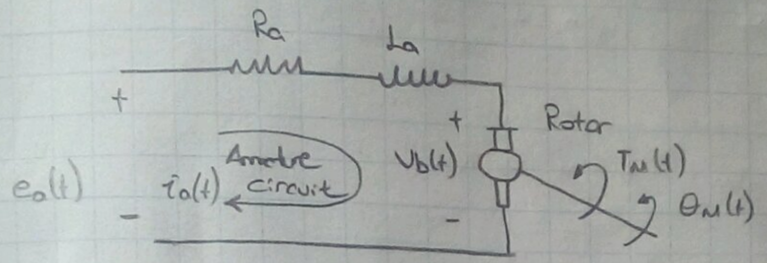
2.42) $v_b(t) = K_b \cdot \frac{d\theta_M(t)}{dt} \longrightarrow V_b(s) = K_b \cdot s \cdot \theta_M(s)$ ①

$v_b(t)$ ↓ electromotive force; K_b ↓ cons.; $\frac{d\theta_M(t)}{dt}$ ↓ angular velocity.



$$R_a \cdot I_a(s) + L_a \cdot s \cdot \underline{I_a(s)} + V_b(s) = E_a(s) \quad ②$$

Armature Current

Torque developed by the motor

$$T_M(s) = K_t \cdot I_a(s)$$

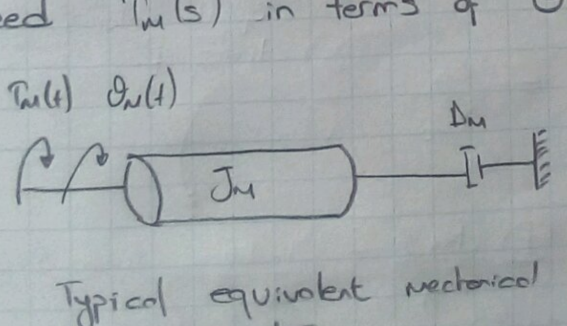
or

$$I_a(s) = \frac{1}{K_t} \cdot T_M(s) \quad ③$$

① + ③ → ② ⟹ Transfer func. of the motor:

$$E_a(s) = \frac{(R_a + L_a \cdot s)\, T_M(s)}{K_t} + K_b \cdot s \cdot \theta_M \quad ④$$

We need $T_M(s)$ in terms of $\theta_M(s)$ ~ $\theta_M(s) / E_a(s)$



Typical equivalent mechanical loading motor

$$T_M(s) = (J_M s^2 + D_M s) \cdot \theta_M(s) \quad ⑤$$

$⑤ \to ④ \Rightarrow \dfrac{(R_a + L_a s).(J_m s^2 + D_m s).\theta_m(s)}{K_t} + K_b s \theta_m(s) = E_a(s)$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a . J_m)}{s\left[s + \frac{1}{J_m} . \left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

Constants

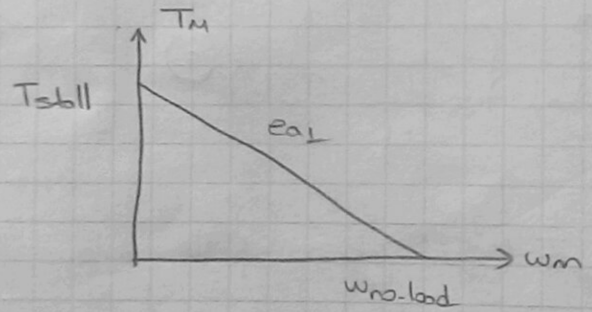
$Eq. 4 \to (L_a = 0)$ $\quad \dfrac{R_a}{K_t} . T_m + K_b . w_m = e_a$

$\left(w_m = s\theta(s) = \dfrac{d\theta_m}{dt}\right)$

angular vel.

