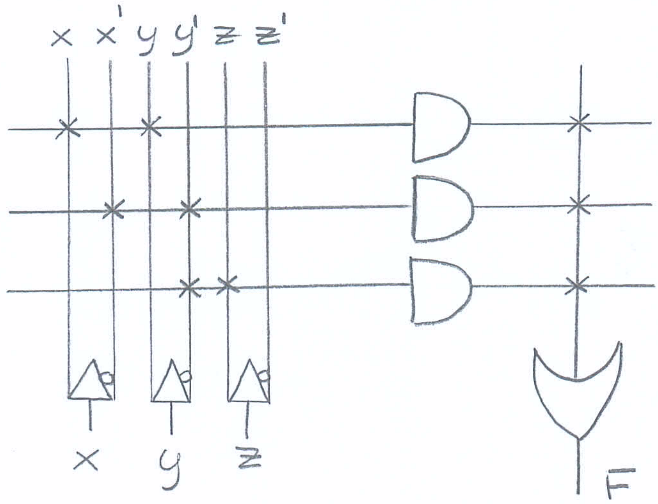
Problem 1) Implement the Boolean function

$$F = xy + x'y' + y'z$$

a. with AND, OR and NOT gates,
b. with OR and NOT gates,
c. with AND and NOT gates,
d. with NAND and NOT gates,
e. with NOR and NOT gates.

Solution.

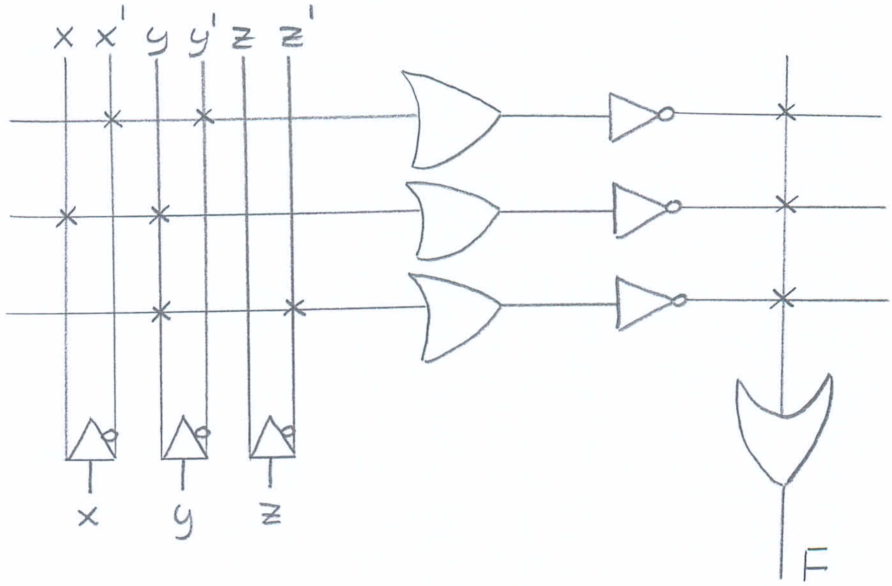
a.



2 NOT gates
3 AND gates
1 OR gate
$\underbrace{\qquad\qquad\qquad}_{\text{used}}$

b. We shall rearrange F as follows:

