Selected Problems - Ib

Problem 1) Convert the following numbers with the indicated bases to decimal:

a. $(4310)_5$

b. $(198)_{12}$

Solution.

a. $(4310)_5 = 0 \cdot 5^0 + 1 \cdot 5^1 + 3 \cdot 5^2 + 4 \cdot 5^3$

$= 0 + 5 + 75 + 500$

$= (580)_{10}$

b. $(198)_{12} = 8 \cdot 12^0 + 9 \cdot 12^1 + 1 \cdot 12^2$

$= 8 + 108 + 144$

$= (260)_{10}$

Problem 2) Determine the base of numbers in each case for the following operations to be correct:

a. $14 / 2 = 5$

b. $24 + 17 = 40$

Solution.

a. We have

$$\frac{(14)_a}{(2)_a} = (5)_a \quad , \quad a > 5$$

$$\Rightarrow (14)_a = 4 \cdot a^0 + 1 \cdot a^1 = 4 + a$$

$\Rightarrow (2)_c = 2 \cdot c^0 = 2$

$\Rightarrow (5)_c = 5 \cdot c^0 = 5$

$\Rightarrow \quad \frac{4+c}{2} = 5 \quad \Rightarrow \quad 4+c = 10 \quad \Rightarrow \quad c = 6$

b. We have

$$(24)_b + (17)_b = 40 \quad , \quad b > 7$$

$$\begin{array}{r} (24)_b \\ +\ (17)_b \\ \hline (40)_b \end{array}$$

$4+7 \equiv 0 \quad \Rightarrow \quad b = 11$

$2+1+1(\text{carry}) \equiv 4$

Problem 3) The solutions to the quadratic equation

$$x^2 - 11x + 22 = 0$$

are $x = 3$ and $x = 6$. What is the base of the numbers?

Solution. Indeed we have

$$(x)_c^2 - (11)_c \cdot (x)_c + (22)_c = 0 \quad , \quad a > 6$$

$\Rightarrow (x)_c = x, \quad (11)_c = 1 \cdot c^0 + 1 \cdot c^1 = 1 + c,$

$(22)_c = 2 \cdot c^0 + 2 \cdot c^1 = 2 + 2c$

$\Rightarrow x^2 - (1+c) \cdot x + (2+2c) = 0$

$x = 3 \Rightarrow 9 - (1+c) \cdot 3 + 2(1+c) = 0$

$\Rightarrow 9 - (1+c)(3-2) = 0$

$\Rightarrow 9 = 1 + c \Rightarrow c = 8$

Indeed;

$x = 6 \Rightarrow 36 - (1+c) \cdot 6 + (2+2c) = 0$

$\Rightarrow 36 - (1+c)(6-2) = 0 \Rightarrow 36 = (1+c) \cdot 4$

$\Rightarrow c = 8$ (justified)

Problem 4) Add and multiply the following numbers without converting them to decimal:

a. Binary numbers 1011 and 101.

b. Hexadecimal numbers 2E and 34.

Solution

a.

$$\begin{array}{r} 1\,1\,1\,1\,0 \\ 1\,0\,1\,1 \\ +\quad 1\,0\,1 \\ \hline 1\,0\,0\,0\,0 \end{array}$$

$$\begin{array}{r} 1\,0\,1\,1 \\ \times\quad 1\,0\,1 \\ \hline 1\,0\,1\,1 \\ 0\,0\,0\,0 \\ +1\,0\,1\,1 \\ \hline 1\,1\,0\,1\,1\,1 \end{array}$$

b.

$$\begin{array}{r} 10 \\ 2E \\ +\ 34 \\ \hline (62)_{16} \end{array} \qquad \begin{array}{r} 2E \\ \times\ 34 \\ \hline B8 \\ +\ 8A \\ \hline (558)_{16} \end{array}$$

Problem 5) Convert the following binary numbers to decimal and hexadecimal:

a. 1.10010

b. 110.010

Explain why the decimal answer in (b) is 4 times that in (a).

