EEEN 322
Communication Engineering

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Spring 2019

Week 4

Last Week

- Fourier Series for periodic signals
- Fourier Transform for aperiodic signals (also for periodic signals)
- Some Fourier Transform pairs
- Properties of Fourier Transform
  - If the signal is real: magnitude spectrum is even, phase spectrum is odd
  - Time delay in the signal causes a linear phase shift in the frequency spectrum
  - Multiplication of the signal by a cosine with ω_0 causes a frequency shift in the frequency spectrum to $\pm\omega_0$ (magnitude halved)
- Signal transmission through a linear system
  - Distortionless transmission
  - Linear distortion due to multipath effects
- Nonlinear distortion
- Ideal versus practical filters - Butterworth filter as a good approximation to ideal low pass filter
- Energy spectral density (ESD), relationship between ESDs of input and output of an LTI system

WEEK 4

Energy Spectral Density (ESD)

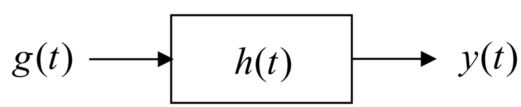
Energy
$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(w)|^2 dw$$
(Parseval)

This is called "Energy Spectral Density" (ESD)

Energy Spectral Density (ESD) $\Psi_g(w) = |G(w)|^2$

Note that you may also write: $E_g = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Psi_g(w)dw$

ESD of input / output:



$$Y(w) = H(w)G(w) \implies |Y(w)|^2 = |H(w)|^2|G(w)|^2$$

$$\Psi_y(w) = |H(w)|^2\Psi_g(w)$$

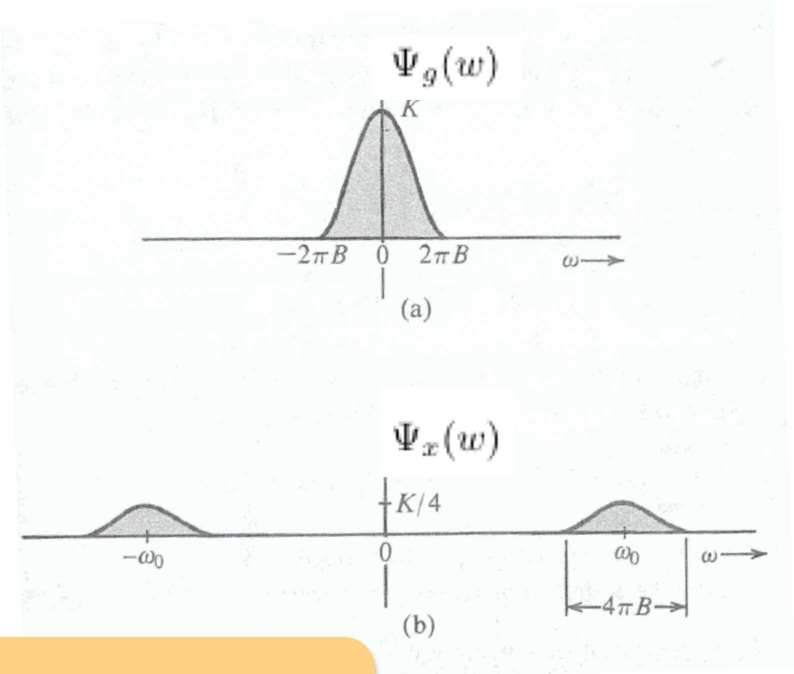
Energy Spectral Density (ESD) of *x(t)=g(t)cosω₀t*

$$x(t) = g(t)cosw_0t$$

$$\Longrightarrow \quad X(w) = \frac{1}{2}[G(w-w_0) + G(w+w_0)]$$

$$\Longrightarrow \quad |X(w)|^2 = \frac{1}{4}[|G(w-w_0)|^2 + |G(w+w_0)|^2]$$

$$\Longrightarrow \quad \Psi_x(w) = \frac{1}{4}[\Psi_g(w-w_0) + \Psi_g(w+w_0)]$$



Note that: $\boxed{E_x = \frac{1}{2}E_g}$

because...

$$\begin{aligned}
E_x &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\Psi_x(w)dw \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\{\frac{1}{4}[\Psi_g(w-w_0) + \Psi_g(w+w_0)]\}dw \\
&= \frac{1}{8\pi}\int_{-\infty}^{\infty}\Psi_g(w-w_0)dw + \frac{1}{8\pi}\int_{-\infty}^{\infty}\Psi_g(w+w_0)dw \\
&= \frac{1}{4}E_g + \frac{1}{4}E_g \\
&= \frac{1}{2}E_g
\end{aligned}$$

$$\boxed{E_g = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Psi_g(w)dw}$$

Energy Spectral Density (ESD) and Power Spectral Density (PSD)

Energy

$$E_g = \int_{-\infty}^{+\infty} |g(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |G(\omega)|^2 d\omega$$ **(Parseval)**

Energy Spectral Density (ESD)

$$\Psi_g(\omega) = |G(\omega)|^2$$

$$\Rightarrow E_g = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Psi_g(\omega) d\omega$$

ESD of Input / Output

$$g(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$\Psi_y(\omega) = |H(\omega)|^2 \Psi_g(\omega)$$

Energy Spectrum of *x(t)=g(t)cosω₀t*

$$\Psi_x(\omega) = \frac{1}{4}\left[\Psi_g(\omega+\omega_0) + \Psi_g(\omega-\omega_0)\right]$$

$$E_x = \frac{1}{2}E_g$$

Power

$$P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} |g(t)|^2 dt$$

Power Spectral Density (PSD)

$$S_g(\omega) = \lim_{T\to\infty} \frac{|G_T(\omega)|^2}{T} \quad (*)$$

The power spectrum can be computed this way as long as the limit exists

$$\Rightarrow P_g = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_g(\omega) d\omega$$

