# Power Distribution Systems

Lecture 5

Alternating Current (A.C.) Distribution

*Part II*

(Unbalanced Loads)

# Introduction

The 3-phase loads that have the same impedance and power factor in each phase are called balanced loads.

When each load phase has different impedance and/or power factor, the loads are unbalanced. In that case, current and power in each phase will be different.

In practice, we may come across the following unbalanced loads :

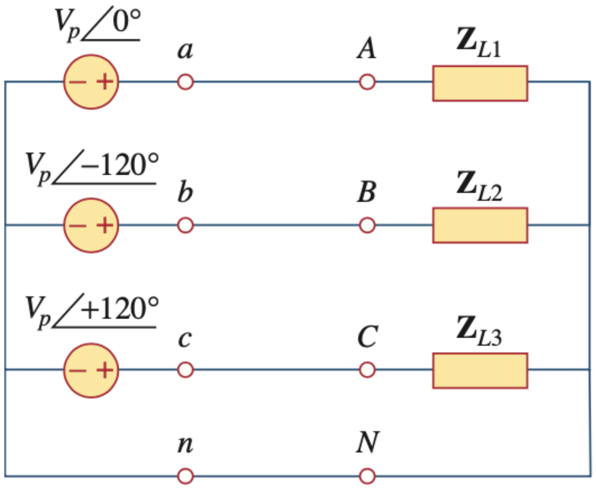
a) Four-wire star-connected unbalanced load (*the most common circumstance*)
b) Unbalanced Δ-connected load
c) Unbalanced 3-wire, *Y*-connected load

The 3-phase, 4-wire system is widely used for distribution of electric power in commercial and industrial buildings.

Firstly, we may take a quick look at 3-phase a.c. systems.

# Balanced Three-Phase Voltages

A three-phase system is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by $120^0$.




Three-phase four-wire system

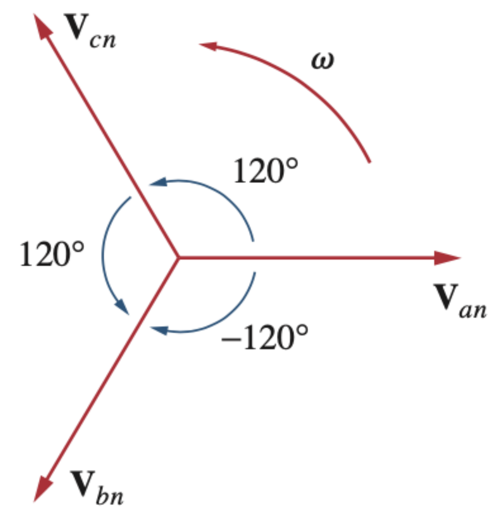
A typical three-phase system consists of three voltage sources connected to loads by three or four wires (or transmission lines). A three-phase system is equivalent to three single-phase circuits. When the neutral line is available, calculation for each phase can be done independently.

# Balanced Three-Phase Voltages

Since the three-phase voltages are $120^0$ out of phase with each other, it can be expressed mathematically as

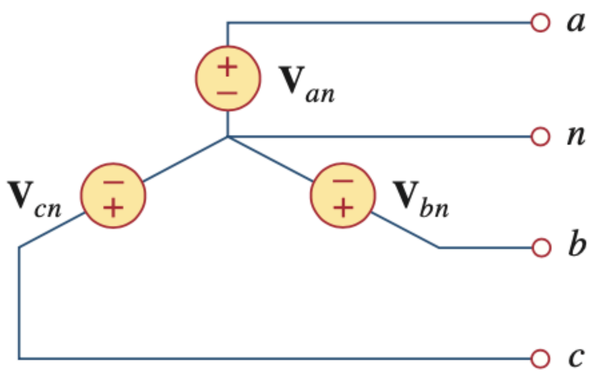
$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -120^\circ$$
$$\mathbf{V}_{cn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

where $V_p$ is the effective or rms value of the phase voltages. This is known as the *abc sequence* or *positive sequence*. In this phase sequence, $\mathbf{V}_{an}$ leads $\mathbf{V}_{bn}$, which in turn leads $\mathbf{V}_{cn}$. This sequence is produced when the vectors rotates counterclockwise.

