42. For the motor, load, and torque-speed curve shown in Figure P2.28, find the transfer function, $G(s) = \theta_L(s)/E_a(s)$. [Section: 2.8]

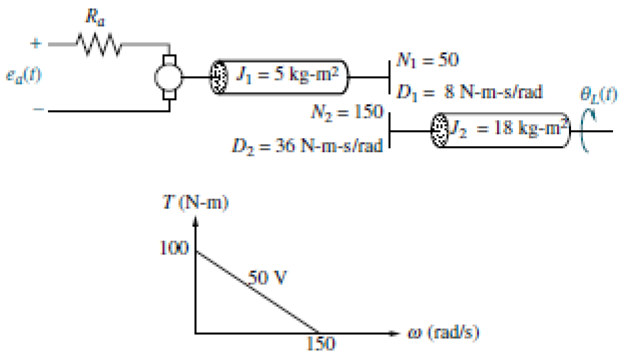


FIGURE P2.28

43. The motor whose torque-speed characteristics are shown in Figure P2.29 drives the load shown in the diagram. Some of the gears have inertia. Find the transfer function, $G(s) = \theta_2(s)/E_a(s)$. [Section: 2.8]

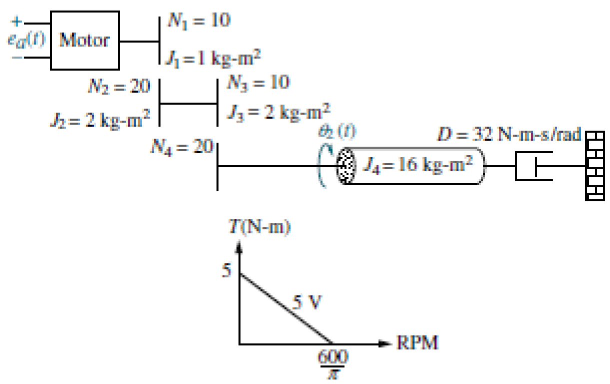


FIGURE P2.29

44. A dc motor develops 55 N-m of torque at a speed of 600 rad/s when 12 volts are applied. It stalls out at this voltage with 100 N-m of torque. If the inertia and damping of the armature are 7 kg-m^2 and 3 N-m-s/rad, respectively, find the transfer function, $G(s) = \theta_L(s)/E_a(s)$, of this motor if it drives an inertia load of 105 kg-m^2 through a gear train, as shown in Figure P2.30. [Section: 2.8]

