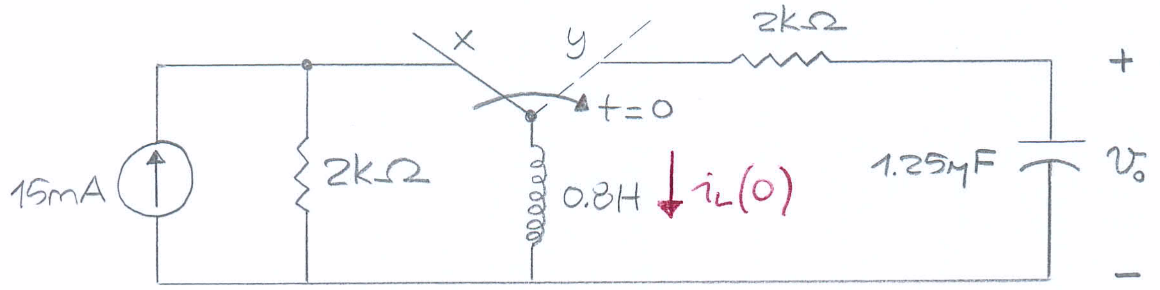
Selected Problems – XIII

Problem 1) The switch has been in position x for a long time in the circuit shown as



At t=0, the switch moves instantaneously to position y.

a. Construct an s-domain circuit for t>0.

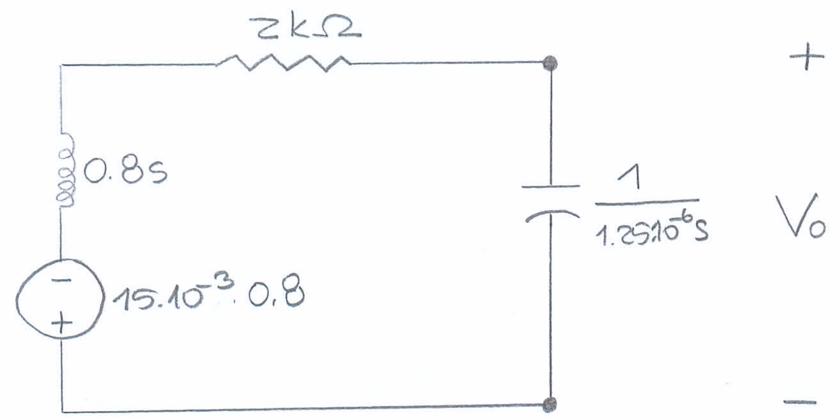
b. Find V_o.

c. Find v_o.

Solution. We find that

$$i_L(0^-) = i_L(0) = i_L(0^+) = 15\text{ mA}$$

a.



b.

$$\frac{V_o + 12 \cdot 10^{-3}}{0.8s + 2000} + \frac{V_o}{1/(1.25 \cdot 10^{-6} s)} = 0$$

$$\Rightarrow V_o + 12 \cdot 10^{-3} + 1.25 \cdot 10^{-6} s \left(0.8s + 2000\right) V_o = 0$$

$$\Rightarrow \left(1 + 10^{-6} s^2 + 2.5 \cdot 10^{-3} s\right) V_o = -12 \cdot 10^{-3}$$

$$\Rightarrow \quad V_o = -\frac{12000}{s^2+2500s+10^6}$$

c. We shall write

$$V_o = -\frac{12000}{(s+500)(s+2000)} = \frac{C_1}{s+500} + \frac{C_2}{s+2000}$$

