

7. A system is described by the following differential equation:

$$\frac{d^3 y}{dt^3} + 3 \frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} + y = \frac{d^3 x}{dt^3} + 4 \frac{d^2 x}{dt^2} + 6 \frac{dx}{dt} + 8x$$

Find the expression for the transfer function of the system,  $Y(s)/X(s)$ . [Section: 2.3]

16. Find the transfer function,  $G(s) = V_o(s)/V_i(s)$ , for each network shown in Figure P2.3. [Section: 2.4]

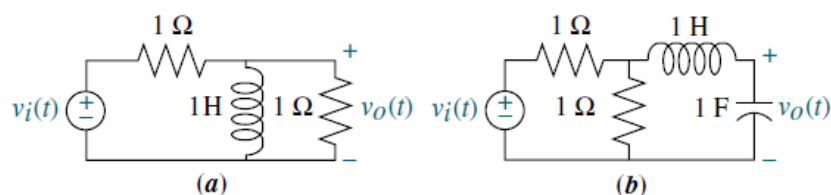


FIGURE P2.3

17. Find the transfer function,  $G(s) = V_L(s)/V(s)$ , for each network shown in Figure P2.4. [Section: 2.4]

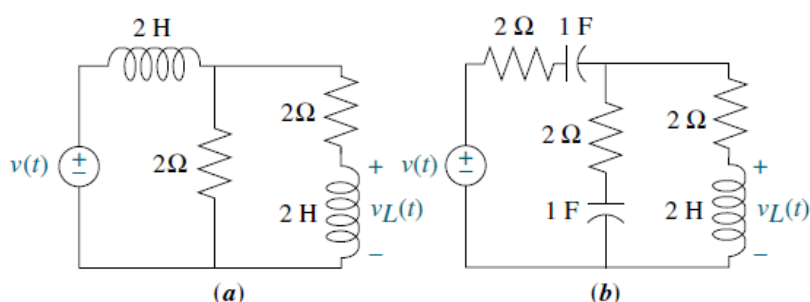
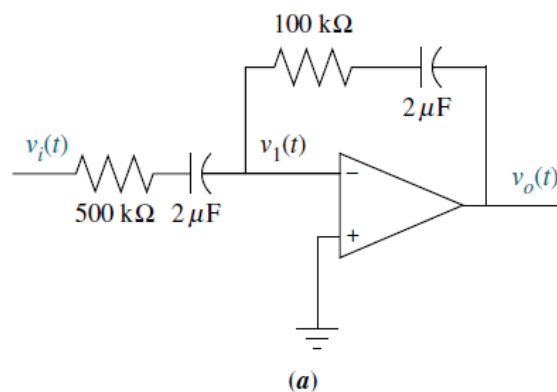
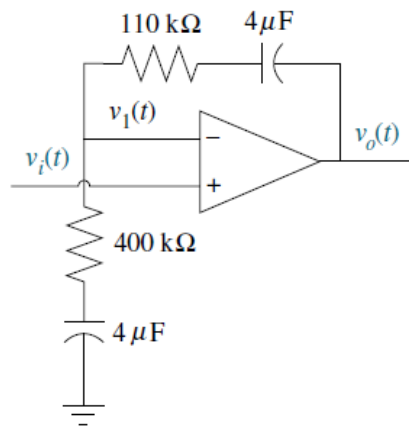


FIGURE P2.4

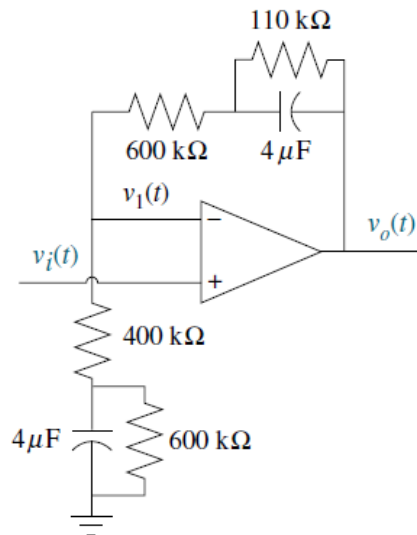
21. Find the transfer function,  $G(s) = V_o(s)/V_i(s)$ , for each operational amplifier circuit shown in Figure P2.7. [Section: 2.4]



22. Find the transfer function,  $G(s) = V_o(s)/V_i(s)$ , for each operational amplifier circuit shown in Figure P2.8. [Section: 2.4]



(a)



(b)

FIGURE P2.8

7.

The Laplace transform of the differential equation, assuming zero initial conditions, is,

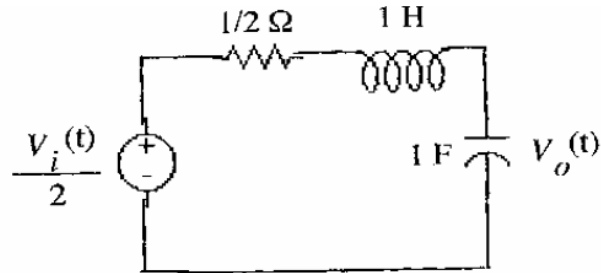
$$(s^3 + 3s^2 + 5s + 1)Y(s) = (s^3 + 4s^2 + 6s + 8)X(s).$$

Solving for the transfer function,  $\frac{Y(s)}{X(s)} = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}.$

16.