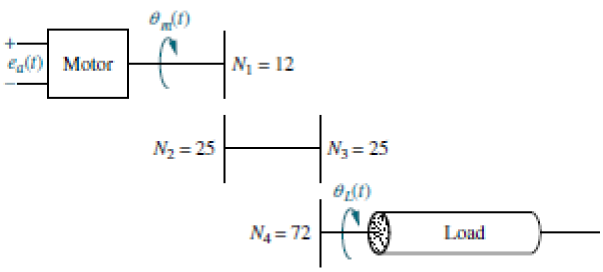


FIGURE P2.30

1. Represent the electrical network shown in Figure P3.1 in state space, where $v_o(t)$ is the output. [Section: 3.4]

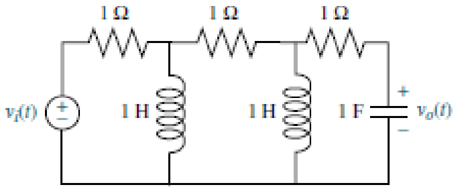


FIGURE P3.1

2. Represent the electrical network shown in Figure P3.2 in state space, where $i_R(t)$ is the output. [Section: 3.4]

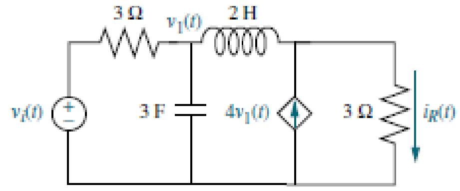


FIGURE P3.2

4. Represent the system shown in Figure P3.4 in state space where the output is $x_3(t)$. [Section: 3.4]

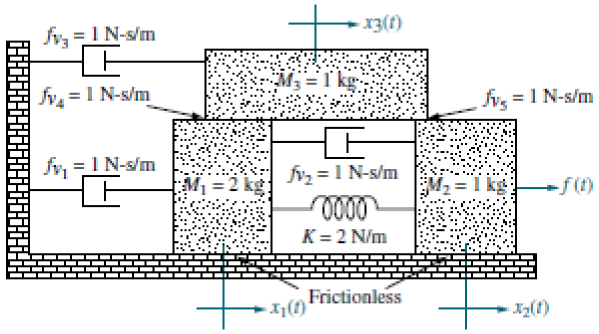


FIGURE P3.4

6. Represent the rotational mechanical system shown in Figure P3.6 in state space, where $\theta_1(t)$ is the output. [Section: 3.4]

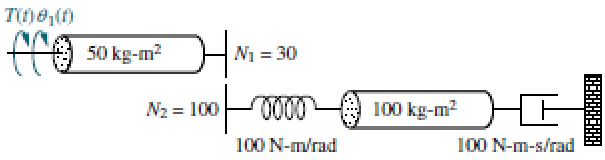


FIGURE P3.6

7. Represent the system shown in Figure P3.7 in state space where the output is $\theta_L(t)$. [Section: 3.4]

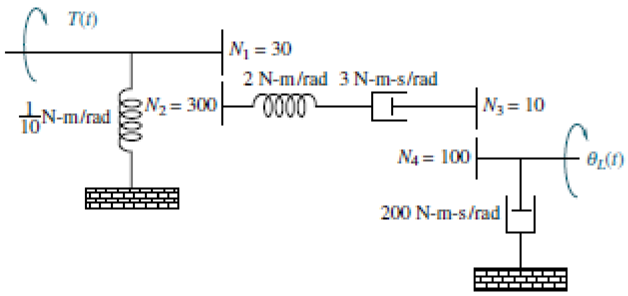


FIGURE P3.7

13. Represent the following transfer function in state space. Give your answer in vector-matrix form. [Section: 3.5]

$$T(s) = \frac{(s^2 + 3s + 8)}{(s+1)(s^2 + 5s + 5)}$$

42.

$$\frac{K_t}{R_a}=\frac{T_{stall}}{E_a}=\frac{100}{50}=2\,;\,K_b=\frac{E_a}{\omega_{no-load}}=\frac{50}{150}=\frac{1}{3}$$

Also,

$$J_m=5+18\left(\frac{1}{3}\right)^2=7;\,D_m=8+36\left(\frac{1}{3}\right)^2=12.$$

Thus,

$$\frac{\theta_m(s)}{E_a(s)}=\frac{2/7}{s(s+\frac{1}{7}(12+\frac{2}{3}))}=\frac{2/7}{s(s+\frac{38}{21})}$$

Since $\theta_L(s)=\frac{1}{3}\,\theta_m(s)$,

$$\frac{\theta_L(s)}{E_a(s)}=\frac{\frac{2}{21}}{s(s+\frac{38}{21})}.$$

43.

The parameters are:

$$\frac{K_t}{R_a}=\frac{T_s}{E_a}=\frac{5}{5}=1;\,K_b=\frac{E_a}{\omega}=\frac{5}{\frac{600}{\pi}2\pi\frac{1}{60}}=$$
$$\frac{1}{4};\;J_m=16\left(\frac{1}{4}\right)^2+4\left(\frac{1}{2}\right)^2+1=3;\;D_m=32\left(\frac{1}{4}\right)^2=2$$

Thus,

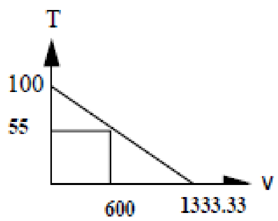
$$\frac{\theta_m(s)}{E_a(s)}=\frac{\frac{1}{3}}{s(s+\frac{1}{3}(2+(1)(\frac{1}{4})))}=\frac{\frac{1}{3}}{s(s+0.75)}$$

Since $\theta_2(s)=\frac{1}{4}\,\theta_m(s)$,

$$\frac{\theta_2(s)}{E_a(s)}=\frac{\frac{1}{12}}{s(s+0.75)}.$$

44.