$$\Rightarrow C_1 = -\frac{12000}{s+2000}\bigg|_{s=-500} = -8$$

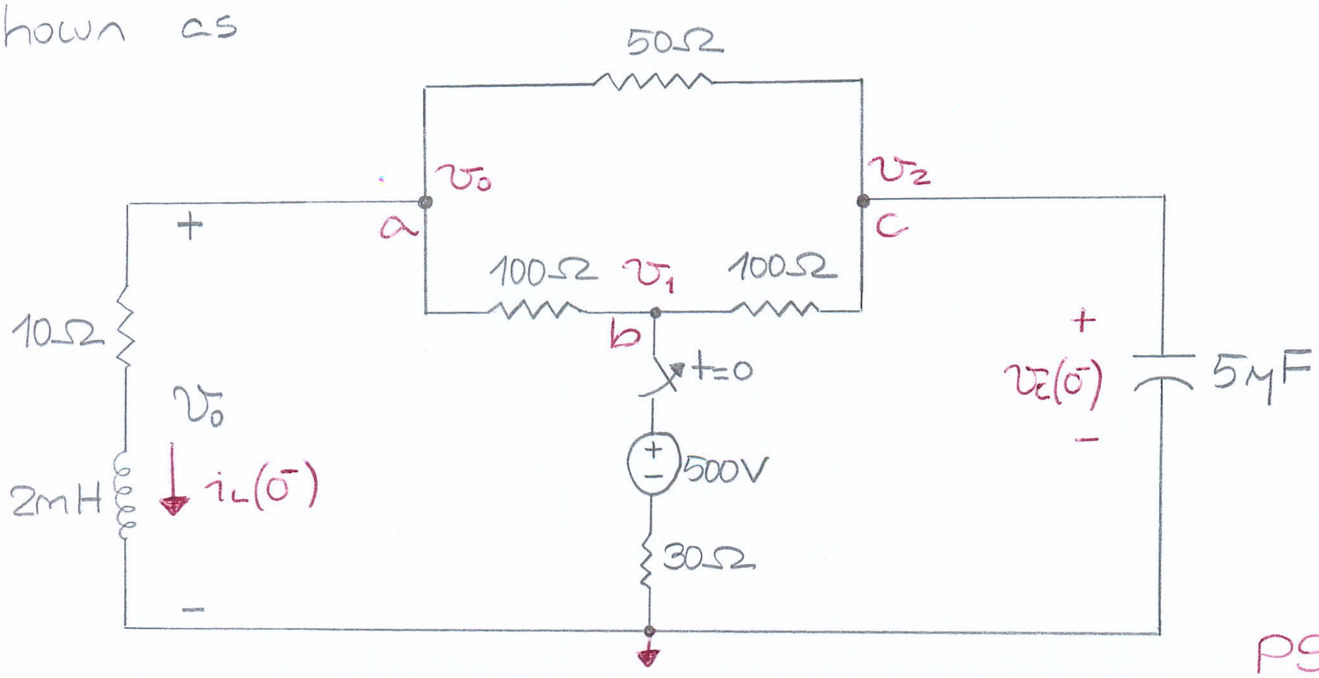
$$C_2 = -\frac{12000}{s+500}\bigg|_{s=-2000} = 8$$

Hence ;

$$v_o = \mathcal{L}^{-1}\left\{-\frac{8}{s+500} + \frac{8}{s+2000}\right\}$$

$$= \left(-8e^{-500t} + 8e^{-2000t}\right)u(t)$$

Problem 2) The switch has been closed for a long time before opening at $t=0$ in the circuit shown as



PS 13.2

a. Construct the s-domain equivalent circuit for $t>0$.

b. Find V_o.

c. Find v_o for $t \geq 0$.

Solution. Before $t=0$, we use node-voltage method to get (inductor is short-circuit, capacitor is open-circuit)

ⓐ :
$$\underset{(10)}{\frac{v_o}{10}} + \underset{(1)}{\frac{v_o - v_1}{100}} + \underset{(2)}{\frac{v_o - v_2}{50}} = 0$$

$$\Rightarrow 13v_o - v_1 - 2v_2 = 0 \qquad (1)$$

ⓑ :
$$\underset{(3)}{\frac{v_1 - v_o}{100}} + \underset{(10)}{\frac{v_1 - 500}{30}} + \underset{(3)}{\frac{v_1 - v_2}{100}} = 0$$

$$\Rightarrow -3v_o + 16v_1 - 3v_2 = 5000 \qquad (2)$$

ⓒ :
$$\underset{(1)}{\frac{v_2 - v_1}{100}} + \underset{(2)}{\frac{v_2 - v_o}{50}} = 0$$

$$\Rightarrow -2v_o - v_1 + 3v_2 = 0 \Rightarrow v_1 = -2v_o + 3v_2 \qquad (3)$$

Hence;

