EEEN 322 Spring 2019

"Angle Modulation 3"

Angle modulation is more immune (less susceptible) to nonlinearities since the amplitude is constant.

Consider a second-order nonlinear device whose input $x(t)$ and output $y(t)$ are related by

$$y(t) = a_1 x(t) + a_2 x^2(t)$$

If $x(t) = \cos[w_c t + \psi(t)]$, then

$$y(t) = a_1 \cos[w_c t + \psi(t)] + a_2 \cos^2[w_c t + \psi(t)]$$

$$= \frac{a_2}{2} + a_1 \cos[w_c t + \psi(t)] + \frac{a^2}{2} \cos[2w_c t + 2\psi(t)]$$

For the FM wave $\psi(t) = k_f \int m(\alpha) d\alpha$, and

$$y(t) = \underbrace{\frac{a_2}{2}}_{\text{DC (filtered out)}} + \underbrace{a_1 \cos[w_c t + k_f \int m(\alpha) d\alpha]}_{\text{FM signal that has the information of } m(t)} + \underbrace{\frac{a_2}{2} \cos[2w_c t + 2k_f \int m(\alpha) d\alpha]}_{\text{another FM signal that has the information of } m(t)}$$

DC (filtered out)

FM signal that has the information of $m(t)$. Carr. freq.: w_c, Mod. constant: k_f; can be obtained by a BPF with center freq. w_c and bandwidth Δw.

another FM signal that has the information of $m(t)$. Carr. freq.: $2w_c$, Mod. constant: $2k_f$. Can be obtained by a BPF with center freq. $2w_c$ and bandwidth $2\Delta w$

$\Rightarrow$ The nonlinearity has not distorted the information in any way.

Because of the property of multiplying the carrier frequency, such nonlinear devices are also called: frequency multipliers

Second order device multiplies the frequency by 2.

Generalize this result: n-th order multiplier (nonlinear device, such as a diode or transistor) multiplies the frequency by n.

Let $y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + \dots + a_n x^n(t)$

If $x(t) = A \cos[w_c t + k_f \int m(\alpha) d\alpha]$, then, using trigonometric identities, ...

$$y(t) = c_0 + c_1 \cos[w_c t + k_f \int m(\alpha)d\alpha] + c_2 \cos[2w_c t + 2k_f \int m(\alpha)d\alpha] + \dots + c_n \cos[n w_c t + n k_f \int m(\alpha)d\alpha]$$

Hence, the output will have spectra at $w_c, 2w_c, \dots, nw_c$, with freq. deviations $\Delta f, 2\Delta f, \dots, n\Delta f$, respectively. Using a bandpass filter centered at w_c, we may obtain the desired signal component $\cos[w_c t + k_f \int m(\alpha)d\alpha]$ without distortion. In fact, any of the terms other than the DC can be obtained and used to extract the information (message). Hence, such devices can be used to increase the carrier frequency as well as the frequency deviation.

A similar nonlinearity in AM causes distortion of the desired signal, and causes unwanted modulation with carrier frequency nw_c.

E.g. (DSB-SC) $x(t) = m(t)\cos w_c t$ passes through a nonlinear system s.t. $y(t) = a x(t) + b x^3(t)$

$$\Rightarrow y(t) = a m(t) \cos w_c t + b m^3(t) \cos^3 w_c t$$

$$= a m(t) \cos w_c t + b m^3(t) \frac{1}{4}[3\cos w_c t + \cos 3w_c t]$$

$$= \underbrace{a m(t) \cos w_c t + \frac{3b}{4} m^3(t) \cos w_c t}_{\text{Passed by the BPF}} + \underbrace{\frac{b}{4} m^3(t) \cos 3w_c t}_{\text{Suppressed by the BPF}}$$

Desired signal : $a m(t)$

Distortion component : $\frac{3b}{4} m^3(t)$ (can not be suppressed by filtering)

Generation of FM waves

① Indirect Method of Armstrong

* First, NBFM is generated as in Fig 5.6(b) (see lecture Note: Angle Modulation 2)
* Then, NBFM is converted to WBFM using frequency multipliers