The following torque-speed curve can be drawn from the data given:



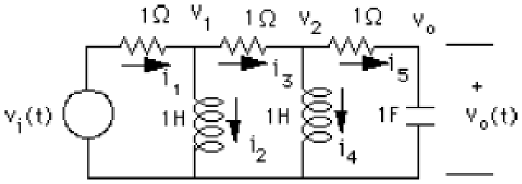
Therefore, $\frac{K_t}{R_a}=\frac{T_{stall}}{E_a}=\frac{100}{12}\,;\,K_b=\frac{E_a}{\omega_{no-load}}=\frac{12}{1333.33}$. Also, $J_m=7+105\left(\frac{1}{6}\right)^2=9.92;\,D_m=$

3. Thus,

$$\frac{\theta_m(s)}{E_a(s)}=\frac{\left(\frac{100}{12}\right)\frac{1}{9.92}}{s(s+\frac{1}{9.92}(3.075))}=\frac{0.84}{s(s+0.31)}.$$ Since $\theta_L(s)=\frac{1}{6}\,\theta_m(s)$, $$\frac{\theta_L(s)}{E_a(s)}=\frac{0.14}{s(s+0.31)}.$$

1.

Add the branch currents and node voltages to the network.



Write the differential equation for each energy storage element.

$$\frac{di_2}{dt} = v_1$$
$$\frac{di_4}{dt} = v_2$$
$$\frac{dv_o}{dt} = i_5$$

Therefore, the state vector is $\mathbf{x} = \begin{bmatrix} i_2 \\ i_4 \\ v_o \end{bmatrix}$

Now obtain v_1, v_2, and i_5 in terms of the state variables. First find i_1 in terms of the state variables.

$-v_i + i_1 + i_3 + i_5 + v_o = 0$

But $i_3 = i_1 - i_2$ and $i_5 = i_3 - i_4$. Thus,

$-v_i + i_1 + (i_1 - i_2) + (i_3 - i_4) + v_o = 0$

Making the substitution for i_3 yields

$-v_i + i_1 + (i_1 - i_2) + ((i_1 - i_2) - i_4) + v_o = 0$

Solving for i_1

$$i_1 = \frac{2}{3}i_2 + \frac{1}{3}i_4 - \frac{1}{3}v_o + \frac{1}{3}v_i$$

Thus,

$$v_1 = v_i - i_1 = -\frac{2}{3}i_2 - \frac{1}{3}i_4 + \frac{1}{3}v_o + \frac{2}{3}v_i$$

Also,

$$i_3 = i_1 - i_2 = -\frac{1}{3}i_2 + \frac{1}{3}i_4 - \frac{1}{3}v_o + \frac{1}{3}v_i$$

and

$$i_5 = i_3 - i_4 = -\frac{1}{3}i_2 - \frac{2}{3}i_4 - \frac{1}{3}v_o + \frac{1}{3}v_i$$

Finally,

$$v_2 = i_5 + v_o = -\frac{1}{3}i_2 - \frac{2}{3}i_4 + \frac{2}{3}v_o + \frac{1}{3}v_i$$

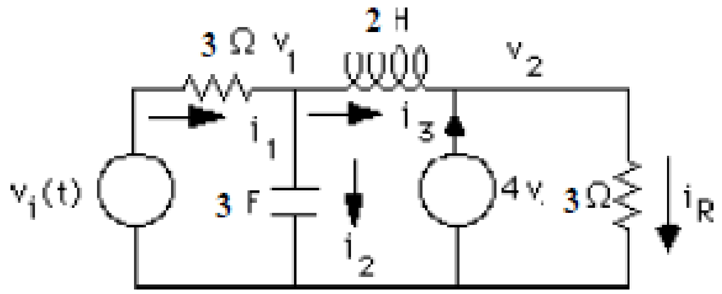
Using v_1, v_2, and i_5, the state equation is

$$\dot{\mathbf{x}} = \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix}\mathbf{x} + \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} v_i$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\mathbf{x}$$

2.

Add branch currents and node voltages to the schematic and obtain,



Write the differential equation for each energy storage element.

4.

