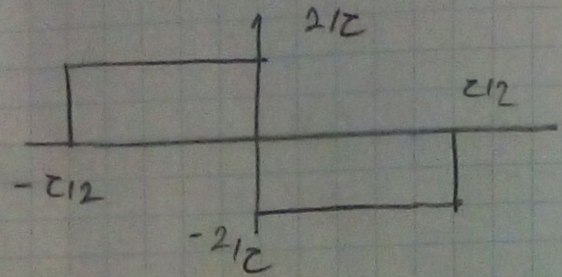
$$\delta_{T_0}(t) \Longleftrightarrow \omega_0 . \delta_{\omega_0}(\omega)$$

Q3) $g(t) = \Delta = \left(\frac{t}{\tau}\right)$



$(\tau/2 - (-\tau/2)) = \tau$

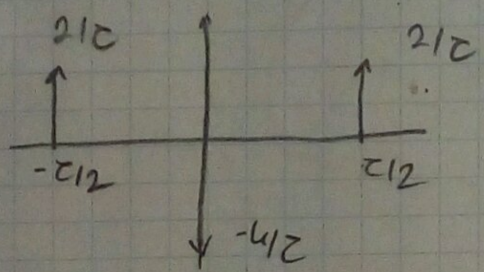
① $dg/dt \rightarrow$ Amount of change in time



$\frac{1-0=1}{t=\tau/2} = 2/\tau$

$\frac{0-1=-1}{t=\tau/2} = -2/\tau$

② d^2g/dt^2



$2/\tau - 0 = 2/\tau$

$-2/\tau - 2/\tau = -4/\tau$

$0 - (-2/\tau) = 2/\tau$

- Der. of a signal at a "jump discontinuity" is an impulse of str. equal to the amount of jump.

$$\frac{d^2g(t)}{dt^2} = \frac{2}{\tau}\left[\delta\left(t+\frac{\tau}{2}\right) - 2\delta(t) + \delta\left(t-\frac{\tau}{2}\right)\right]$$

Diff. Prop. $\rightarrow \frac{d^2g(t)}{dt^2} \Longleftrightarrow (j\omega)^2 \cdot G(\omega) = -\omega^2 \cdot G(\omega)$

Time-Sh. Prop. $\rightarrow \delta(t-t_0) \Longleftrightarrow e^{-j\omega t_0}$

$$-\omega^2 G(\omega) = \frac{2}{\tau}\cdot\left(e^{j\omega\frac{\tau}{2}} - 2 + e^{-j\omega\frac{\tau}{2}}\right)$$

$$2\cos\left(\frac{\omega\tau}{2}\right)$$

$$= \frac{4}{\tau}\underbrace{\left(\cos\frac{\omega\tau}{2} - 1\right)}$$

$$\sin^2(x) = \frac{1}{2}\left[1 - \cos(2x)\right]$$

$$2\cdot\sin^2(x) = 1 - \cos(2x)$$

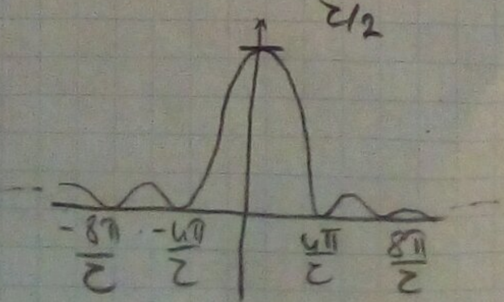
$$-\omega^2 \cdot G(\omega) = -\frac{8}{\tau}\cdot\sin^2\left(\frac{\omega\tau}{4}\right)$$

$$\longrightarrow \quad G(\omega) = \frac{8}{\omega^2\cdot\tau}\cdot\sin^2\left(\frac{\omega\tau}{4}\right)$$

$$= \underbrace{\frac{4^2}{\omega^2\cdot\tau^2}\cdot\frac{\tau}{2}}_{\left(\frac{8}{\omega^2\tau}\right)}\cdot\sin^2\left(\frac{\omega\tau}{4}\right)$$

$$= \frac{\tau}{2}\cdot\Bigg[\underbrace{\frac{\sin\left(\frac{\omega\tau}{4}\right)}{(\omega\tau/4)}}_{=\operatorname{sinc}(\omega\tau/4)}\Bigg]^2$$

$$G(\omega) = \underbrace{\frac{\tau}{2}\cdot\operatorname{sinc}(\omega\tau/4)}$$



