CMPE 352
Signal Processing & Algorithms

Spring 2019

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Review Questions (1)

- What is a signal?

  A function of time containing information about the behavior or nature of some phenomenon

- What is a system?

  An “object” that responds to particular signals by producing other signals or some desired/observed behavior

- What is a continuous-time signal?

  A signal whose domain (time) is a continuum (e.g., a connected interval of the reals). That is, the function's domain is an uncountable set.

  − Does the function need to be continuous?

    No

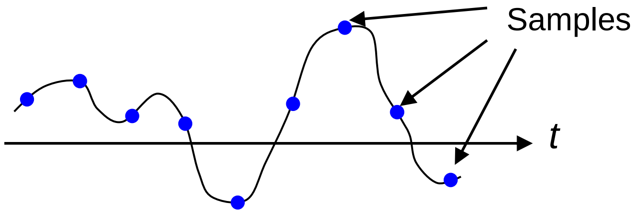
Review Questions (2)

- What is a discrete-time signal?

  It is a time-series consisting of a sequence of quantities. In other words, it is a time-series that is a function over a domain of integers.

- What operation needs to be performed to convert a continous-time signal to a discrete-time signal?

  Sampling



- What operation needs to be performed to convert a discrete-time signal to a continuous-time signal?

  Interpolation

- What is a continuous-amplitude signal?

  A signal whose range (amplitude) is a continuum. That is, the function's range is an uncountable set.

Review Questions (3)

- What is an <u>embedded</u> system?

  An embedded system is a computer system with a dedicated function within a larger system, often with real-time computing constraints (embedded as part of a complete device often including hardware and other (mechanical, electrical, etc.) parts).

- What is the main difference between a <u>microprocessor</u> and a <u>DSP</u>?

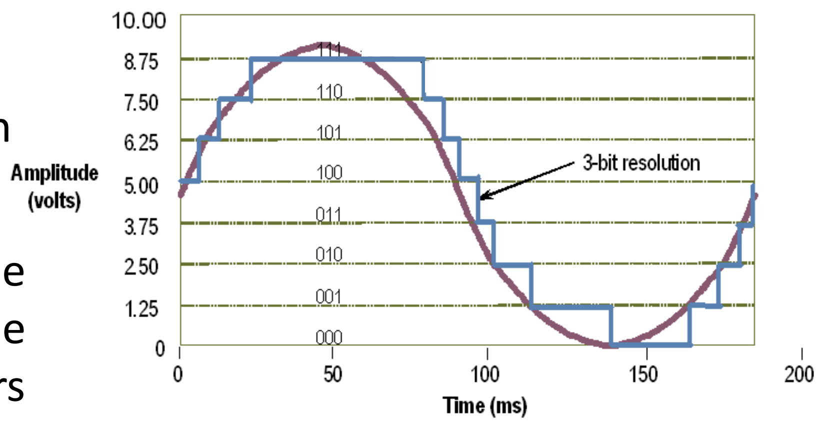
  A microprocessor is a general purpose device (using external chips for memory and peripheral interface circuits, etc.) whereas a DSP (Digital Signal Processor) is a device that is specifically designed to achieve high performance in implementing signal processing functions.

Signals and Systems: Why are We interested?

- In general, computer engineers are interested in the digital processing of signals for applications such as sensor processing, digital image processing, digital signal processing for telecommunications, control systems, computer architecture, data storage, biomedical engineering, robotics and artificial intelligence, seismology, etc.
- Some more specific examples are
  - Encoding music in MP3 format
  - Compression of picture in JPEG format
  - Adobe Photoshop relies heavily on digital signal processing
  - Voice recognition, face recognition (e.g., tag faces in Facebook)
  - SIRI, Instagram
  - ...
- Engineers usually study digital signals in one of the following two domains: <u>time domain and frequency domain</u>.

From Analog to Digital

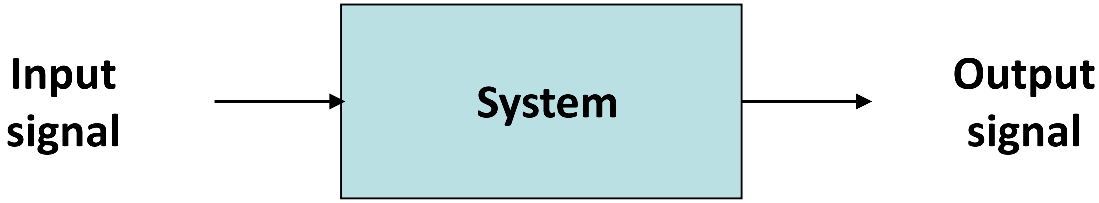
- To process and manipulate an analog signal by a computer/processor, this signal must be converted into digital form ("digitized"): "voltage-to-bits"
- Digitization is realized by an analog-to-digital converter (ADC).
- Analog-to-digital conversion is usually carried out in two stage: sampling and quantization
  - **Sampling** means that the signal is converted into a discrete-time signal. After sampling, it may be necessary to hold the signal because the quantization operation is not instantaneous.
  - **Quantization** means that each amplitude measurement is approximated by a value from a finite set. (Rounding real numbers to integers is an example of quantization.)



- The inverse operation "bits-to-voltage" is called digital-to-analog conversion, which is realized by a device called a digital-to-analog converter (DAC).

System Representation

- A system transforms an input signal to an output signal

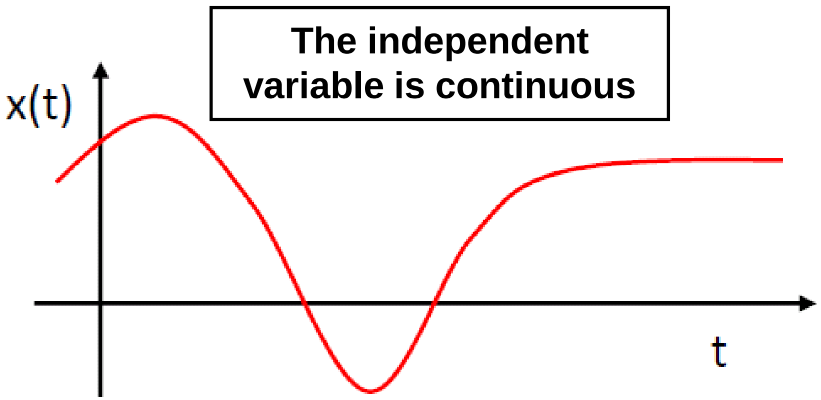


- Stated very generally, a system can be represented as the ratio of the output signal to the input signal

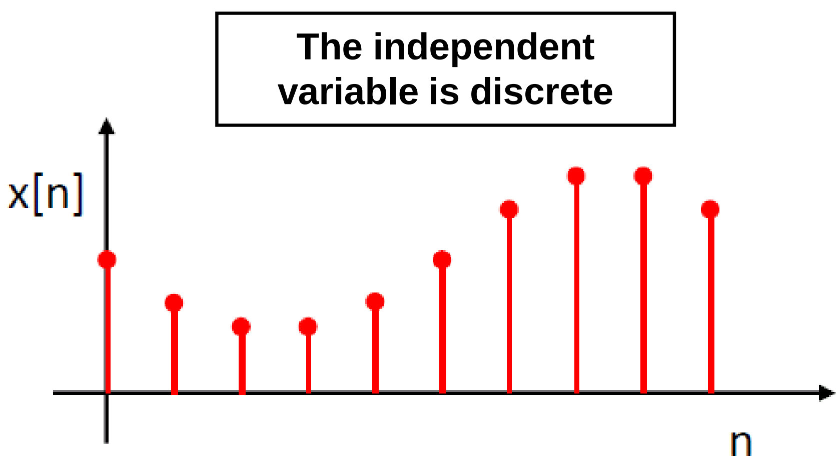
$$\text{System} = \frac{\text{output signal}}{\text{input signal}}$$

