

1)

$$a) G_c(s) = K$$

$$G_p(s) = \frac{1}{(s+2)(s+5)(s+10)}$$

$$H(s) = 10$$

$$\begin{aligned} \text{T.F. } \frac{C(s)}{R(s)} &= \frac{G_c(s) \cdot G_p(s)}{1 + G_c(s) \cdot G_p(s) \cdot H(s)} \\ &= \frac{K / (s+2)(s+5)(s+10)}{1 + \frac{10K}{(s+2)(s+5)(s+10)}} \end{aligned}$$

$$\boxed{T(s) = \frac{C(s)}{R(s)} = \frac{K}{(s+2) \cdot (s+5) \cdot (s+10) + 10K}}$$

$$\begin{aligned} \frac{C(s)}{R(s)} &= \frac{K}{(s^2+7s+10) \cdot (s+10) + 10K} \\ &= \frac{K}{s^3 + 10s^2 + 7s^2 + 70s + 10s + 100 + 10K} \end{aligned}$$

$$\frac{C(s)}{R(s)} = \frac{K}{s^3 + 17s^2 + 80s + 100 + 10K}$$

for stability

take R.H. criterion

The characteristic equation (C.E.) $= 1 + G_c(s) \cdot G_p(s) \cdot H(s) = 0$

$=$ Denominator of $\frac{C(s)}{R(s)} = T(s) = \text{T.F.}$

$$s^3 + 17s^2 + 80s + 100 + 10K = 0$$

$$\begin{array}{l|ll} s^3 & 1 & 80 \\ s^2 & 17 & 10K+100 \\ s^1 & \frac{1360-10K-100}{17} & > 0 \rightarrow 1260-10K > 0 \\ s^0 & 10K+100 & > 0 \rightarrow K > -10 \end{array} \quad K < 126$$

for stability the range of K is ;

$$\boxed{-10 < K < 126}$$

b) for marginally (damped oscillation)

The k value is $k = 126$

c) frequency of oscillations.