- a. Writing the node equations,  $\frac{V_o - V_i}{s} + \frac{V_o}{s} + V_o = 0$ . Solve for  $\frac{V_o}{V_i} = \frac{1}{s+2}$ .

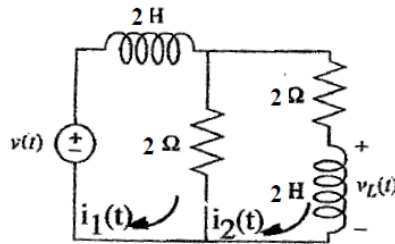
- b. Thevenizing,



Using voltage division,  $V_o(s) = \frac{V_i(s)}{2} \frac{\frac{1}{s}}{\frac{1}{2} + s + \frac{1}{s}}$ . Thus,  $\frac{V_o(s)}{V_i(s)} = \frac{1}{2s^2 + s + 2}$

17.

- a.



Writing mesh equations

$$(2s+2)I_1(s) - 2 I_2(s) = V_i(s)$$

$$-2I_1(s) + (2s+4)I_2(s) = 0$$

But from the second equation,  $I_1(s) = (s+2)I_2(s)$ . Substituting this in the first equation yields,

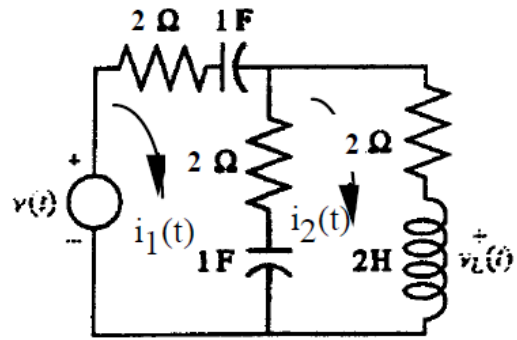
$$(2s+2)(s+2)I_2(s) - 2 I_2(s) = V_i(s)$$

or

$$I_2(s)/V_i(s) = 1/(2s^2 + 4s + 2)$$

But,  $V_L(s) = sI_2(s)$ . Therefore,  $V_L(s)/V_i(s) = s/(2s^2 + 4s + 2)$ .

b.



$$\left(4 + \frac{2}{s}\right)I_1(s) - \left(2 + \frac{1}{s}\right)I_2(s) = V(s)$$

$$-\left(2 + \frac{1}{s}\right)I_1(s) + \left(4 + \frac{1}{s} + 2s\right)I_2(s) = 0$$

Solving for  $I_2(s)$ :

$$I_2(s) = \frac{\begin{vmatrix} \frac{4s+2}{s} & V(s) \\ -\frac{(2s+1)}{s} & 0 \end{vmatrix}}{\begin{vmatrix} \frac{4s+2}{s} & -\frac{(2s+1)}{s} \\ -\frac{(2s+1)}{s} & (2s^2+4s+1) \end{vmatrix}} = \frac{sV(s)}{4s^2 + 6s + 1}$$

Therefore,  $\frac{V_L(s)}{V(s)} = \frac{2sI_2(s)}{V(s)} = \frac{2s^2}{4s^2 + 6s + 1}$

**21.**

**a.**

$$Z_1(s) = 5 \times 10^5 + \frac{1}{2 \times 10^{-6} s}$$

$$Z_2(s) = 10^5 + \frac{1}{2 \times 10^{-6} s}$$

Therefore,

$$-\frac{Z_2(s)}{Z_1(s)} = -\frac{1}{5} \frac{(s+5)}{(s+1)}$$

**b.**

$$Z_1(s) = 10^5 \left( \frac{5}{s} + 1 \right) = 10^5 \frac{(s+5)}{s}$$

$$Z_2(s) = 10^5 \left( 1 + \frac{5}{s+5} \right) = 10^5 \frac{(s+10)}{(s+5)}$$

Therefore,

$$-\frac{Z_2(s)}{Z_1(s)} = -\frac{s(s+10)}{(s+5)^2}$$

22.

a.

$$Z_1(s) = 4 \times 10^5 + \frac{1}{4 \times 10^{-6} s}$$

$$Z_2(s) = 1.1 \times 10^5 + \frac{1}{4 \times 10^{-6} s}$$

Therefore,

$$G(s) = \frac{Z_1(s) + Z_2(s)}{Z_1(s)} = 1.275 \frac{(s + 0.98)}{(s + 0.625)}$$

b.

$$Z_1(s) = 4 \times 10^5 + \frac{\frac{10^{11}}{s}}{4 \times 10^5 + \frac{0.25 \times 10^6}{s}}$$

$$Z_2(s) = 6 \times 10^5 + \frac{27.5 \frac{10^9}{s}}{110 \times 10^3 + \frac{0.25 \times 10^6}{s}}$$

Therefore,

$$\frac{Z_1(s) + Z_2(s)}{Z_1(s)} = \frac{2640s^2 + 8420s + 4275}{1056s^2 + 3500s + 2500}$$