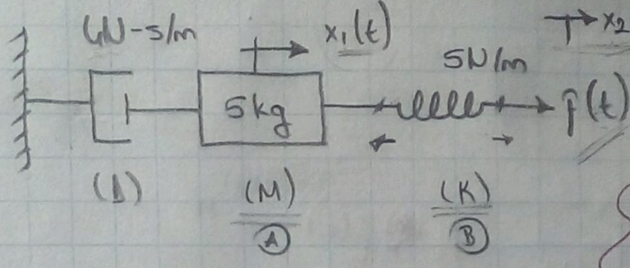
EEEN - 352 (PS-4)

2-23) Find the transfer function, $G(s) = X_1(s)/F(s)$, for the translational mechanical system



1 Force dist?

Spring → $K.\,x_2$

Mass → $M.\,\frac{d^2x_1}{dt^2}$

Damper → $D.\,\frac{dx_1}{dt}$

$\frac{dx}{dt} = v$

$\frac{d^2x}{dt^2} = a$

$\frac{dv}{dt} = a$

1 Equations of motion.

(A) $M.\,\frac{d^2x_1}{dt^2} + D.\,\frac{dx_1}{dt} + K.x_1 - K.x_2(t) = 0$

(B) $-K.\,x_1(t) + K.x_2(t) = f(t)$

↳ $5s^2X_1(s) + 4.s\,X_1(s) + 5.\,X_1(s) - 5.\,X_2(s) = 0$

(A) $(5s^2+4s+5)\,X_1(s) - 5X_2(s) = 0$

↳ (B) $-5\,X_1(s) + 5X_2(s) = F(s)$

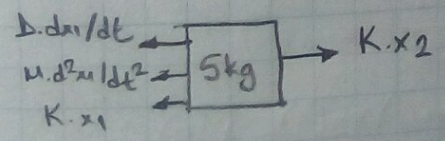
+

$(5s^2+4s)\,X_1(s) = F(s)$

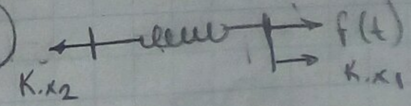
$G(s) = X_1(s)/F(s) = \frac{1}{5s^2+4s}$

2 Free-body diagram

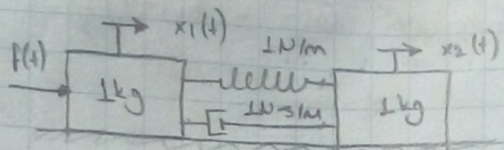
(A)



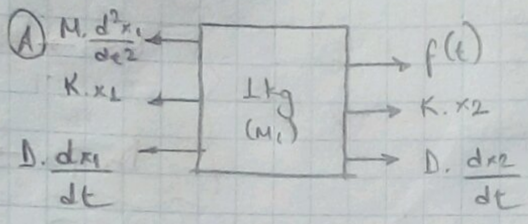
(B)



2-24) Find the transfer f., $G(s) = X_2(s)/F(s)$, for the translational mechanical network.



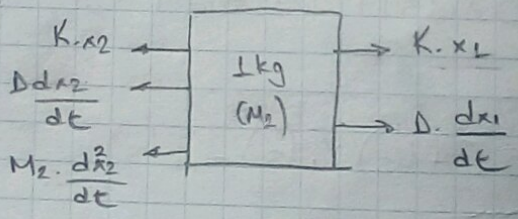
① Free-body d.



(A) $(1).\frac{d^2x_1}{dt^2} + (1).\frac{dx_1}{dt} + (1)x_1 - (1)x_2 - (1)\frac{dx_2}{dt} = f(s)$

↳ $(s^2+s+1)X_1(s) - (s+1).X_2(s) = F(s)$

② Free-body d.



(B) $(1).\frac{d^2x_2}{dt^2} + (1)\frac{dx_2}{dt} + (1)x_2 - (1)x_1 - (1)\frac{dx_1}{dt} = 0$

