**2.** Find the output response, $c(t)$, for each of the systems shown in Figure P4.1. Also find the time constant, rise time, and settling time for each case. [Sections: 4.2, 4.3]

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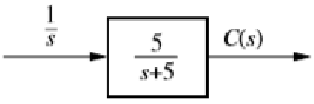
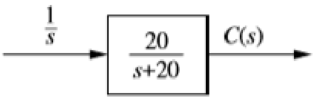



**(a)**




**(b)**

**4.** Find the capacitor voltage in the network shown in Figure P4.2 if the switch closes at $t = 0$. Assume zero initial conditions. Also find the time constant, rise time, and settling time for the capacitor voltage. [Sections: 4.2, 4.3]

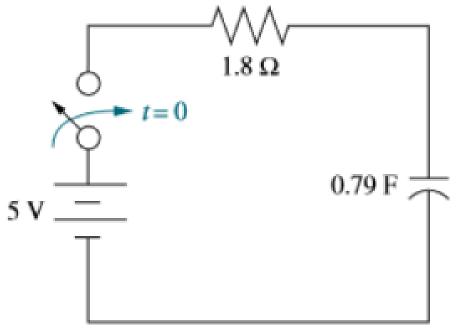



**FIGURE P4.2**

**6.** For the system shown in Figure P4.3, (a) find an equation that relates settling time of the velocity of the mass to $M$; (b) find an equation that relates rise time of the velocity of the mass to $M$. [Sections: 4.2, 4.3]

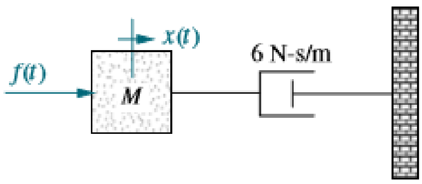



**FIGURE P4.3**

**8.** For each of the transfer functions shown below, find the locations of the poles and zeros, plot them on the $s$-plane, and then write an expression for the general form of the step response without solving for the inverse Laplace transform. State the nature of each response (overdamped, underdamped, and so on). [Sections: 4.3, 4.4]

**a.** $T(s) = \dfrac{2}{s+2}$

**b.** $T(s) = \dfrac{5}{(s+3)(s+6)}$

**c.** $T(s) = \dfrac{10(s+7)}{(s+10)(s+20)}$

**d.** $T(s) = \dfrac{20}{s^2+6s+144}$

**e.** $T(s) = \dfrac{s+2}{s^2+9}$

**f.** $T(s) = \dfrac{(s+5)}{(s+10)^2}$

**20.** For each of the second-order systems that follow, find $\zeta$, $\omega_n$, $T_s$, $T_p$, $T_r$, and $\%OS$. [Section: 4.6].

**a.** $T(s) = \dfrac{16}{s^2+3s+16}$

**b.** $T(s) = \dfrac{0.04}{s^2+0.02s+0.04}$

**c.** $T(s) = \dfrac{1.05\times10^7}{s^2+1.6\times10^3 s+1.05\times10^7}$

**25.** For the system shown in Figure P4.7, do the following: [Section: 4.6]

**a.** Find the transfer function $G(s) = X(s)/F(s)$.

**b.** Find $\zeta$, $\omega_n$, $\%OS$, $T_s$, $T_p$, and $T_r$.

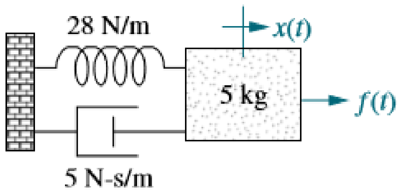



**FIGURE P4.7**

**26.** For the system shown in Figure P4.8, a step torque is applied at $\theta_1(t)$. Find

**a.** The transfer function, $G(s) = \theta_2(s)/T(s)$.

**b.** The percent overshoot, settling time, and peak time for $\theta_2(t)$. [Section: 4.6]

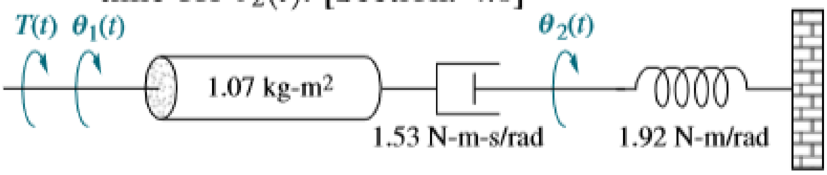



**FIGURE P4.8**

**2.**

**a.** $C(s) = \frac{5}{s(s+5)} = \frac{1}{s} - \frac{1}{s+5}$. Therefore, $c(t) = 1 - e^{-5t}$.

Also, $T = \frac{1}{5}$, $T_r = \frac{2.2}{a} = \frac{2.2}{5} = 0.44$, $T_s = \frac{4}{a} = \frac{4}{5} = 0.8$.

**b.** $C(s) = \frac{20}{s(s+20)} = \frac{1}{s} - \frac{1}{s+20}$. Therefore, $c(t) = 1 - e^{-20t}$. Also, $T = \frac{1}{20}$,

$T_r = \frac{2.2}{a} = \frac{2.2}{20} = 0.11$, $T_s = \frac{4}{a} = \frac{4}{20} = 0.2$.

