Problem 1)

a.

$$Y + x'z + xy' = Y + (x'z + x)(x'z + Y) \quad \text{A.3}$$
$$= Y + (x' + x)(z + x)(x'z + Y') \quad \text{A.6}$$
$$= Y + (z + x)(x'z + Y') \quad \text{A.3}$$
$$= (Y + z + x)(Y + x'z + Y') \quad \text{A.6}$$
$$= X + Y + \overline{z}$$

⑩

b.

$$A'B' + B'C + AC + AB + BC' = A'B' + AC + AB + BC' \quad \text{Consensus theorem}$$

$$\underbrace{A'B' + AC + B'C}$$

$$= A'B' + AC + AB + BC' \quad \text{Consensus theorem}$$

$$\underbrace{AC + BC' + AB}$$

$$= A'B' + AC + BC'$$

⑮

Question 2   Solution

a) $f(x_2, x_1, x_0) = x_0 x_1 x_2 + x_0 x_1' x_2 + x_0' x_1 x_2$ $\boxed{5\text{ pts}}$

b)

x_0	x_1	x_2	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$\boxed{7\text{ pts}}$

c)

$\boxed{8\text{ pts}}$

$f = x_0 x_2 + x_1 x_2$

d)
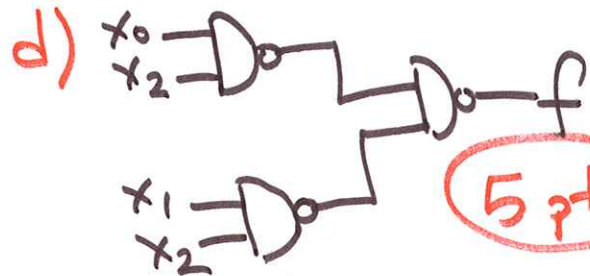
$\boxed{5\text{ pts}}$

$$F(A, B, C, D) = \bar{A}\bar{B}\bar{D} + \bar{A}C + B\bar{C}\bar{D} + BD + AB\bar{C}$$

$\boxed{15\text{pts}}$

$$ex: \left((A + B + D)' + (A + C')' \right)'$$

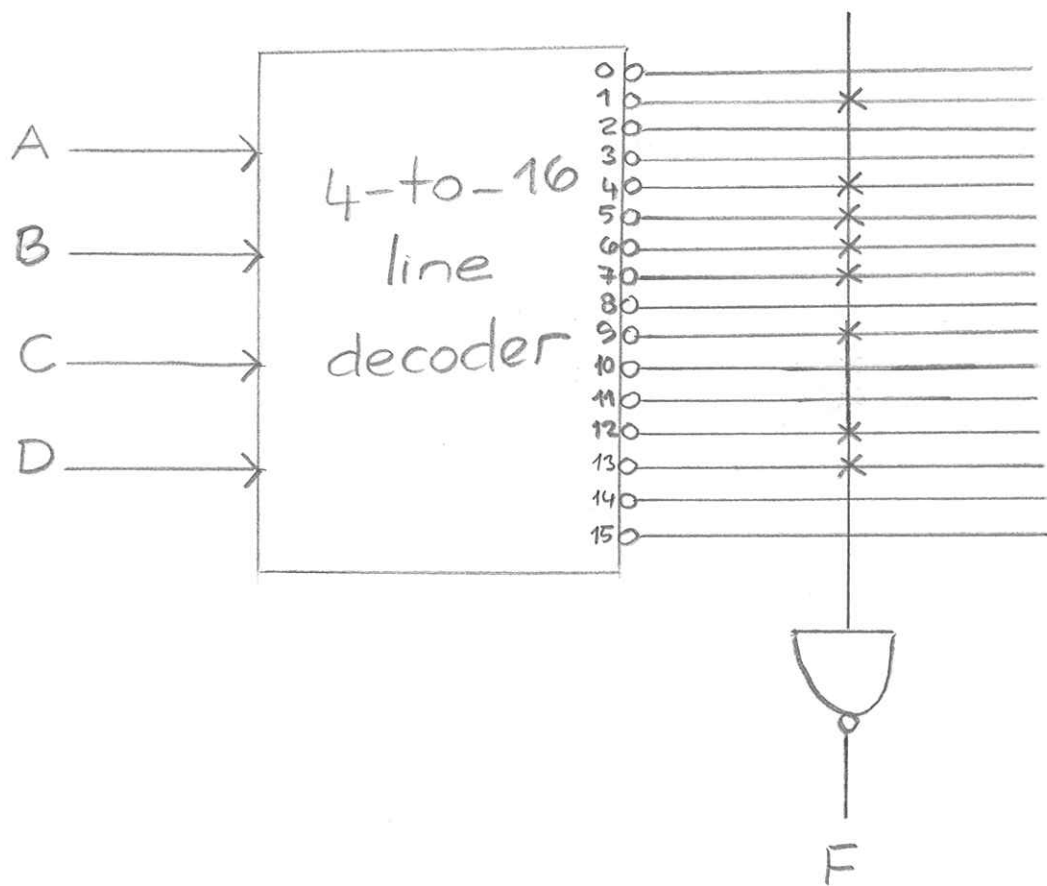**b)** A A' B B' C C' D D'

