EEEN 460
Optimal Control

2020 Spring

Lecture III
State-Space Modeling

Eigenvalues of an $n \times n$ Matrix A. The eigenvalues of an $n \times n$ matrix $\mathbf{A}$ are the roots of the characteristic equation

$$|\lambda\mathbf{I} - \mathbf{A}| = 0$$

The eigenvalues are also called the characteristic roots.

Consider, for example, the following matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

The characteristic equation is

$$|\lambda\mathbf{I} - \mathbf{A}| = \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda + 6 \end{vmatrix}$$

$$= \lambda^3 + 6\lambda^2 + 11\lambda + 6$$

$$= (\lambda + 1)(\lambda + 2)(\lambda + 3) = 0$$

The eigenvalues of $\mathbf{A}$ are the roots of the characteristic equation, or -1, -2, and -3.

Diagonalization of $n \times n$ Matrix. Note that if an $n \times n$ matrix $\mathbf{A}$ with distinct eigenvalues is given by

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}$$

the transformation $\mathbf{x} = \mathbf{P}\mathbf{z}$, where

$$\mathbf{P} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \cdots & \lambda_n^2 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \cdots & \lambda_n^{n-1} \end{bmatrix}$$

$\lambda_1, \lambda_2, \ldots, \lambda_n = n$ distinct eigenvalues of $\mathbf{A}$

will transform $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ into the diagonal matrix, or

$$\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{bmatrix} \lambda_1 & & & & & 0 \\ & \lambda_2 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ 0 & & & & & \lambda_n \end{bmatrix}$$

Example

Consider the following state-space representation of a system.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$y = \mathbf{C}\mathbf{x}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

The eigenvalues of matrix **A** are

$$\lambda_1 = -1, \qquad \lambda_2 = -2, \qquad \lambda_3 = -3$$

Thus, three eigenvalues are distinct. If we define a set of new state variables z_1, z_2, and z_3 by the transformation

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

or

$$\mathbf{x} = \mathbf{P}\mathbf{z}$$

where

$$\mathbf{P} = \begin{bmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}$$

then, by substituting into Equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$\mathbf{P}\dot{\mathbf{z}} = \mathbf{A}\mathbf{P}\mathbf{z} + \mathbf{B}u$$

$$\mathbf{P}\dot{\mathbf{z}} = \mathbf{A}\mathbf{P}\mathbf{z} + \mathbf{B}u$$

By premultiplying both sides of this last equation by $\mathbf{P}^{-1}$, we get

$$\dot{\mathbf{z}} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}\mathbf{z} + \mathbf{P}^{-1}\mathbf{B}u$$

or

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 3 & 2.5 & 0.5 \\ -3 & -4 & -1 \\ 1 & 1.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$+ \begin{bmatrix} 3 & 2.5 & 0.5 \\ -3 & -4 & -1 \\ 1 & 1.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} u$$

Simplifying gives

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix} u$$

The output equation,

$$y = \mathbf{CPz}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

State space equation

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

Invariance of Eigenvalues. To prove the invariance of the eigenvalues under a linear transformation, we must show that the characteristic polynomials $|\lambda\mathbf{I} - \mathbf{A}|$ and $|\lambda\mathbf{I} - \mathbf{P}^{-1}\mathbf{A}\mathbf{P}|$ are identical.

Since the determinant of a product is the product of the determinants, we obtain

$$\begin{aligned}|\lambda\mathbf{I} - \mathbf{P}^{-1}\mathbf{A}\mathbf{P}| &= |\lambda\mathbf{P}^{-1}\mathbf{P} - \mathbf{P}^{-1}\mathbf{A}\mathbf{P}| \\ &= |\mathbf{P}^{-1}(\lambda\mathbf{I} - \mathbf{A})\mathbf{P}| \\ &= |\mathbf{P}^{-1}||\lambda\mathbf{I} - \mathbf{A}||\mathbf{P}| \\ &= |\mathbf{P}^{-1}||\mathbf{P}||\lambda\mathbf{I} - \mathbf{A}|\end{aligned}$$

Noting that the product of the determinants $|\mathbf{P}^{-1}|$ and $|\mathbf{P}|$ is the determinant of the product $|\mathbf{P}^{-1}\mathbf{P}|$, we obtain

$$\begin{aligned}|\lambda\mathbf{I} - \mathbf{P}^{-1}\mathbf{A}\mathbf{P}| &= |\mathbf{P}^{-1}\mathbf{P}||\lambda\mathbf{I} - \mathbf{A}| \\ &= |\lambda\mathbf{I} - \mathbf{A}|\end{aligned}$$

Thus, we have proved that the eigenvalues of $\mathbf{A}$ are invariant under a linear transformation.

