MATH 233
Fall 2018
Class Week 6

Students:
In all questions, think about the sample space, try writing the sample space (as we did in the lectures, not all elements but the beginning few elements and the last element, in a kind of ordering). Describe the experiment. Describe the event you are interested in. List the outcomes corresponding to the event you are interested in.

1. **What is the probability that when a coin is flipped six times in a row, it lands heads up every time?**

Experiment: A fair coin is tossed six times.

Sample Space = { HHHHHH, HHHHHT, …. TTTTTT}

$\text{Event}_{\text{all heads}}$ = { HHHHHH }

| Sample Space | = 2^6 = 64

| $\text{Event}_{\text{all heads}}$ | = 1

P ($\text{Event}_{\text{all heads}}$) = 1 / 64

2. What is the probability that a five-card poker hand contains the two of diamonds and the three of spades?

(Remember that there are 13 kinds and 4 suits of each kind in a deck of 52 cards)

Experiment : Select 5 cards out of a 52-card deck.

Sample Space = { {1C,1H,1S,1D,2A}, {1C,1H,1S,1D,2S}, ….. {13C,13H,13S,13D,12A}}

$\text{Event}_{2,3}$: Two diamonds and three spades among 5 cards

$\text{Event}_{2,3}$ = {{1D,2D,1S,2S,3S}, {1D,3D,1S,2S,3S}, ….. {12D,13D,11S,12S,13S}}

|Sample Space | = C(52,5)

| $\text{Event}_{2,3}$ | = ?

C(13,2) is the number of ways to choose 2 diamonds from 13 diamonds. = 78
C(13,3) is the number of ways to choose 3 spades from 13 spades. = 286

C(13,2) C(13,3) = 78. 286 = 22308 = | $\text{Event}_{2,3}$ |

P($\text{Event}_{2,3}$) = | $\text{Event}_{2,3}$ | / |Sample Space | = 22308 / 2598960 = 0.00858

3. What is the probability that a five-card poker hand contains a flush, that is, five cards of the same suit?

(Remember that there are 13 kinds and 4 suits of each kind in a deck of 52 cards)

Experiment : Select 5 cards out of a 52-card deck.

Sample Space = { {1C,1H,1S,1D,2A}, {1C,1H,1S,1D,2S}, ….. {13C,13H,13S,13D,12A}}

$Event_{flush}$: Five cards of the same suit

$Event_{flush}$ = {{1D,2D,3D,4D,5D}, {1D,2D,3D,4D,6D}, …. {1H,2H,3H,4H,5H}, ….. }

|Sample Space | = C(52,5)

C(4,1) is the ways to select a suite.

C(13,5) is the number of ways to select 5 cards from a suite.

| $Event_{flush}$ | = C(4,1) C(13,5) = 4 . 1287 = 5148

5148 outcomes of the sample space are 5 cards of the same suite.

P($Event_{flush}$) = 5148 / C(52,5) = 5148 / 2598960 = 0.00198

4. What is the probability that a fair die never comes up an even number when it is rolled six times?

Experiment : Roll a fair die six times

Sample Space = { 1-1-1-1-1-1, 1-1-1-1-1-2, ….. 6-6-6-6-6-6}
(Note that each value in this set is an ordered list, not a set)

$Event_{AllOdds}$: All six outcomes are odd numbers

$Event_{AllOdds}$ = { 1-1-1-1-1-1, 1-1-1-1-1-3, ….. 5-5-5-5-5-5}

|Sample Space| = 6^6 = 46656

|$Event_{AllOdds}$ | = ?

The set $Event_{AllOdds}$ is equivalent to all possible six-letter words out of the letters {1,3,5}. Thus,
|$Event_{AllOdds}$ | = 3^6 = 729

P($Event_{AllOdds}$) = 729 / 46656 = 0.015625

5. What is the probability that a positive integer not exceeding 100 selected at random is divisible by 3?

Experiment : A positive integer between 1 and 100 is chosen (including 1 and 100).

Event3 : The number between [1,100] is divisable by 3.

Sample space = {1, 2, … , 100}   (like rolling a die with 100 faces)

|Sample Space | = 100

Event3 = {3,6,9,12,15,18,…. 99}

Any number is iether congruent to 0, 1 or 2 in mod 3. Event3 is the numbers that are congruent to 0 in mod 3. Therefore, 1/3 of the numbers between 3 and 98 (including them) are congruent to 0. to those add {99,1,2} out of which only one is congruent to 0. There are 98-3+1 = 96 numbers in [3,98] and 96/3 = 32 of them are congruent to 0. There is one number in {99,1,2} congruent to 0. Therefore there is a total of 33 numbers which are congruent to 0 in [1,100].

|Event3| = 33

P(Event3) = |Event3| / |Sample Space | = 33/100 = 0.33

6. In a superlottery, a player selects 7 numbers out of the first 80 positive integers. What is the probability that a person wins the grand prize by picking 7 numbers that are among the 11 numbers selected at random by a computer.

Sample space is all possible 7-nuber picks.

Sample Space = {{1,2,3,4,5,6,7}, {1,2,3,4,5,6,8}, ….. }
|Sample Space | = C(80,7) = 3,176,716,400

EventGP : The chosen 7 numbers is among the chosen 11 numbers

For every 11-number set, there are C(11, 7) = 330   7-number sets that are included in the 11 elements.

Thus, |EventGP| = 330

How many different 7-combinations are there?

C(80,7) = 3,176,716,400

P(EventGP) = |EventGP| / |Sample Space| = 330 / 3,176,716,400 = 0.00000010388