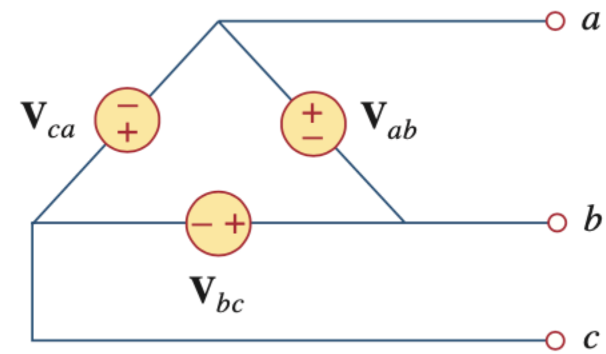


# Balanced Three-Phase Voltages

The voltage sources can be either wye-connected or delta-connected.




Y-connected three-phase voltage sources




Δ-connected three-phase voltage sources

Let us consider the wye-connected voltages. The voltages $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, and $\mathbf{V}_{cn}$ are respectively between lines *a*, *b*, and *c*, and the neutral line *n*. If the voltage sources have the *same amplitude and frequency* ω and are out of phase with each other *by 120⁰*, the voltages are said to be ***balanced***. This implies that

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

# Balanced Three-Phase Voltages

**Example 5.1** *Given that* $V_{bn} = 110\angle 30^0$ *V, find* $V_{an}$ *and* $V_{cn}$*, assuming a positive (abc) sequence.*

# Unbalanced Three-Phase Systems

An unbalanced system is caused by two possible situations:

(1) the source voltages are not equal in magnitude and/or differ in phase by angles that are unequal,

(2) load impedances are unequal.

In this lecture, we will assume balanced source voltages, but an unbalanced load.

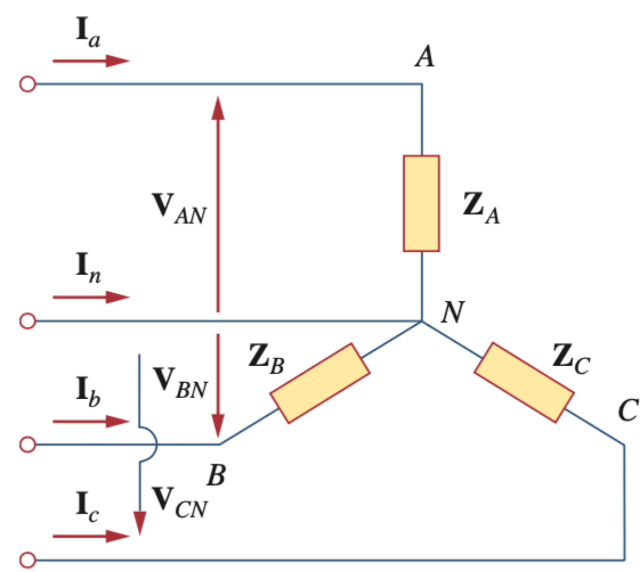
The line currents are determined by Ohm's law as

$$\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_A}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B}, \qquad \mathbf{I}_c = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_C}$$

