Tuğberk Göç
115200084

1)

a) Given $G(s) = \dfrac{K}{s.(s+10)}$

Closed loop T.F. for a negative feedback;

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)} = \frac{K/s.(s+10)}{1+\frac{K}{s.(s+10)} \times 1} = \frac{K}{s(s+10)+K}$$

$$\frac{C(s)}{R(s)} = \frac{K}{s^2+10s+K}$$

Characteristic eqn; $s^2+10s+K=0$

$s^2+2\zeta\omega_n s+\omega_n^2=0$

To obtain a 12% overshoot in a unit step function

$$\text{Max.P.O.} = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.12$$

$$\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} = \ln(0.12) = -2.12$$

$$\pi.\zeta = 2.12 \times \sqrt{1-\zeta^2}$$

$$9.87\zeta^2 = (2.12)^2.(1-\zeta^2)$$

$$14.36\zeta^2 = 4.494$$

$$\zeta^2 = 0.313$$

$$\boxed{\zeta \cong 0.56}$$

So, from characteristic eqn; $2\zeta\omega_n = 10$

$$\boxed{\omega_n = 8.93 \text{ rad/s}}$$

$$K = \omega_n^2 = (8.93)^2 = 79.74$$

$$\boxed{K = 79.74}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{79.74}{s^2+10s+79.74}}$$

b) Two-times faster in terms of settling time means that;

$$T_s' = \frac{T_s}{2} = \frac{4/\zeta.w_n}{2} = \frac{2}{\zeta.w_n} = 0.399 \cong 0.4sn.$$

Char. eqn. $\rightarrow s^2 + 2\zeta w_n s + w_n^2$

$$0.4 = \frac{4}{\zeta.w_n} \rightarrow \zeta w_n = 10 \quad (\zeta = 0.5594)$$

$$w_n = \frac{10}{0.5594} = 17.88 \text{ rad/sn}$$

$$K = w_n^2 = 17.88^2 = 319.694 \cong 320$$

$$\boxed{\text{Characteristic T.F.};\ \frac{K}{s^2 + 2\zeta w_n s + w_n^2} = \frac{320}{s^2 + 20s + 320}}$$

c) Unit-step response; $C(s) = R(s).G(s) \quad (R(s) = 1/s)$

$$C(s) = \frac{1}{s} \cdot \frac{w_n^2}{s^2 + 2\zeta w_n^2 s + w_n^2} = \frac{1}{s} \cdot \frac{w_n^2}{\underbrace{s^2 + 2\zeta w_n s + \zeta^2 w_n^2}_{(s+\zeta w_n)^2} \underbrace{- \zeta^2 w_n^2 + w_n^2}_{w_n^2(1-\zeta^2)}}$$

added

$$C(s) = \frac{w_n^2}{(s+\zeta w_n)^2 + w_n^2.(1-\zeta^2)} \cdot \frac{1}{s}$$

$$\mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left[\frac{1}{s} - \underbrace{\frac{s+\zeta w_n}{(s+\zeta w_n)^2 + w_d^2}}_{e^{-\zeta w_n t}.\cos w_d t} - \frac{\zeta w_n}{w_d} \underbrace{\frac{w_d}{(s+\zeta w_n)^2 + w_d^2}}_{e^{-\zeta w_n t}.\sin w_d t}\right]$$

$$\mathcal{L}^{-1}\left[\frac{s+\alpha}{(s+\alpha)^2 + w^2}\right] = e^{-\alpha t}.\cos wt \qquad \mathcal{L}^{-1}\left[\frac{w}{(s+\alpha)^2 + w^2}\right] = e^{-\alpha t}.\sin wt$$