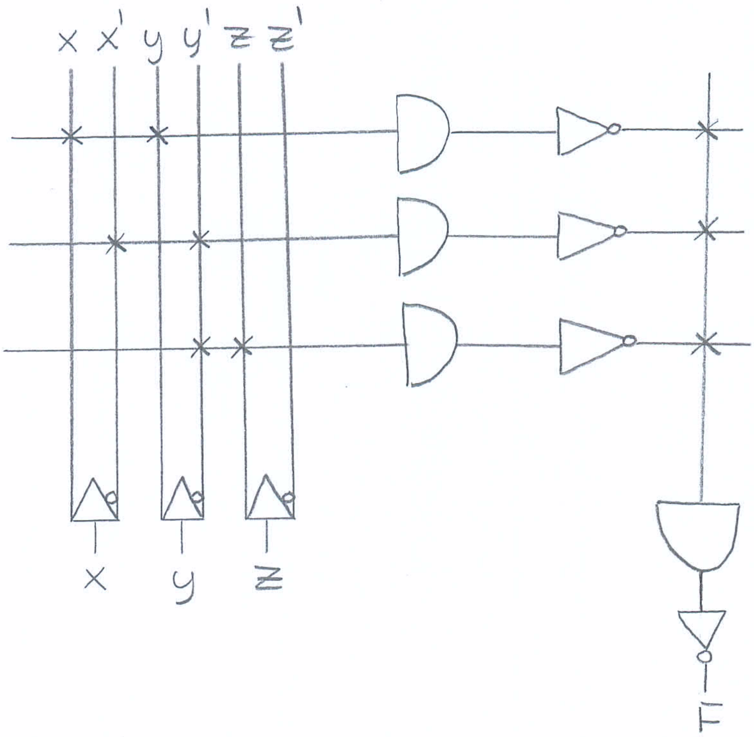
$$F = (x'+y')' + (x+y)' + (y+z')'$$



6 NOT gates
4 OR gates
$\underbrace{\qquad\qquad\qquad}_{\text{used}}$

c. We shall rearrange F as follows:

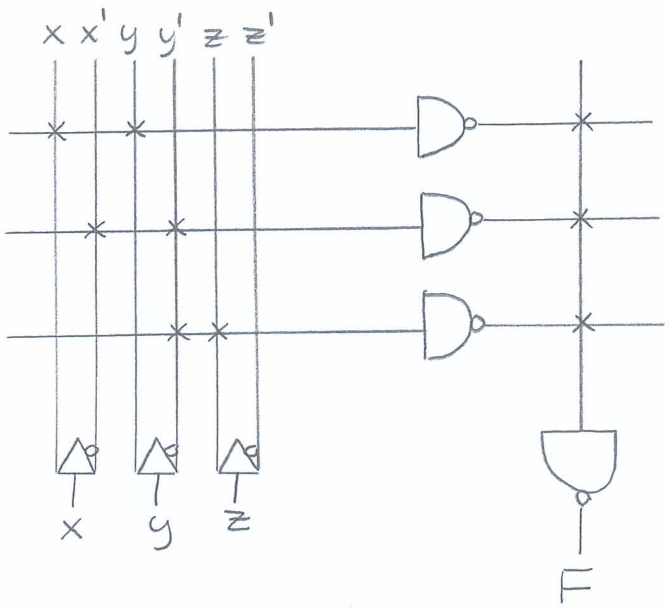
$$F = \left[(xy)' \cdot (x'y')' \cdot (y'z)' \right]'$$