Applying KCL at node *N* gives the neutral line current as

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c)$$

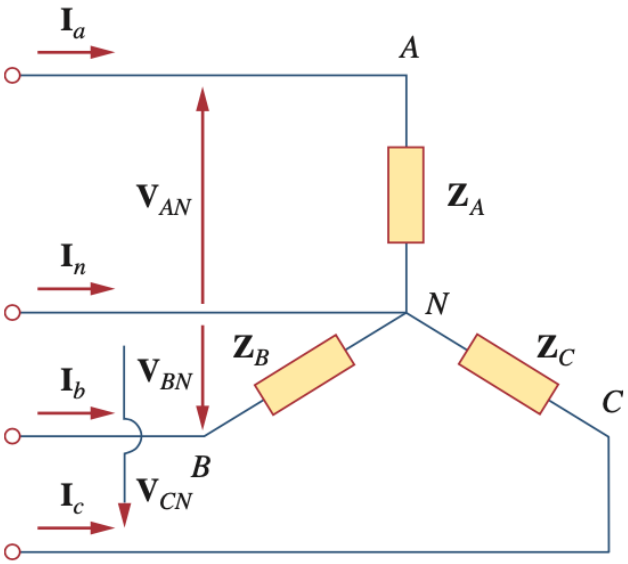
where $Z_A$, $Z_B$, and $Z_C$ are not equal.




Unbalanced three-phase Y-connected load with balanced voltage sources

# Unbalanced Three-Phase Systems

**Example 5.2** *The unbalanced wye-load of figure has balanced voltages of 100 V and the abc sequence. Calculate the line currents and the neutral current. Take* $\mathbf{Z}_A$*=15 Ω,* $\mathbf{Z}_B$ *=10+j5 Ω,* $\mathbf{Z}_C$ *=6-j8 Ω.*



# Four-Wire Star Connected Unbalanced Loads

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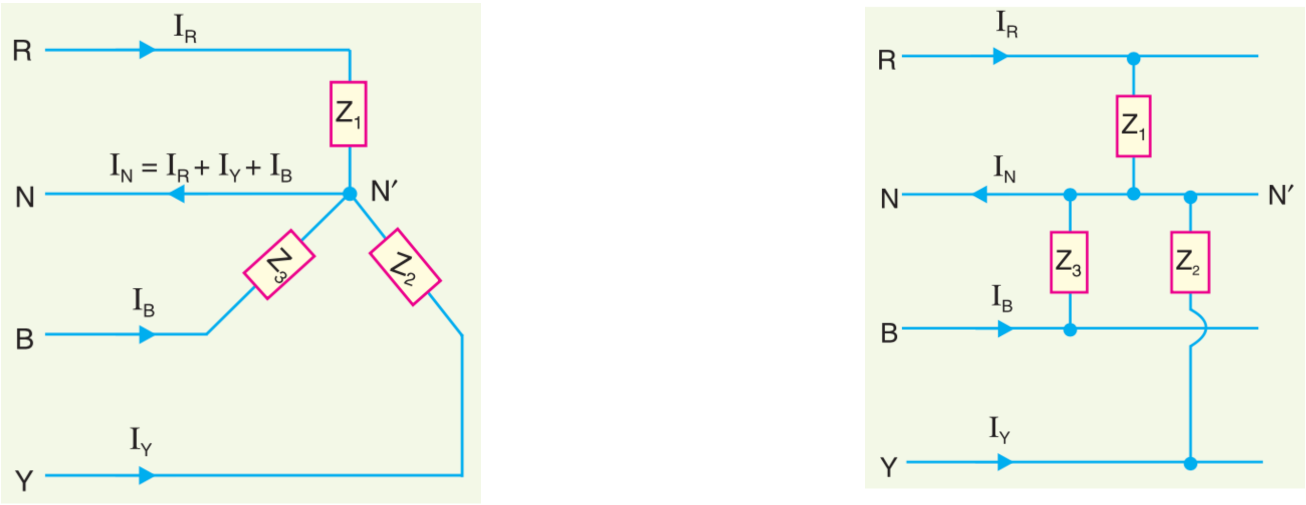
There are two ways to obtain four-wire star connected unbalanced loads;

a) Three-phase unbalanced load (*rare*)

b) Three single-phase loads (*common*)




(a) (b)

R: Red
B: Blue
Y: Yellow
N: Neutral

Load impedances $Z_1$, $Z_2$ and $Z_3$ are unequal. Each phase voltage is equal, i.e. balanced three-phase source.