- Thus, when the “system” is “multiplied by the input signal” the output signal is obtained

Continuous-time & discrete-time signals



- Large portion of signals representing physical quantities are continuous-time
  - voltage, current in electrical circuits; air pressure variation (sound); velocity of a body; temperature; etc.
- Notation: x(t), y(t), …



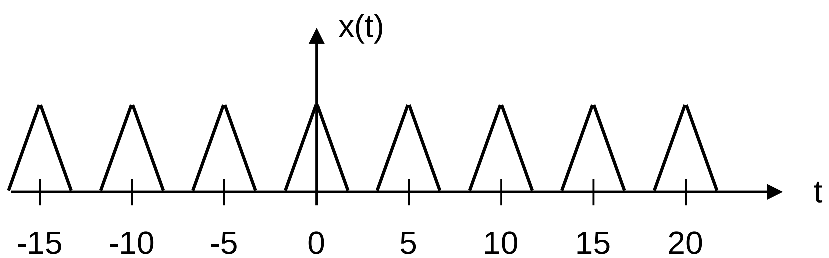
- Many signals are inherently discrete
  - daily stock-market values; yearly inflation values; sampled signals; etc.
- Notation: x[n], x_n , …
- For sampled signals x[n] = x(nT), with T denoting the sampling interval

Periodic signals

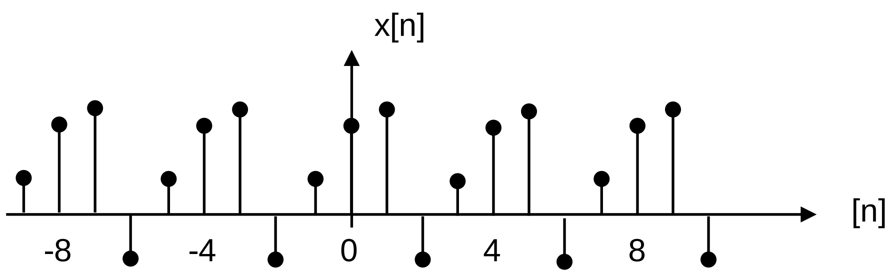
- For such signals, there exists a positive value T (respectively, N) such that for k = …, -2, -1, 0, +1, +2, …

$x(t) = x(t+kT)$ T = period of the continuous-time signal

$x[n] = x[n+kN]$ N = period of the discrete-time signal



A periodic time-continuous signal of period T = ?

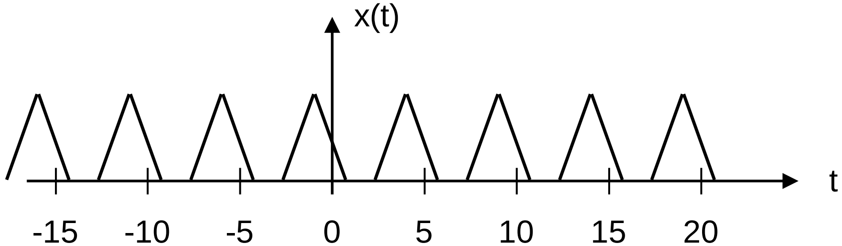


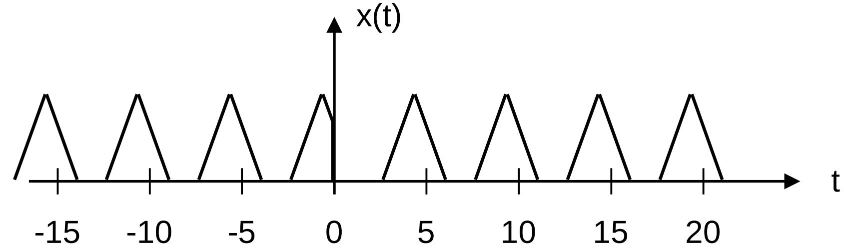
A periodic time-discrete signal of period N = ?

- Note: signals which are not periodic are called <u>aperiodic</u> (nonperiodic) signals

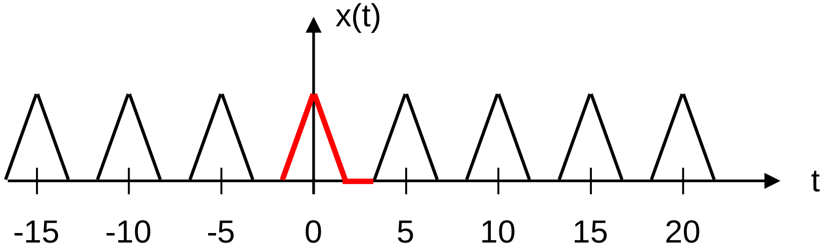
Periodic signals (2)

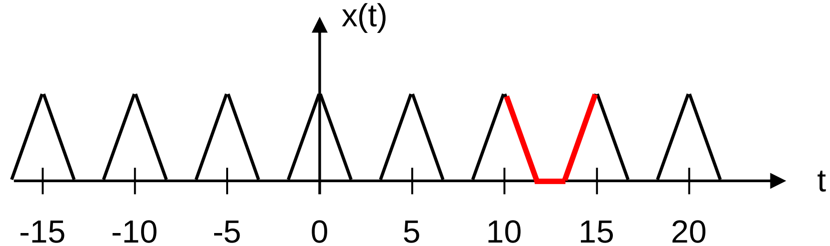
- Are these signals periodic?





- A periodic signal x(t) of period T can be generated by periodic extension of any segment of x(t) of duration T





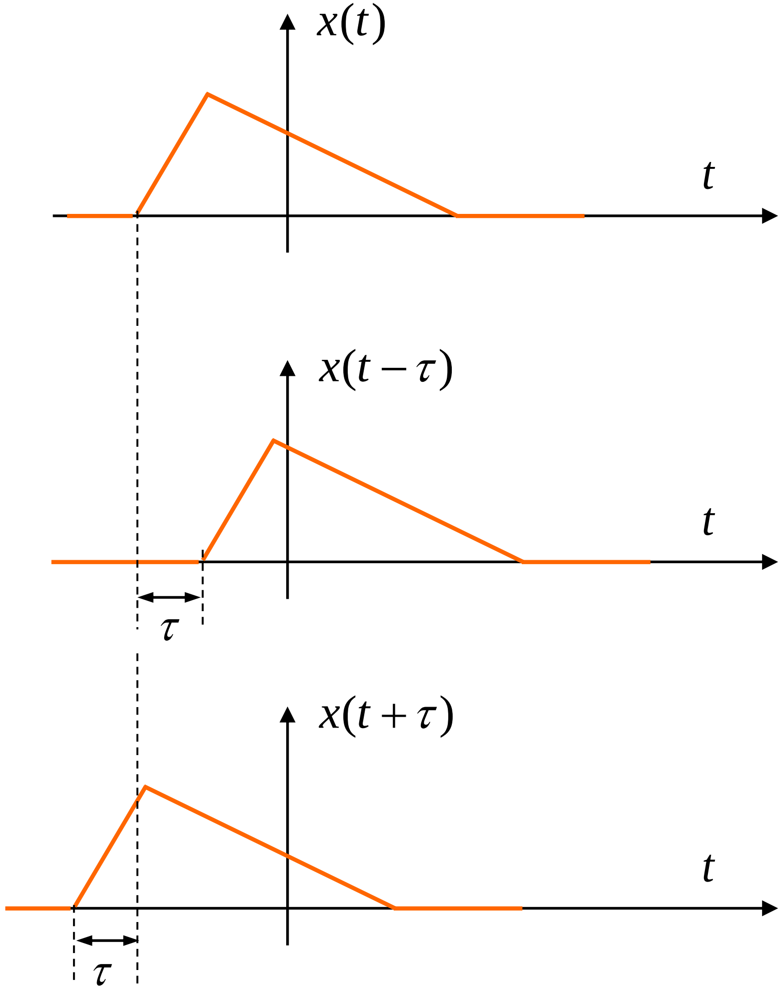
- The area under $x(t)$ <u>over any interval of duration T</u> is the same

$$\int_{a}^{a+T} x(t)\, dt = A \quad \forall a$$

Some useful signal operations

- There are three basic signal operations
  - Time shifting
  - Time scaling
  - Time reversal
- The three operations can be combined