6 NOT gates
4 AND gates
used

d. We have

$$F = \left[(xy)' \cdot (x'y')' \cdot (y'z)' \right]'$$



2 NOT gates
4 NAND gates
used

e. We shall convert F into product of sums form as

$$F = (xy + x')(xy + y') + y'z$$

$$= \underbrace{(x+x')}_{1}(y+x')(x+y')\underbrace{(y+y')}_{1} + y'z$$

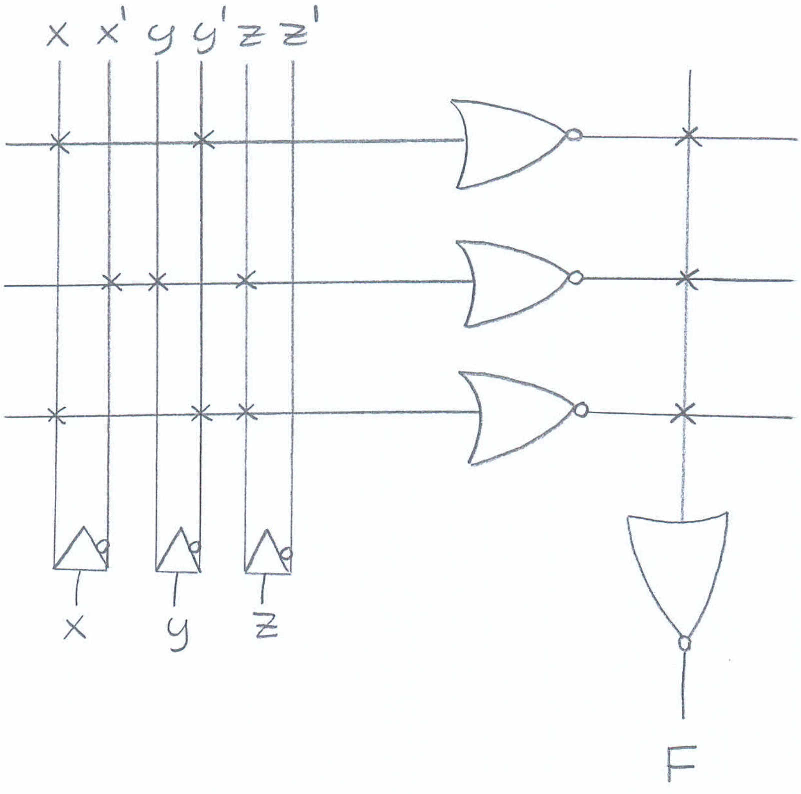
$$= (x'+y)(x+y') + y'z$$

$$= \left[(x'+y)(x+y') + y'\right]\left[(x'+y)(x+y') + z\right]$$

$$= \underbrace{(x'+y+y')}_{1}(x+y'+y')(x'+y+z)(x+y'+z)$$

$$= (x+y')(x'+y+z)(x+y'+z)$$

$$= \left[(x+y')' + (x'+y+z)' + (x+y'+z)'\right]'$$



$\underbrace{\text{2 NOT gates} \quad \text{4 NOR gates}}_{\text{used}}$

Problem 2) Show that a positive logic NAND gate is a negative logic NOR gate and vice versa.

Solution. We consider a gate with

x	y	z
L	L	H
L	H	H
H	L	H
H	H	L

Positive logic : $L \equiv 0$, $H \equiv 1$

x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

$\Rightarrow$ a positive logic NAND gate

Negative logic : $L \equiv 1$, $H \equiv 0$

x	y	z
1	1	0
1	0	0
0	1	0
0	0	1

$\Rightarrow$ a negative logic NOR gate

Moreover ;

