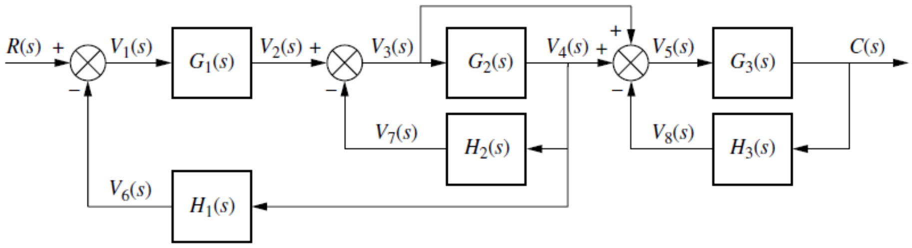
PROBLEM: Reduce the system shown in Figure 5.11 to a single transfer function.



SOLUTION: In this example we make use of the equivalent forms shown in Figures 5.7 and 5.8. First, move $G_2(s)$ to the left past the pickoff point to create parallel subsystems, and reduce the feedback system consisting of $G_3(s)$ and $H_3(s)$. This result is shown in Figure 5.12(a).

Second, reduce the parallel pair consisting of $1/G_2(s)$ and unity, and push $G_1(s)$ to the right past the summing junction, creating parallel subsystems in the feedback. These results are shown in Figure 5.12(b).