Some useful signal operations: time shifting



Original signal

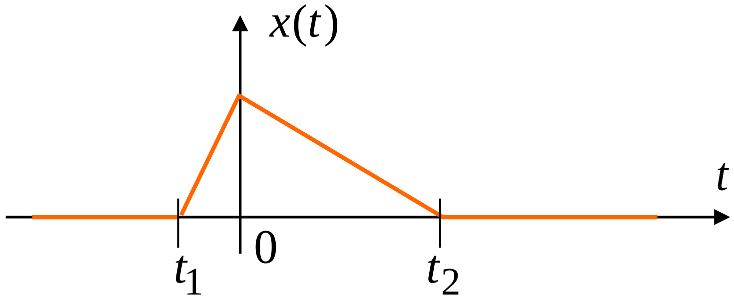
$$x(t) \rightarrow x(t-\tau)$$

Right-shifting

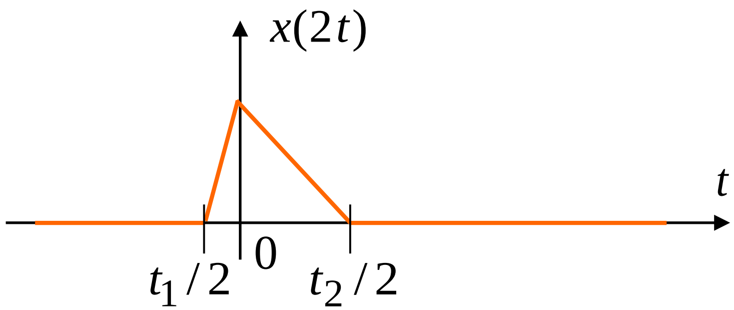
$$x(t) \rightarrow x(t+\tau)$$

Left-shifting

Some useful signal operations: time scaling

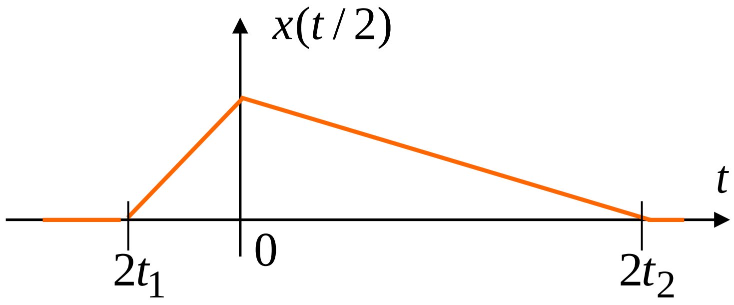


Original signal



$x(t) \rightarrow x(at) \qquad (a>1)$

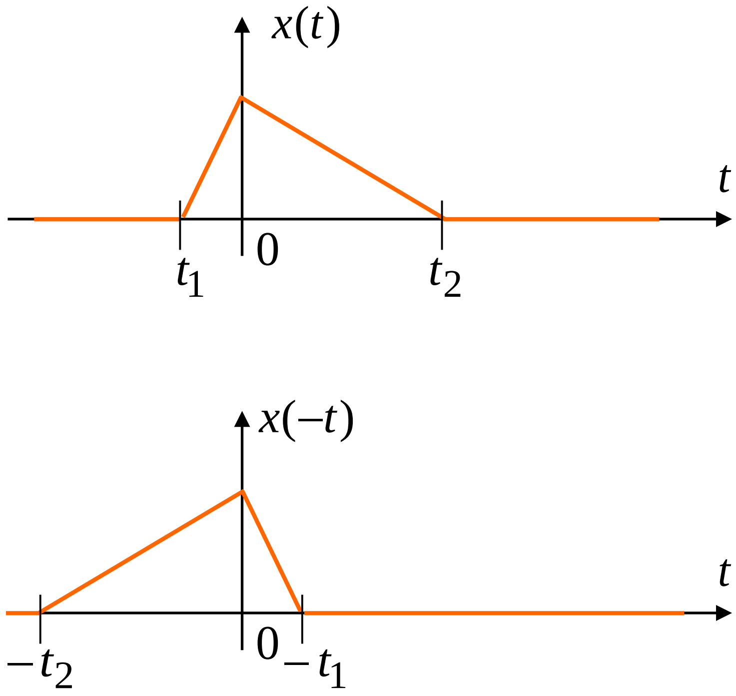
Compression by a factor a



$x(t) \rightarrow x(t/a) \qquad (a>1)$

Expansion by a factor a

Some useful signal operations: time reversal



Original signal

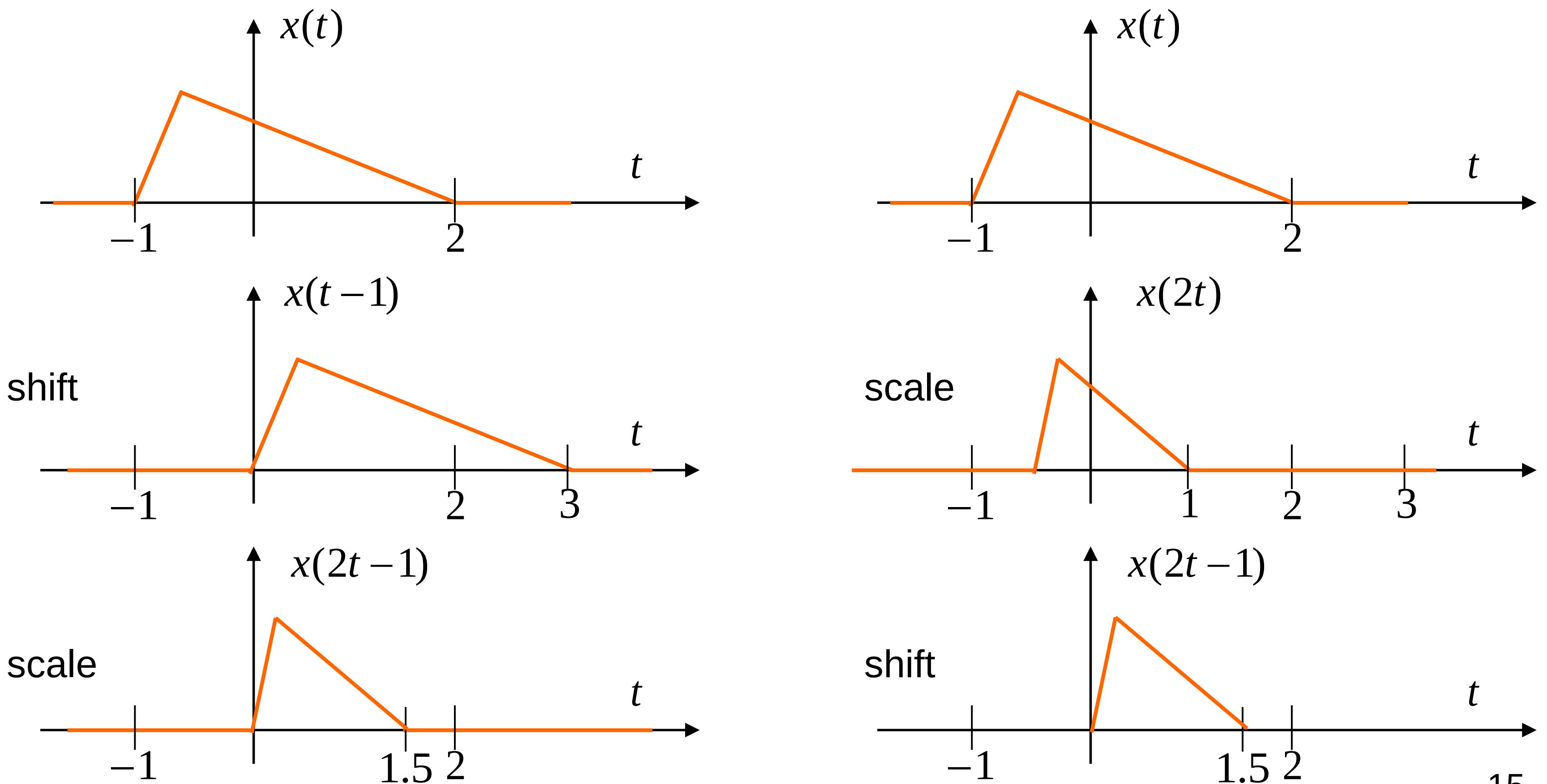
$$x(t) \rightarrow x(-t)$$

Time reversal

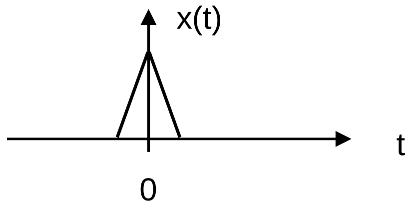
Some useful signal operations: combination

- The three basic signal operations can be combined:

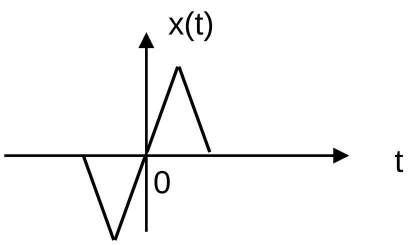
$$\boxed{x(t) \quad \rightarrow \quad x(at-b)}$$



Even and odd signals



An even (time-continuous) signal



An odd (time-continuous) signal

- Hence the even/odd property is a symmetry property under time reversal.