Nonuniqueness of a Set of State Variables. It has been stated that a set of state variables is not unique for a given system. Suppose that $x_1, x_2, \ldots, x_n$ are a set of state variables.

Then we may take as another set of state variables any set of functions

$$\begin{aligned}
\hat{x}_1 &= X_1(x_1, x_2, \ldots, x_n) \\
\hat{x}_2 &= X_2(x_1, x_2, \ldots, x_n) \\
&\ \ \vdots \\
\hat{x}_n &= X_n(x_1, x_2, \ldots, x_n)
\end{aligned}$$

provided that, for every set of values $\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n$, there corresponds a unique set of values $x_1, x_2, \ldots, x_n$, and vice versa. Thus, if $\mathbf{x}$ is a state vector, then $\hat{\mathbf{x}}$, where

$$\hat{\mathbf{x}} = \mathbf{P}\mathbf{x}$$

is also a state vector, provided the matrix $\mathbf{P}$ is nonsingular. Different state vectors convey the same information about the system behavior.

SOLVING THE TIME-INVARIANT STATE EQUATION

In this section, we shall obtain the general solution of the linear time-invariant state equation. We shall first consider the homogeneous case and then the nonhomogeneous case.

Solution of Homogeneous State Equations. Before we solve vector-matrix differential equations, let us review the solution of the scalar differential equation

$$\dot{x} = ax$$

In solving this equation, we may assume a solution $x(t)$ of the form

$$x(t) = b_0 + b_1 t + b_2 t^2 + \cdots + b_k t^k + \cdots$$

By substituting this assumed solution into Equation above we obtain

$$\begin{aligned} &b_1 + 2b_2 t + 3b_3 t^2 + \cdots + kb_k t^{k-1} + \cdots \\ &= a(b_0 + b_1 t + b_2 t^2 + \cdots + b_k t^k + \cdots) \end{aligned}$$

$$\dot{x} = ax$$

In solving this equation, we may assume a solution $x(t)$ of the form

$$x(t) = b_0 + b_1 t + b_2 t^2 + \cdots + b_k t^k + \cdots$$

By substituting this assumed solution into Equation above we obtain

$$b_1 + 2b_2 t + 3b_3 t^2 + \cdots + kb_k t^{k-1} + \cdots$$
$$= a(b_0 + b_1 t + b_2 t^2 + \cdots + b_k t^k + \cdots)$$

If the assumed solution is to be the true solution, Equation above must hold for any t. Hence, equating the coefficients of the equal powers of t, we obtain

$$b_1 = ab_0$$
$$b_2 = \frac{1}{2}ab_1 = \frac{1}{2}a^2 b_0$$
$$b_3 = \frac{1}{3}ab_2 = \frac{1}{3 \times 2}a^3 b_0$$
$$\vdots$$
$$b_k = \frac{1}{k!}a^k b_0$$

The value of b_0 is determined by substituting $t = 0$ into Equation above or

$$x(0) = b_0$$

Hence, the solution $x(t)$ can be written as

$$x(t) = \left(1 + at + \frac{1}{2!}a^2 t^2 + \cdots + \frac{1}{k!}a^k t^k + \cdots\right)x(0)$$
$$= e^{at}x(0)$$

We shall now solve the vector-matrix differential equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

where $\mathbf{x} = n$-vector

$\mathbf{A} = n \times n$ constant matrix

By analogy with the scalar case, we assume that the solution is in the form of a vector power series in t, or

$$\mathbf{x}(t) = \mathbf{b}_0 + \mathbf{b}_1 t + \mathbf{b}_2 t^2 + \cdots + \mathbf{b}_k t^k + \cdots$$

$$\mathbf{b}_1 + 2\mathbf{b}_2 t + 3\mathbf{b}_3 t^2 + \cdots + k\mathbf{b}_k t^{k-1} + \cdots$$
$$= \mathbf{A}\left(\mathbf{b}_0 + \mathbf{b}_1 t + \mathbf{b}_2 t^2 + \cdots + \mathbf{b}_k t^k + \cdots\right)$$

$$\mathbf{b}_1 + 2\mathbf{b}_2 t + 3\mathbf{b}_3 t^2 + \cdots + k\mathbf{b}_k t^{k-1} + \cdots$$