PSD of Input / Output

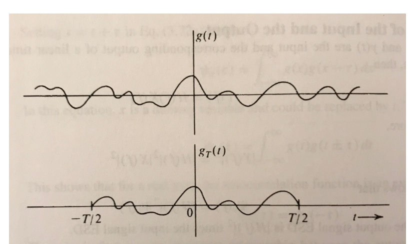
$$g(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$S_y(\omega) = |H(\omega)|^2 S_g(\omega)$$

Power Spectrum of *x(t)=g(t)cosω₀t*

$$S_x(\omega) = \frac{1}{4}\left[S_g(\omega+\omega_0) + S_g(\omega-\omega_0)\right]$$

$$P_x = \frac{1}{2}P_g$$



$(*) g_T(t)$ is $g(t)$ truncated to $\left[-\frac{T}{2}, \frac{T}{2}\right]$

Essential Bandwidth of a Signal

- Because the energy of a practical signal is finite, the signal spectrum must approach zero as the frequency goes to infinity
- Most of the signal energy is contained within a certain band of ***B*** Hz, and the energy content of the higher frequencies can be negligible
- The bandwidth ***B*** is called the **essential bandwidth** of the signal
- The criterion for selecting *B* depends on the error tolerance in a particular application (examples are, 90%, 95%, 99%, etc)

Example: Rectangular pulse

$$g(t) = rect\frac{t}{T}$$

$$rect\frac{t}{T} \leftrightarrow Tsinc\frac{\omega T}{2}$$

➔ Energy spectrum: $\Psi_g(\omega) = |G(\omega)|^2 = T^2 sinc^2\frac{\omega T}{2}$

Energy within a bandwidth of W: $E_W = \frac{1}{2\pi}\int_{-W}^{W}\Psi_g(\omega)d\omega = \frac{T^2}{2\pi}\int_{-W}^{W} sinc^2\frac{\omega T}{2}\,d\omega$

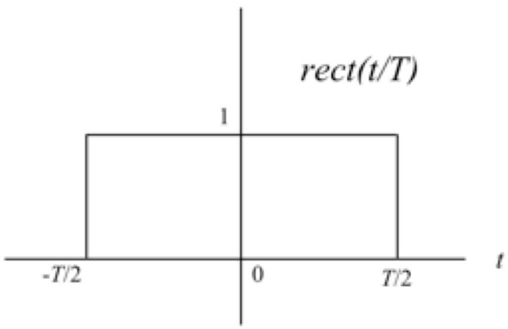
$$\frac{E_W}{E_g} = \underbrace{\frac{T}{2\pi}\int_{-W}^{W} sinc^2\frac{\omega T}{2}\,d\omega}_{\text{Compute numerically}}$$

$$\boxed{\Rightarrow \frac{E_W}{E_g} = 90\% \text{ if } W = \frac{2\pi}{T}}$$

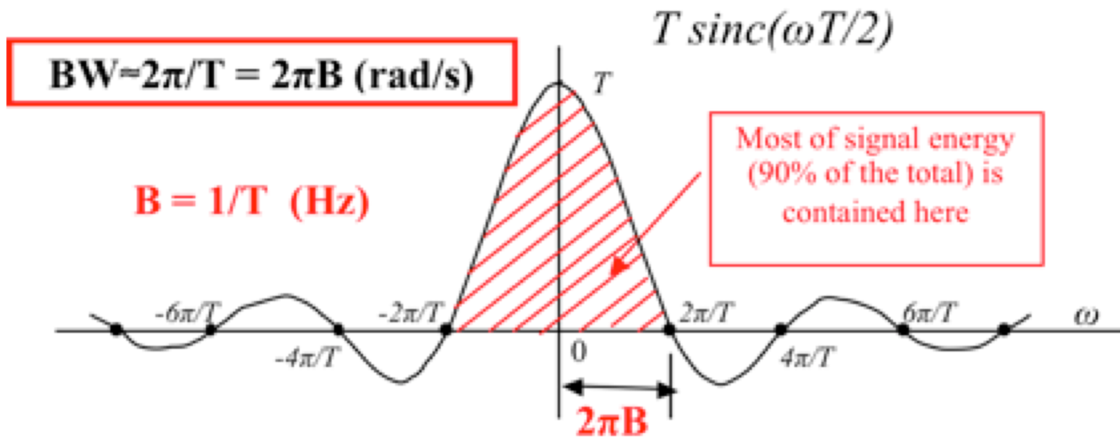
90% of signal energy is contained within the bandwidth of B = 1 / T

Essential bandwidth of a rectangular pulse
(by the 90% criterion)

Time



Frequency



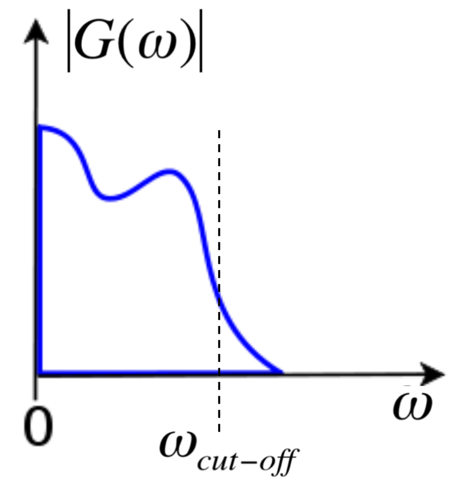
We define bandwidth (**B**) considering **positive** frequencies only.