  How can we express it mathematically?

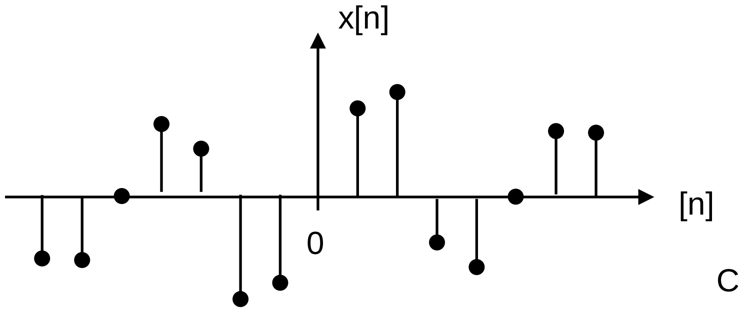
$\boxed{x(t) = x(-t) \quad \text{even signal}}$

$\boxed{x(t) = -x(-t) \quad \text{odd signal}}$

- This property also exists for discrete-time signals

$\boxed{x[n] = x[-n] \quad \text{even signal}}$

$\boxed{x[n] = -x[-n] \quad \text{odd signal}}$



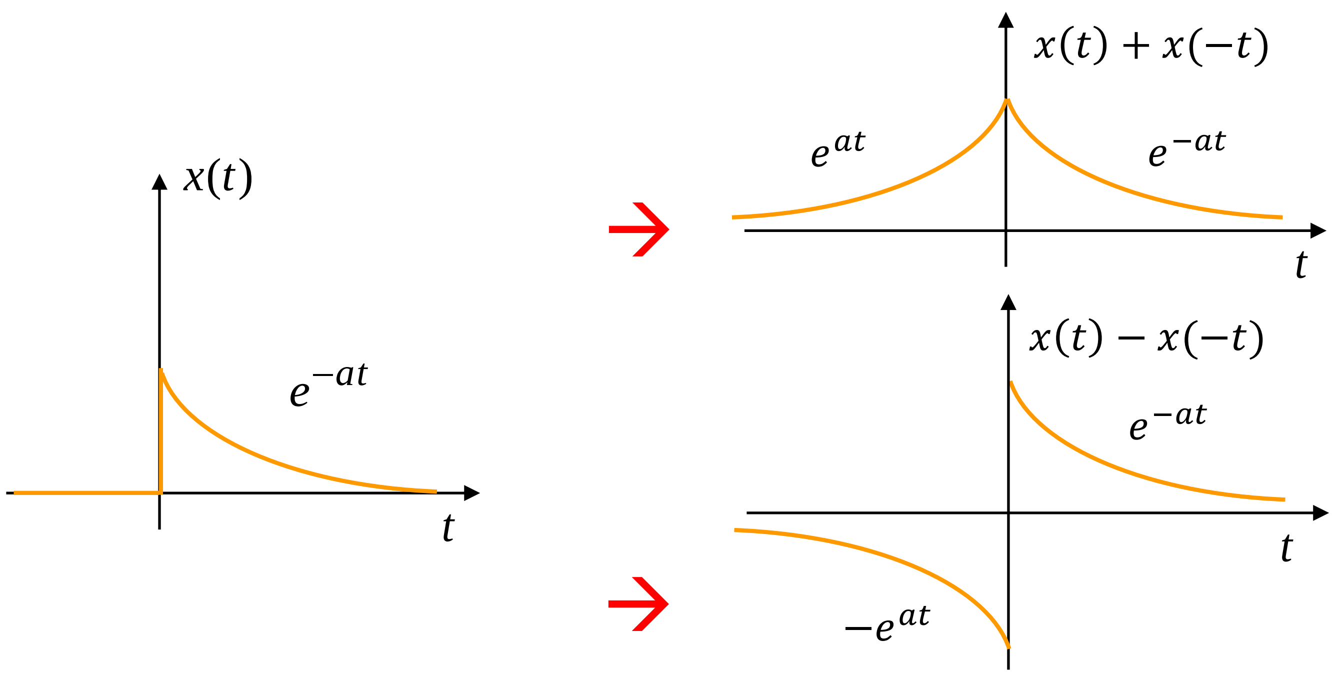
An odd discrete-time signal

Even and odd signals (2)

- Property: Take any signal $x(t)$, add to it its time-reversed version $x(-t)$, then you will get an even signal: $x_e(t) = x(t) + x(-t)$

- Property: Take any signal $x(t)$, subtract from it its time-reversed version $x(-t)$, then you will get an odd signal: $x_o(t) = x(t) - x(-t)$

- Example



Even and odd signals (3)

- Any signal can be decomposed as the sum of two signals, one of which is even and the other odd

Given x(t) $\rightarrow \quad x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

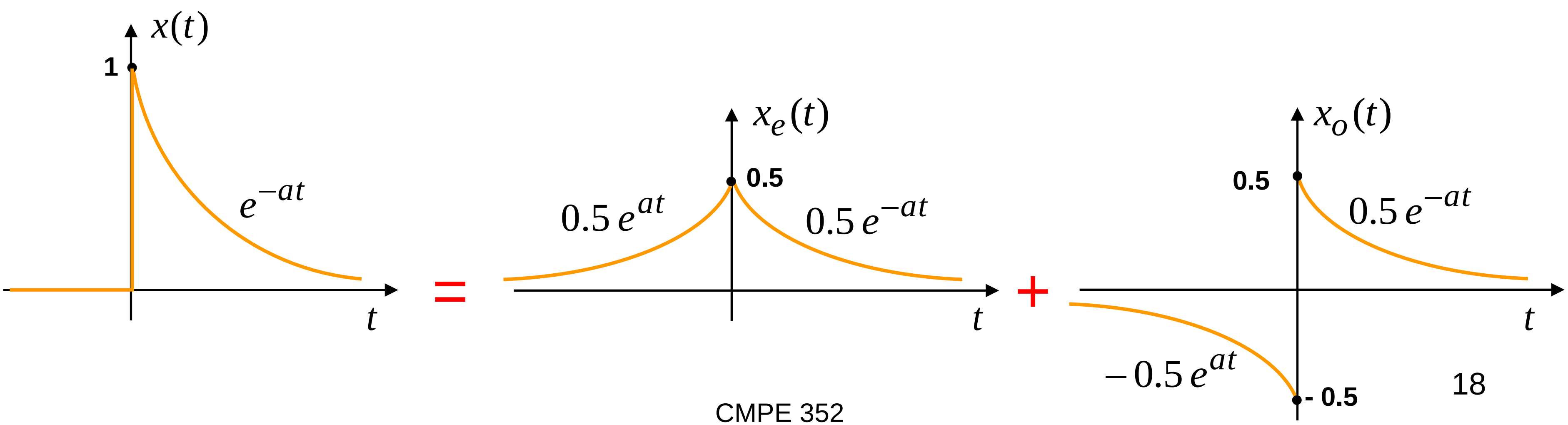
$\rightarrow \quad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$$\boxed{x(t) = x_e(t) + x_o(t)}$$