to justify vice versa situation, we consider

x	y	z
L	L	H
L	H	L
H	L	L
H	H	L

x	y	z
0	0	1
0	1	0
1	0	0
1	1	0

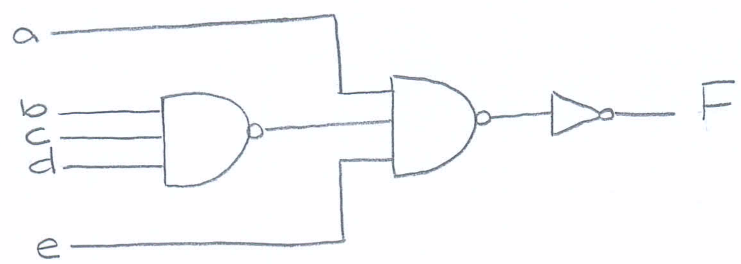
⇒ a positive logic NOR gate

x	y	z
1	1	0
1	0	1
0	1	1
0	0	1

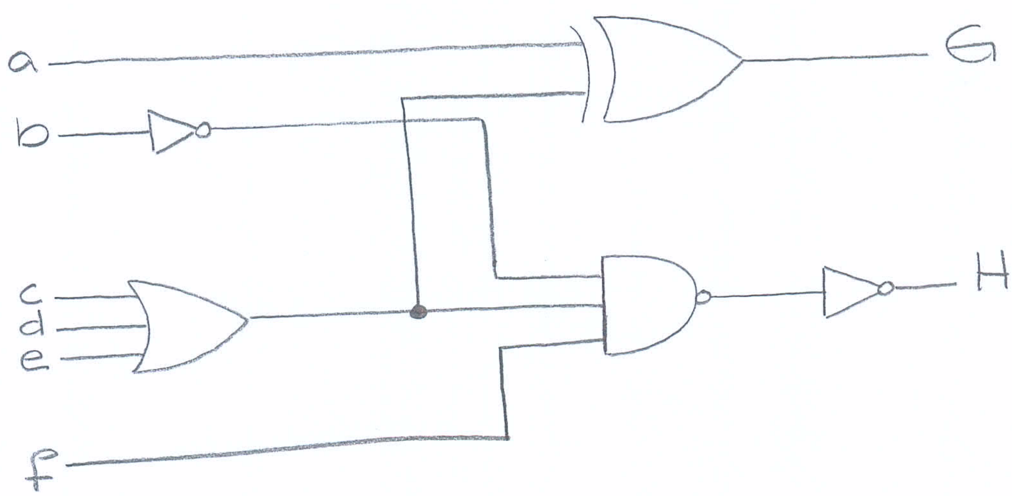
⇒ a negative logic NAND gate

Problem 3) Write Boolean expressions and construct the truth tables describing the outputs of the circuits represented by the following logic diagrams:

a.



b.



Solution.

a. $$F = \left(\left(a\cdot(b\cdot c\cdot d)'\cdot e\right)'\right)'$$

$$= a\cdot(b'+c'+d')e$$

$$= ab'e + ac'e + ad'e$$

$$= \sum m(17, 19, 21, 23, 25, 27, 29)$$

a	b	c	d	e	F
0	0	0	0	0	0
		⋮			⋮
					0
1	0	0	0	1	1
1	0	0	1	0	0
1	0	0	1	1	1
1	0	1	0	0	0
1	0	1	0	1	1
1	0	1	1	0	0
1	0	1	1	1	1
1	1	0	0	0	0
1	1	0	0	1	1
1	1	0	1	0	0
1	1	0	1	1	1
1	1	1	0	0	0
1	1	1	0	1	1
1	1	1	1	0	0
1	1	1	1	1	0

b.

$$G = a \oplus (c+d+e)$$

$$= a'(c+d+e) + a(c+d+e)'$$

$$= a'c + a'd + a'e + ac'd'e'$$

$$= \sum m(\underbrace{8-31}_{a'c}, \underbrace{4-31}_{a'd}, \underbrace{2-31}_{a'e}, \underbrace{32, 33, 48, 49}_{ac'd'e'})$$

$$= \sum m(2-33, 48, 49)$$

a	b	c	d	e	f	G
0	0	0	0	0	0	0
0	0	0	0	0	1	0
0	0	0	0	1	0	1
		⋮				⋮
1	0	0	0	0	1	1
1	0	0	0	1	0	0
		⋮				⋮
1	0	1	1	1	1	0
1	1	0	0	0	0	1
1	1	0	0	0	1	0
		⋮				
1	1	1	1	1	1	0

$$H = \left(\left(b' \cdot (c+d+e) \cdot f\right)'\right)'$$

$$= b' \cdot (c+d+e) \cdot f$$

$$= b'cf + b'df + b'ef$$

$$= \sum m(\underbrace{9, 11, 13, 15, 41, 43, 45, 47}_{b'cf},$$

$$\underbrace{5, 7, 13, 15, 37, 39, 45, 47}_{b'df},$$

$$\underbrace{3, 7, 11, 15, 35, 39, 43, 47}_{b'ef})$$

$$= \sum m(3, 5, 7, 9, 11, 13, 15, 35, 37, 39, 41, 43,$$

$$45, 47)$$

PS 3.8

a	b	c	d	e	f	H
0	0	0	0	0	0	0
			⋮			⋮
0	0	0	0	1	1	1
0	0	0	1	0	0	0
0	0	0	1	0	1	1
0	0	0	1	1	0	0
0	0	0	1	1	1	1
0	0	1	0	0	0	0
0	0	1	0	0	1	1
0	0	1	0	1	0	0
0	0	1	0	1	1	1
0	0	1	1	0	0	0
0	0	1	1	0	1	1
0	0	1	1	1	0	0
0	0	1	1	1	1	1
			⋮			⋮
1	0	0	0	1	1	1
1	0	0	1	0	0	0
1	0	0	1	0	1	1
1	0	0	1	1	0	0
1	0	0	1	1	1	1
1	0	1	0	0	0	0
1	0	1	0	0	1	1
1	0	1	0	1	0	0
1	0	1	0	1	1	1
1	0	1	1	0	0	0
1	0	1	1	0	1	1
1	0	1	1	1	0	0
1	0	1	1	1	1	1

PS 3.9

↓ remaining lines → 0

Problem 4) Use the tabulation method to simplify the following Boolean function:

$$F(x,y,z) = \Sigma m(0,1,3,4,5)$$

Solution.