Baseband Signal

A **baseband signal** is a signal that can include frequencies that are very near zero, by comparison with its highest frequency

A **signal at baseband** is usually considered to include frequencies from near 0 Hz up to the highest (cut-off) frequency in the signal (with significant energy/power)

Baseband communication: no shift of the spectrum of the signal to be transmitted (the baseband signal) to the higher frequencies by modulation



cutoff frequency

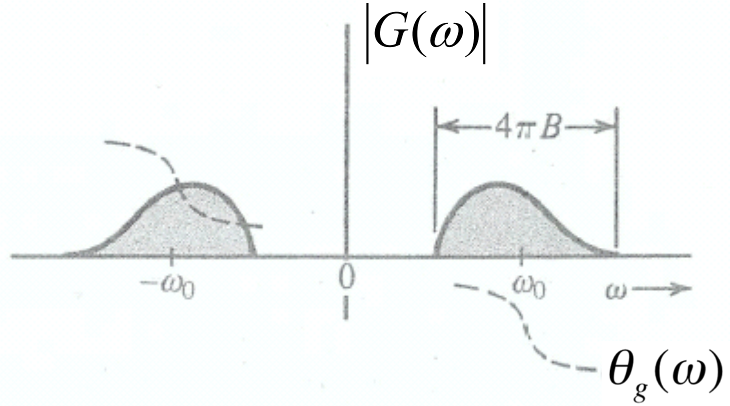
Note: *G(0)* is the "DC content" of the signal

Bandpass Signal

A **bandpass signal** is a signal *g(t)* whose frequency domain representation *G(ω)* is nonzero for frequencies in a usually small neighborhood of some high frequency ω_0, that is:

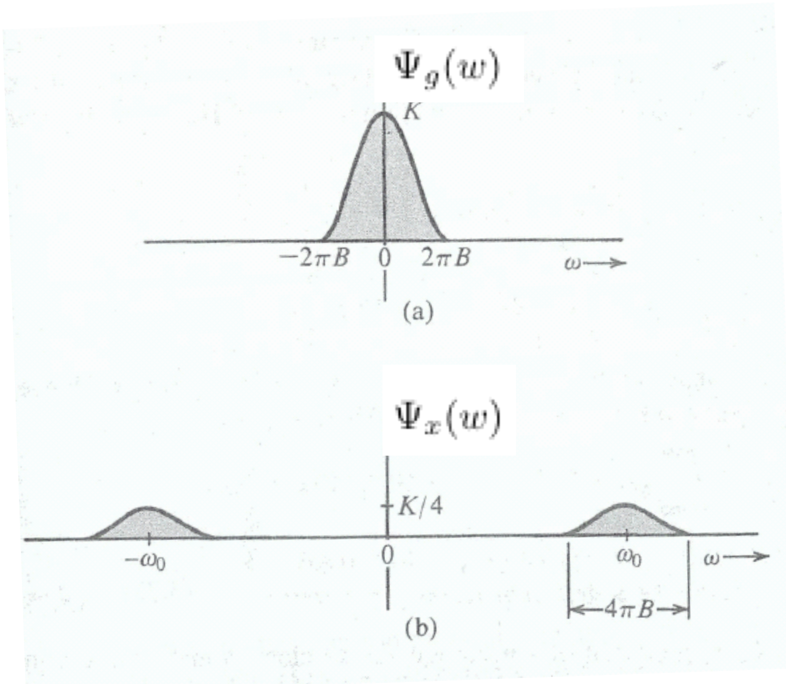
$$G(\omega) \equiv 0 \quad \text{for} \quad |\omega - \omega_0| > 2\pi B \qquad (\omega_0 >> 2\pi B)$$

$$G(\omega) = |G(\omega)| e^{j\theta_g(\omega)}$$



ω_0: **center frequency**

$$x(t) = g(t)\cos\omega_0 t \leftrightarrow X(\omega) = (\frac{1}{2}\left[G(\omega-\omega_0) + G(\omega+\omega_0)\right]$$



g(t) (the **message** signal to be modulated) is a baseband signal

Bandwidth: $2\pi B$ rad/s (B Hz)