(3) in (1) : $13v_o - (-2v_o + 3v_2) - 2v_2 = 0$

$$\Rightarrow 15v_o - 5v_2 = 0 \Rightarrow v_2 = 3v_o \qquad (4)$$

(3) in (2) : $-3v_o + 16(-2v_o + 3v_2) - 3v_2 = 5000$

$$\Rightarrow -35v_o + 45v_2 = 5000 \qquad (5)$$

(4) in (5) : $-35v_o + 45 \cdot (3v_o) = 5000$

$$\Rightarrow v_o = 50\,V, \quad v_2 = 150\,V$$

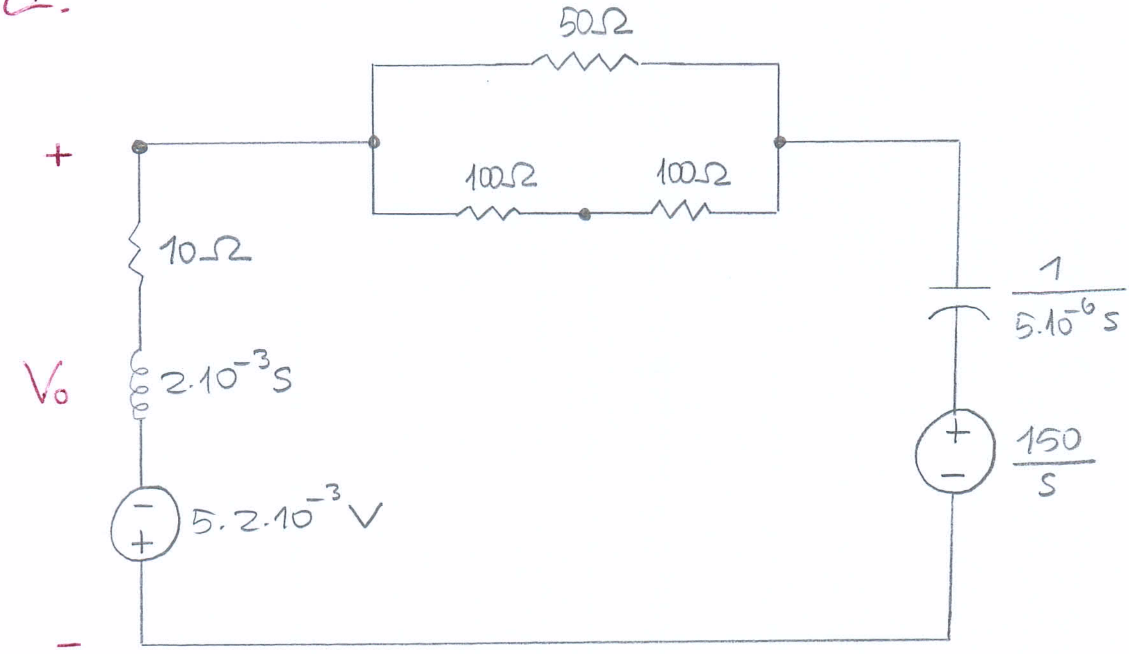
PS 13.3

Hence ;

$$i_L(0^-) = i_L(0) = i_L(0^+) = \frac{v_o}{10} = \frac{50}{10} = 5A$$

$$v_c(0^-) = v_c(0) = v_c(0^+) = v_2 = 150V$$

a.



$$50\Omega \parallel (100\Omega \sim 100\Omega) \Rightarrow R_{eq} = \frac{50.200}{50+200} = 40\Omega$$

b.

Hence;

$$\frac{V_o + 0.01}{2.10^{-3}s + 10} + \frac{V_o - (150/s)}{(2.10^5/s) + 40} = 0$$

$$\Rightarrow \frac{500V_o + 5}{s + 5000} + \frac{sV_o - 150}{40s + 2.10^5} = 0$$

$$\Rightarrow \frac{500V_o + 5}{s+5000} + \frac{s(V_o/40) - (15/4)}{s + 5000} = 0$$