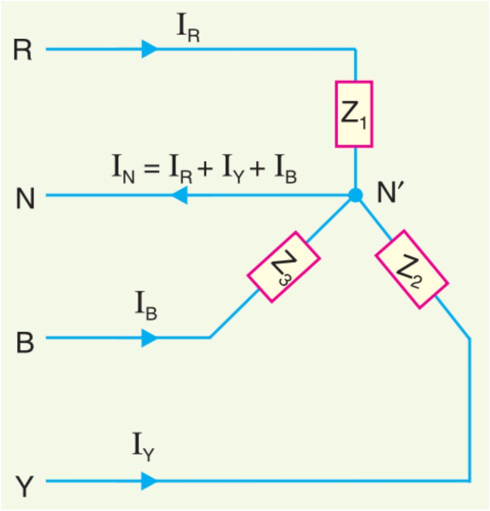
In actual practice, most of the 3-phase loads (*e.g.* 3-phase motors) are 3-phase, 3-wire and are balanced loads. However, when it is added single phase loads (*e.g.* lights, fans etc.), the balance is lost.

# Four-Wire Star Connected Unbalanced Loads



Since the load is unbalanced, the line currents will be different in magnitude and displaced from one another by unequal angles. The current in the neutral wire will be the phasor sum of the three line currents *i.e*.

Current in neutral wire, $I_N = I_R + I_Y + I_B$ (phasor sum)

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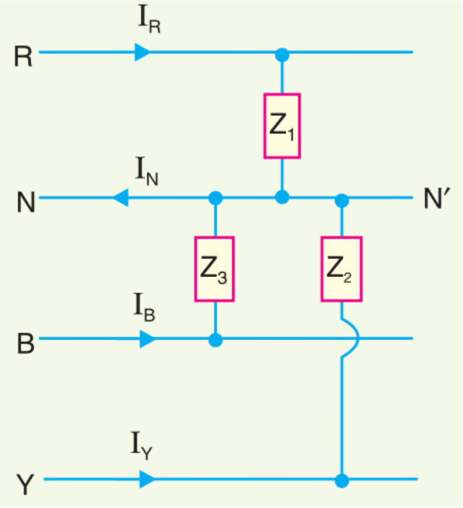


Two circuits are essentially equivalent

# Four-Wire Star Connected Unbalanced Loads

It should be noted that;

- The neutral wire has negligible resistance, thus supply neutral $N$ and load neutral $N'$ will be at the same potential.
- The amount of current flowing in the neutral wire will depend upon the magnitudes of line currents and their phasor relations. In most circuits encountered in practice, the neutral current is equal to or smaller than one of the line currents.

# Four-Wire Star Connected Unbalanced Loads

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<u>**Example 5.3**</u> *Non-reactive loads of 10 kW, 8 kW and 5 kW are connected between the neutral and the red, yellow and blue phases respectively of a 3-phase, 4-wire system. The line (phase-to-phase) voltage is 400 V. Calculate*

*a) the current in each line and*

*b) the current in the neutral wire.*

# Four-Wire Star Connected Unbalanced Loads

**Example 5.4** *The three line leads of a 400/230 V, 3-phase, 4-wire supply are designated as R, Y and B respectively. The fourth wire or neutral wire is designated as N. The phase sequence is RYB.*

*a) Compute the currents in the four wires when the following loads are connected to this supply :*

*From R to N : 20 kW, unity power factor*
*From Y to N : 28·75 kVA, 0·866 lag*
*From B to N : 28·75 kVA, 0·866 lead*

*b) If the load from B to N is removed, what will be the value of currents in the four wires ?*

# Four-Wire Star Connected Unbalanced Loads

**Example 5.5** *A three-phase, four-wire distributor supplies a balanced voltage of 400/230 V to a load consisting of 30 A at p.f. 0.866 lagging for R-phase, 30 A at p.f. 0.866 leading for Y phase and 30 A at unity p.f. for B phase. The resistance of each line conductor is 0.2 Ω. The area of X-section of neutral is half of any line conductor. Calculate the supply end voltage for R phase. The phase sequence is RYB.*