EEEN 202
Electrical and Electronic Circuits II

MIDTERM EXAM, Spring 2014-2015 ***Duration: 100 minutes***

Problem 1 (25 points)

There is no energy stored in the circuit in Figure P1 at the time the source is energized.

a. Find the s-domain expression for $V_o(s)$. **(10 points)**

b. Use the s-domain expression derived in (a) to predict the initial- and final-values of $v_o(t)$. **(5 points)**

c. Find the time domain expression for $v_0(t)$ for $t \geq 0$. **(10 points)**

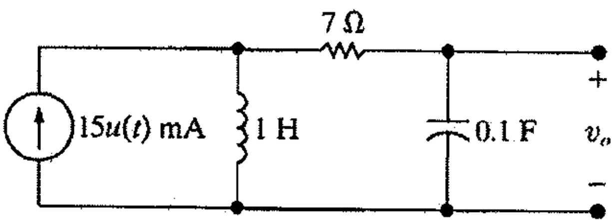


Figure P1

Problem 2) (25 points)

The op amp in the noninverting amplifier circuit of Figure P2 has an input resistance of 440 kΩ, an output resistance of 5 kΩ, and an open-loop gain of 100,000. Assume that the amplifier is operating in its linear region.

a. Calculate the voltage gain (v_0 / v_g) of the amplifier. **(15 points)**

b. Find the inverting and noninverting input voltages v_n and v_p in microvolts when $v_g = 1\,\text{V}$. **(10 points)**

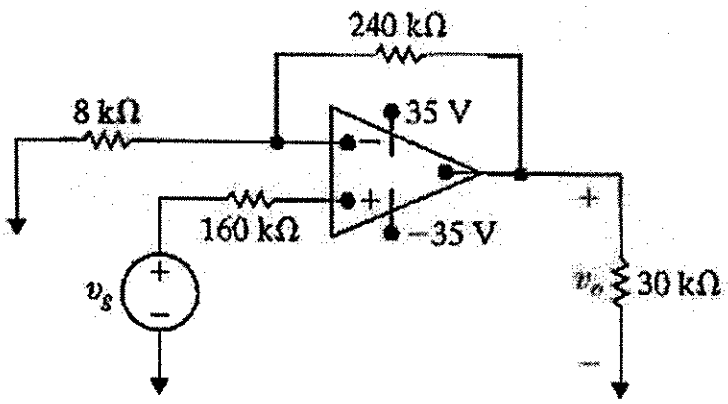


Figure P2

Problem 3) (25 points)

Consider the following circuit in Figure P3.

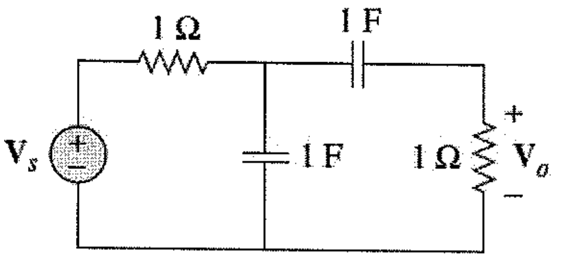


Figure P3

- **a.** Derive the transfer function of the filter, that is $\frac{V_0(s)}{V_s(s)}$. **(10 points)**
- **b.** Determine the type of the filter. Justify your answer. **(5 points)**
- **c.** Calculate the cutoff and center frequencies, bandwidth and quality factor of the filter. **(10 points)**

Problem 4) (25 points)

Design a passive bandreject filter with a quality factor of $\frac{2}{3}$ and a center frequency of 4 krad/sec using an 80nF capacitor.

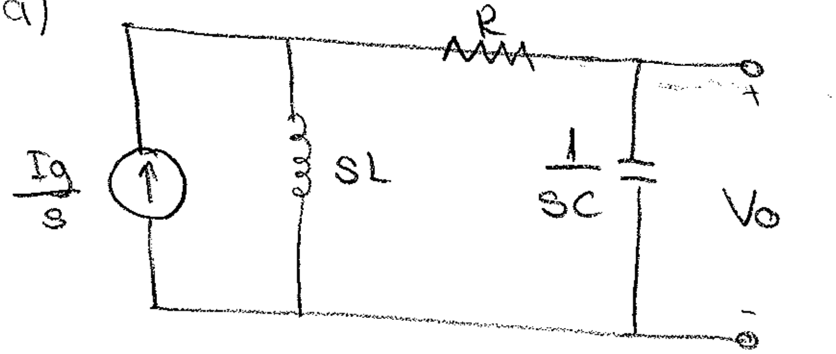
- **a.** Draw your circuit by labeling the component values and output voltage. **(10 points)**
- **b.** Derive the transfer function of the bandreject filter. **(10 points)**
- **c.** For the passive bandreject filter designed in part (a), calculate the bandwidth and the two cut off frequencies. **(5 points)**