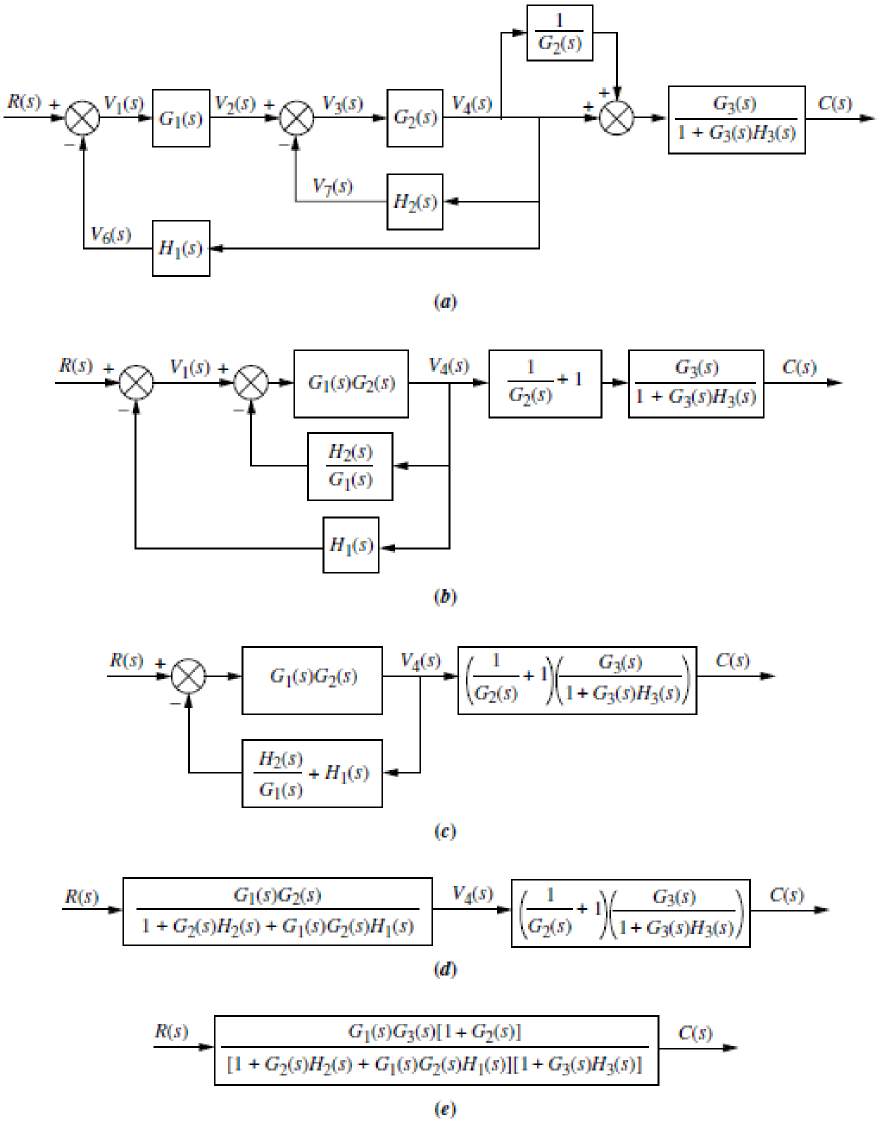


FIGURE 5.12 Steps in the block diagram reduction for Example 5.2

3. Find the equivalent transfer function, $T(s) = C(s)/R(s)$, for the system shown in Figure P5.3. [Section: 5.2]

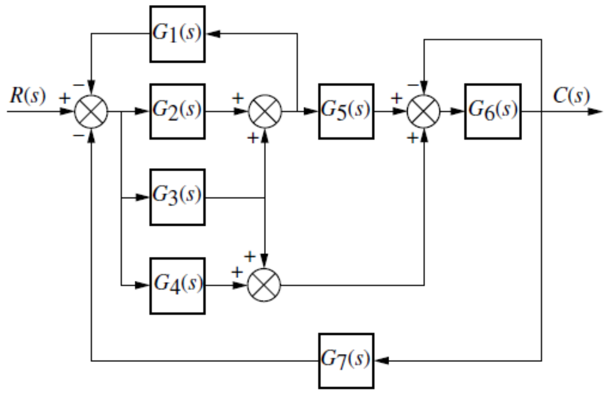
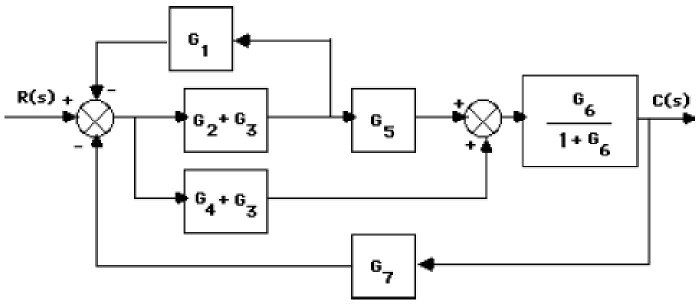


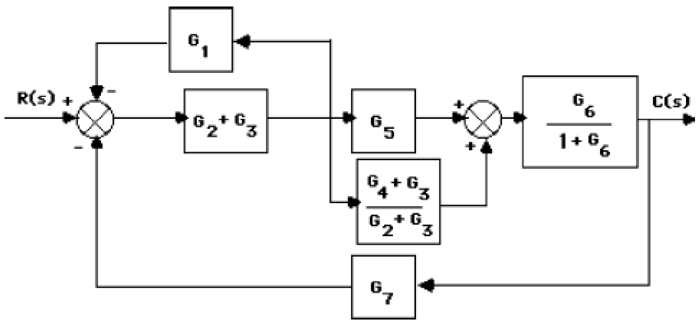
FIGURE P5.3

3.

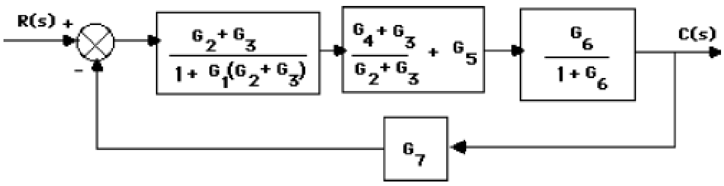
a. Split G_3 and combine with G_2 and G_4. Also use feedback formula on G_6 loop.



Push $G_2 + G_3$ to the left past the pickoff point.



Using the feedback formula and combining parallel blocks,



Multiplying the blocks of the forward path and applying the feedback formula,

$$T(s) = \frac{G_6G_4 + G_6G_3 + G_6G_5G_3 + G_6G_5G_2}{1 + G_6 + G_3G_1 + G_2G_1 + G_7G_6G_4 + G_7G_6G_3 + G_7G_6G_5G_3 + G_7G_6G_5G_2 + G_6G_3G_1 + G_6G_2G_1}$$

12. For the system shown in Figure P5.12, find the output, $c(t)$, if the input, $r(t)$, is a unit step. [Section: 5.3]

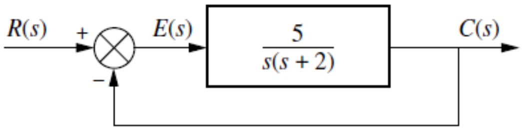


FIGURE P5.12

$$C(s) = \frac{5}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}$$

$$A = 1$$

$$5 = s^2 + 2s + 5 + Bs^2 + Cs$$

$$\therefore B = -1,\ C = -2$$

$$C(s) = \frac{1}{s} - \frac{s+2}{s^2+2s+5} = \frac{1}{s} - \frac{s+2}{(s+1)^2+4}$$

$$= \frac{1}{s} - \frac{(s+1)+1}{(s+1)^2+4} = \frac{1}{s} - \frac{(s+1)+\frac{1}{2}2}{(s+1)^2+4}$$

$$c(t) = 1 - e^{-t}(\cos 2t + \frac{1}{2}\sin 2t)$$

16. For the system of Figure P5.16, find the values of K_1 and K_2 to yield a peak time of 1.5 second and a settling time of 3.2 seconds for the closed-loop system's step response. [Section: 5.3]