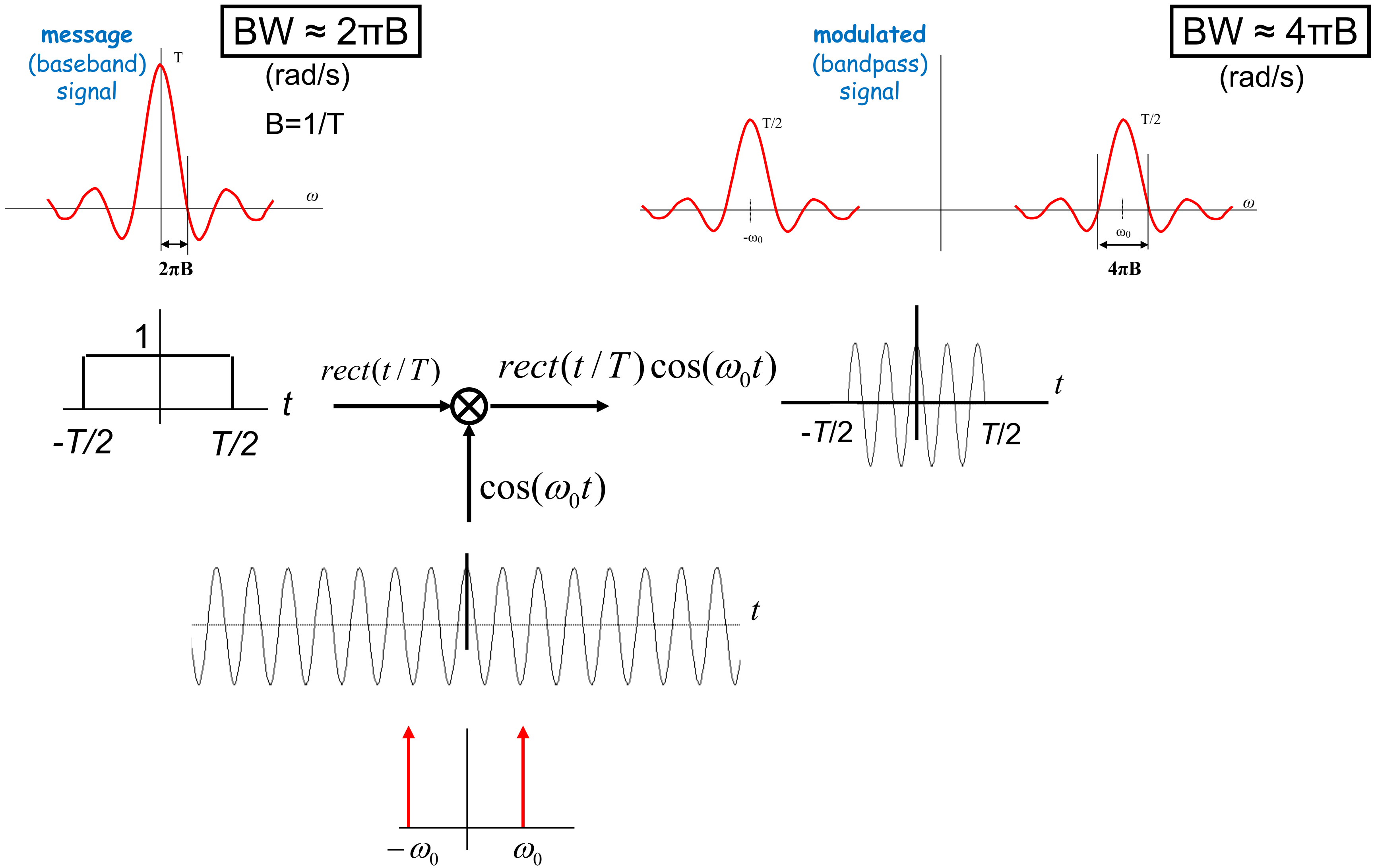
x(t) (the **modulated** signal to be sent via the antenna) is a bandpass signal

Bandwidth: $4\pi B$ rad/s ($2B$ Hz)

- ω_0: center frequency
- In this context it may also be called *“carrier frequency”*

*The modulation example shown here is **DSB-SC** modulation.*

Example: $g(t)\cos\omega_0 t \leftrightarrow \frac{1}{2}\left[G(\omega-\omega_0)+G(\omega+\omega_0)\right]$ where *g(t)* is a rectangular function



*The modulation example shown here is **DSB-SC** modulation.*

Communication System with Signal Modulation & Demodulation

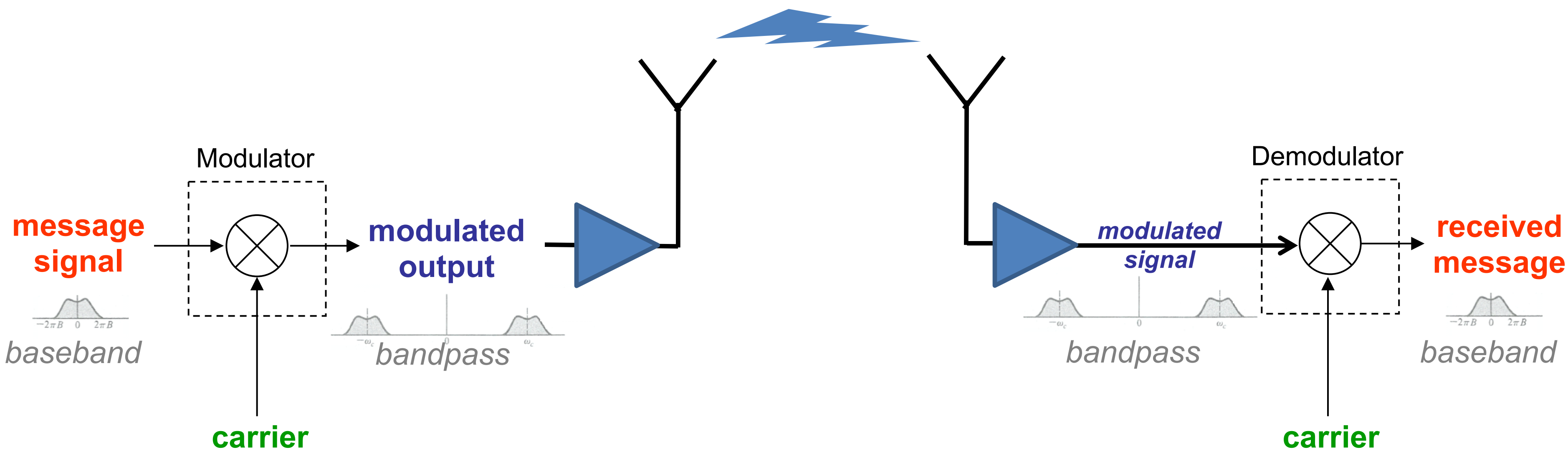


*Illustration is for the case of **DSB-SC** modulation with **coherent** demodulation.*

Modulation and Demodulation

Modulation: process of impressing an analog signal on the amplitude, frequency or phase of a sinusoidal carrier.