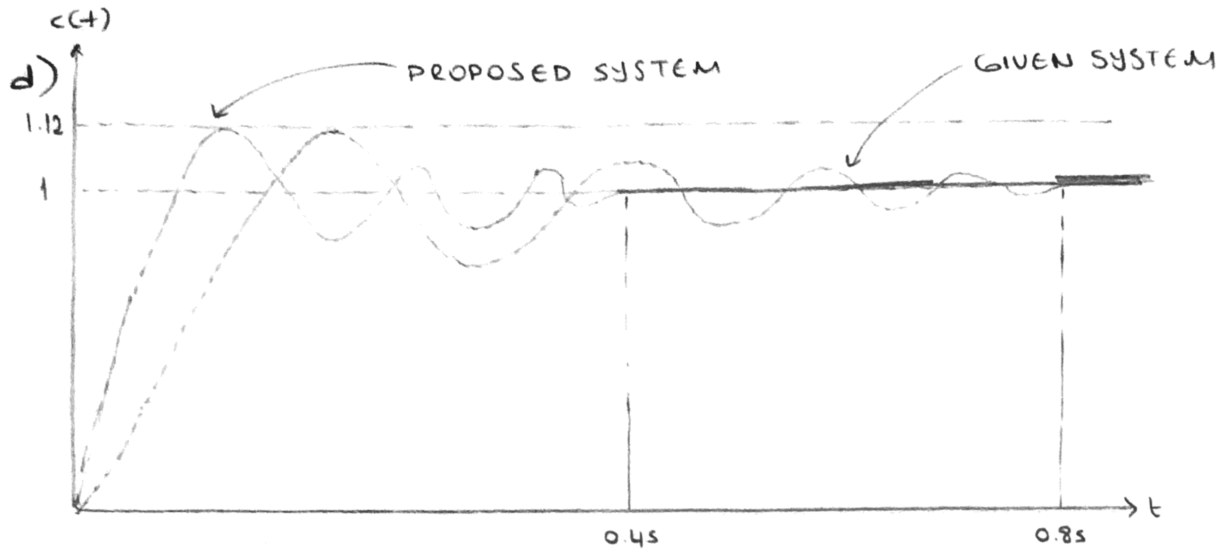
$$c(t) = 1 - e^{-\zeta w_n t}.\cos w_d t - e^{-\zeta w_n t}.\sin w_d t.\frac{\zeta w_n}{w_d}$$

$$c(t) = 1 - \frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} \underbrace{(\sqrt{1-\zeta^2}\cos w_d t + \zeta \sin w_d t)}_{\sin(w_d t + \theta_1)}$$

$$\boxed{c(t) = 1 - \frac{e^{-\zeta w_d t}}{\sqrt{1-\zeta^2}}.\sin(w_d t + \theta_1)} \quad \text{where } \theta_1 = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

$$w_d = w_n\sqrt{1-\zeta^2}$$

d)



2) From given control system

$$G(s) = \frac{k}{s^2+s+1}, \quad H(s) = 1 - \frac{1}{k} = \frac{k-1}{k}$$

$$T.F. = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}$$

$$= \frac{\frac{k}{s^2+s+1}}{1+\frac{\cancel{k}}{s^2+s+1}\cdot\frac{k-1}{\cancel{k}}} = \frac{k}{s^2+s+1+(k-1)}$$

$$\frac{C(s)}{R(s)} = \frac{k}{s^2+s+k} = \frac{w_n^2}{s^2+2\delta w_n s+w_n^2}, \quad w_n = \sqrt{k}, \quad 2\delta w_n = 1$$

a) Given $M.P\% = 20\% \longrightarrow M.P = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} = 0.2$

$$\frac{-\pi\delta}{\sqrt{1-\delta^2}} = \ln(0.2) = 1.609$$

$$\frac{\pi^2.\delta^2}{1-\delta^2} = 2.5902$$

$$\delta^2(9.859+2.5902) = 2.5902$$

$$\delta^2 = 0.208$$

$$\delta = 0.456 \text{ (Damping Ratio)}$$

$$2\delta w_n = 1 \longrightarrow 2.(0.456).w_n = 1$$

$$w_n = 1.096$$

$$\boxed{k = w_n^2 = (1.096)^2 = 1.202}$$

b) Peak Time (t_p) $= \dfrac{\pi}{w_n \cdot \sqrt{1-\delta^2}} = \dfrac{\pi}{1.096 \cdot \sqrt{1-(0.456)^2}} = \dfrac{\pi}{0.975}$

$$\boxed{T_p = 3.219 \text{ secs}}$$

c) Settling Time (T_s) $= \dfrac{4}{\delta w_n} = \dfrac{4}{(0.456).(1.096)}$

$$\boxed{T_s = 8.0035 \text{ sec.}}$$

d) Given fastest response without any oscillation (Critically Damping)