F'

F
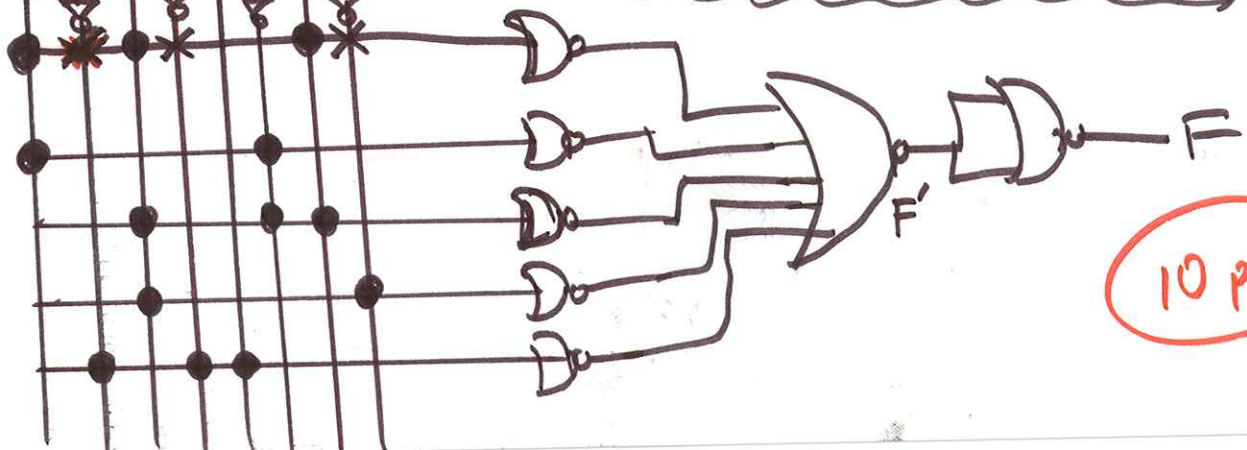
$\boxed{10\text{ pts}}$

Problem 4) We have

$$F(A,B,C,D) = AB + BC'D' + A'BC + C'D$$

$$= AB(C+C')(D+D') + (A+A')BC'D'$$
$$+ A'BC(D+D') + (A+A')(B+B')C'D$$

$$= \sum m(1,4,5,6,7,9,12,13,14,15)$$

a. We consider a decoder that generates maxterms of a four variable Boolean fn., that is having complemented outputs with a minimum size of 4-to-16 line