↳ $(s^2+s+1).X_2(s) - (s+1)X_1(s) = 0$

(A) $(s^2+s+1)X_1(s) - (s+1)X_2(s) = F(s)$

(B) $(s^2+s+1)X_2(s) - (s+1)X_1(s) = 0$

↳

$X_1(s) = X_2(s)\frac{(s^2+s+1)}{(s+1)} \rightarrow \underline{A}$

$$(s^2+s+1)\cdot\frac{(s^2+s+1)}{(s+1)}\cdot X_2(s) - (s+1)\cdot X_2(s) = F(s)$$

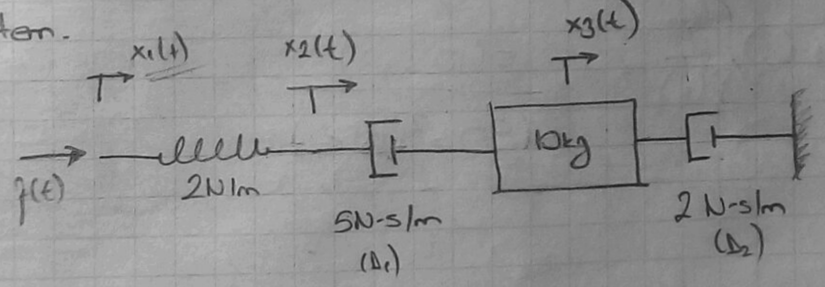
$$\Rightarrow \left[\frac{s^4+s^3+s^2+s^3+s^2+s+s^2+s+1 - (s+1)\cdot(s+1)}{s+1}\right] X_2(s) = F(s)$$

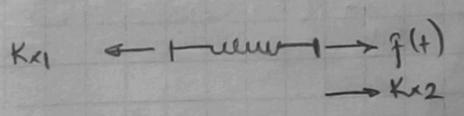
$$\Rightarrow \left(\frac{s^4+2s^3+3s^2+\cancel{2s}+\cancel{1}-\cancel{s^2}-\cancel{2s}-\cancel{1}}{s+1}\right) X_2(s) = F(s)$$

$$\frac{(s^4+2s^3+2s^2)}{(s+1)} X_2(s) = F(s)$$

$$G(s) = X_2(s)/F(s) = \frac{s+1}{s^4+2s^3+2s^2}$$

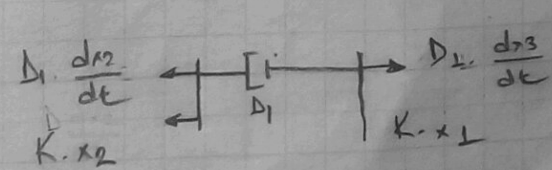
2-25) Find the tra. f., $G(s) = X_2(s)/F(s)$, for the translational system.



① 

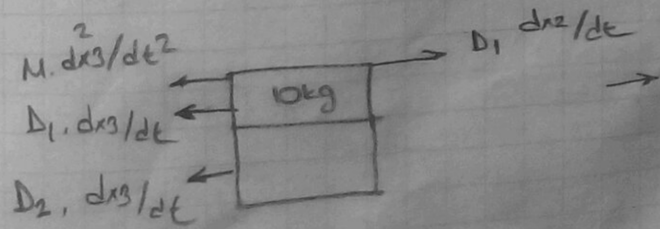
$\rightarrow$ $Kx_1 - Kx_2 = f(t)$

$2x_1(s) - 2x_2(s) = F(s)$

② 

$$\rightarrow (5)\cdot\frac{dx_2}{dt} + 2\cdot x_2 - 2\cdot x_1 - 5\cdot\frac{dx_3}{dt} = 0$$

$$\rightarrow (5s+2)x_2(s) - 2x_1(s) - 5s\,x_3(s) = 0$$

③ 

$$\rightarrow (10)\cdot\frac{d^2x_3}{dt^2} + (5)\cdot\frac{dx_3}{dt} + (2)\cdot\frac{dx_3}{dt} - (2)\frac{dx_2}{dt} = 0$$

$$\rightarrow (10s^2+7s)\cdot X_3(s) - (2s)x_2(s) = 0$$

1) $x_1(s) = \dfrac{F(s) + 2x_2(s)}{2}$

3) $X_3(s) = \dfrac{5s\,X_2(s)}{10s^2+7s}$

2) $-\cancel{2}\left[\dfrac{F(s)+2x_2(s)}{\cancel{2}}\right] + (5s+2).X_2(s) - (5s).\dfrac{(5s.X_2(s))}{10s^2+7s} = 0$

$$F(s) = -2x_2(s) + (5s+2).X_2(s) - \frac{(25s^2).X_2(s)}{10s^2+7s}$$

$$F(s) = X_2(s)\left[(5s+\cancel{2})-\cancel{2}\right] - \frac{(25s^2).X_2(s)}{10s^2+7s}$$

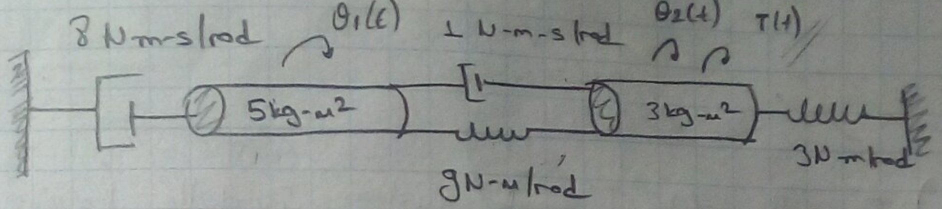
$$F(s) = X_2(s)\left[\frac{50s^3+35s^2-25s^2}{10s^2+7s}\right]$$

$$G(s) = X_2(s)/F(s) = \frac{\cancel{s}(10s+7)}{\cancel{s}(50s^2+10s)} = \frac{10s+7}{50s^2+10s}$$

2-30) For each of the rotational mechanical sys., write, but do not solve the equations of motion.