(See Fig. 10 Armstrong Indirect FM Transmitter)

GOAL:

1) Carrier freq. of 91.2 MHz (FM range is determined to be 88 MHz – 108 MHz by FCC)
2) Freq. deviation of $\Delta f = 75$ kHz (determined by FCC)

(FCC: Federal Communications Commission)

$f_{c_1} = 200$ kHz is because it is easy to construct stable crystal oscillators as well as balanced modulators at this frequency

$\Delta f_1 = 25$ Hz because we want to maintain $\beta \ll 1$ for NBFM and the baseband spectrum ranges from 50Hz to 15kHz ($\beta = 0.5$ for the worst possible case, $f_m = 50$ Hz)

75 kHz / 25 Hz = 3000, this can be done by two multiplier stages of 64 and 48 ($64 \times 48 = 3072$)

$\Rightarrow$ output of $\Delta f = 25 \times 3072 = 76.8$ kHz

200 kHz $\times$ 3072 = 600 MHz $\Rightarrow$ use a freq. converter (mixer) after the first freq. multiplier.

200kHz $\times$ 64 = 12.8 MHz (First freq. multiplier)

12.8 MHz – 10.9 MHz = 1.9 MHz (Freq. converter)

1.9 MHz × 48 = 91.2 MHz (Second freq. multiplier)

(Remember frequency converters (mixers). Here, $\omega_{mix} = 10.9$ MHz therefore it is "down-conversion")

⊕ Frequency stability

⊖ Distortion because of the approximation done for NBFM. Amplitude limiting in the frequency multipliers removes most of this distortion.

⊖ Inherent noise caused by excessive multiplication

⊖ Distortion at lower modulating frequencies, where $\Delta f / f$ is not small enough.

② Direct Generation

* Use a voltage-controlled oscillator (VCO), where the freq. is controlled by an external voltage ($m(t)$ here)

(i) May use an opamp and a hysteric comparator (e.g. Schmitt trigger circuit)

(ii) May use a resonant circuit and vary one of the reactive components (C or L) by $m(t)$

E.g. a reverse biased semiconductor diode acts as a capacitor whose capacitance varies with the bias voltage (Such diodes are commercially known as varicaps, varactors, or voltacaps)

The freq. of oscillation: $\omega_0 = \frac{1}{\sqrt{LC}}$

If $C = C_0 - k\,m(t)$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L(C_0 - k\,m(t))}} = \frac{1}{\sqrt{LC_0\left(1 - \frac{k\,m(t)}{C_0}\right)}} = \frac{1}{\sqrt{LC_0}\left[1 - \frac{k\,m(t)}{C_0}\right]^{1/2}}$$

$$= \frac{1}{\sqrt{LC_0}}\left[1 - \frac{km(t)}{C_0}\right]^{-1/2} \approx \frac{1}{\sqrt{LC_0}}\left[1 + \frac{km(t)}{2C_0}\right] \quad \text{if } \frac{km(t)}{C_0} \ll 1$$

[Taylor Series approximation $(1+x)^n \approx 1 + nx$ for $|x| \ll 1$]

$$w_0 \approx \frac{1}{\sqrt{LC_0}} + \frac{km(t)}{2C_0\sqrt{LC_0}} = w_c + \frac{w_c\, k\, m(t)}{2C_0} = w_c + k_f\, m(t)$$

$\Rightarrow w_0 \approx w_c + k_f\, m(t)$ where $w_c = \frac{1}{\sqrt{LC_0}}$ and $k_f = \frac{k w_c}{2C_0}$

⊕ Produces sufficient freq. deviation and requires little freq. multiplication

⊖ Poor freq. stability (can be corrected by using feedback)

Demodulation of FM

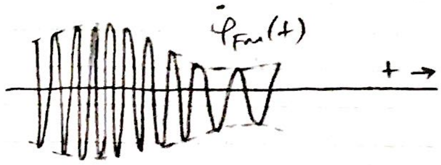
(1) A zero-crossing detector (frequency counters designed to measure the instantaneous freq. by the number of zero crossings) can be used. The rate of zero crossings gives the instantaneous frequency of the input signal. Before a freq. counter, a hard limiter should be used.

