MATH 233
Fall 2018
Class Worksheet for Week #4

0. Solutions of Quiz#1 (A and B sections; posted on http://learn.bilgi.edu.tr).

1. One hundred tickets, numbered 1, 2, 3, . . . , 100, are sold to 100 different people for a drawing. Four different prizes are awarded, including a grand prize (a trip toTahiti).How many ways are there to award the prizes if

a) there are no restrictions?
b) the person holding ticket 47 wins the grand prize?
c) the person holding ticket 47 wins one of the prizes?
d) the person holding ticket 47 does not win a prize?
e) the people holding tickets 19 and 47 both win prizes?
f) the people holding tickets 19, 47, and 73 all win prizes?
g) the people holding tickets 19, 47, 73, and 97 all win prizes?
h) none of the people holding tickets 19, 47, 73, and 97 wins a prize?
i) the grand prize winner is a person holding ticket 19, 47, 73, or 97?
j) the people holding tickets 19 and 47 win prizes, but the people holding tickets 73 and 97 do not win prizes?

Solution: Let the 100 tickets be T1, T2, …. T100. And let the four prizes be P1, P2, P3, P4.

a) First prize can be given to 100 different people. 2nd prize to 99, … and 4th to 96. The result is 4-permutations from a set of 100 items. I.e., P(100,4)

b) T47 wins the grand prize. Then remaining 3 prizes is P(99,3) because it is choosing 3-permutations from a set of 99 items.

c) There are four ways that T47 wins one of the prizes. And in each case the remaining prizes can be in P(99,3) ways. Therefore, the answer is 4.P(99,3)

d) If T47 does not win, then we consider a set with 99 elements, where 4-permutations are choosen. Thus, P(99,4)

e) T19 and T47 can win prizes in P(4, 2) ways. The remaining 98 can win in P(98,2) ways. Therefore, the answer is P(4,2) P(98, 2)

f) P(4,3) is the number of ways T19 and T47 and T73 wins 3 of the 4 prizes. The remaining prizes can be won by any of the 97 tickets. The asnwer is P(4,3) 97

g) P(4, 4) = 4! (number of ways of arranging 4 distinct items)

h) In this case, the problem is finding 4-permutations out of 96 people: P(96, 4)

i) Grand prize winner is T19 then, P(99,3). 3 other such cases. Therefore, the asnwer is 4 P(99,3)

j) P(4,2) (as in e) is the number of ways T19 and T47 wins prizes. P(96, 2) is the number of ways remaining 96 people win prizes. Thus, the asnwer is P(4,2) P(96, 2)

2. A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?

Solution: Let 40 questions be Q1, Q2, …. Q40. Each Qi can be true or false.
17 of the Qi's are true.

The number of ways these 17 true eustions can be choosen is equivalent to:

Number of 40-bit strings with 17 ones. The answer is C(40,17)

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Class (Week 5)

1. A batch of 100 manufactured items is checked by an inspector. The inspector examines 10 items selected at random. If none of the 10 items is defective, he accepts the whole batch. Otherwise the batch is subjected to further inspection. How many ways can the inspector select these 10 items?

A: The problem is selection 10-combination from a set of 100 elements.
C (100, 10) = 100! / (10! 90!) = 17,310,309,456,440

2. A kindergarted in Canada has 30 students. 5 students are Asian, 10 students are European, and the rest of the students are African. Assume the teacher wants to select a football team of size 11. In how many ways can the selection be made, so that there are **at least 5 European students** in the team and **at least 5 African students?**

A. List all possible team arrangements of 11 (F : African, E : European, A : Asian)
1. (5F, 5E, A)
2. (5F, 6E)
3. (6F, 5E)

# of ways for selecting (5F, 5E, A) = C(15, 5).C(10, 5). C(5, 1)
= 15! / (10! 5!) . 10! / (5! 5!). 5
= 5. 15! / (5! 5! 5!)
= 3783780

# of ways for selecting (5F, 6E) = C(15, 5). C(10, 6)
= 15! / (10! 5!) . 10! / (6! 4!)
= 630630

# of ways for selecting (6F, 5E) = C(15, 6). C(10, 5)
= 15! / (9! 6!) . 10! / (5! 5!)
= 1261260

Total # of ways of selecting the 11-student team = 3783780 + 630630 + 1261260 = 5,675,670

3 How many ways are there to distribute six objects to five boxes if

a) both the objects and boxes are labeled?
b) the objects are unlabeled, but the boxes are labeled?

a) O1, O2, O3, O4, O5, O6 are objects. B1, B2, B3, B4, B5 are the boxes. Think about the possible placements :

B1(O1, O2), B2(O3), B3(O4), B4(O5,O6), B5()
B1(O2, O3), B2(O1), B3(O4), B4(O5,O6), B5()
B1(O1, O2, O3, O4, O5, O6), B2(), B3(), B4(), B5()

A box can be filled in 64 (2^6) different ways.