$$= \mathbf{A}\left(\mathbf{b}_0 + \mathbf{b}_1 t + \mathbf{b}_2 t^2 + \cdots + \mathbf{b}_k t^k + \cdots\right)$$

$$\mathbf{b}_1 = \mathbf{A}\mathbf{b}_0$$

$$\mathbf{b}_2 = \frac{1}{2}\mathbf{A}\mathbf{b}_1 = \frac{1}{2}\mathbf{A}^2\mathbf{b}_0$$

$$\mathbf{b}_3 = \frac{1}{3}\mathbf{A}\mathbf{b}_2 = \frac{1}{3 \times 2}\mathbf{A}^3\mathbf{b}_0$$

$$\vdots$$

$$\mathbf{b}_k = \frac{1}{k!}\mathbf{A}^k\mathbf{b}_0$$

Thus, the solution $\mathbf{x}(t)$ can be written as

$$\mathbf{x}(t) = \left(\mathbf{I} + \mathbf{A}t + \frac{1}{2!}\mathbf{A}^2 t^2 + \cdots + \frac{1}{k!}\mathbf{A}^k t^k + \cdots\right)\mathbf{x}(0)$$

$$\mathbf{I} + \mathbf{A}t + \frac{1}{2!}\mathbf{A}^2 t^2 + \cdots + \frac{1}{k!}\mathbf{A}^k t^k + \cdots = e^{\mathbf{A}t}$$

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0)$$

Laplace Transform Approach to the Solution of Homogeneous State Equations

Let us first consider the scalar case:

$$\dot{x} = ax$$

Taking the Laplace transform of Equation we obtain

$$sX(s) - x(0) = aX(s)$$

where $X(s) = \mathcal{L}[x]$. Solving Equation for $X(s)$ gives

$$X(s) = \frac{x(0)}{s - a} = (s - a)^{-1}x(0)$$

The inverse Laplace transform of this last equation gives the solution

$$x(t) = e^{at}x(0)$$

The foregoing approach to the solution of the homogeneous scalar differential equation can be extended to the homogeneous state equation:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)$$

Taking the Laplace transform of both sides of Equation , we obtain

$$s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s)$$

where $\mathbf{X}(s) = \mathscr{L}[\mathbf{x}]$. Hence,

$$(s\mathbf{I} - \mathbf{A})\mathbf{X}(s) = \mathbf{x}(0)$$

Premultiplying both sides of this last equation by $(s\mathbf{I} - \mathbf{A})^{-1}$, we obtain

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}(0)$$

The inverse Laplace transform of $\mathbf{X}(s)$ gives the solution $\mathbf{x}(t)$. Thus,

$$\mathbf{x}(t) = \mathscr{L}^{-1}\left[(s\mathbf{I} - \mathbf{A})^{-1}\right]\mathbf{x}(0)$$

Note that

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{\mathbf{I}}{s} + \frac{\mathbf{A}}{s^2} + \frac{\mathbf{A}^2}{s^3} + \cdots$$

Hence, the inverse Laplace transform of $(s\mathbf{I} - \mathbf{A})^{-1}$ gives

$$\mathscr{L}^{-1}\left[(s\mathbf{I} - \mathbf{A})^{-1}\right] = \mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2t^2}{2!} + \frac{\mathbf{A}^3t^3}{3!} + \cdots = e^{\mathbf{A}t}$$

$$\boxed{\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0)}$$

State-Transition Matrix

We can write the solution of the homogeneous state equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

as

$$\mathbf{x}(t) = \mathbf{\Phi}(t)\mathbf{x}(0)$$

where $\mathbf{\Phi}(t)$ is an $n \times n$ matrix and is the unique solution of

$$\dot{\mathbf{\Phi}}(t) = \mathbf{A}\mathbf{\Phi}(t), \qquad \mathbf{\Phi}(0) = \mathbf{I}$$

To verify this, note that

$$\mathbf{x}(0) = \mathbf{\Phi}(0)\mathbf{x}(0) = \mathbf{x}(0)$$

and

$$\dot{\mathbf{x}}(t) = \dot{\mathbf{\Phi}}(t)\mathbf{x}(0) = \mathbf{A}\mathbf{\Phi}(t)\mathbf{x}(0) = \mathbf{A}\mathbf{x}(t)$$

$$\mathbf{\Phi}(t) = e^{\mathbf{A}t} = \mathcal{L}^{-1}\left[(s\mathbf{I} - \mathbf{A})^{-1}\right]$$