LIST #1

m_i	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
4	1	0	0	✓
3	0	1	1	✓
5	1	0	1	✓

LIST #2

Product terms	x	y	z	
0,1	0	0	–	✓
0,4	–	0	0	✓
1,3	0	–	1	PI2 $\equiv x'z$
1,5	–	0	1	✓
4,5	1	0	–	✓

LIST # 3

Product terms	x	y	z	
0,1,4,5	–	0	–	PI1 ≡ y'
0,4,1,5	–	0	–	

– No more list !

PI_i / m_i	0	1	3	4	5
PI1*	X	X		X	X
PI2*		X	X		

– Both PI1 and PI2 are essential prime implicants

Hence ;

$$F(x, y, z) = PI1 + PI2$$

$$= y' + x'z$$

Problem 5) Find all the prime implicants for the following Boolean function and determine which are essential:

$$F(A,B,C,D) = \Sigma m(0,2,3,5,7,8,10,11,14,15)$$

Solution.

LIST #1

m_i	A	B	C	D	
0	0	0	0	0	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
3	0	0	1	1	✓
5	0	1	0	1	✓
10	1	0	1	0	✓
7	0	1	1	1	✓
11	1	0	1	1	✓
14	1	1	1	0	✓
15	1	1	1	1	✓

LIST #2

Product terms	A	B	C	D	
0,2	0	0	–	0	✓
0,8	–	0	0	0	✓
2,3	0	0	1	–	✓
2,10	–	0	1	0	✓
8,10	1	0	–	0	✓
3,7	0	–	1	1	✓
3,11	–	0	1	1	✓
5,7	0	1	–	1	PI5
10,11	1	0	1	–	✓
10,14	1	–	1	0	✓
7,15	–	1	1	1	✓
11,15	1	–	1	1	✓
14,15	1	1	1	–	✓

LIST #3

Product terms	A	B	C	D	
0,2,8,10	–	0	–	0	PI1 ≡ B'D'
0,8,2,10	–	0	–	0	
2,3,10,11	–	0	1	–	PI2 ≡ B'C
2,10,3,11	–	0	1	–	
3,7,11,15	–	–	1	1	PI3 ≡ CD
3,11,7,15	–	–	1	1	
10,11,14,15	1	–	1	–	PI4 ≡ AC
10,14,11,15	1	–	1	–	

- No more list !

PI_i / m_i	0 ✓	2 ✓	3	5 ✓	7 ✓	8 ✓	10 ✓	11 ✓	14 ✓	15 ✓
PI 1 *	X	X				X	X			
PI 2		X	X				X	X		
PI 3			X					X		X
PI 4 *							X	X	X	X
PI 5 *				X	X					

PI1, PI4, PI5 are all essential PI's

Reduced PI chart

PI_i / m_i	3
PI 2	X
PI 3	X

$\Rightarrow$ we can choose either PI 2 or PI 3 as essential

Hence;

PI's : $B'D'$, $B'C$, CD, AC, $A'BD$

Essential PI's : $B'D$, AC, $A'BD$, $B'C$ (or CD)

Problem 6) Using the tabulation method, simplify the following Boolean function F together with the don't care conditions, d and then express the simplified function in sum-of-minterms form:

$$F(A,B,C,D) = \sum m\,(0,6,8,13,14)$$

$$d(A,B,C,D) = \sum m\,(2,4,10)$$

Draw a 2-level NAND gate logic circuit diagram for F (A, B, C, D).

Solution.