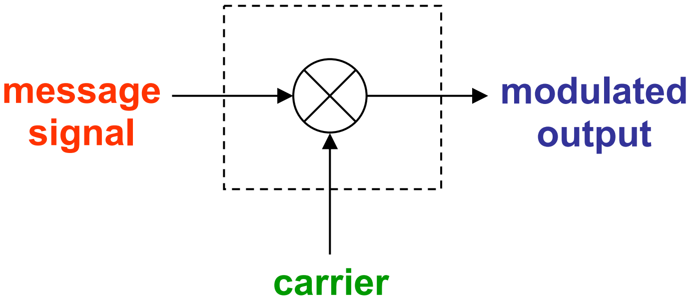
Amplitude Modulation (AM)
Frequency Modulation (FM)
Phase Modulation (PM)

Demodulation: process of recovering the signal that was impresssed on the sinusoidal carrier.

Amplitude Modulation (AM)

	Suppressed Carrier	With Carrier
Double Sideband (DSB)	**DSB-SC**	**DSB+C** **(a.k.a. AM)**
Single Sideband (SSB)	**SSB-SC**	**SSB+C**
Vestigial Sideband (VSB)	**VSB-SC**	**VSB+C**

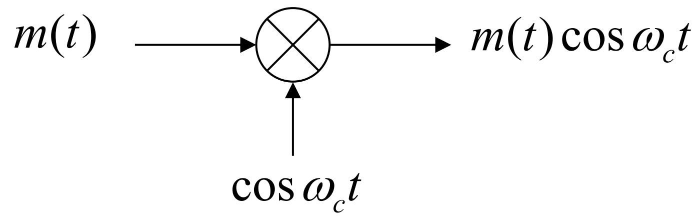
In general: The **amplitude** of the **carrier** is changed by the **message signal**



$$\varphi_{DSB-SC}(t) = m(t)\cos\omega_c t$$

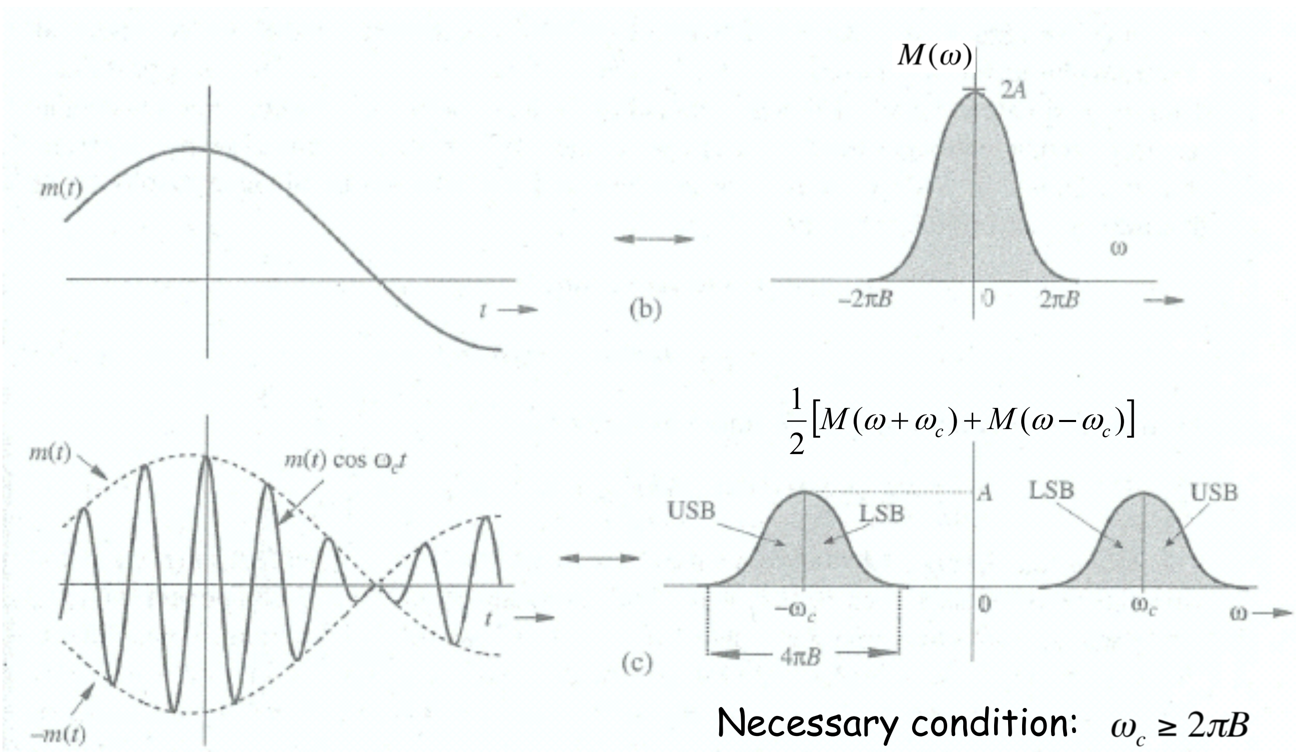
Double Sideband Suppressed Carrier (DSB-SC) Modulation

Modulating baseband signal (Message)



