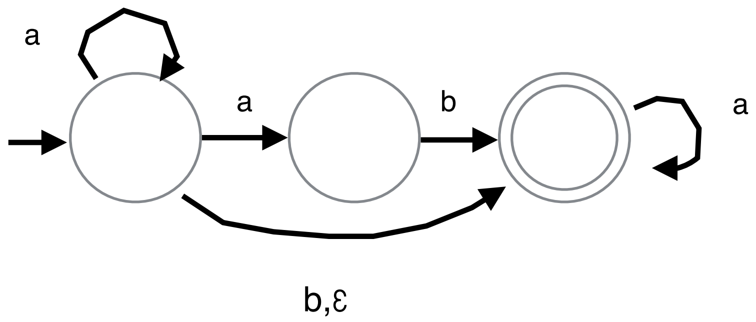
CMPE 222
Spring 2018
Midterm Exam Solutions

(30 pts)
1. Consider the following regular expression:

a*(ab U b U ε)a*

a) Design a non-deterministic finite-state automaton that recognizes the language this regular expression describes.
b) Write a **regular grammar** for the language this regular expression describes.



The regular grammar is:

Naming the states as variables in the regular grammar, S,X,Y beginning from the initial state:

S —> aS | aX | bY | Y
X —> bY
Y —> aY | ε

Now replacing the rule S —> Y with S —> aY | ε (since S —> Y is not a regular grammar rule), gives us the following right-regular grammar:

S —> aS | aX | bY | aY | ε
X —> bY
Y —> aY | ε

(40 pts)
2. Let D = {w I w contains an even number of a's and an odd number of b's and does not contain the substring ab}. (Σ = {a,b})

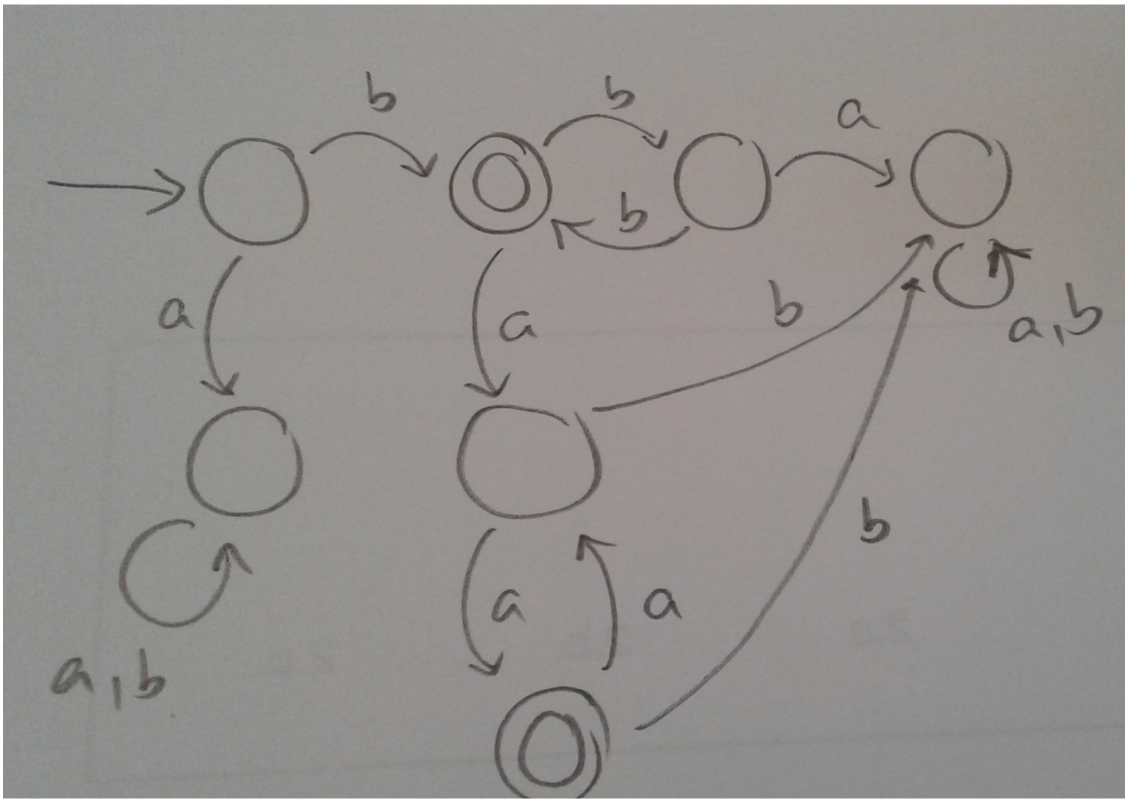
a) Design a deterministic finite state automaton the recognizes D. (You do **not** have to write it as a five-tuple.)
b) Give a regular expression that generates D.