⑩

b. We reconsider $F(A,B,C,D)$ as follows:

$$F(A,B,C,D) = AB(C+C') + BC'D' + A'BC + (B+B')C'D$$

$$= A(BC) + A(BC') + D'(BC') + A'(BC)$$
$$+ D(BC') + D(B'C')$$
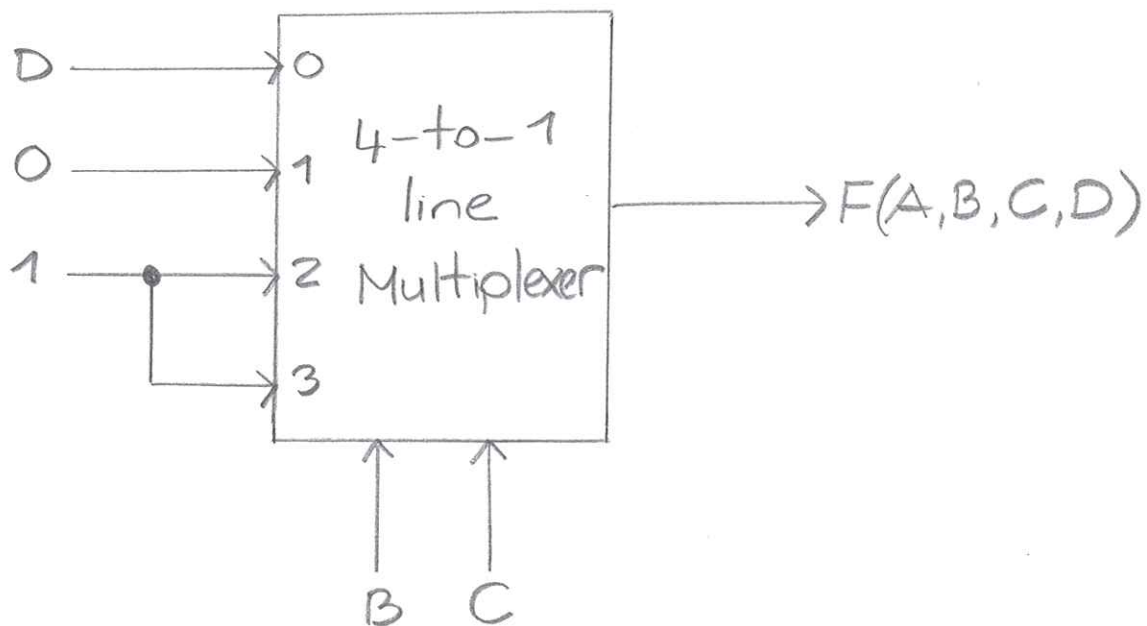
$$= (A+A')BC + (A+D'+D)(BC')$$
$$+ D(B'C')$$

$$= 1 \cdot BC + 1 \cdot BC' + D \cdot B'C'$$

$$= 1 \cdot m_3 + 1 \cdot m_2 + D \cdot m_0 + 0 \cdot m_1$$

where $m_i, i=0,1,2,3$ denotes minterms of a Boolean fn. with input variables B and C.

Alternatively;
- if we employ A and B as select (control)
inputs, then we have

$$F(A,B,C,D) = 1 \cdot (AB) + (A+A')BC'D' + A'BC$$
$$+ (A+A')(B+B')C'D$$

$$= 1 \cdot (AB) + C'D' \cdot (AB) + C'D' \cdot (A'B) + C \cdot (A'B)$$
$$+ C'D \cdot (A'B') + C'D \cdot (A'B) + C'D \cdot (AB')$$
$$+ C'D \cdot (AB)$$

$$= C'D \cdot (A'B') + \overbrace{(C'D' + C + C'D)}^{=1} \cdot (A'B)$$
$$+ C'D(AB') + (1 + C'D' + C'D) \cdot (AB)$$

$$= C'D \cdot m_0 + 1 \cdot m_1 + C'D \cdot m_2 + 1 \cdot m_3$$

where $m_i, i = 0,1,2,3$ denotes the minterms of
a Boolean function with input variables A
and B

Hence;