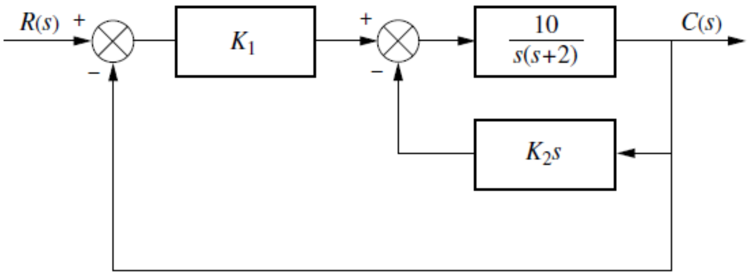


FIGURE P5.16

16.

The equivalent forward-path transfer function is $G(s)=\dfrac{10K_1}{s[s+(10K_2+2)]}$. Hence,

$T(s)=\dfrac{G(s)}{1+G(s)}=\dfrac{10K_1}{s^2+(10K_2+2)s+10K_1}$. Since

$T_s=\dfrac{4}{\text{Re}}=3.2, \ \therefore \text{Re}=1.25;$ and $T_p=\dfrac{\pi}{\text{Im}}=1.5, \ \therefore \text{Im}=2.09$. The poles are thus at

$-1.25+j2.09$. Hence, $\omega_n=\sqrt{1.25^2+2.09^2}=\sqrt{10K_1}$. Thus, $K_1=0.593$. Also, $(10K_2+2)/2=\text{Re}$ $=1.25$. Hence, $K_2=0.05$.

17. Find the following for the system shown in Figure P5.17: [Section: 5.3]

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a. The equivalent single block that represents the transfer function, $T(s)=C(s)/R(s)$.

b. The damping ratio, natural frequency, percent overshoot, settling time, peak time, rise time, and damped frequency of oscillation.

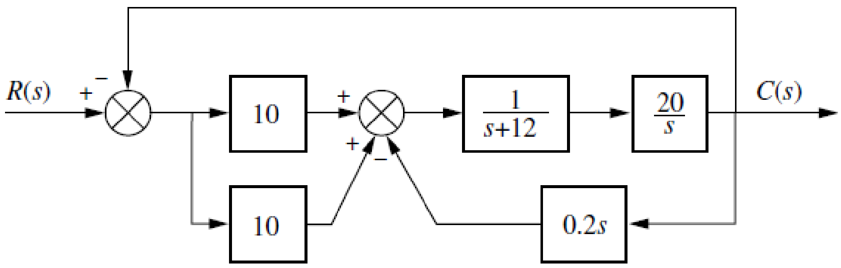

FIGURE P5.17

17.

a. For the inner loop, $G_e(s)=\dfrac{20}{s(s+12)}$, and $H_e(s)=0.2s$. Therefore, $T_e(s)=\dfrac{G_e(s)}{1+G_e(s)H_e(s)}=$

$\dfrac{20}{s(s+16)}$. Combining with the equivalent transfer function of the parallel pair, $G_p(s)=20$, the system is reduced to an equivalent unity feedback system with $G(s)=G_p(s)\,T_e(s)=\dfrac{400}{s(s+16)}$. Hence, $T(s)=$

$\dfrac{G(s)}{1+G(s)}=\dfrac{400}{s^2+16s+400}$.

b. $\omega_n^2=400$; $2\zeta\omega_n=16$. Therefore, $\omega_n=20$, and $\zeta=0.4$. $\%OS=e^{-\zeta\pi/\sqrt{1-\zeta^2}}x100=25.38$;

$T_s=\dfrac{4}{\zeta\omega_n}=0.5$; $T_p=\dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}=0.171$. From Figure 4.16, $\omega_n T_r=1.463$. Hence, $T_r=0.0732$.

$\omega_d=\text{Im}=\omega_n\sqrt{1-\zeta^2}=18.33$.