(2) If we apply $\varphi_{FM}(t)$ to an ideal differentiator, the output is

$$\dot{\varphi}_{FM}(t) = \frac{d}{dt}\left\{A\cos\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]\right\}$$

$$= -A\left[w_c + k_f m(t)\right]\sin\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]$$

The signal $\dot{\varphi}_{FM}(t)$ is both amplitude and frequency modulated,

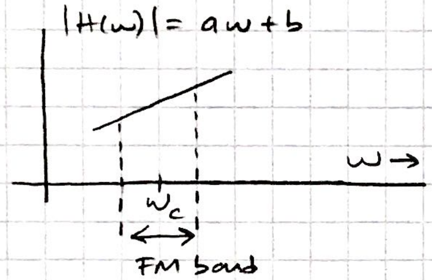


the envelope being $A[w_c + k_f m(t)]$. Because $\Delta w = k_f m_p < w_c$,

$\omega_c + k_f m(t) > 0 \;\; \forall t$, and $m(t)$ can be obtained by envelope detection of $\dot{\varphi}_{FM}(t)$.

The transfer function of a differentiator (e.g. an opamp differentiator) is $j\omega$, $H(\omega) = j\omega \Rightarrow |H(\omega)| = \omega$

In fact, any frequency-selective network with a transfer fct. of the form $|H(\omega)| = a\omega + b$ over the FM band would yield an output proportional to the instantaneous freq.



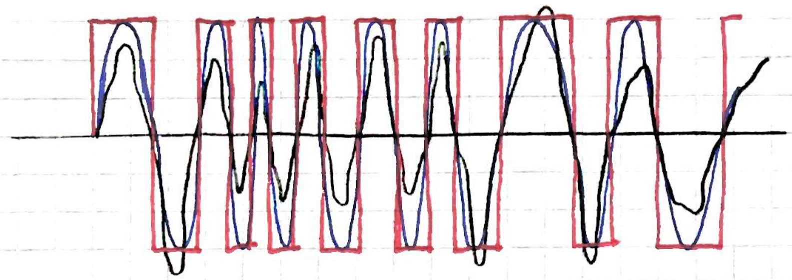
$$\varphi_{FM}(t) \longrightarrow \boxed{\frac{d}{dt}} \xrightarrow{\dot{\varphi}_{FM}(t)} \boxed{\text{Envelope detector}} \longrightarrow A[\omega_c + k_f m(t)]$$

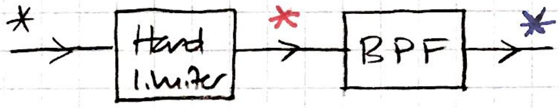
Several factors, such as channel noise, fading, and so on, cause A to vary. This variation in A should be removed before applying the signal to the FM detector.

Bandpass Limiter

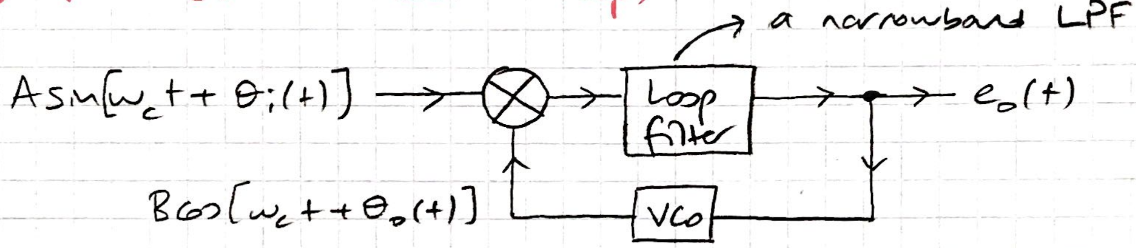
The amplitude variations of an angle-modulated carrier can be eliminated by what is known as a bandpass limiter, which consists of a hard limiter followed by a bandpass filter. Input-output relationship of a hard limiter:

$$v_o = \begin{cases} +1 & \text{if } v_i > 0 \\ -1 & \text{if } v_i < 0 \end{cases}$$