**4.**

Using voltage division, $\frac{V_C(s)}{V_i(s)} = \frac{1/RC}{S + \frac{1}{RC}} = \frac{0.703}{s+0.703}$. Since $V_i(s) = \frac{5}{s}$

$$V_c(s) = \frac{5}{s}\left(\frac{0.703}{s+0.703}\right) = \frac{5}{s} - \frac{5}{s+0.703}.$$

Therefore $v_c(t) = 5 - 5e^{-0.703t}$. Also,

$$T = \frac{1}{0.703} = 1.422;\; T_r = \frac{2.2}{0.703} = 3.129;\; T_s = \frac{4}{0.703} = 5.69.$$

**6.**

Writing the equation of motion,

$$(Ms^2 + 6s)X(s) = F(s)$$

Thus, the transfer function is,

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + 6s}$$

Differentiating to yield the transfer function in terms of velocity,

$$\frac{sX(s)}{F(s)} = \frac{1}{Ms + 6} = \frac{1/M}{s + \frac{6}{M}}$$

Thus, the settling time, $T_s$, and the rise time, $T_r$, are given by

$$T_s = \frac{4}{6/M} = \frac{2}{3}M = 0.667M; \quad T_r = \frac{2.2}{6/M} = \frac{1.1}{3}M = 0.367M$$

**8.**

**a.** Pole: -2; $c(t) = A + Be^{-2t}$ ; first-order response.

**b.** Poles: -3, -6; $c(t) = A + Be^{-3t} + Ce^{-6t}$; overdamped response.

**c.** Poles: -10, -20; Zero: -7; $c(t) = A + Be^{-10t} + Ce^{-20t}$; overdamped response.

**d.** Poles: $(-3+j3\sqrt{15})$, $(-3-j3\sqrt{15})$ ; $c(t) = A + Be^{-3t}\cos(3\sqrt{15}\ t + \phi)$; underdamped.

**e.** Poles: j3, -j3; Zero: -2; $c(t) = A + B\cos(3t + \phi)$; undamped.

**f.** Poles: -10, -10; Zero: -5; $c(t) = A + Be^{-10t} + Cte^{-10t}$; critically damped.

**20.**

**a.** $\omega_n^2 = 16$ r/s, $2\zeta\omega_n = 3$. Therefore $\zeta = 0.375$, $\omega_n = 4$. $T_s = \frac{4}{\zeta\omega_n} = 2.667$ s; $T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.8472$ s; $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 28.06\ \%$; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1) = 1.4238$; therefore, $T_r = 0.356$ s.

**b.** $\omega_n^2 = 0.04$ r/s, $2\zeta\omega_n = 0.02$. Therefore $\zeta = 0.05$, $\omega_n = 0.2$. $T_s = \frac{4}{\zeta\omega_n} = 400$ s; $T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 15.73$ s; $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 85.45\ \%$; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1)$; therefore, $T_r = 5.26$ s.

**c.** $\omega_n^2 = 1.05 \times 10^7$ r/s, $2\zeta\omega_n = 1.6 \times 10^3$. Therefore $\zeta = 0.247$, $\omega_n = 3240$. $T_s = \frac{4}{\zeta\omega_n} = 0.005$ s; $T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.001$ s; $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 44.92\ \%$; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1)$; therefore, $T_r = 3.88\times10^{-4}$ s.

**25.**

**a.** Writing the equation of motion yields, $(5s^2+5s+28)X(s)=F(s)$

Solving for the transfer function,

$$\frac{X(s)}{F(s)}=\frac{1/5}{s^2+s+\dfrac{28}{5}}$$

**b.** $\omega_n^2 = 28/5$ r/s, $2\zeta\omega_n = 1$. Therefore $\zeta = 0.211$, $\omega_n = 2.37$. $T_s = \dfrac{4}{\zeta\omega_n} = 8.01$ s; $Tp = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} =$ 1.36 s; $\%OS = e^{-\zeta\pi / \sqrt{1-\zeta^2}} \times 100 = 50.7$ %; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1)$; therefore, $T_r = 0.514$ s.

**26.**

Writing the loop equations,

$$(1.07s^2+1.53s)\theta_1(s)-1.53\theta_2(s)=T(s)$$
$$-1.53s\theta_1(s)+(1.53s+1.92)\theta_2(s)=0$$

Solving for $\theta_2(s)$,

$$\theta_2(s)=\frac{\begin{vmatrix}(1.07s^2+1.53s) & T(s)\\ -1.53s & 0\end{vmatrix}}{\begin{vmatrix}(1.07s^2+1.53s) & -1.53s\\ -1.53s & (1.53s+1.92)\end{vmatrix}}=\frac{0.935T(s)}{s^2+1.25s+1.79}$$

Forming the transfer function,

$$\frac{\theta_2(s)}{T(s)}=\frac{0.935}{s^2+1.25s+1.79}$$

Thus $\omega_n = 1.34$, $2\zeta\omega_n = 1.25$. Thus, $\zeta = 0.467$. From Eq. (4.38), %OS = 19.0%. From Eq. (4.42), $T_s$ = 6.4 seconds. From Eq. (4.34), $T_p$ = 2.66 seconds.