Damping Ratio (δ) = 1

We know that ; $2\delta w_n = 1$

$w_n = 0.5$

$K = w_n^2 = (0.5)^2 = 1/4$

$$\boxed{K = 0.25}$$

3) From the blue line in the figure ;

a) T.F. of underdamped system ;

$$T.F = \frac{w_n^2}{s^2 + 2\delta w_n s + w_n^2}$$

$\cancel{T_p} = \dfrac{\pi}{w_n\sqrt{1-\delta^2}}$, $T_s = \dfrac{4}{\delta w_n}$

b) % Peak Over Shoot $= e^{\frac{-\delta\pi}{\sqrt{1-\delta^2}}} \times 100\%$

(Amplitude – Time)

$$\frac{2.4 - 2}{2} = e^{\frac{-\delta\pi}{\sqrt{1-\delta^2}}}$$

(Time)

$$0.2 = e^{\frac{-\delta\pi}{\sqrt{1-\delta^2}}}$$

$$\frac{\ln(0.2)}{\pi} = \frac{-\delta}{\sqrt{1-\delta^2}} \longrightarrow \boxed{\delta = 0.4563}$$

$$6 = \frac{4}{\delta w_n} \longrightarrow w_n = \frac{4}{6 \times (0.4563)} = 1.46$$

$$\boxed{w_n = 1.46}$$

$$\boxed{T.F = \frac{2.1316}{s^2 + 1.335 + 2.1316} = G_1}$$

c) G_3 is the second order system and also it is critically damped system.

d) G_3 (red coloured) is the critically damped system ($\xi = 1$) becouse it requiress less time than G_2. (G_1 : underdamped, G_4 : overdamped)

$w_n = 1.46$, $\xi = 1$;

$$T.F. = \frac{w_n^2}{s^2 + 2\xi w_n + w_n^2}$$

$$\boxed{T.F. \text{ of } G_3 = \frac{2.13}{s^2 + 2.92s + 2.13}}$$

4) Let us use Mason's Gain formula

$$T.F. = \frac{C(s)}{R(s)} = \frac{\sum_{i=0}^{N} P_i \Delta_i}{\Delta}$$

P_i : ith forward path gain

Δ : 1 − [(sum of all individual loop gains) + (sum of gain products of all possible two non-touching loops) − (sum of gain products of all possible three non-touching loops)]

Δ_i : obtained from Δ by removing the loops which touches ith forward path.

Answer: we have two forward path

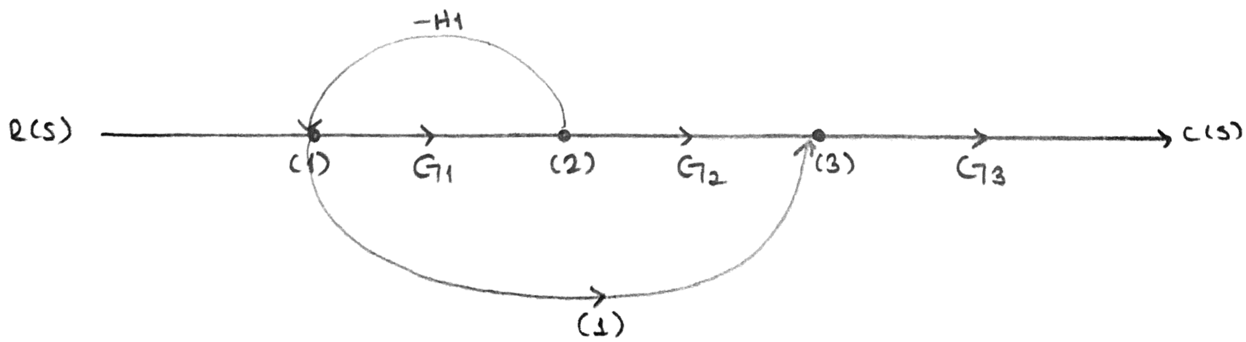
(i) $R(s) \rightarrow G_1 \rightarrow G_2 \rightarrow G_3 \rightarrow C(s)$, $P_1 = G_1 G_2 G_3$

(ii) $R(s) \longrightarrow \otimes \longrightarrow \boxed{G_3} \longrightarrow C(s)$, $P_2 = G_3$