$\frac{\omega\tau}{4} = \pi$

$\rightarrow \omega = \frac{4\pi}{\tau}$

5

$\frac{\omega\tau}{4} = 2\pi \rightarrow \omega = \frac{8\pi}{\tau}$

9 a)

$$\Delta\left(\frac{t}{2\pi}\right) \rightarrow \pi-(-\pi)=2\pi$$

Carrier $\rightarrow \cos 10t$

$$g(t)=\Delta\left(\frac{t}{2\pi}\right)\cdot\cos 10t$$

$g(t).\cos w_0 t \rightarrow$ freq. Shift

$\Longleftrightarrow \frac{1}{2}\left[G(w-w_0)+G(w+w_0)\right]$

Table 3.1 $\rightarrow \Delta\left(\frac{t}{2\pi}\right) \Longleftrightarrow \pi.\operatorname{sinc}^2\left(\frac{\pi w}{2}\right)$

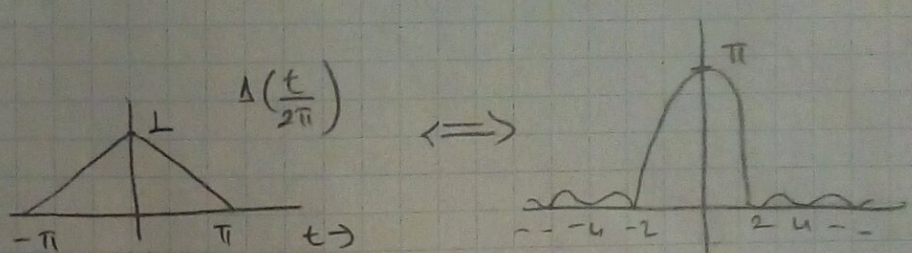
$$\text{Mod. Prop.} \rightarrow g(t).\cos(\underbrace{10t}_{w_0}) \Longleftrightarrow \pi.\frac{1}{2}\left[\operatorname{sinc}^2\left[\frac{\pi(w-10)}{2}\right]+\operatorname{sinc}^2\left[\frac{\pi(w+10)}{2}\right]\right]$$

b) Delayed by $2\pi \rightarrow$ Time-shift

$$(a).e^{-jw(2\pi)}$$

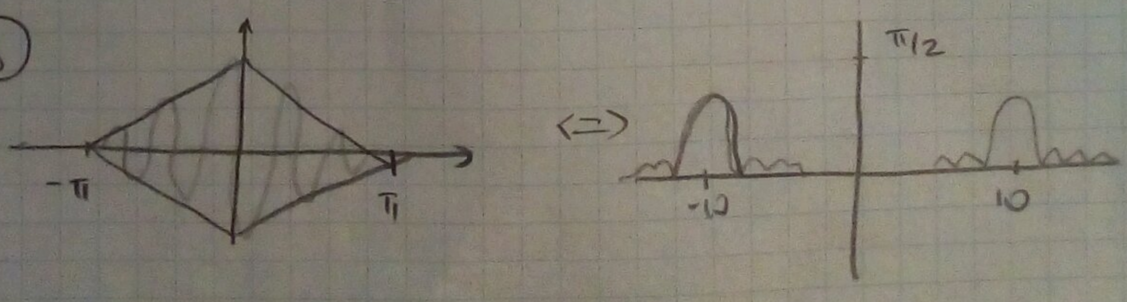
c)