Equations of motion in Laplace:

$$(2s^2+3s+2)X_1(s)-(s+2)X_2(s)-sX_3(s)=0$$
$$-(s+2)X_1(s)+(s^2+2s+2)X_2(s)-sX_3(s)=F(s)$$
$$-sX_1(s)-sX_2(s)+(s^2+3s)X_3(s)=0$$

Equations of motion in the time domain:

$$2\frac{d^2x_1}{dt^2}+3\frac{dx_1}{dt}+2x_1-\frac{dx_2}{dt}-2x_2-\frac{dx_3}{dt}=0$$
$$-\frac{dx_1}{dt}-2x_1+\frac{d^2x_2}{dt^2}+2\frac{dx_2}{dt}+2x_2-\frac{dx_3}{dt}=f(t)$$
$$-\frac{dx_1}{dt}-\frac{dx_2}{dt}+\frac{d^2x_3}{dt^2}+3\frac{dx_3}{dt}=0$$

Define state variables:

$$z_1 = x_1 \quad \text{or} \quad x_1 = z_1 \tag{1}$$

$$z_2 = \frac{dx_1}{dt} \quad \text{or} \quad \frac{dx_1}{dt} = z_2 \tag{2}$$

$$z_3 = x_2 \quad \text{or} \quad x_2 = z_3 \tag{3}$$

$$z_4 = \frac{dx_2}{dt} \quad \text{or} \quad \frac{dx_2}{dt} = z_4 \tag{4}$$

$$z_5 = x_3 \quad \text{or} \quad x_3 = z_5 \tag{5}$$

$$z_6 = \frac{dx_3}{dt} \quad \text{or} \quad \frac{dx_3}{dt} = z_6 \tag{6}$$

Substituting Eq. (1) in (2), (3) in (4), and (5) in (6), we obtain, respectively:

$$\frac{dz_1}{dt} = z_2 \tag{7}$$

$$\frac{dz_3}{dt} = z_4 \tag{8}$$

$$\frac{dz_5}{dt} = z_6 \tag{9}$$

Substituting Eqs. (1) through (6) into the equations of motion in the time domain and solving for the derivatives of the state variables and using Eqs. (7) through (9) yields the state equations:

$$\begin{aligned}
\frac{dz_1}{dt} &= z_2 \\
\frac{dz_2}{dt} &= -z_1 - \frac{3}{2}z_2 + z_3 + \frac{1}{2}z_4 + \frac{1}{2}z_6 \\
\frac{dz_3}{dt} &= z_4 \\
\frac{dz_4}{dt} &= 2z_1 + z_2 - 2z_3 - 2z_4 + z_6 + f(t) \\
\frac{dz_5}{dt} &= z_6 \\
\frac{dz_6}{dt} &= z_2 + z_4 - 3z_6
\end{aligned}$$

The output is $x_3 = z_5$.

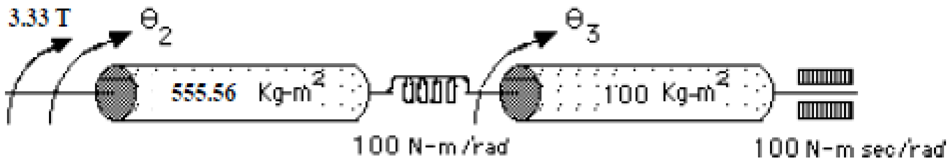
In vector-matrix form:

$$\dot{\mathbf{Z}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1.5 & 1 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & -2 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & -3 \end{bmatrix} \mathbf{Z} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} f(t)$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{Z}$$

6.

Drawing the equivalent network,



Writing the equations of motion,

$$(555.56s^2+100)\theta_2-100\theta_3=3.33T$$
$$-100\theta_2+(100s^2+100s+100)\theta_3=0$$

Taking the inverse Laplace transform and simplifying,

$$\ddot{\theta}_2+0.18\theta_2-0.18\theta_3=0.006T$$
$$-\theta_2+\ddot{\theta}_3+\dot{\theta}_3+\theta_3=0$$

Defining the state variables as

$$x_1=\theta_2,\ x_2=\dot{\theta}_2,\ x_3=\theta_3,\ x_4=\dot{\theta}_3$$

Writing the state equations using the equations of motion and the definitions of the state variables