Carrier

Modulated bandpass signal

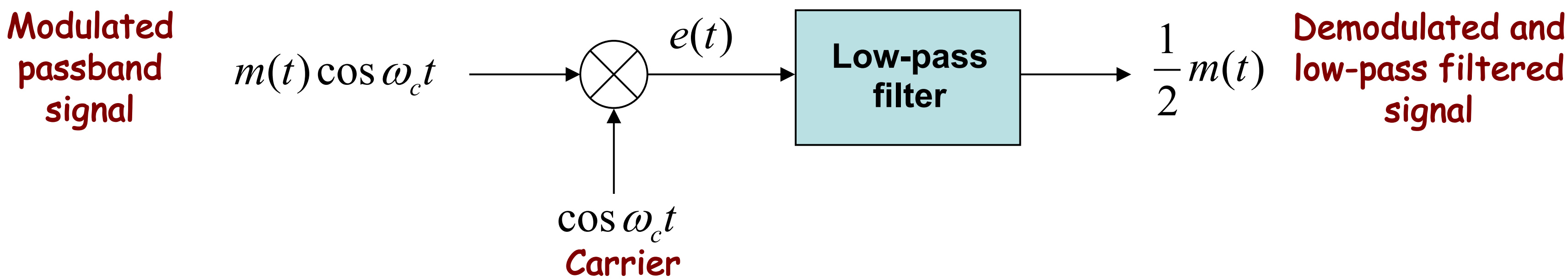


Bandwidth: B

Bandwidth: 2B

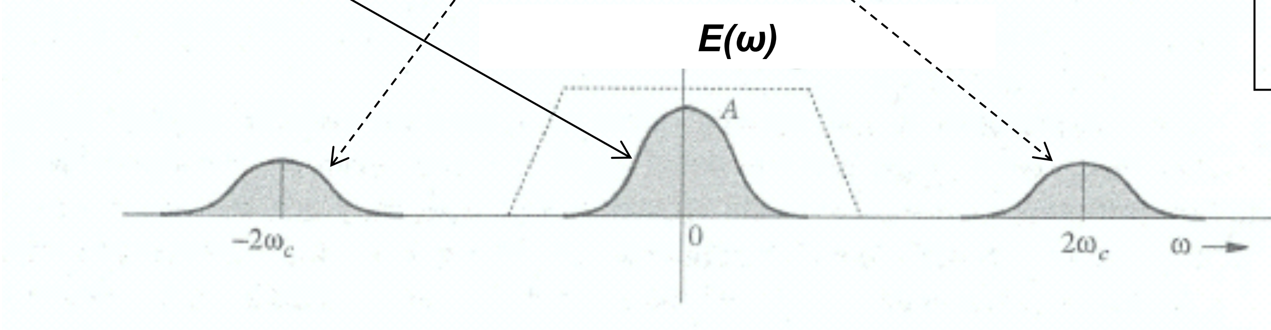
Necessary condition: $\omega_c \geq 2\pi B$

Double Sideband Suppressed Carrier (DSB-SC) Demodulation



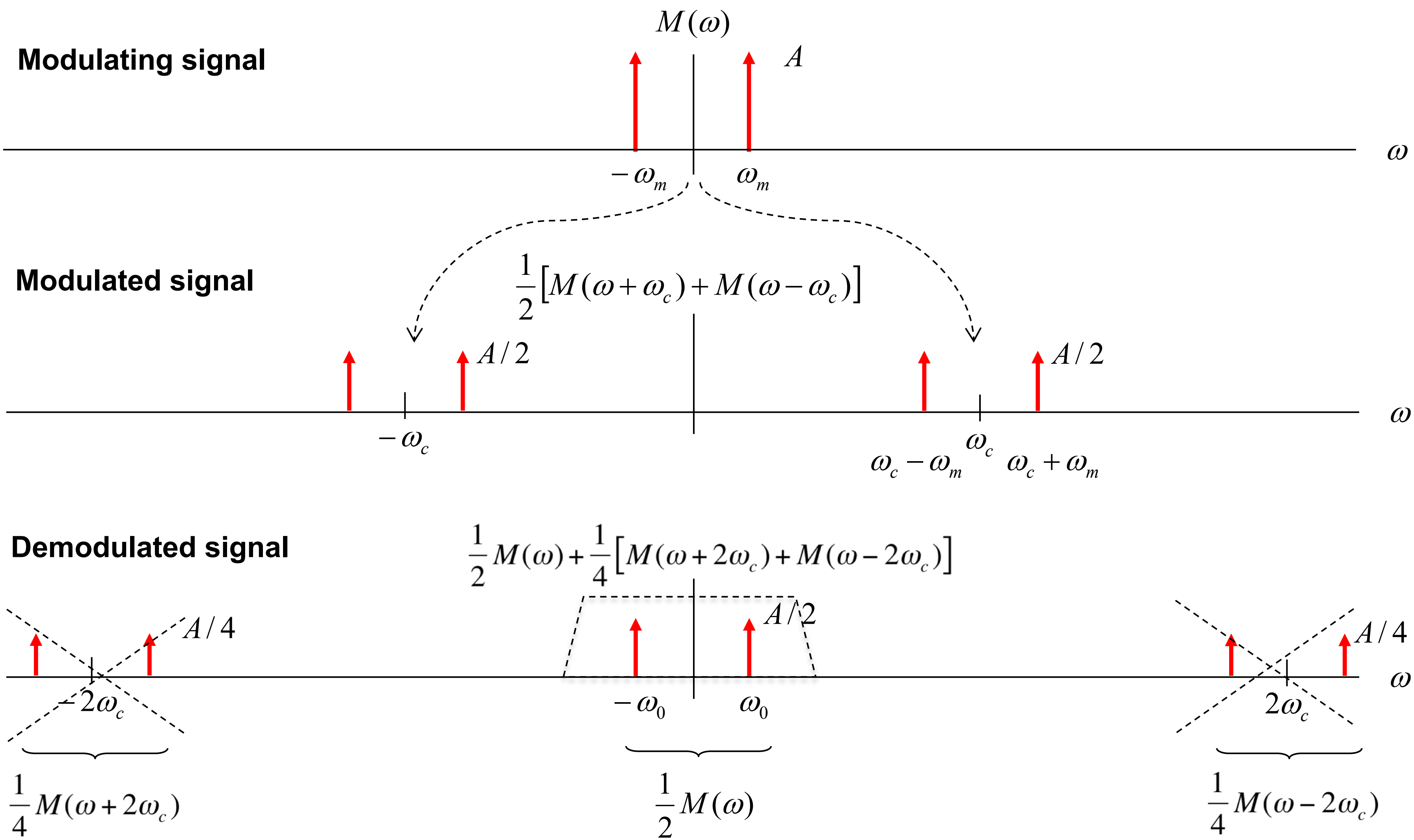
$$e(t) = m(t)\cos^2\omega_c t = m(t)\left[\frac{1+\cos 2\omega_c t}{2}\right] = \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2\omega_c t$$