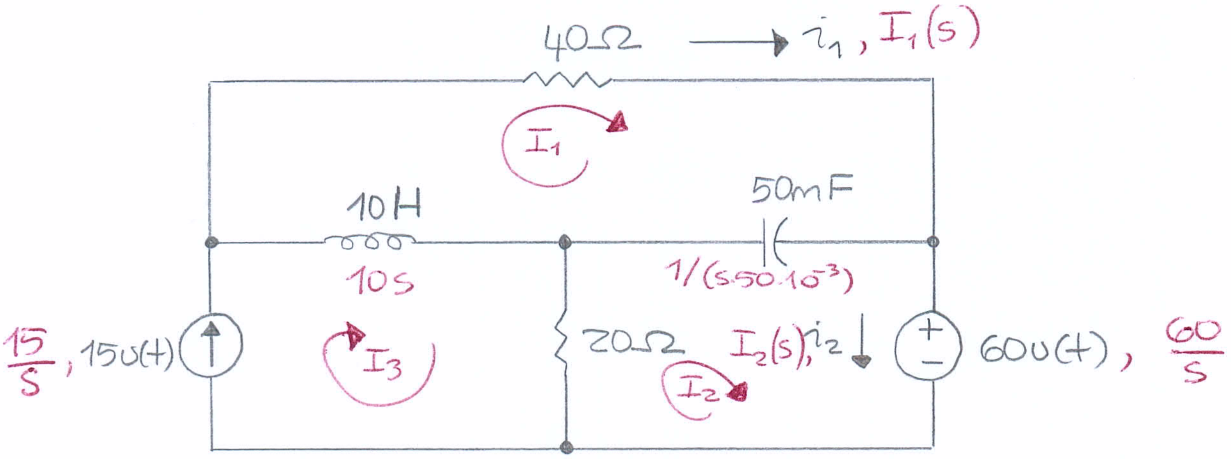
$$\Rightarrow \left(\frac{s}{40} + 500\right)V_o + \frac{5}{4} = 0 \Rightarrow V_o = -\frac{50}{s+20000}$$

PS 13.4

Therefore;

c. $v_o = \mathcal{L}^{-1}\{V_o\} = -50e^{-20000t}u(t)$

Problem 3) There is no energy stored at the time the sources are energized in the circuit shown as



a. Find $I_1(s)$ and $I_2(s)$.

b. Use the initial- and final-value theorems to check the initial- and final-values of $i_1(t)$ and $i_2(t)$.

c. Find $i_1(t)$ and $i_2(t)$ for $t \geq 0$.

Solution. Note that

$$I_3 = \frac{15}{s}$$

a. We first write mesh equations as follows

Mesh 1 :

$$40I_1 + \frac{20}{s}(I_1 - I_2) + 10s\left(I_1 - \frac{15}{s}\right) = 0$$

$$\Rightarrow \left(40 + \frac{20}{s} + 10s\right)I_1 - \frac{20}{s}I_2 = 150 \qquad (1)$$

Mesh 2 :

$$20\left(I_2-\frac{15}{s}\right)+\frac{20}{s}(I_2-I_1)+\frac{60}{s}=0$$

$$\Rightarrow -\frac{20}{s}I_1+\left(20+\frac{20}{s}\right)I_2=\frac{240}{s} \qquad (2)$$

-we elaborately obtain

$$(s+1)/\ (s^2+4s+2)I_1-2I_2=15s$$

$$2/\ -I_1+(s+1)I_2=12$$

$$\Rightarrow \left[(s+1)(s^2+4s+2)-2\right]I_1=15s(s+1)+24$$

$$\Rightarrow (s^3+5s^2+6s)I_1=15s^2+15s+24$$

$$\Rightarrow \qquad I_1=\frac{15s^2+15s+24}{s(s+2)(s+3)}$$

and

$$I_2=\frac{1}{s+1}\left[12+\frac{15s^2+15s+24}{s^3+5s^2+6s}\right]$$

$$=\frac{12s^3+75s^2+87s+24}{s(s+1)(s+2)(s+3)}$$

$$=\frac{\cancel{(s+1)}(12s^2+63s+24)}{s\cancel{(s+1)}(s+2)(s+3)}$$

$$=\frac{12s^2+63s+24}{s(s+2)(s+3)}$$

b.