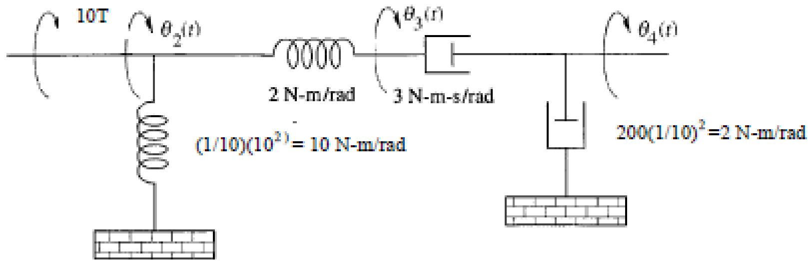
$$\dot{x}_1=x_2$$
$$\dot{x}_2=\ddot{\theta}_2=-0.18\theta_2+0.18\theta_3+0.006T=-0.18x_1+0.18x_3+0.006T$$
$$\dot{x}_3=x_4$$
$$\dot{x}_4=\ddot{\theta}_3=\theta_2-\theta_3-\dot{\theta}_3=x_1-x_3-x_4$$
$$y=3.33\theta_2=3.33x_1$$

In vector-matrix form,

$$\dot{\mathbf{x}}=\begin{bmatrix}0&1&0&0\\-0.18&0&0.18&0\\0&0&0&1\\1&0&-1&-1\end{bmatrix}\mathbf{x}+\begin{bmatrix}0\\0.006\\0\\0\end{bmatrix}T$$
$$y=\begin{bmatrix}3.33&0&0&0\end{bmatrix}\mathbf{x}$$

7.

Drawing the equivalent circuit,



Writing the equations of motion,

$$12\theta_2(s) - 2\theta_3(s) = 10T(s)$$
$$-2\theta_2(s) + (3s+2)\theta_3(s) - 3s\theta_4(s) = 0$$
$$-3s\theta_3(s) + 5s\theta_4(s) = 0$$

Taking the inverse Laplace transform,

$$12\theta_2(t) - 2\theta_3(t) = 10T(t) \qquad (1)$$

$$-2\theta_2(t) + 3\dot{\theta}_3(t) + 2\theta_3 - 3\dot{\theta}_4(t) = 0 \qquad (2)$$

$$-3\dot{\theta}_3(t) + 5\dot{\theta}_4(t) = 0 \qquad (3)$$

From (3),

$$\dot{\theta}_3(t) = \frac{5}{3}\dot{\theta}_4(t) \text{ and } \theta_3(t) = \frac{5}{3}\theta_4(t) \qquad (4)$$

assuming zero initial conditions.

From (1)

$$\theta_2(t) = \frac{1}{6}\theta_3(t) + \frac{5}{6}T(t) = \frac{5}{18}\theta_4(t) + \frac{5}{6}T(t) \qquad (5)$$

Substituting (4) and (5) into (2) yields the state equation (notice there is only one equation),

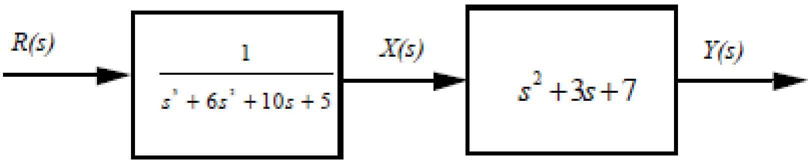
$$\dot{\theta}_4(t) = -\frac{25}{18}\theta_4(t) + \frac{5}{6}T(t)$$

The output equation is given by,

$$\theta_L(t) = \frac{1}{10}\theta_4(t)$$

13.

The transfer function can be represented as a block diagram as follows:



Writing the differential equation for the first box:

$$\dddot{x} + 6\ddot{x} + 10\dot{x} + 5x = r(t)$$

Defining the state variables:

$$x_1 = x$$
$$x_2 = \dot{x}$$
$$x_3 = \ddot{x}$$

Thus,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -5x - 10\dot{x} - 6\ddot{x} + r(t) = -5x_1 - 10x_2 - 6x_3 + r(t)$$

From the second box,

$$y = \ddot{x} + 3\dot{x} + 8x = 8x_1 + 3x_2 + x_3$$

In vector-matrix form:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -10 & -6 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$y = \begin{bmatrix} 8 & 3 & 1 \end{bmatrix} \mathbf{x}$$