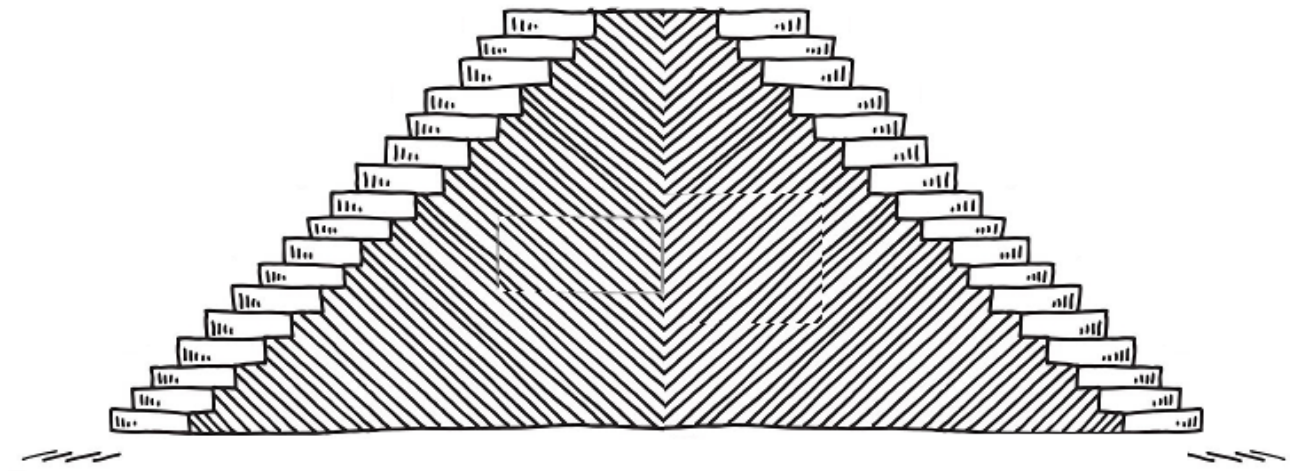


# FINAL AND MIDTERM EXAM QUESTIONS OF MATH 233 (FROM PREVIOUS YEARS)

1. Consider the following stairway-peak where one can climb up using the stairs on the left and then climb down the peak using the same number of stairs on the right. Assume that one can climb the peak with **one stair**, or **two stairs** in one step, and similarly one can climb down the peak **one stair**, **two stairs** in one step.