a

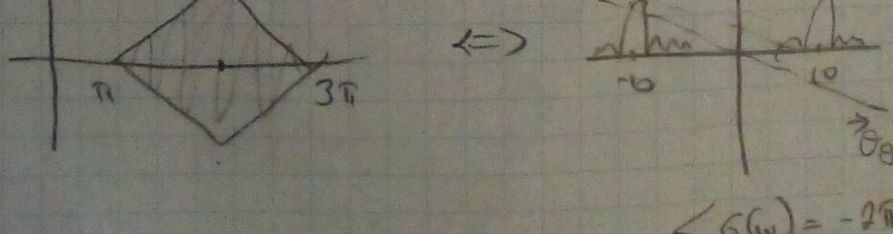


$\frac{\pi w}{2}=\pi \rightarrow w=2$

b



c

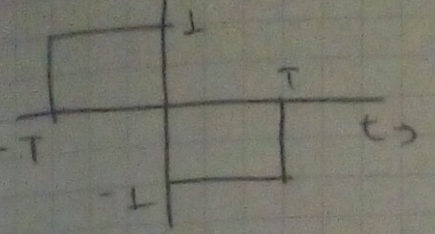


b

$\theta_g(w)=-2\pi w$

$\angle G(w)=\underline{-2\pi w}$

Q5)



a) $$G(\omega)= \int_{-\infty}^{\infty} g(t)\cdot e^{-j\omega t}\, dt$$

$$G(\omega)= \int_{-T}^{0} e^{-j\omega t}\, dt + \int_{0}^{T} (-1)\, e^{-j\omega t}\, dt$$

$$= \left(\frac{e^{-j\omega t}}{-j\omega} \Big|_{-T}^{0} \right) - \left(\frac{e^{-j\omega t}}{-j\omega} \Big|_{0}^{T} \right)$$

$$= \left[\frac{1}{-j\omega} - \left(\frac{e^{j\omega T}}{-j\omega} \right) \right] - \left[\frac{e^{-j\omega T}}{-j\omega} - \frac{1}{-j\omega} \right]$$

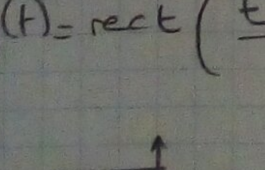
$$= -\frac{2}{j\omega} + \frac{1}{j\omega} \cdot \underbrace{\left(e^{j\omega T} + e^{-j\omega T} \right)}_{= 2\cos(\omega T)}$$

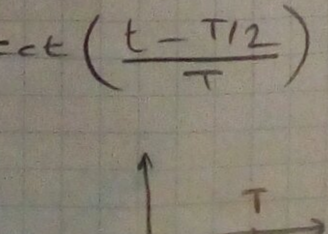
$$= -\frac{2}{j\omega} \left[1 - \cos(\omega T) \right]$$

$$\longrightarrow 1 - \cos 2A = 2\sin^2 A$$

$$= \frac{j4}{\omega} \cdot \sin^2\left(\frac{\omega T}{2} \right)$$

b) $$g(t) = \text{rect}\left(\frac{t + T/2}{T} \right) - \text{rect}\left(\frac{t - T/2}{T} \right)$$





$$\text{rect}\left(\frac{t}{T}\right) \Longleftrightarrow T \cdot \text{sinc}\left(\frac{\omega T}{2}\right) \quad \text{(Table 3.1)}$$

$$\text{rect}\left(\frac{t \pm T/2}{T}\right) \Longleftrightarrow T \cdot \text{sinc}\left(\frac{\omega T}{2}\right) \cdot e^{\pm j\omega T/2}$$

$$G(\omega) = T \cdot \text{sinc}\left(\frac{\omega T}{2}\right) \cdot \left[e^{j\omega T/2} - e^{-j\omega T/2}\right]$$

$2j$

$$= T \cdot \text{sinc}\left(\frac{\omega T}{2}\right) \cdot 2j \cdot \sin\left(\frac{\omega T}{2}\right)$$

$$= \frac{\sin(\omega T/2)}{(\omega T/2)}$$

$$= \frac{j4}{\omega} \sin^2\left(\frac{\omega T}{2}\right)$$

c) $\delta(t) \Longleftrightarrow 1$

$1 - \cos 2x = 2 \cdot \sin^2 x$

$$\frac{dg(t)}{dt} = \delta(t+T) - 2\delta(t) + \delta(t-T)$$

$$(j\omega) \cdot G(\omega) = e^{j\omega T} - 2 + e^{-j\omega T}$$

$$= -2 + 2\cos(\omega T)$$

$$= -2\left[1 - \cos(\omega T)\right]$$

$$(j\omega) G(\omega) = -4\sin^2\left(\frac{\omega T}{2}\right)$$

8

$$G(\omega) = \frac{j4}{\omega} \cdot \sin^2\left(\frac{\omega T}{2}\right)$$