EEEN 202 Midterm Exam

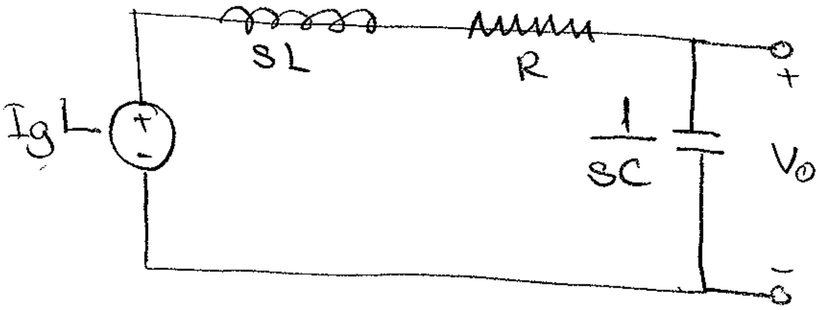
- Solutions -

1) s-domain equivalent circuit:

a)



Applying source transformation:



$$V_o = \frac{LI_g}{\left(sL + R + \frac{1}{sC}\right)} \times \frac{1}{sC} = \frac{LI_g}{\frac{LCs^2 + RCs + 1}{sC}} \, \frac{1}{sC}$$

$$= \frac{LI_g}{LCs^2 + RCs + 1} = \frac{I_g/C}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$= \frac{(0.015/0.1)}{s^2 + \left(\frac{7}{1}\right)s + \left(\frac{1}{1 \cdot 0.1}\right)} = \frac{0.15}{s^2 + 7s + 10} = \frac{0.15}{(s+2)(s+5)}$$

$$\boxed{V_o(s) = \frac{0.15}{s^2 + 7s + 10}}$$

b) Using initial-value theorem:

$$\lim_{t\to 0^+} v_0(t) = \lim_{s\to\infty} s V_0(s) = \lim_{s\to\infty} \frac{0.15s}{s^2+7s+10} = 0$$

$$\boxed{v_0(0^+) = 0}$$

Using final-value theorem:

$$\lim_{t\to\infty} v_0(t) = \lim_{s\to 0} s V_0(s) = \lim_{s\to 0} \frac{0.15s}{s^2+7s+10} = 0$$

$$\boxed{v_0(\infty) = 0}$$

c) $$V_0(s) = \frac{0.15}{(s+2)(s+5)} = \frac{K_1}{(s+2)} + \frac{K_2}{(s+5)}$$

$$K_1 = (s+2)V_0(s)\Big|_{s=-2} = \frac{0.15}{(s+5)}\Big|_{s=-2} = \frac{0.15}{3} = 0.05$$

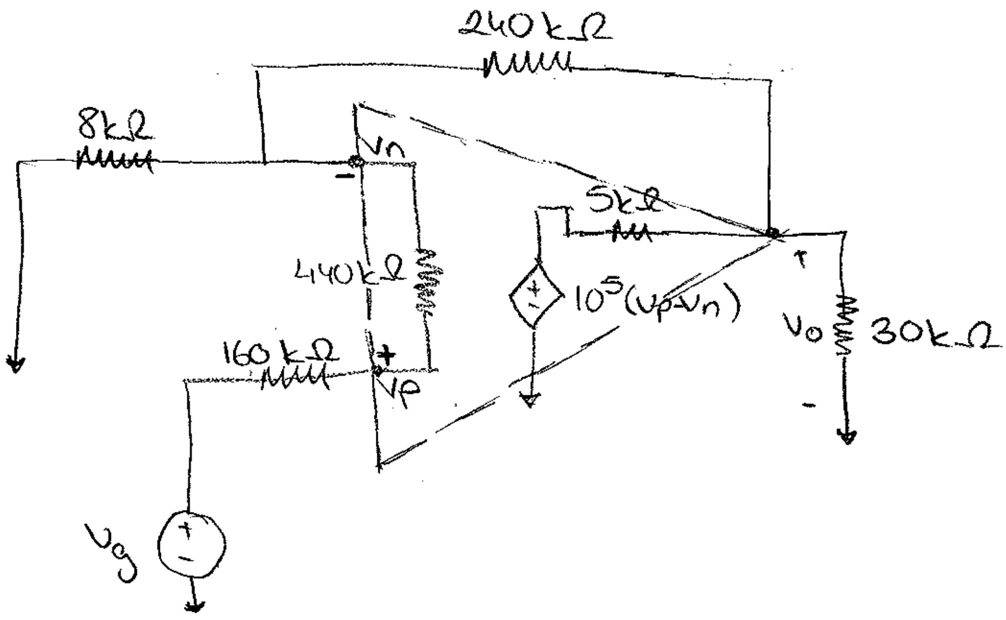
$$K_2 = (s+5)V_0(s)\Big|_{s=-5} = \frac{0.15}{(s+2)}\Big|_{s=-5} = \frac{0.15}{-3} = -0.05$$

$$V_0(s) = \frac{0.05}{s+2} - \frac{0.05}{s+5} =$$

$$\boxed{v_0(t) = \left[0.05\, e^{-2t} - 0.05\, e^{-5t}\right] u(t) \ \text{V}}$$

2)

a)