$$i_1(0) = \lim_{s\to\infty} s\, I_1(s)$$

$$= \lim_{s\to\infty} \cancel{s}\, \frac{15s^2+15s+24}{\cancel{s}(s+2)(s+3)}$$

$$= \lim_{s\to\infty} \frac{\cancel{s^2}\left(15+\frac{15}{s}+\frac{24}{s^2}\right)}{\cancel{s^2}\left(1+\frac{5}{s}+\frac{6}{s^2}\right)}$$

$$= 15$$

$$i_1(\infty) = \lim_{s\to 0} s\, I_1(s)$$

$$= \lim_{s\to 0} \cancel{s}\, \frac{15s^2+15s+24}{\cancel{s}(s+2)(s+3)}$$

$$= \frac{24}{2\cdot 3}$$

$$= 4$$

and

$$i_2(0) = \lim_{s\to\infty} s\, I_2(s)$$

$$= \lim_{s\to\infty} \cancel{s}\, \frac{12s^2+63s+24}{\cancel{s}(s+2)(s+3)}$$

$$= \lim_{s\to\infty} \frac{\cancel{s^2}\left(12+\frac{63}{s}+\frac{24}{s^2}\right)}{\cancel{s^2}\left(1+\frac{5}{s}+\frac{6}{s^2}\right)}$$

$$= 12$$

$$i_2(\infty) = \lim_{s\to 0} s\, I_2(s)$$

$$= \lim_{s\to 0} \cancel{s}\, \frac{12s^2+63s+24}{\cancel{s}(s+2)(s+3)}$$

$$= \frac{24}{2\cdot 3}$$

$$= 4$$

c. We find that

$$I_1(s) = \frac{15s^2+15s+24}{s(s+2)(s+3)} = \frac{C_1}{s} + \frac{C_2}{s+2} + \frac{C_3}{s+3}$$

$$\Rightarrow C_1 = \left.\frac{15s^2+15s+24}{(s+2)(s+3)}\right|_{s=0} = \frac{24}{2\cdot 3} = 4$$

$$C_2 = \left.\frac{15s^2+15s+24}{s(s+3)}\right|_{s=-2} = \frac{60-30+24}{(-2)\cdot 1} = -27$$

$$C_3 = \left.\frac{15s^2+15s+24}{s(s+2)}\right|_{s=-3} = \frac{135-45+24}{(-3)\cdot(-1)} = 38$$

Hence;

$$i_1(t) = \mathcal{L}^{-1}\{I_1(s)\} = \left(4-27e^{-2t}+38e^{-3t}\right)u(t)$$

and

$$I_2(s) = \frac{12s^2+63s+24}{s(s+2)(s+3)} = \frac{d_1}{s} + \frac{d_2}{s+2} + \frac{d_3}{s+3}$$

$$\Rightarrow d_1 = \left.\frac{12s^2+63s+24}{(s+2)(s+3)}\right|_{s=0} = \frac{24}{2\cdot 3} = 4$$

$$d_2 = \left.\frac{12s^2+63s+24}{s(s+3)}\right|_{s=-2} = \frac{48-126+24}{(-2)\cdot 1} = 27$$

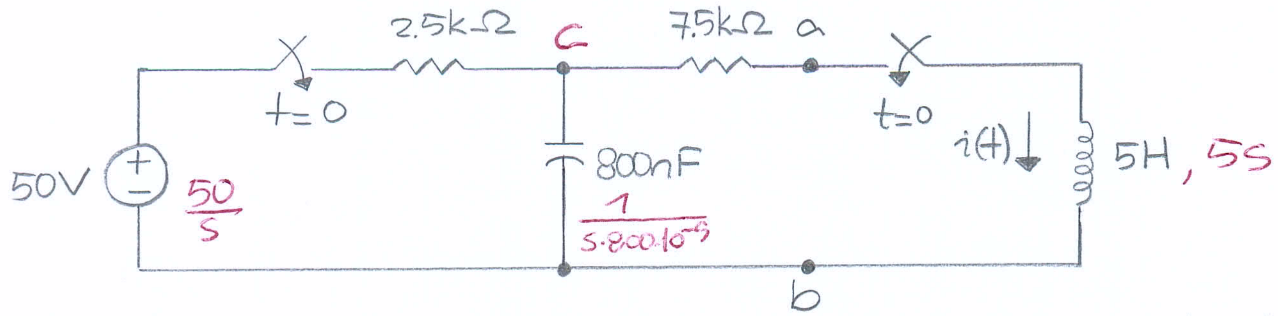
$$d_3 = \left.\frac{12s^2+63s+24}{s(s+2)}\right|_{s=-3} = \frac{108-189+24}{(-3)\cdot(-1)} = -19$$

Thus;

PS 13.8

$$i_2(t) = \mathcal{L}^{-1}\{I_2(s)\} = \left(4+27e^{-2t}-19e^{-3t}\right)u(t)$$

Problem 4) The two switches operate simultaneously in the circuit shown as



There is no energy stored in the circuit at the instant the switches close. Find $i(t)$ for $t \geq 0^+$ by first finding the s-domain Thévenin equivalent of the circuit to the left of the terminals a, b.