(Suggestion: Describe D more simply)

When we try to write strings in D, we see that if there is no ab substring, then, either strings can only have a's or b's followed by a's or just b's. But, when also we take into account the fact that a's are even, whereas b's are odd, the following regular expression results:

b(bb)*(aa)*

The DFA (based on the regular expression) is:



(30 pts)

3. For any string $w = w_1w_2 \cdots w_n$, the reverse of w, written w^R, is the string w in reverse order, $w_n \cdots w_2w_1$. For any language A, let $A^R = \{w^R \mid w \in A\}$. Show that if A is regular, so is A^R.

(Hint: Remember that a language is ***regular*** if and only if it is recognized by a finite-state automaton).

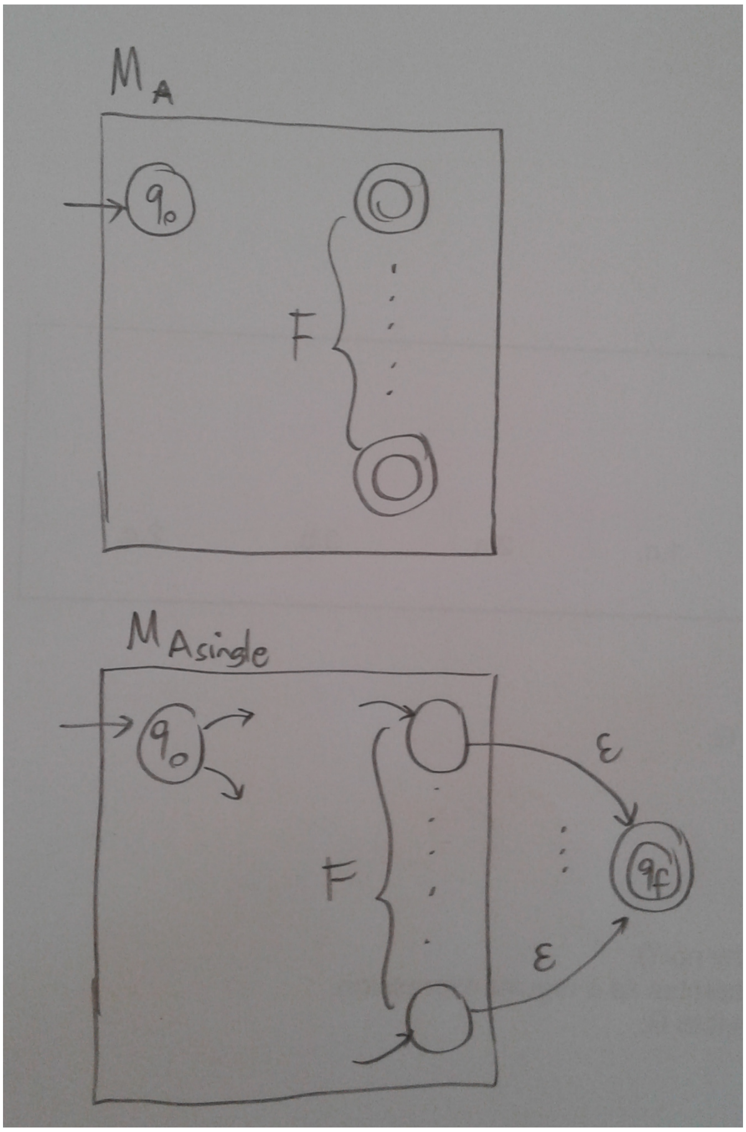
Let $M_A = (Q, \Sigma, \delta, q_0, F)$ be the FSA that recognizes A.