Solution.

a. $(1.10010)_2 = 1.2^0 + 1.2^{-1} + 0.2^{-2} + 0.2^{-3} + 1.2^{-4} + 0.2^{-5}$

$= 1 + 0.5 + 0.0625$

$= 1.5625$

$(1.10010)_2 \equiv (\underbrace{0001}.\underbrace{1001}\underbrace{0000})_2$

$= (1.90)_{16}$

b. $(110.010)_2 = 0.2^0 + 1.2^1 + 1.2^2 + 0.2^{-1} + 1.2^{-2} + 0.2^{-3}$

$= 2 + 4 + 0.25$

$= 6.25$

$(110.010)_2 \equiv (0110.0100)_2$

$= (6.4)_{10}$

-Note that the binary fraction point in (a) is shifted two digit positions to the right

-Since single digit position shift to the right for a binary fraction point is multiplication by 2, the 4 times enlargement is thus justified by 2 digit position shift to the right!

Problem 6) Obtain (r-1)'s and r's complement of the following numbers:

a. $(11011010)_2$

b. $(52784630)_{10}$

Solution.

a. 1's complement of $(11011010)_2$

$= (00100101)_2$

2's complement of $(11011010)_2$

$=$ 1's complement of $(11011010)_2$

$+ 1$

$= (00100101)_2 + 1$

$= (00100110)_2$

b. 9's complement of $(52784630)_{10}$

$= (47215369)_{10}$

10's complement of $(52784630)_{10}$

$= (47215369)_{10} + 1$

$= (47215370)_{10}$

Problem 7) Convert decimal (27.315) to binary.

Solution. We convert the integer part and fractional part seperately:

$27 - 16 = 11$, 2^4
$11 - 8 = 3$, 2^3
$3 - 2 = 1$, 2^1
$1 - 1 = 0$, 2^0

$(27)_{10} = (11011)_2$

$0.315 \times 2 = \boxed{0}.630$

$0.630 \times 2 = \boxed{1}.260$

$0.260 \times 2 = \boxed{0}.520$

$0.520 \times 2 = \boxed{1}.040$

$0.040 \times 2 = \boxed{0}.080$

$0.080 \times 2 = \boxed{0}.160$

$0.160 \times 2 = \boxed{0}.32$

$0.32 \times 2 = \boxed{0}.640$

$0.640 \times 2 = \boxed{1}.280$

$0.280 \times 2 = \boxed{0}.56$

$0.56 \times 2 = \boxed{1}.12$

$\vdots$

$(0.315) \cong (0.0101\ldots)_2$

Hence ;

$(27.315)_{10} \cong (11011.0101)_2$

((truncating with 4 digits

Problem 8) Perform the subtraction on the following numbers using the r's complement of the subtrahend.

PS 1.13

a. $1631 - 745 = ?$

b. $110000 - 10101 = ?$

c. $125 - 1800 = ?$

d. $1001 - 101000 = ?$

Solution.

a.
$$\begin{array}{r} 0110 \\ 1631 \\ -\ 745 \\ \hline 886 \end{array}$$

$$\begin{array}{r} 1631 \\ 254 \\ +\ \quad 1 \\ \hline [1]886 \end{array}$$

254 → 9's complement) 10's comp.

[1] ↓ discarded

b.
$$\begin{array}{r} 11110 \\ 110000 \\ -\ 10101 \\ \hline 011011 \end{array}$$

$$\begin{array}{r} 110000 \\ 01010 \\ +\ \quad 1 \\ \hline [1]11011 \end{array}$$

[1] ↓

c.
$$\begin{array}{r} 125 \\ -\ 1800 \\ \hline \Downarrow \\ 0010 \\ 1800 \\ -\ 125 \\ \hline -(1675)_{10} \end{array}$$

$$\begin{array}{r} 125 \\ 8199 \\ +\ \quad 1 \\ \hline 8325 \end{array}$$

⇒ no carry out ⇓ 10's complement

$$\begin{array}{r} 1674 \\ +\ \quad 1 \\ \hline \end{array}$$

⇒ $-(1675)_{10}$

PS 1.14

d. 1 1 0 0 1

1 0 1 0 0 0

$=$

$\Downarrow$

0 1 0

1 0 1 0 0 0

1 0 0 1

$-$

$-(100011)_2$

0 0 1 0 0 1

0 1 0 1 1 1

$+$ 1

0 1 1 1 0 1 $\Rightarrow$ 2's comp.

$\downarrow$

no carry out

1 0 0 0 1 0

$+$ 1

$-(100011)_2$

$\uparrow$

a minus sign is appended!