- Similarly for time-discrete signals:

$$\boxed{x[n] = x_e[n] + x_o[n]}$$

- Example



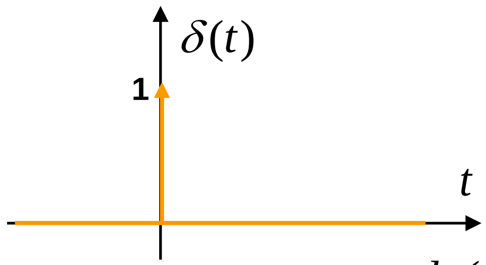
Elementary signals

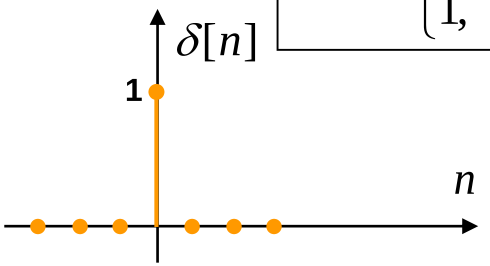
- Unit impulse
- Unit step
- Exponential
- Sinusoidal

The unit impulse and the unit step

Unit impulse

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$





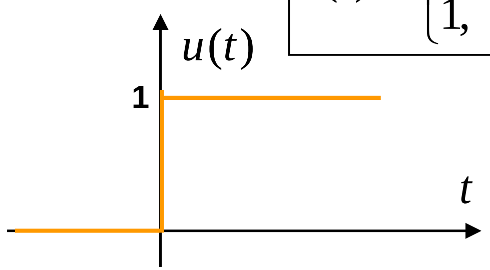
$$\delta(t) = \frac{du(t)}{dt}$$

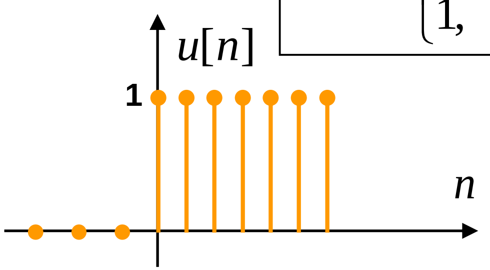
$$\delta[n] = u[n] - u[n-1]$$

Unit step

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$





$$u(t) = \int_{-\infty}^{t} \delta(t)\, dt$$

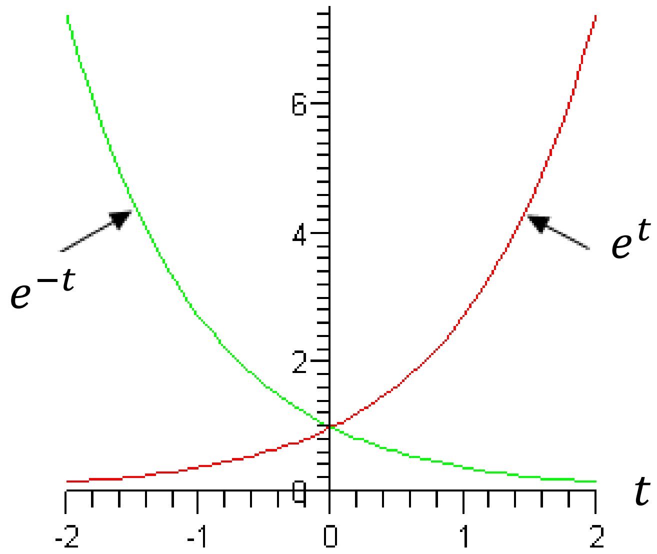
$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

Continuous-time exponential signal

$$x(t) = Be^{at}$$

$\boxed{a > 0:}$ Growing exponential

$\boxed{a < 0:}$ Decaying exponential

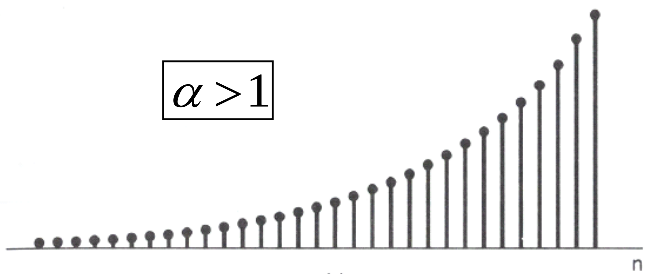


Discrete-time exponential signal

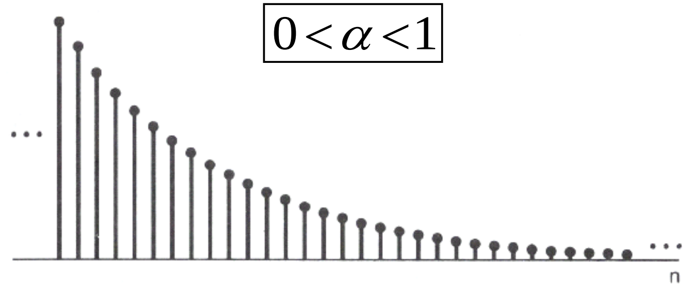
$$\boxed{x[n] = \alpha^n}$$

If α is real ➔ <u>real exponential</u>:

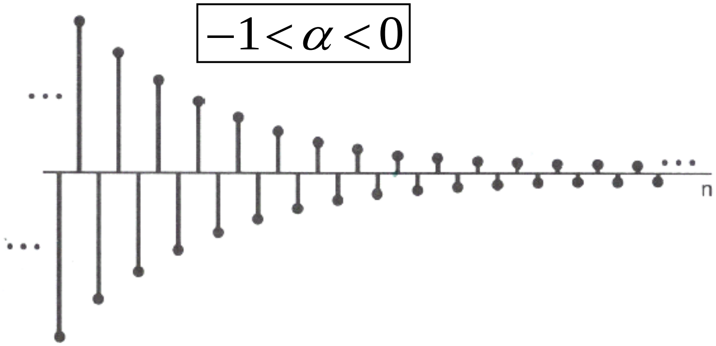
$\alpha > 1$



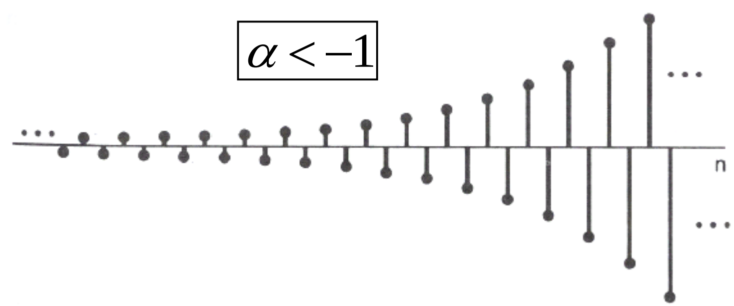
$0 < \alpha < 1$



$-1 < \alpha < 0$



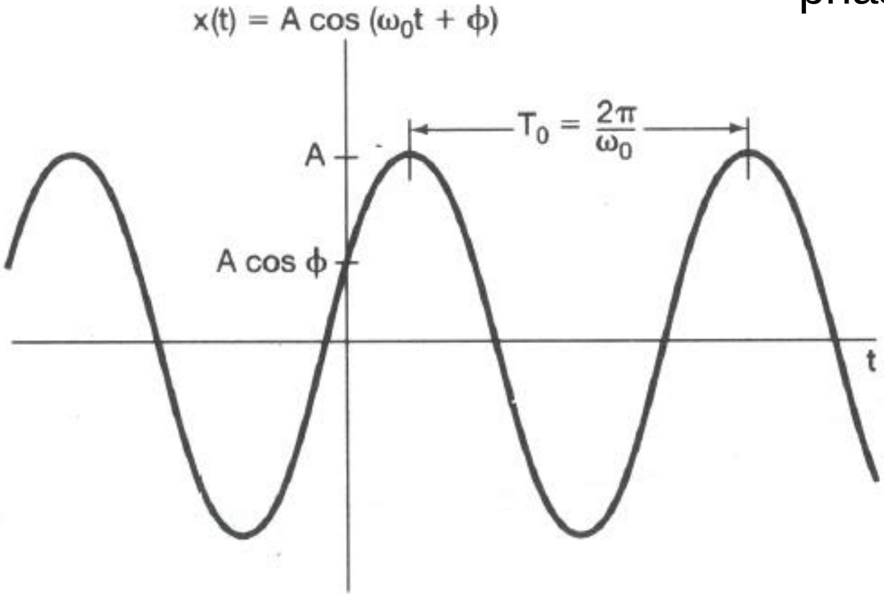
$\alpha < -1$



Sinusoidal signal

$$x(t) = A\cos(\omega_0 t + \phi)$$

$$y(t) = B\sin(\underbrace{\omega_0 t + \phi}_{\text{phase}})$$



f_0: frequency (Hz)

$\omega_0 = 2\pi f_0$: (radian) frequency

T_0: period

$$f_0 = \frac{1}{T_0}$$

Heinrich Hertz

Heinrich Rudolf Hertz (1857-1894) was a German physicist who first conclusively proved the existence of electromagnetic waves theorized by James Clerk Maxwell's electromagnetic theory of light. The unit of frequency – cycle per second – was named the “Hertz" in his honor. This unit is written as Hz.

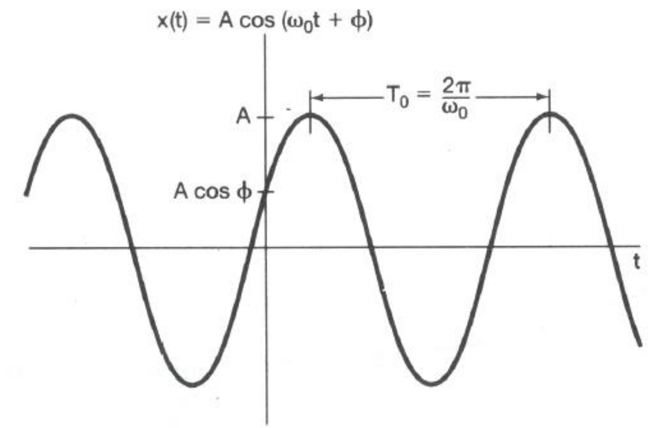
(Hz, kHz, MHz, GHz, etc...)

Sinusoidal signal and harmonics

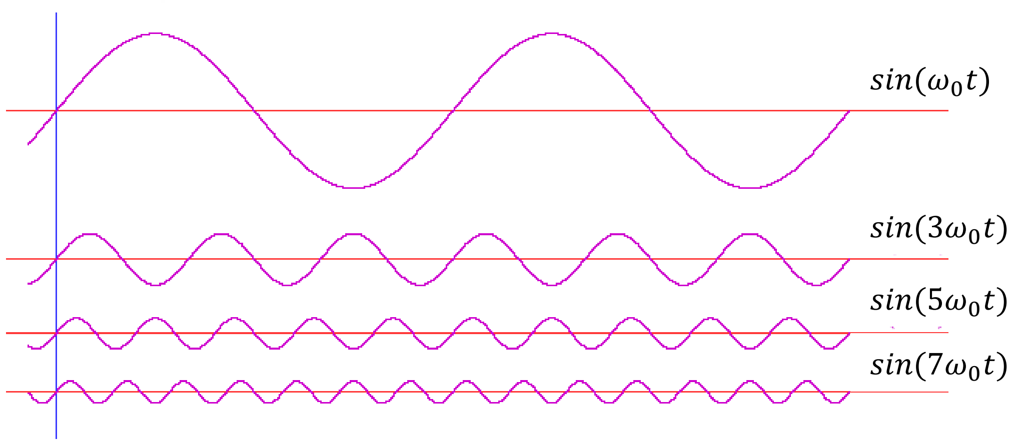
$$x(t) = A\cos(\omega_0 t + \phi)$$
$$y(t) = B\sin(\omega_0 t + \phi)$$

$$x_k(t) = A\cos(k\omega_0 t + \phi)$$
$$y_k(t) = B\sin(k\omega_0 t + \phi)$$

$x_k(t)$, $y_k(t)$: k-th harmonic (k = 2, 3, ...)



$\omega_0 = 2\pi f_0$: fundamental frequency



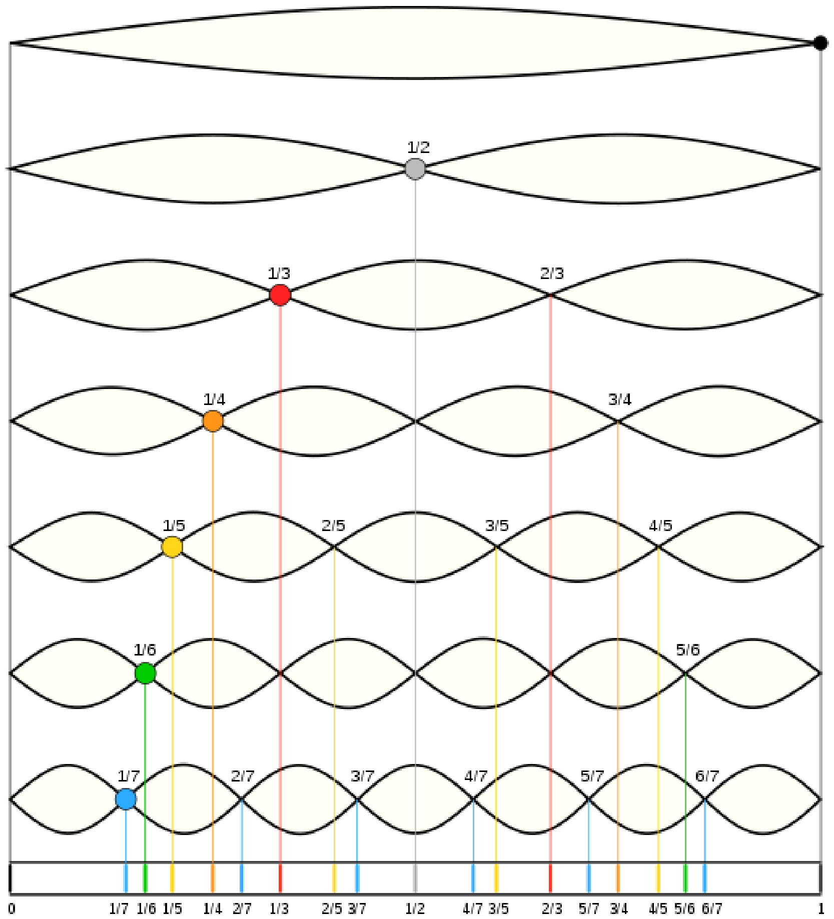
fundamental (wave)