(mass → Inertia, M → J)

ⓐ.



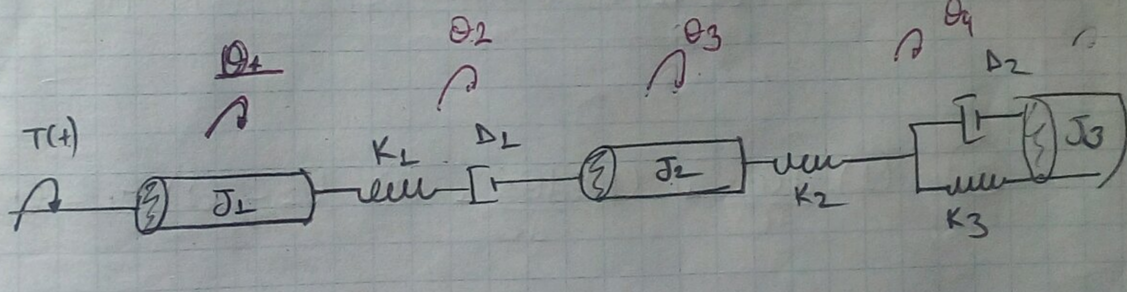
① $5 \cdot \frac{d^2\theta_1}{dt^2} + 8 \frac{d\theta_1}{dt} + 1 \cdot \frac{d\theta_1}{dt} + 9 \cdot \theta_1 - 1 \cdot \frac{d\theta_2}{dt} - 9 \cdot \theta_2 = 0$

$(5s^2 + 8s + 1)\theta_1(s) - (s+9)\theta_2(s) = 0$

② $3\frac{d^2\theta_2}{dt^2} + 3 \cdot \theta_2 + 9 \cdot \theta_2 + 1 \cdot \frac{d\theta_2}{dt} - 1 \cdot \frac{d\theta_1}{dt} - 9 \cdot \theta_1 = T(t)$

$(3s^2 + 12 + s)\theta_2(s) - (s+9)\theta_1(s) = T(s)$

ⓑ



① $J_1 \cdot \frac{d^2\theta_1}{dt} + K_1\theta_1 - K_1 \cdot \theta_2 = T(t)$

$\rightarrow (J_1 s^2 + K_1)\theta_1(s) - K_1\theta_2(s) = T(s)$

② $K_1 \cdot \theta_2 + D_1 \cdot \frac{d\theta_2}{dt} - K_1 \cdot \theta_1 - D_1 \frac{d\theta_3}{dt} = 0$

$\rightarrow (D_1 s + K_1)\theta_2(s) - K_1\theta_1(s) - D_1 s \cdot \theta_3(s) = 0$

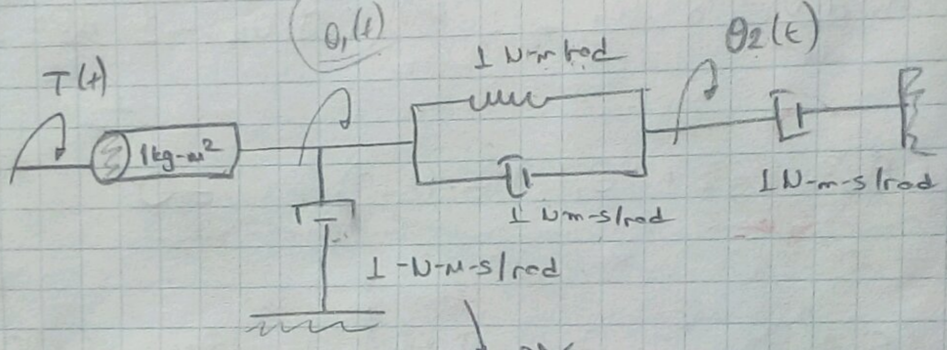
③ $J_2 \cdot \frac{d^2\theta_3}{dt^2} + D_1 \cdot \frac{d\theta_3}{dt} + K_2\theta_3 - D_1 \cdot \frac{d\theta_2}{dt} - K_2 \cdot \theta_4 = 0$