③ PLL (Phase-locked-loop)



(Remember) VCO: $\omega(t) = \omega_c + c e_o(t)$

$$\Rightarrow \text{phase } \phi(t) = \omega_c t + \underbrace{c\int_{-\infty}^{t} e_o(\tau)\,d\tau}_{\theta_o(t)}$$

$$\Rightarrow \dot{\theta}_o(t) = c e_o(t)$$

$$\Rightarrow e_o(t) = \dot{\theta}_o(t)/c$$

The input of the loop filter is:

$$AB\sin[\omega_c t+\theta_i(t)]\cos[\omega_c t+\theta_o(t)]$$

$$= \underbrace{\frac{AB}{2}[\sin(\theta_i-\theta_o)}_{e_o(t)} + \underbrace{\sin(2\omega_c t+\theta_i+\theta_o)]}_{\text{suppressed by the loop filter}}$$

when $\theta_i \nearrow$, $\theta_i-\theta_o \nearrow$, $e_o \nearrow$, $\theta_o \nearrow$, then $\theta_i-\theta_o \searrow \Rightarrow \theta_o \approx \theta_i$,

then the loop is locked, i.e., PLL tracks the input sinusoid

If FM $\quad \theta_i(t) = k_f\int_{-\infty}^{t} m(\alpha)\,d\alpha$

when the PLL is locked, $\theta_o(t) \approx \theta_i(t)$

$$\Rightarrow \theta_o(t) \approx k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha$$

$$\Rightarrow \dot{\theta}_o(t) \approx k_f\, m(t)$$

$$\Rightarrow e_o(t) \approx \frac{k_f}{c}\, m(t)$$ (the output of PLL is some constant times the message $m(t)$)

If PM $\theta_i(t) = k_p m(t)$

when the PLL is locked $\theta_o(t) \approx \theta_i(t)$

$$\Rightarrow \theta_o(t) \approx k_p\, m(t)$$

$$\Rightarrow \dot{\theta}_o(t) \approx k_p\, \dot{m}(t)$$

$$\Rightarrow e_o(t) \approx \frac{k_p}{c}\, \dot{m}(t)$$ (The output of PLL is some constant times the derivative of $m(t)$. Then, integrate it in order to recover the message $m(t)$)

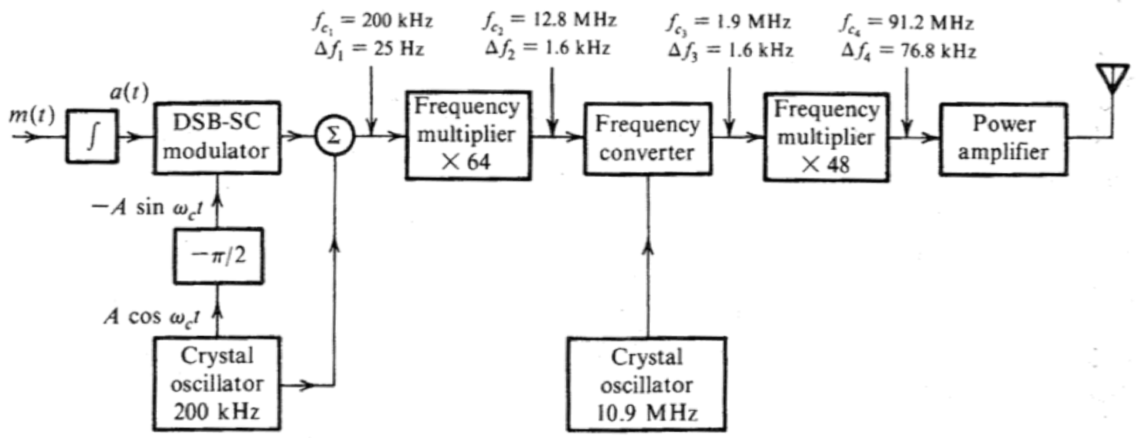


Figure 5.10 Armstrong indirect FM transmitter.