3rd harmonic

5th harmonic

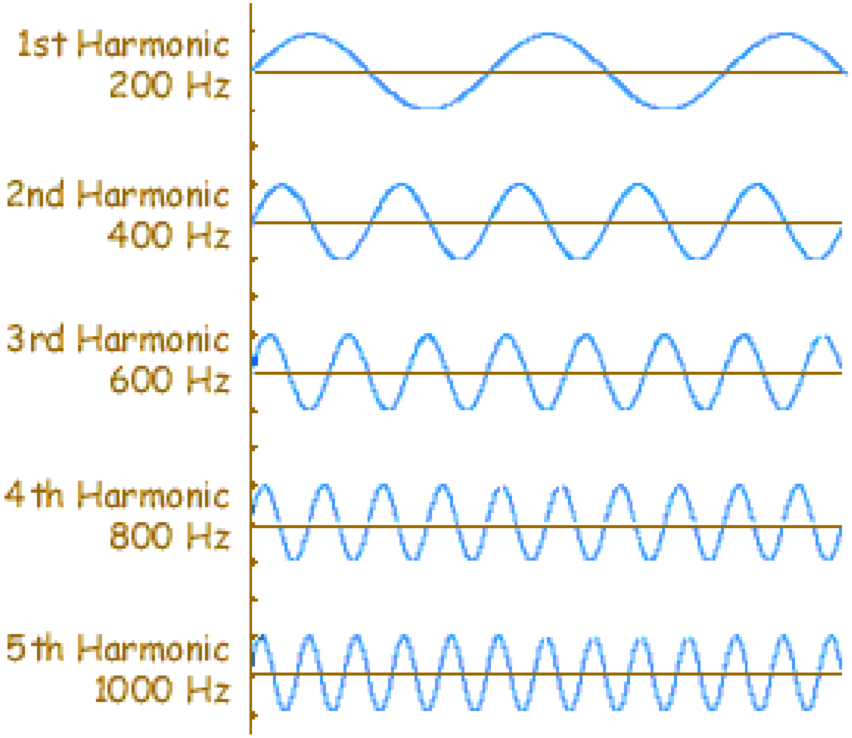
7th harmonic

Sinusoidal signal and harmonics



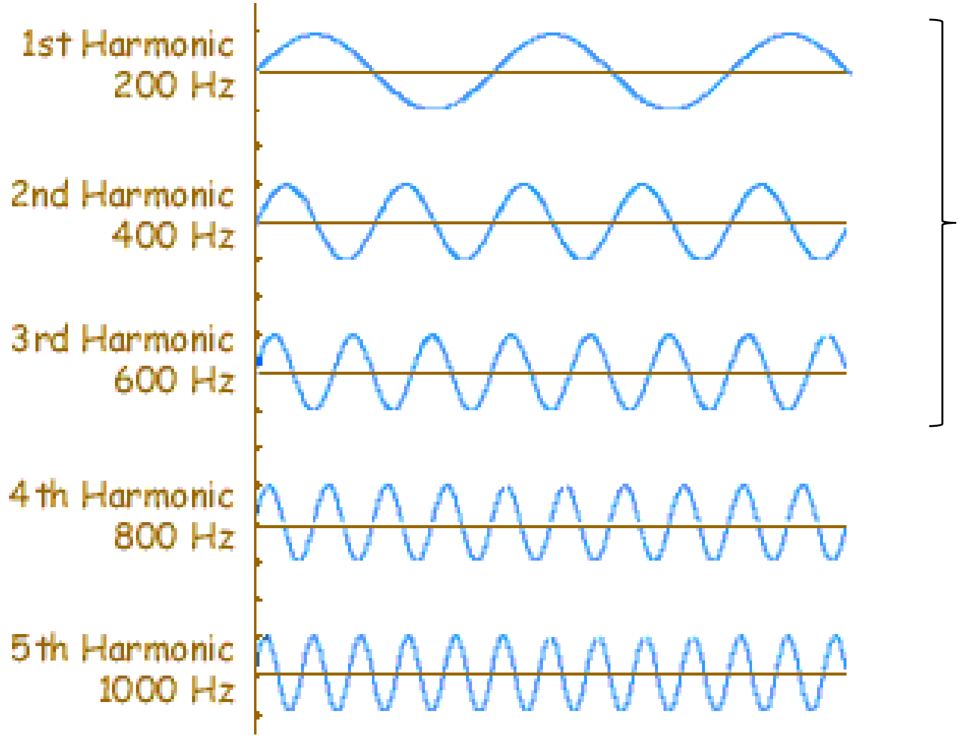
The nodes of a vibrating string are harmonics.

Sinusoidal signal and harmonics



By adding these 2 sounds (sinusoids) we could create a new sound

Sinusoidal signal and harmonics



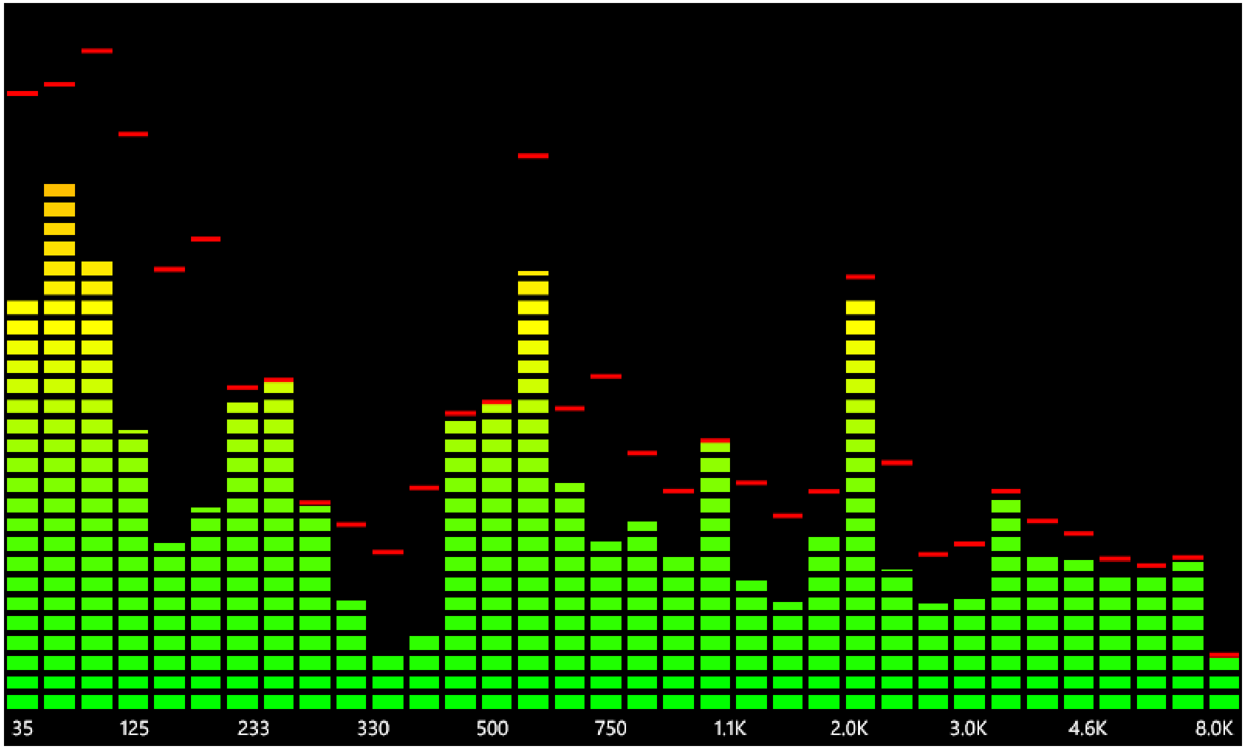
By adding these 3 sounds (sinusoids) we could create a new sound

We can synthesize sounds by adding pure sinusoids!