Set of objects **Set of boxes**

Think of a six-letter word where each letter can be one of B1, … B5.

B1 B2 B1 B1 B1 B1 is the placement where O2 is placed in B2 and all others are in B1.

B1 B2 B3 B4 B5 B5 is the placement where O1 is placed in B1, O2 in B2, O3 in B3, O4 in B3 and O5 and O6 in B5

Thus each letter can take 6 different values (each object can go to 6 different boxes). The result is 5^6 = 15625

This also corresponds to the number of functions from a set of 6 elements to a set of 5 elements.

b) O, O, O, O, O, O are objects. B1, B2, B3, B4, B5 are the boxes. Here are two placements:

B1(O, O), B2(O), B3(O), B4(O,O), B5()
B1(), B2(O, O, O, O, O, O), B3(), B4(), B5()

Encode the two placements as:

OO I O I O I OO I
I OOOOOO III

Thus, the problem reduces to placing six Os and 4 Is in 10-letter word (made of the letters O and I). Thus number of bitstrings of size 10 with 4 ones in it. C(10, 4)

4. What is the probability that the sum of the numbers on two dice is even when they are rolled? What is the sample space?

Sample Space = { (1,1), (1,2), … (1,6),
(2,1), (2,2), …(2,6),
….
(6,1), (6,2), …. (6,6) } (list of all possible outcomes)
Thus, size of the sample space is = 36

Event that sum of two dice is even is the following subset of the sample space:
{ (1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6.4), (6,6) }

Size of the outcomes in this event is 18.

P(E) = lEl / lS.Spacel = 18 / 36 = 1/2

5. Which is more likely: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled?

Experiment 1 : Rolling two dice
Experiment 2 : Rolling three dice

Sample Space 1: { (1,1), (1,2), … (1,6),
(2,1), (2,2), …(2,6),
….
(6,1), (6,2), …. (6,6) }

Sample Space 2 : { (1,1,1), (1,1,2), … (1,1,6),
(1,2,1), (1,2,2), …(1,2,6),
…
…
….
(6,6,1), (6,6,2), …. (6,6,6) }

Event 1 : A total of 8 when two dice are rolled = {(2,6), (3,5), (4,4), (5,3), (6,2) }
Event 2 : A total of 8 when three dice are rolled = {(1,1,6), (1,2,5), (1,3,4), (1,4,3), (1,5,2), (1,6,1), (2,1,5), (2,2,3), (2,3,3), (2,4,2), (2,5,1), (3,1,4), (3,2,3), (3,3,2), (3,4,1), (4,1,3), (4,2,2), (4,3,1), (5,1,2), (5,2,1), (6,1,1)}

l Event 1 l = 5
l Event 2 l = 21

P (Event 1) = 5 / 36
P (Event 2)= 21 / 216 = 7 / 72

Event 1 is more likely since 5 / 36 is greater than 7 / 72

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Class Week 6

Students:
In all questions, think about the sample space, try writing the sample space (as we did in the lectures, not all elements but the beginning few elements and the last element, in a kind of ordering). Describe the experiment. Describe the event you are interested in. List the outcomes corresponding to the event you are interested in.

1. What is the probability that when a coin is flipped six times in a row, it lands heads up every time?

Experiment: A fair coin is tossed six times.

Sample Space = { HHHHHH, HHHHHT, …. TTTTTT}

$\text{Event}_{\text{all heads}}$ = { HHHHHH }

| Sample Space | = 2^6 = 64

| $\text{Event}_{\text{all heads}}$ | = 1

P ($\text{Event}_{\text{all heads}}$) = 1 / 64

2. What is the probability that a five-card poker hand contains the two of diamonds and the three of spades?

(Remember that there are 13 kinds and 4 suits of each kind in a deck of 52 cards)

Experiment : Select 5 cards out of a 52-card deck.