$$\Rightarrow E(\omega) = \underbrace{\frac{1}{2}M(\omega)} + \underbrace{\frac{1}{4}\left[M(\omega+2\omega_c) + M(\omega-2\omega_c)\right]}$$



Coherent demodulation uses a carrier of the *same* frequency and phase (homodyne) as that for modulation when $m(t)$ itself is a cosine as well (with angular frequency ω_m)

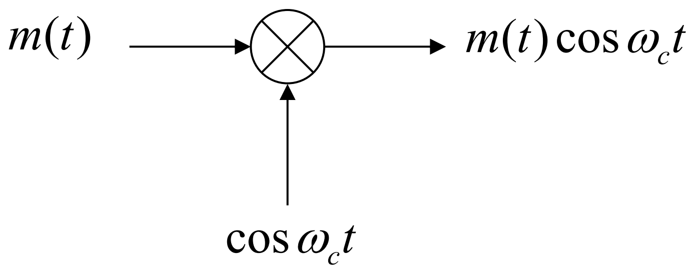
DSB-SC Tone Modulation-Demodulation



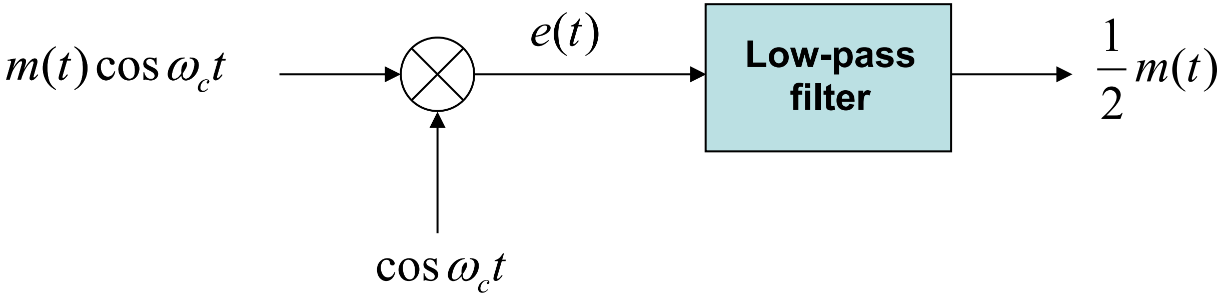
Dashed lines depict the low-pass filtering operation

Modulation - Demodulation Circuits for DSB-SC Modulation

Modulation



Demodulation



➔All the modulators discussed can also be used as demodulators (provided that proper output filtering is performed)

Modulation - Demodulation Circuits for DSB-SC Modulation

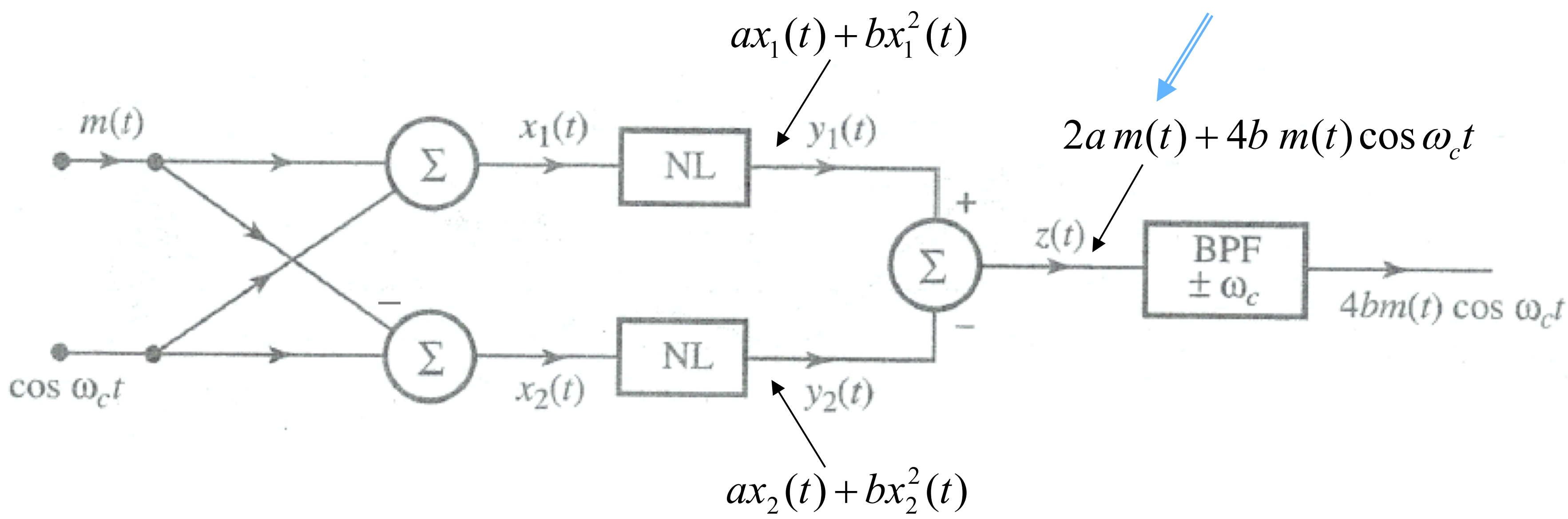
- **Modulation:**
  - **Multiplier modulator**
  - **Nonlinear modulator**
  - **Switching modulator**
    - **Diode-bridge modulator**
    - **Ring modulator**
- **Demodulation**
  - **All the modulators above**