$$T_m = -\frac{K_b . K_t}{R_a} . w_m + \frac{K_t}{R_a} . e_a \quad ⑥$$

Eq. ⑥ is a straight line



Torque speed curve

$T_{stall} = \dfrac{K_t}{R_a} . e_a$

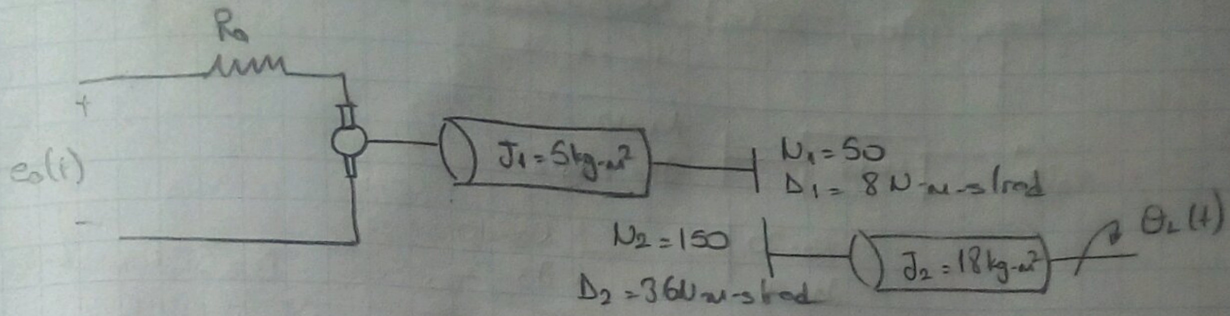
$w_{no-load} = \dfrac{e_a}{K_b}$

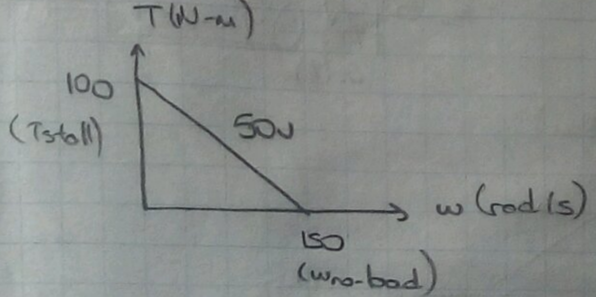
Electrical constants of the motor's trans. F

① $\dfrac{K_t}{R_a} = \dfrac{T_{stall}}{e_a}$ $\quad (w_m = 0$ @ 6)

② $K_b = \dfrac{e_a}{w_{no-load}}$ $\quad (T_m = 0$ @ 6)

2.42) For the motor, load, and torque-speed curve, find the transfer func., $G(s) = \theta_L(s) / E_a(s)$





$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a} = \frac{100}{50} = \underline{\underline{2}}$$

$$\frac{E_a}{w_{no\text{-}load}} = \frac{50}{150} = \underline{\underline{1/3}} = K_b$$

$$J_m = 5 + 18 \cdot \left(\frac{50}{150}\right)^2 = 7$$

$$D_m = 8 + 36 \cdot \left(\frac{50}{150}\right)^2 = 12$$

$$\frac{\theta_M(s)}{E_a(s)} = \frac{K_t / (R_a \cdot J_m)}{s\left[s + \frac{1}{J_m} \cdot \left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

$$= \frac{2/7}{s\left[s + \frac{1}{7} \cdot \left(12 + \frac{2}{3}\right)\right]}$$

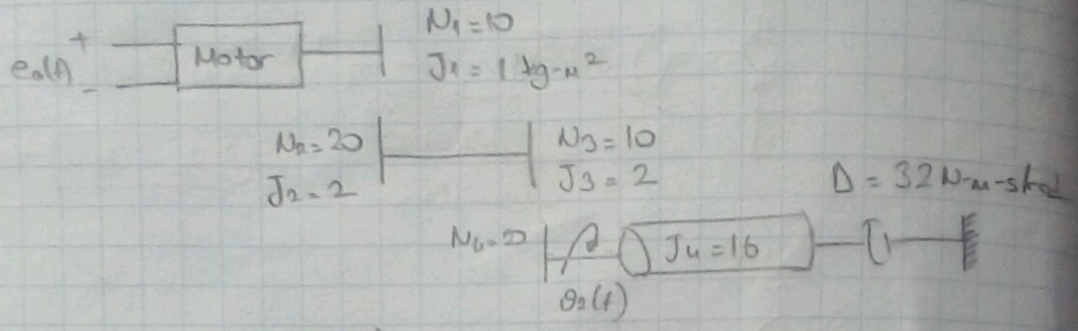
$$\frac{\theta_M}{E_a} = \frac{2/7}{s\left(s + \frac{38}{21}\right)}$$

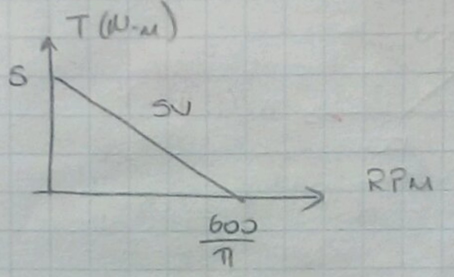
$$\theta_L = \theta_M \left(\frac{50}{150}\right)$$

$$\theta_L = \frac{1}{3} \theta_M$$

$$\boxed{\frac{\theta_L}{E_a} = \frac{2/21}{s\left(s + \frac{38}{21}\right)}}$$

2.43) The motor whose torque-speed char. are shown in fig. drives the load shown in the diagram. Find the tran. func., $G(s) = \theta_2(s) / E_a(s)$