Note that

$$\mathbf{\Phi}^{-1}(t) = e^{-\mathbf{A}t} = \mathbf{\Phi}(-t)$$

If the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ of the matrix $\mathbf{A}$ are distinct, than $\mathbf{\Phi}(t)$ will contain the n exponentials

$$e^{\lambda_1 t}, e^{\lambda_2 t}, \ldots, e^{\lambda_n t}$$

In particular, if the matrix $\mathbf{A}$ is diagonal, then

$$\mathbf{\Phi}(t) = e^{\mathbf{A}t} = \begin{bmatrix} e^{\lambda_1 t} & & & & & 0 \\ & e^{\lambda_2 t} & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ 0 & & & & & e^{\lambda_n t} \end{bmatrix} \qquad (\mathbf{A}\text{: diagonal})$$

Properties of State-Transition Matrices

We shall now summarize the important properties of the state-transition matrix $\Phi(t)$

For the time-invariant system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

for which

$$\Phi(t) = e^{\mathbf{A}t}$$

we have the following:

1. $\Phi(0) = e^{\mathbf{A}0} = \mathbf{I}$
2. $\Phi(t) = e^{\mathbf{A}t} = \left(e^{-\mathbf{A}t}\right)^{-1} = \left[\Phi(-t)\right]^{-1}$ or $\Phi^{-1}(t) = \Phi(-t)$
3. $\Phi(t_1 + t_2) = e^{\mathbf{A}(t_1+t_2)} = e^{\mathbf{A}t_1}e^{\mathbf{A}t_2} = \Phi(t_1)\Phi(t_2) = \Phi(t_2)\Phi(t_1)$
4. $\left[\Phi(t)\right]^n = \Phi(nt)$
5. $\Phi(t_2 - t_1)\Phi(t_1 - t_0) = \Phi(t_2 - t_0) = \Phi(t_1 - t_0)\Phi(t_2 - t_1)$

EXAMPLE

Obtain the state-transition matrix $\Phi(t)$ of the following system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Obtain also the inverse of the state-transition matrix, $\Phi^{-1}(t)$.

SOLUTION

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

For this system,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

The state-transition matrix $\mathbf{\Phi}(t)$ is given by

$$\mathbf{\Phi}(t) = e^{\mathbf{A}t} = \mathscr{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]$$

Since

$$s\mathbf{I} - \mathbf{A} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

the inverse of $(s\mathbf{I} - \mathbf{A})$ is given by

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{1}{(s+1)(s+2)} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{s+3}{(s+1)(s+2)} & \dfrac{1}{(s+1)(s+2)} \\ \dfrac{-2}{(s+1)(s+2)} & \dfrac{s}{(s+1)(s+2)} \end{bmatrix}$$

Hence,

$$\mathbf{\Phi}(t) = e^{\mathbf{A}t} = \mathcal{L}^{-1}\left[(s\mathbf{I} - \mathbf{A})^{-1}\right]$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

Noting that $\mathbf{\Phi}^{-1}(t) = \mathbf{\Phi}(-t)$, we obtain the inverse of the state-transition matrix as follows:

$$\mathbf{\Phi}^{-1}(t) = e^{-\mathbf{A}t} = \begin{bmatrix} 2e^{t} - e^{2t} & e^{t} - e^{2t} \\ -2e^{t} + 2e^{2t} & -e^{t} + 2e^{2t} \end{bmatrix}$$

Finding the State Transition Matrix with Matlab

```
% Find State transition matrix
% Given ---------------------------------------------------------------------
MatrixA = [0 1;-2 -3];
MatrixI = [1 0;0 1];
% Create Symbolic Object s ---------------------------------------------------
syms s;
% ---------------------------------------------------------------------------
ILT = (((s*MatrixI) - MatrixA)^-1);
MatrixSTM = [ilaplace(ILT(1,1)) ilaplace(ILT(1,2));ilaplace(ILT(2,1))....
ilaplace(ILT(2,2))];
simplify(MatrixSTM);
% Display State Transition Matrix -------------------------------------------
disp('State Transition Matrix is = ');
disp(MatrixSTM);
```

```
>> Untitled
State Transition Matrix is =
[   2*exp(-t) - exp(-2*t),    exp(-t) - exp(-2*t)]
[ 2*exp(-2*t) - 2*exp(-t), 2*exp(-2*t) - exp(-t)]
```

End of Lecture III