MATH 233
Fall 2018
Quiz #1 B Solutions

Duration: ***50 minutes.***
Remark: ***Show your thinking/work. Do not just write a number as a result.***

1. Consider all bitstrings of length 16. A bitstring is made up of bits that are either 0 or 1. For example, 00100111 is a bitstring of length 8.

a) How many possible bitstrings of length 16 are there?
b) How many of bitstrings of length 16 contain a single 1 or a single 0?

a) Each bit can be either 0 or 1, Therefore for 16 bits there are 2^{16} = 65536 different choices.

b) Bitstrings that contain a single 0 are:

0111111111111111
1011111111111111
…
1111111111111110

There are 16 such bitstrings.

Similarly there are 16 bitstrings that contain a single 1.

Thus, in total there are **32 bitstrings** of length 16 contain a single 1 or a single 0

2. Prove the following identity **using induction**:

$$1 + 8 + 27 + \dots + n^3 = (n^2.(n+1)^2\ ) / 4$$

a) **Base case:** For n=1,

The left hand side of the equation is $1^3 = 1$.
The right hand side of the equation is $1.2^2 / 4 = 1$

Thus, the assertion holds for n=1.

b) **Inductive step:** Assume the assertion holds for k,
i.e., $1 + 8 + 27 + \ldots + k^3 = (k^2.(k+1)^2\ ) / 4$

Show that it holds for k+1. That is to say, show that $1 + 8 + 27 + \ldots + k^3 + (k+1)^3 = ((k+1)^2.(k+2)^2\ ) / 4$

$$\begin{aligned} 1 + 8 + 27 + \ldots + k^3 + (k+1)^3 &= (k^2.(k+1)^2\ ) / 4 + (k+1)^3 \\ &= (k+1)^2 (k^2 + 4k + 4) / 4 \\ &= (k+1)^2 (k + 2)^2 / 4 \end{aligned}$$

Base case and inductive steps show that the assertion holds for all n larger than or equal to 1.