LIST # 1

mi	A	B	C	D	
0	0	0	0	0	✓
2	0	0	1	0	✓
4	0	1	0	0	✓
8	1	0	0	0	✓
6	0	1	1	0	✓
10	1	0	1	0	✓
13	1	1	0	1	PI 4
14	1	1	1	0	✓

LIST # 2

Product terms	A	B	C	D	
0,2	0	0	–	0	✓
0,4	0	–	0	0	✓
0,8	–	0	0	0	✓
2,6	0	–	1	0	✓
2,10	–	0	1	0	✓
4,6	0	1	–	0	✓
8,10	1	0	–	0	✓
6,14	–	1	1	0	✓
10,14	1	–	1	0	✓

LIST # 3

Product terms	A	B	C	D	
0,2,4,6	0	–	–	0	PI1 ≡ $A'D'$
0,4,2,6	0	–	–	0	
0,2,8,10	–	0	–	0	PI2 ≡ $B'D'$
0,8,2,10	–	0	–	0	
2,6,10,14	–	–	1	0	PI3 ≡ CD'
2,10,6,14	–	–	1	0	

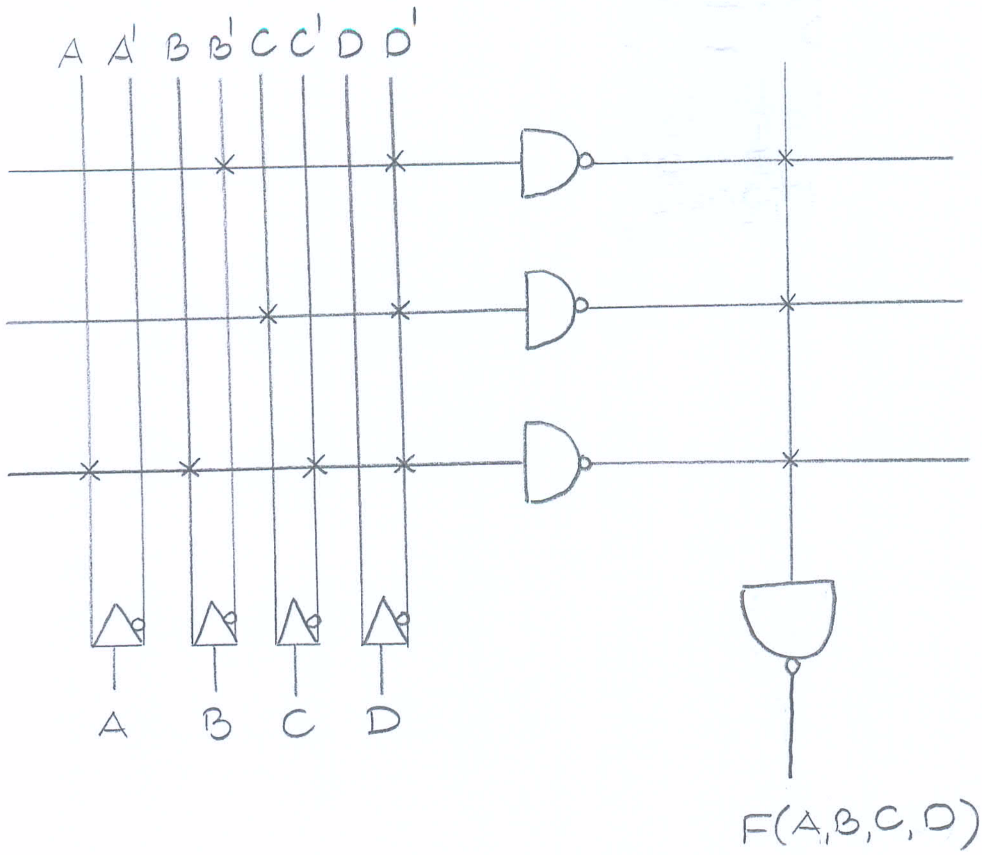
– No more list!

PI_i / m_i	0✓	6✓	8✓	13✓	14✓
PI1	X	X			
PI2*	X		X		
PI3*		X			X
PI4*				X	

PI2, PI3, PI4 are essential PI's

– Since all minterms are covered by the essential PI's determined

$$F(A,B,C,D) = PI2 + PI3 + PI4$$
$$= B'D' + CD' + ABC'D'$$



2-level NAND gate logic circuit diagram