Sample Space = { {1C,1H,1S,1D,2A}, {1C,1H,1S,1D,2S}, ….. {13C,13H,13S,13D,12A}}

$\text{Event}_{2,3}$: Two diamonds and three spades among 5 cards

$\text{Event}_{2,3}$ = {{1D,2D,1S,2S,3S}, {1D,3D,1S,2S,3S}, ….. {12D,13D,11S,12S,13S}}

|Sample Space | = C(52,5)

| $\text{Event}_{2,3}$ | = ?

C(13,2) is the number of ways to choose 2 diamonds from 13 diamonds. = 78
C(13,3) is the number of ways to choose 3 spades from 13 spades. = 286

C(13,2) C(13,3) = 78. 286 = 22308 = | $\text{Event}_{2,3}$ |

P($\text{Event}_{2,3}$) = | $\text{Event}_{2,3}$ | / |Sample Space | = 22308 / 2598960 = 0.00858

3. What is the probability that a five-card poker hand contains a flush, that is, five cards of the same suit?

(Remember that there are 13 kinds and 4 suits of each kind in a deck of 52 cards)

Experiment : Select 5 cards out of a 52-card deck.

Sample Space = { {1C,1H,1S,1D,2A}, {1C,1H,1S,1D,2S}, ….. {13C,13H,13S,13D,12A}}

$Event_{flush}$: Five cards of the same suit

$Event_{flush}$ = {{1D,2D,3D,4D,5D}, {1D,2D,3D,4D,6D}, …. {1H,2H,3H,4H,5H}, ….. }

|Sample Space | = C(52,5)

C(4,1) is the ways to select a suite.

C(13,5) is the number of ways to select 5 cards from a suite.

| $Event_{flush}$ | = C(4,1) C(13,5) = 4 . 1287 = 5148

5148 outcomes of the sample space are 5 cards of the same suite.

P($Event_{flush}$) = 5148 / C(52,5) = 5148 / 2598960 = 0.00198

4. What is the probability that a fair die never comes up an even number when it is rolled six times?

Experiment : Roll a fair die six times

Sample Space = { 1-1-1-1-1-1, 1-1-1-1-1-2, ….. 6-6-6-6-6-6}
(Note that each value in this set is an ordered list, not a set)

$Event_{AllOdds}$: All six outcomes are odd numbers

$Event_{AllOdds}$ = { 1-1-1-1-1-1, 1-1-1-1-1-3, ….. 5-5-5-5-5-5}

|Sample Space| = 6^6 = 46656

|$Event_{AllOdds}$ | = ?

The set $Event_{AllOdds}$ is equivalent to all possible six-letter words out of the letters {1,3,5}. Thus,
|$Event_{AllOdds}$ | = 3^6 = 729

P($Event_{AllOdds}$) = 729 / 46656 = 0.015625

5. What is the probability that a positive integer not exceeding 100 selected at random is divisible by 3?

Experiment : A positive integer between 1 and 100 is chosen (including 1 and 100).

Event3 : The number between [1,100] is divisable by 3.

Sample space = {1, 2, … , 100}   (like rolling a die with 100 faces)

|Sample Space | = 100

Event3 = {3,6,9,12,15,18,…. 99}

Any number is iether congruent to 0, 1 or 2 in mod 3. Event3 is the numbers that are congruent to 0 in mod 3. Therefore, 1/3 of the numbers between 3 and 98 (including them) are congruent to 0. to those add {99,1,2} out of which only one is congruent to 0. There are 98-3+1 = 96 numbers in [3,98] and 96/3 = 32 of them are congruent to 0. There is one number in {99,1,2} congruent to 0. Therefore there is a total of 33 numbers which are congruent to 0 in [1,100].

|Event3| = 33

P(Event3) = |Event3| / |Sample Space | = 33/100 = 0.33

6. In a superlottery, a player selects 7 numbers out of the first 80 positive integers. What is the probability that a person wins the grand prize by picking 7 numbers that are among the 11 numbers selected at random by a computer.

Sample space is all possible 7-nuber picks.

Sample Space = {{1,2,3,4,5,6,7}, {1,2,3,4,5,6,8}, ….. }
|Sample Space | = C(80,7) = 3,176,716,400

EventGP : The chosen 7 numbers is among the chosen 11 numbers

For every 11-number set, there are C(11, 7) = 330   7-number sets that are included in the 11 elements.

Thus, |EventGP| = 330

How many different 7-combinations are there?

C(80,7) = 3,176,716,400

P(EventGP) = |EventGP| / |Sample Space| = 330 / 3,176,716,400 = 0.00000010388