Solution. We notice that

$$V_{Th} = V_{ab} = V_{cd} = \frac{50}{s} \frac{10^7/8s}{2500 + \frac{10^7}{8s}}$$

$$= \frac{5.10^8}{s(20000s + 10^7)} = \frac{5.10^4}{s(2s+1000)} = \frac{25000}{s(s+500)}$$

- for the Thévenin impedance, we calculate the equivalent impedance seen through a, b:

$$2500\,\Omega \parallel \frac{10^7}{8s} \Rightarrow Z_{eq1} = \frac{2500 \cdot 10^7/8s}{2500 + \frac{10^7}{8s}}$$

$$\Rightarrow Z_{eq1} = \frac{25.10^7}{20000s + 10^7}$$

$$Z_{eq1} \sim 7500\,\Omega \Rightarrow Z_{Th} = 7500 + \frac{25.10^9}{20000s + 10^7}$$

$$= 7500 + \frac{25.10^5}{2s + 1000}$$

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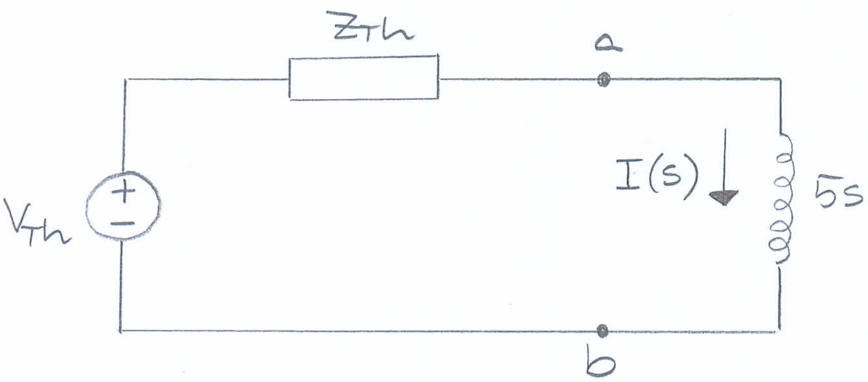
Hence;

$$Z_{Th} = \frac{15000s + 75\cdot 10^5 + 25\cdot 10^5}{2s + 1000}$$

$$= \frac{7500s + 5\cdot 10^6}{s + 500}$$

- we thus have



$$I(s) = \frac{V_{Th}}{Z_{Th} + 5s} = \frac{25000 / [s(s+500)]}{\frac{7500s + 5\cdot 10^6}{s+500} + 5s}$$

$$= \frac{25000}{s(7500s + 5\cdot 10^6) + 5s^2(s+500)}$$

$$= \frac{25000}{5s^3 + 10000s^2 + 5\cdot 10^6 s}$$

$$= \frac{\overset{5}{\cancel{25}}000}{\cancel{5}s(s^2 + 2000s + 10^6)}$$

$$= \frac{5\cdot 10^3}{s(s+1000)^2}$$

$$= \frac{C_1}{s} + \frac{C_2}{(s+1000)^2} + \frac{C_3}{s+1000}$$

$$\Rightarrow \quad C_1 = \left.\frac{5.10^3}{(s+1000)^2}\right|_{s=0} = 5.10^{-3}$$

$$C_2 = \left.\frac{5.10^3}{s}\right|_{s=-1000} = -5$$

$$C_3 = \left.\frac{d}{ds}\left(\frac{5.10^3}{s}\right)\right|_{s=-1000} = -\left.\frac{5.10^3}{s^2}\right|_{s=-1000} = -5.10^{-3}$$

Hence;

$$\mathcal{L}^{-1}\{I(s)\} = i(t) = 5.10^{-3} - 5te^{-1000t} - 5.10^{-3}e^{-1000t}, \; t \geq 0$$

$$\Rightarrow \quad i(t) = 5\left(1 - 1000te^{-1000t} - e^{-1000t}\right)u(t) \text{ mA}$$