Get the Auxillary equation from R-H criteria

$$A(s) = 0$$

$$A(s) = 17s^2 + 10k + 100 = 0$$

$$\text{put } k = 126$$

$$17s^2 + 1260 + 100 = 0$$

$$17s^2 = -1360$$

$$s = \pm j \sqrt{\frac{1360}{17}}$$

$$j\omega = \pm j \cdot 8.94 \quad (s = j\omega)$$

$$\omega = 8.94 \text{ rad/sec.}$$

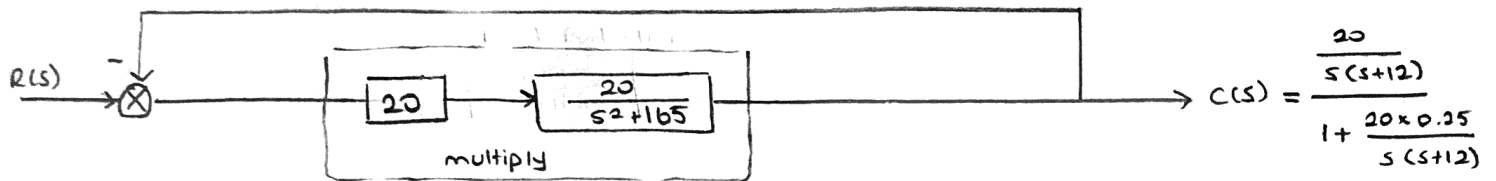
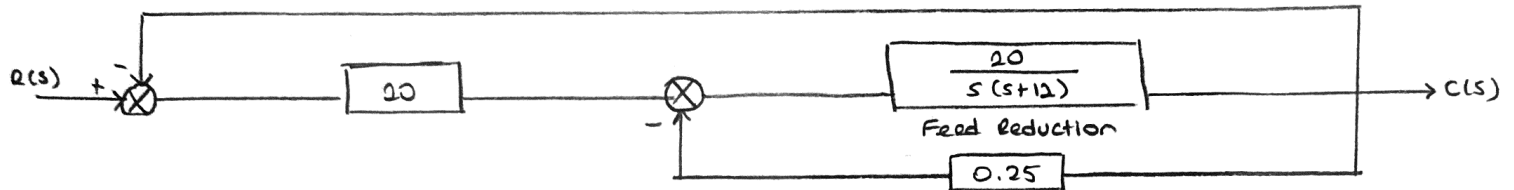
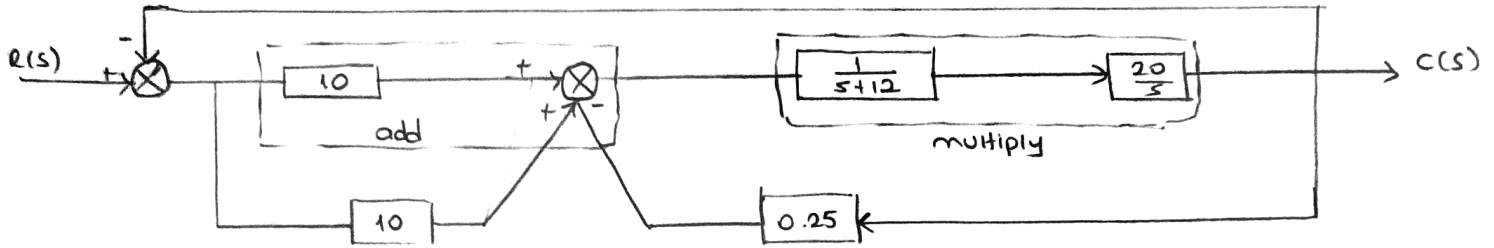
$$2\pi f = 8.36$$

$$f = \frac{8.36}{2\pi}$$

$$f = 1.42 \text{ Hz}$$

Damped oscillation with frequency $f = 1.42 \text{ Hz}$ where k is 126.

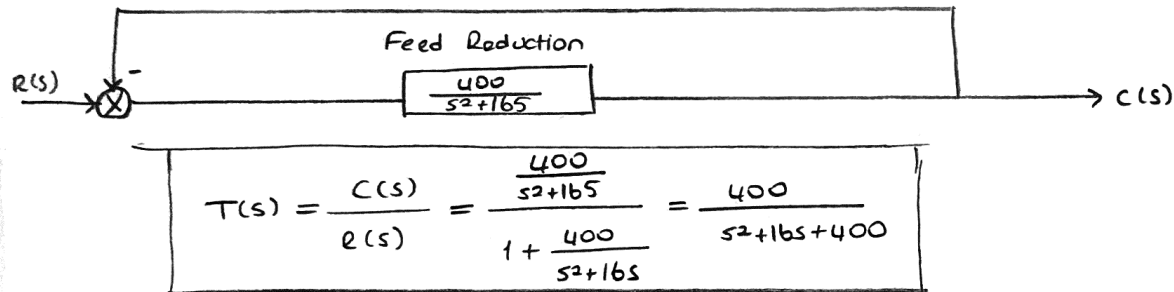
2)



$$C(s) = \frac{\frac{20}{s(s+12)}}{1 + \frac{20 \times 0.25}{s(s+12)}}$$

$$= \frac{20}{s(s+12) + 45}$$

$$= \frac{20}{s^2 + 16s + 400}$$



$$T(s) = \frac{C(s)}{R(s)} = \frac{\frac{400}{s^2 + 16s}}{1 + \frac{400}{s^2 + 16s}} = \frac{400}{s^2 + 16s + 400}$$

Characteristic equation; $s^2 + 16s + 400$

Standard eq. $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$$2\zeta\omega_n = 16 \quad \omega_n^2 = 400$$

$$\zeta = \frac{16}{2\omega_n} = \frac{16}{40} \quad \boxed{\omega_n = \sqrt{400} = 20 \text{ rad/s}}$$

$$\boxed{\zeta = 0.4}$$

$$\boxed{\text{Settling Time } (T_s) = \frac{4}{\zeta\omega_n} = \frac{4}{(0.4 \times 20)} = 0.5 \text{ sec.}}$$