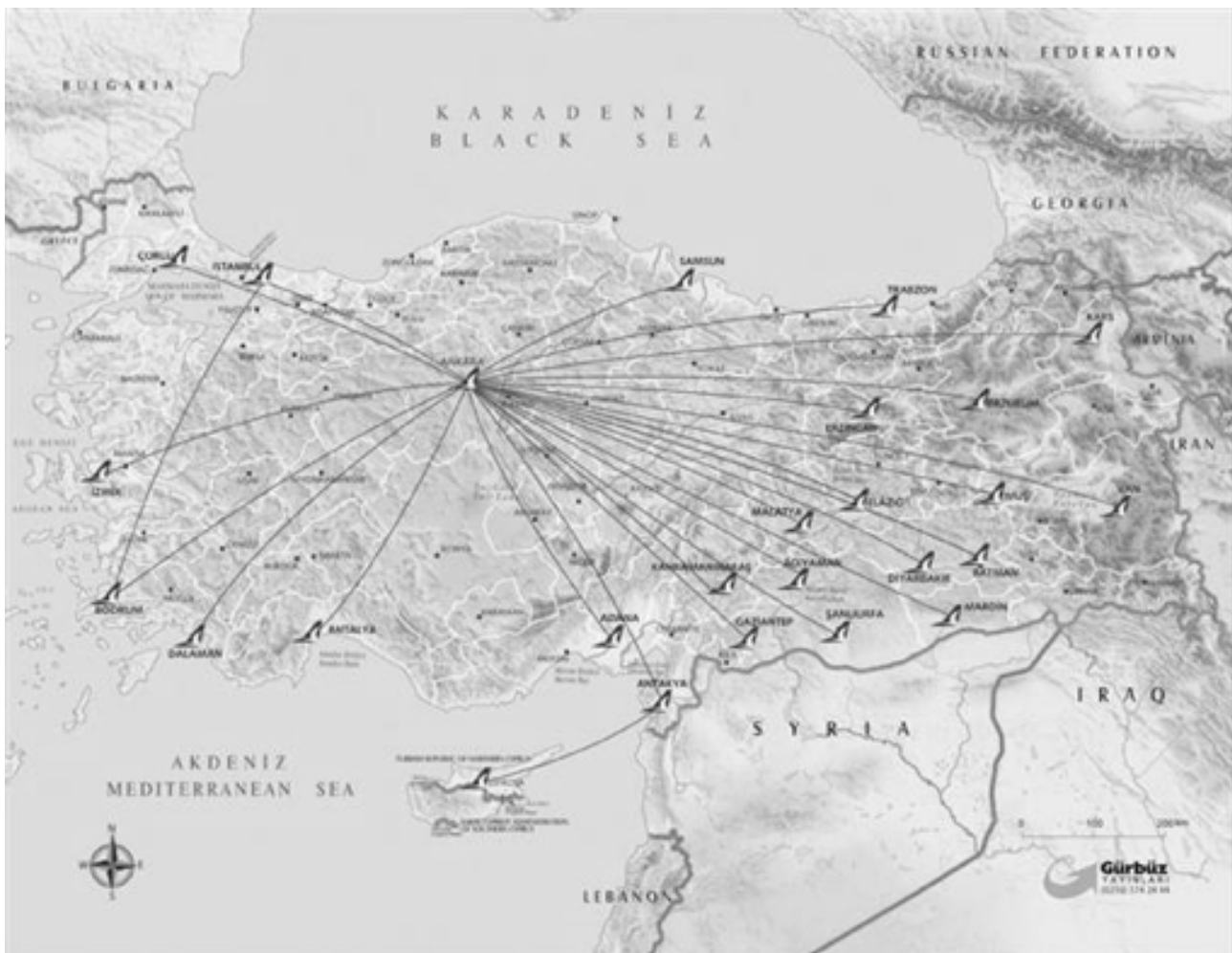
- a) Write a **recurrence relation** that gives the number of ways of climbing up and then down such a peak.
- b) What are the **initial conditions**?
- c) Assume that the peak has 99 stairs up and down. In how many ways can one do the fastest climb up and down, if each step takes the same amount of time (regardless of the number of stairs covered in a step)?

2. What is the probability that a 13-card bridge hand contains

- a) all 13 hearts?
- b) 13 cards of the same suit?
- c) seven spades and six clubs?
- d) seven cards of one suit and six cards of a second suit?
- e) four diamonds, six hearts, two spades, and one club?
- f) four cards of one suit, six cards of a second suit, two cards of a third suit, and one card of the fourth suit?

(Remember that in a **deck of 52 cards**, there are **13 kinds** (A,2,3,4,...,10,J,Q,K) and **4 suites** (spade, club, heart and diamond) from each kind.)

3. The following map shows the flight routes of an airline in Turkey. The airline is based in Ankara, the capital of Turkey, close to the geographical center of Turkey.



- How would you model these flight routes as a graph? Draw it as a graph.
- What is the vertex with the largest degree? Why?
- Are edges in **your graph** directed or undirected? Why?

4. A sequence of 10 bits (a bit string of length 10) is randomly generated.

- What is the probability that **at most three** of these bits is 0?
- What is the probability that the number of 0s and 1s are equal?

5. Give a formula for the coefficient of  $x^k$  in the expansion of  $(x + 1/x)^{100}$ , where  $k$  is an integer. Remember the Binomial Theorem:

$$(x + y)^n = \sum_{j=0}^n \binom{n}{j} x^{n-j} y^j$$

6. There are 10 questions on discrete mathematics final exam. How many ways are there to assign scores to the problem if the sum of the scores is 100 and

a) Each question can have any points from 0 to 100 (Yes, it is stupid to assign 0 points to a question, but assume that this can be the case).

b) Each question is at least 5 points.

7. How many ways are there to put five temporary employees into four identical offices if

a) An office can take at most two employees.

b) An office can host take any number of employees.

8. We are interested in finding the likelihood of having a poker hand contains a full house, that is to say 3 of one kind and 2 of another. Remember that a deck of 52 cards contains 13 kinds (A, 2,3,..., Q,K) and 4 suites (spade, club, diamond, heart)

a) What is the experiment?

b) What is the sample space?

c) What is the size of the sample space?

d) What is the event (of having a full house)?

e) What is the size of the event?

f) What is the probability of having a hand containing full house?

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