Multiplier Modulator for DSB-SC Modulation

- Modulation is achieved directly by multiplying *m(t)* by $\cos\omega_c t$ using an analog multiplier
  - Rather difficult to maintain linearity
  - Rather expensive
  - → Best to avoid them if possible

Nonlinear Modulator for DSB-SC Modulation

The circuit is "balanced"
with respect to *cos( )*
<u>but not</u> with respect to *m( )*
→ *"single balanced"*

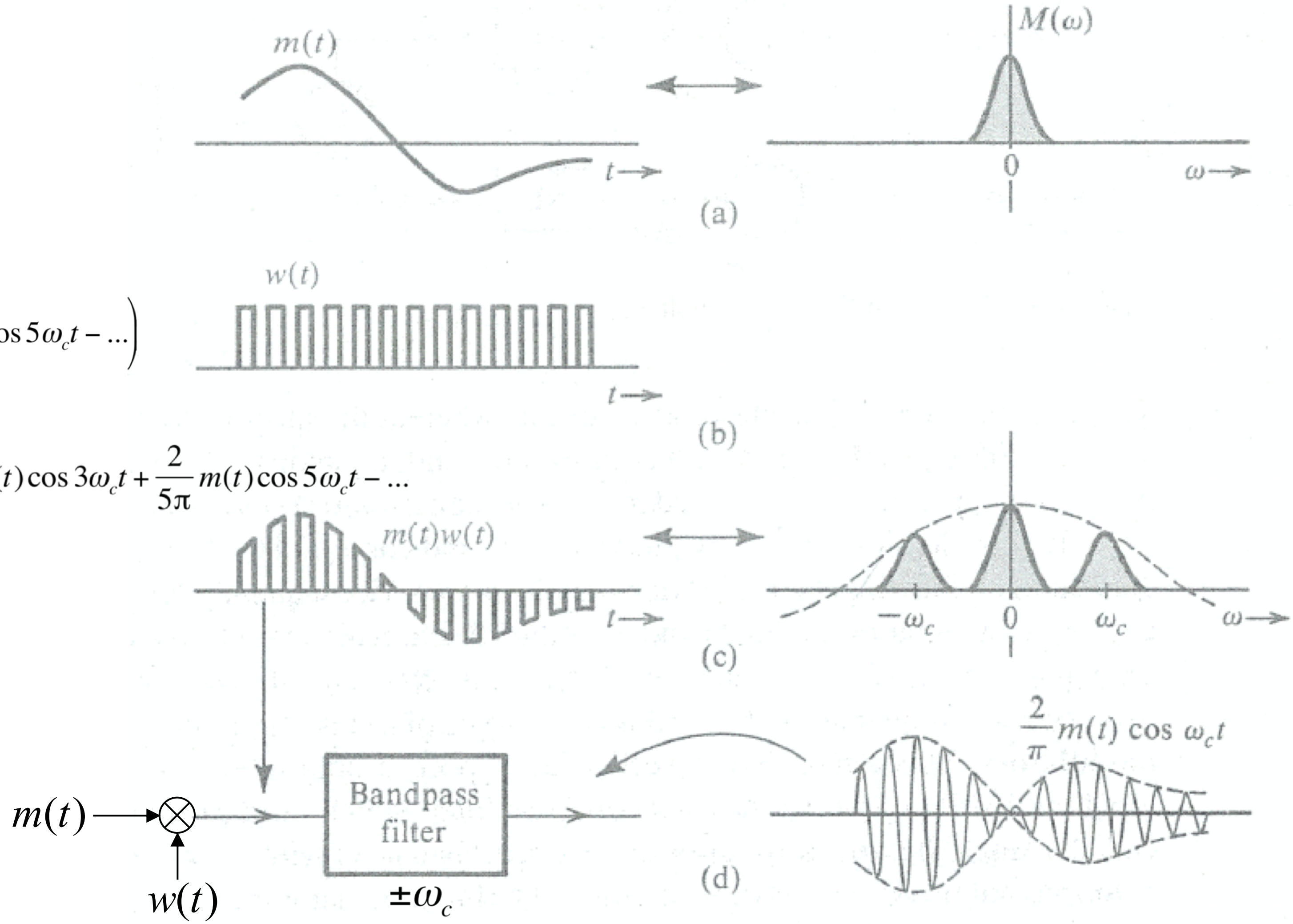


Single-balanced modulator

Switching Modulator for DSB-SC Modulation

$$\frac{1}{2}+\frac{2}{\pi}\left(\cos\omega_c t-\frac{1}{3}\cos 3\omega_c t+\frac{1}{5}\cos 5\omega_c t-\ldots\right)$$

$$\frac{1}{2}m(t)+\frac{2}{\pi}m(t)\cos\omega_c t-\frac{2}{3\pi}m(t)\cos 3\omega_c t+\frac{2}{5\pi}m(t)\cos 5\omega_c t-\ldots$$



Single-balanced modulator