$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a} = \frac{5}{5} = 1$$

$$K_b = \frac{e_a}{\omega_{no\text{-}load}} = \frac{5}{\underbrace{\frac{600}{\pi} \cdot \frac{1}{60}}_{\text{RPS} \sim \text{freq.}} \cdot 2\pi} = \frac{1}{4}$$

$w = 2\pi f$

$$J_m = 1 + 2 \cdot \left(\frac{10}{20}\right)^2 + 2 \cdot \left(\frac{10}{20}\right)^2 + 16 \left(\frac{1}{2} \cdot \frac{1}{2}\right)^2 = 3$$

$$D_m = 32 \left(\frac{1}{2} \cdot \frac{1}{2}\right)^2 = 2$$

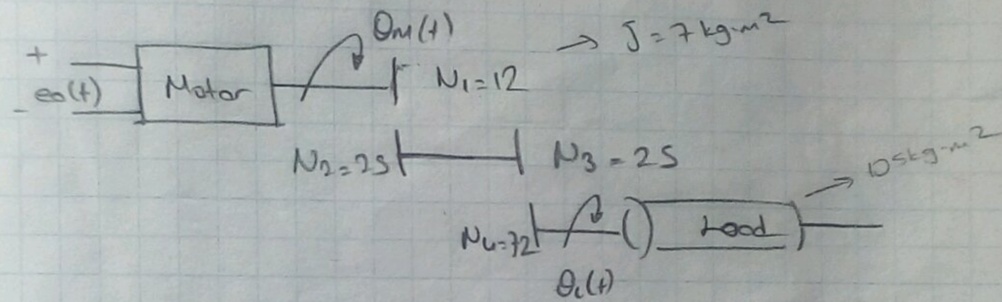
$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t / (R_a J_m)}{s\left[s + \frac{1}{J_m} \left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

$$= \frac{1/3}{s\left[s + \frac{1}{3}\left(2 + \frac{1}{4}\right)\right]} = \frac{1/3}{s(s + 0.75)}$$

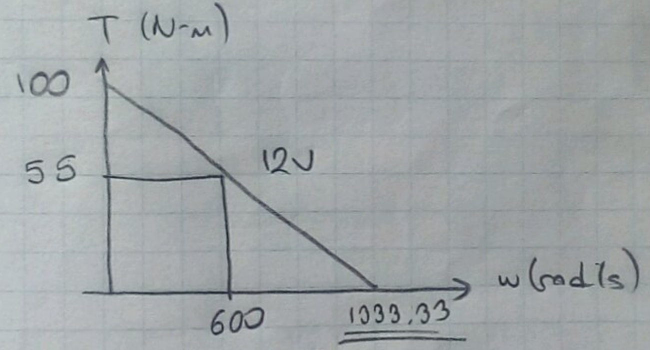
$\theta_2(s) = \frac{1}{4}\theta_2(s)$

$$\boxed{\frac{\theta_2}{E_a} = \frac{1/12}{s(s+0.75)}}$$

2.45) A dc-motor develops 55 N-m of torque at a speed of 600 rad/s when 12 V are applied. It stalls out at this voltage with 100 N-m (T_{stall}) of torque. If the inertia and damping of the armature are 7 kg-m^2 and 3 N-m s/rad (D) find the tran. f. $G(s) = \theta_L(s)/E_a(s)$ of this motor if it drives an inertia load of 105 kg-m^2 through a gear train.



Sol. →



$$\frac{K_t}{R_a} = \frac{T_{stall}}{E_a} = \frac{100}{12}$$

$$K_b = \frac{E_a}{w_{no\text{-}load}} = \frac{12}{1333.33}$$

$$J_M = 7 + 105\left(\frac{25}{72} \cdot \frac{12}{25}\right)^2 = 9.92 \qquad D = 3$$

$$\frac{\theta_M}{E_a} = \frac{100/12 . (9.92)}{s\left(s + \frac{1}{9.92}\left(3 + \frac{100}{12} \cdot \frac{12}{1333}\right)\right)} = \frac{0.84}{s(s+0.31)}$$

$$\theta_L = \left(\frac{25}{72} \cdot \frac{12}{25}\right)\theta_m = \frac{1}{6}\theta_m$$

$$\boxed{G(s) = \frac{\theta_L}{E_a} = \frac{0.14}{s(s+0.31)}}$$