$\rightarrow (J_2 s^2 + D_1 s + K_2)\theta_3(s) - D_1 s\theta_2(s) - K_2\theta_4(s) = 0$

(4) $J_3 \frac{d^2\theta_4}{dt^2} + D_2 \frac{d\theta_4}{dt} + K_3 . \theta_4 + K_2\theta_4 - K_2\theta_3 = 0$

$\rightarrow (J_3 s^2 + D_2 s + (K_3 + K_4))\theta_4(s) - K_2 . \theta_3(s) = 0$

2-31) For the rotational mech. system, find the transfer f., $G(s) = \theta_2(s)/T(s)$



$\downarrow$ $+(1) \frac{d\theta_1}{dt}$

(1) $(1) . \frac{d^2\theta_1}{dt^2} + (1) . \theta_1 + (1) . \frac{d\theta_1}{dt} - (1) . \theta_2 - (1) \frac{d\theta_2}{dt} = T(s)$

$\rightarrow \underline{(s^2 + 2s + 1)\theta_1(s) - (s+1)\theta_2(s) = T(s)}$

(2) $(1) . \frac{d\theta_2}{dt} + (1) . \frac{d\theta_2}{dt} + (1) . \theta_2 - (1) . \theta_1 - (1) . \frac{d\theta_1}{dt} = 0$

$\rightarrow \underline{(s + s + 1)\theta_2(s) - (s+1)\theta_1(s) = 0}$

$\downarrow$

$\theta_1(s) = \frac{(2s+1)}{(s+1)} \theta_2(s)$

$\rightarrow$ (1)

$(s^2 + 2s + 1) . \frac{(2s+1)}{(s+1)} \theta_2(s) - (s+1) . \theta_2(s) = T(s)$

$$\frac{\left[(2s^3 + s^2 + 4s^2 + 2s + 2s + 1) - (s^2 + 2s + 1)\right]\theta_2(s)}{s+1} = T(s)$$

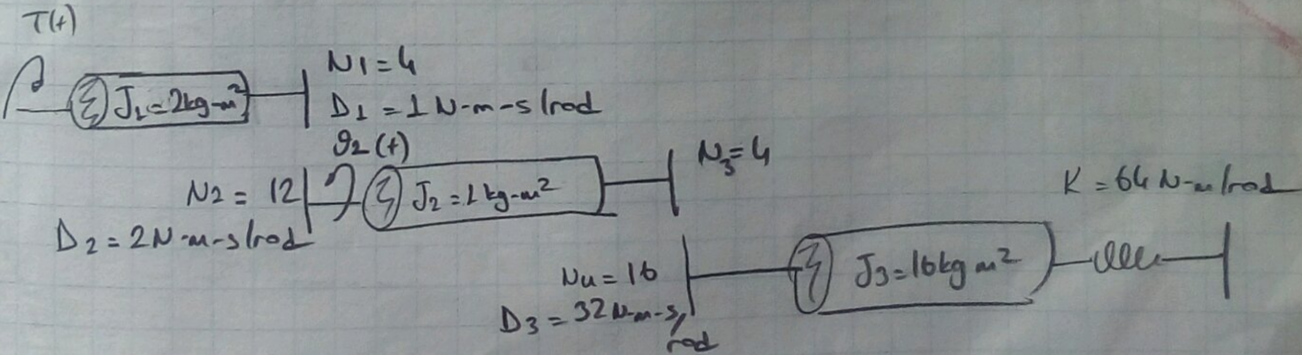
$$\frac{(2s^3 + 4s^2 + 2s)}{s+1}\,\theta_2(s) = T(s)$$

$$\frac{2s(s^2 + 2s + 1)}{s+1}\,\theta_2(s) = T(s)$$

$$\frac{2s(s+1)^2}{s+1}\,\theta_2(s) = T(s)$$

$$G(s) = \theta_2(s)/T(s) = \frac{1}{2s(s+1)}$$

2-33) for the rotational system, find the transfer f., $G(s) = \theta_2(s)/T(s)$



$$\left\{ s^2\left[\left(\frac{N_2}{N_1}\right)^2 J_1 + J_2 + \left(\frac{N_3}{N_4}\right)^2 \cdot J_3\right] + s\left[D_2 + D_1\left(\frac{N_2}{N_1}\right)^2 + D_3\left(\frac{N_3}{N_4}\right)^2\right] + \left[K \cdot \left(\frac{N_3}{N_4}\right)^2\right]\right\}\theta_2(s) = T(s)\cdot\left(\frac{N_2}{N_1}\right)$$

$$\frac{\theta_2(s)}{T(s)} = \frac{3}{20s^2 + 13s + 4}$$