(iii) $\Delta_1 = 1$

(iv) $\Delta_2 = 1$

(v) $\Delta = 1 + G_1 H_1$



$$\therefore T.F. = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

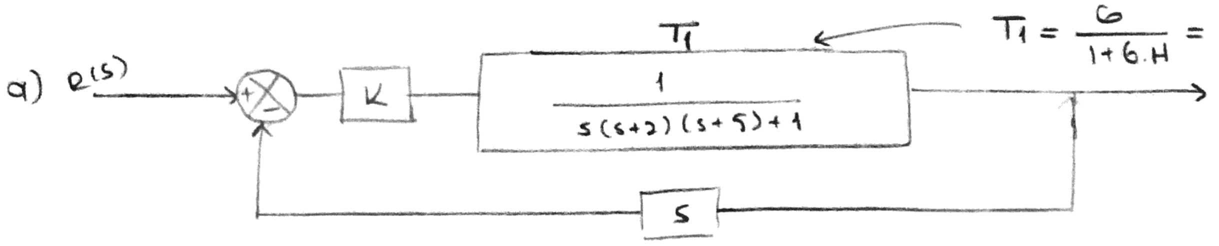
$$= \frac{G_1 G_2 G_3 \times 1 + G_3 \times 1}{1 + G_1 H_1}$$

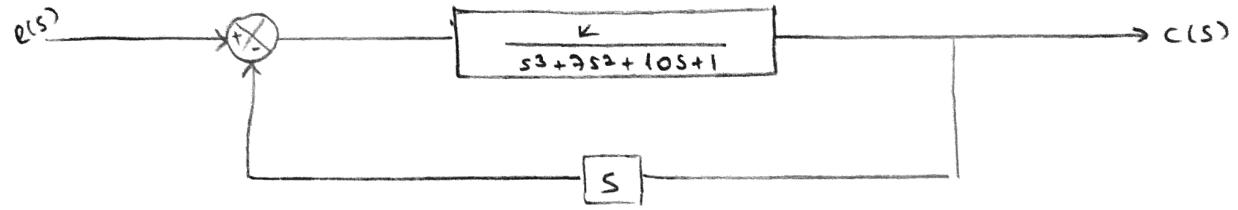
$$\boxed{\therefore T.F. = \frac{G_1 G_2 G_3 + G_3}{1 + G_1 H_1} = \frac{G_3 (G_1 G_2 + 1)}{(1 + G_1 H_1)}}$$

5)

$$T_1 = \frac{G}{1+G.H} = \frac{1/s.(s+2).(s+5)}{1+1/s.(s+2).(s+5)\times 1}$$

a)





Applying T.F. formula;

$$\boxed{\frac{C(s)}{R(s)} = \frac{K/s^3+7s^2+10s+1}{1+\dfrac{K}{s^3+7s^2+10s+1}\times s} = \frac{K}{s^3+7s^2+(10+K)s+1}}$$

b) Characteristic Equation is: $s^3+7s^2+(10+K)s+1=0$

By R-H criteria;

s^3	1	$10+K$
s^2	7	1
s^1	$\dfrac{7(10+K)-1\times 1}{7}$	0
s^0	1	0

System becomes stable, the condition is $\dfrac{7.(10+K)-1}{7} > 0$

$70+7K-1>0 \longrightarrow 7K > -69$

$K > -69/7 \cong -9.86$

$$\boxed{K > -69/7 \quad (\text{or} \quad K > -9.86)}$$

c) Open loop transfer function;

$$G(s) = \frac{\text{Numerator}}{\text{Denominator}-\text{Numerator}} = \frac{K}{s^3+7s^2+(10+K)s+1-K} \quad (K=20)$$

When $K=20$; $G(s) = \dfrac{20}{s^3+7s^2+30s-19}$

∴ No. of open loop poles at origin = 0

∴ Type = 0

d) $E_{ss} = \frac{5}{1+K_p}$; $K_p = \lim_{s \to 0} G(s)$

$$K_p = \lim_{s \to 0} \frac{20}{s^3+70s^2+30s-19}$$

$$K_p = -20/19$$

$$\boxed{E_{ss} = \frac{5}{1-\frac{20}{19}} = -95}$$

e) $E_{ss} = \frac{5}{K_v}$; $K_v = \lim_{s \to 0} s.G(s)$

$$K_v = \lim_{s \to 0} \frac{20}{s^3+70s^2+30s-19} = 0$$

$$\boxed{E_{ss} = \frac{5}{0} = \infty}$$