How can we construct $M_{A^R} = (Q', \Sigma', \delta'', q_0', F')$?

How is M_{A^R} different from M_A ?

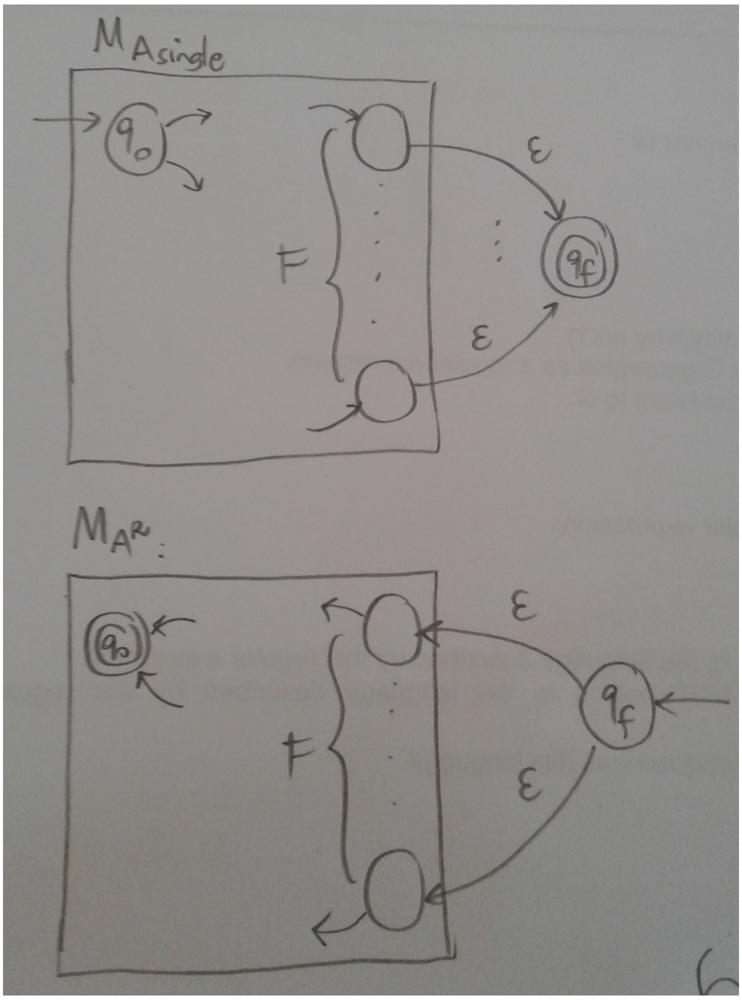
Firstly, convert M_A to an NFA with a single accept state as depicted in the following figure.

$M_{Asingle} = (Q \cup \{q_f\}, \Sigma, \delta', q_0, \{q_f\})$
and
$\delta'(q,r) = \delta(q,r)$ if q is in Q and
$\delta'(q,r) = \delta(q,r) \cup \{q_f\}$ is q is in F and $r = \varepsilon$.



How will How is M_{A^R} different from $M_{Asingle}$?

Well, the direction of all arrows will be reversed. Hence, the initial state of M_{A^R} will be the final state of $M_{Asingle}$, i.e. q_f. The final state of M_{A^R} will be the initial state of $M_{Asingle}$, i.e. q_0. And also all transitions where $\delta'(q,r) = s$ in $M_{Asingle}$ will be as $\delta''(s,r) = q$ in M_{A^R}.



$M_{A^R} = (Q', \Sigma, \delta'', q_0', F')$ is going to be:

$Q' = Q \cup \{q_f\}$

$\delta''(s,r) = q$ if $\delta'(q,r) = s$

$q_0' = q_f$

$F' = \{q_0\}$