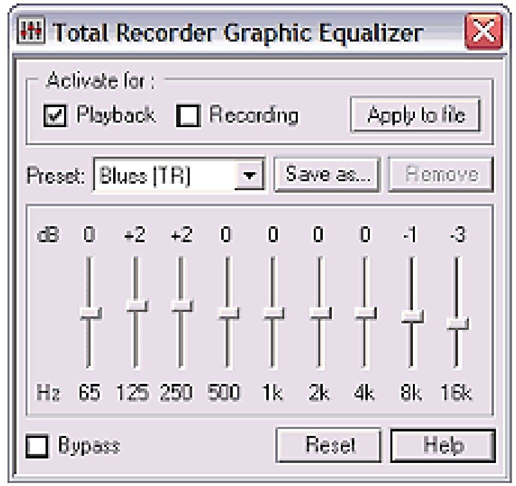
Conversely, any sound (any signal) can be decomposed into a sum of pure sinusoids.

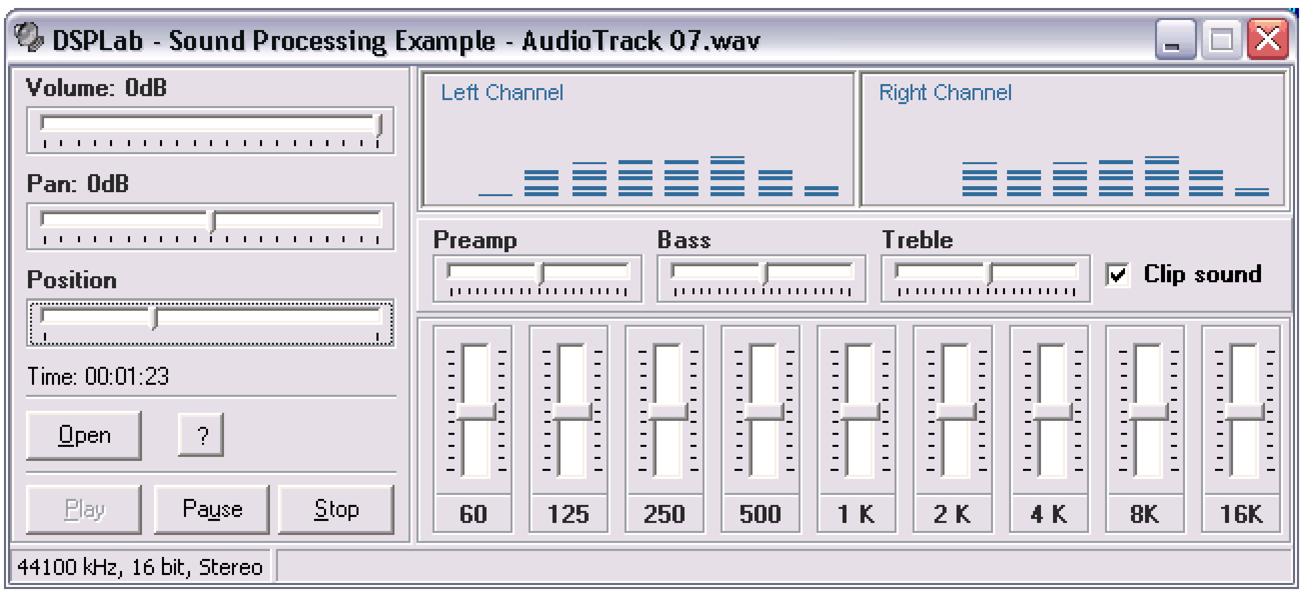
Really any signal?

Equalizer



Equalizer

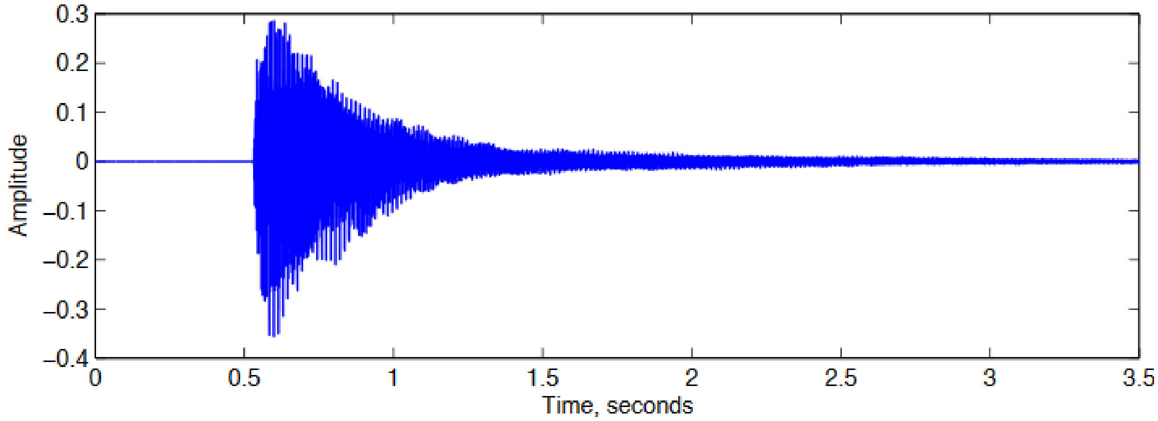

Total Recorder Graphic Equalizer
Activate for :
Playback
Recording
Apply to file
Preset:
Blues (TR)
Save as...
Remove
dB 0 +2 +2 0 0 0 0 -1 -3
Hz 65 125 250 500 1k 2k 4k 8k 16k
Bypass
Reset
Help

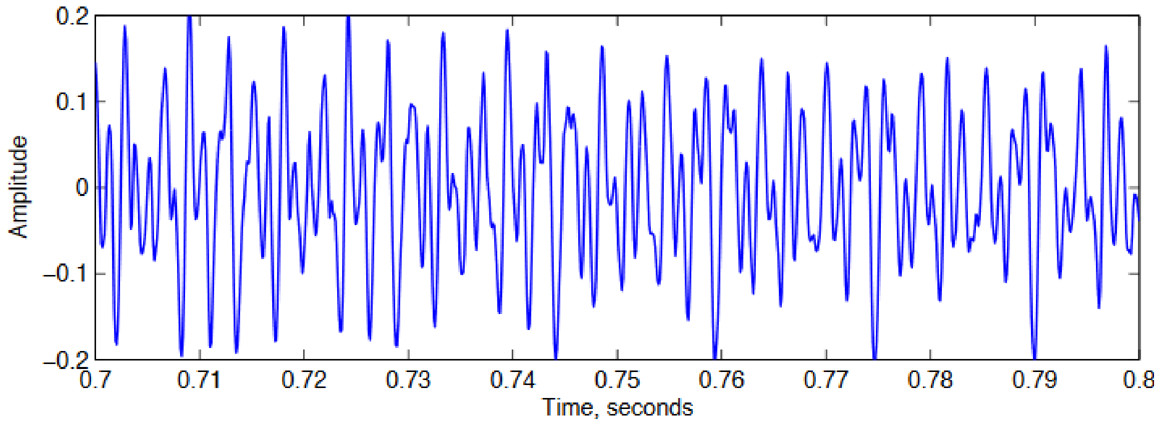

DSPLab - Sound Processing Example - AudioTrack 07.wav
Volume: 0dB
Pan: 0dB
Position
Time: 00:01:23
Open
?
Play
Pause
Stop
Left Channel
Right Channel
Preamp
Bass
Treble
Clip sound
60 125 250 500 1 K 2 K 4 K 8K 16K
44100 kHz, 16 bit, Stereo

Piano Chord Sound

- Listen to the piano sound. You hear several notes being struck and fading away. This corresponds to the time-domain waveform plotted on the right:

- The time-series plot shows the time the chord starts, and its decay, but it is difficult tell what the notes are from the waveform.





Piano Chord

If we represent the waveform as a sum of sinusoids at different frequencies, and plot the amplitude at each frequency, the plot is much simpler to understand.