Applying KCL at non-inverting input terminal:

$$\frac{V_p - V_g}{160\times10^3} + \frac{V_p - V_n}{440\times10^3} = 0$$

$$\frac{11V_p - 11V_g + 4V_p - 4V_n}{1760\times10^3} = 0 \Rightarrow 15V_p - 4V_n - 11V_g = 0 \quad (\mathrm{I})$$

Applying KCL at inverting input terminal:

$$\frac{V_n}{8\times10^3} + \frac{V_n - V_o}{240\times10^3} + \frac{V_n - V_p}{440\times10^3} = 0$$

$$\frac{330V_n + 11V_n - 11V_o + 6V_n - 6V_p}{2640\times10^3} = 0 \Rightarrow -6V_p + 347V_n - 11V_o = 0$$

(II)

Applying KCL at output terminal:

$$\frac{V_o - V_n}{240 \times 10^3} + \frac{V_o}{30 \times 10^3} + \frac{V_o - 10^5(V_p - V_n)}{5 \times 10^3} = 0$$

$$\frac{V_o - V_n + 8V_o + 48V_o - 48 \cdot 10^5(V_p - V_n)}{240 \times 10^3} = 0 \Rightarrow -48 \cdot 10^5 V_p + 48 \cdot 10^5 V_n + 57 V_o = 0 \quad (\text{III})$$

Using (I) and (II):

$$\begin{aligned} 15 V_p - 4V_n - 11 V_g &= 0 \\ 6 V_p + 347 V_n - 11 V_o &= 0 \end{aligned} \Rightarrow \begin{aligned} 30 V_p - 8 V_n - 22 V_g &= 0 \\ -30 V_p + 1735 V_n - 55 V_o &= 0 \end{aligned}$$

$$1727 V_n - 22 V_g - 55 V_o = 0$$

$$\Rightarrow 157 V_n - 2 V_g - 5 V_o = 0 \quad (\text{IV})$$

From (IV): $V_n = \dfrac{2V_g + 5V_o}{157}$ (V)

From (I): $15 \cdot V_p - 4 V_n - 11 V_g = 15 V_p - \dfrac{8V_g + 20V_o}{157} - 11 V_g = 0$

$$\Rightarrow V_p = \frac{1735 V_g + 20 V_o}{157 \cdot 15}$$

Using (V) and (VI) in (III):

$$-48 \cdot 10^5 \left(\frac{1735 V_g + 20 V_o}{157 \cdot 15}\right) + 48 \cdot 10^5 \left(\frac{30 V_g + 75 V_o}{15 \cdot 157}\right) + 57 V_o = 0$$

$$\Rightarrow (-960 \times 10^5 + 3600 \times 10^5 + 134235) V_o = (83280 \cdot 10^5 - 1440 \times 10^5) V_g$$

$$(2640 \times 10^5 + 134235) V_o = 81840 \times 10^5 V_g$$

$$26413423 5 V_o = 8184000000 V_g$$

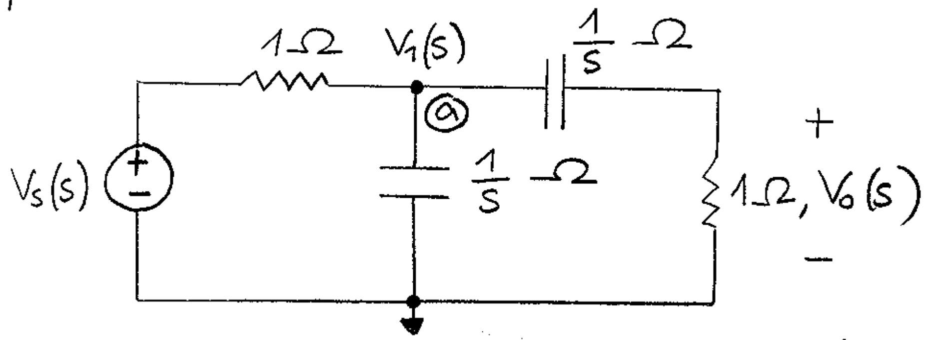
$$\frac{V_o}{V_g} = \frac{8184000000}{264134235} = 30.98 \Rightarrow \boxed{\frac{V_o}{V_g} = 30.98}$$

b) $V_o = 30.98 \cdot V_g = 30.98 \text{ V}$

$$V_n = \frac{2 V_g + 5 V_o}{157} = \frac{2 + 154.9}{157} = 0.99936 \text{ V} \Rightarrow \boxed{V_n = 999.36 \text{mV}}$$

$$V_p = \frac{1735 V_g + 20 V_o}{157 \cdot 15} = \frac{1735 + 619.6}{2355} = 0.99983 \text{V} \Rightarrow \boxed{V_p = 999.83 \text{mV}}$$

Problem 3) We shall first draw the s-domain equivalent circuit



a. We consider KCL at node a :

$$\frac{V_1(s)-V_s(s)}{1}+\frac{V_1(s)}{1/s}+\frac{V_1(s)}{(1/s)+1}=0$$

$$\Rightarrow \quad V_1(s)-V_s(s)+sV_1(s)+\frac{sV_1(s)}{1+s}=0$$

$$\Rightarrow \quad \left(1+s+\frac{s}{1+s}\right)V_1(s)=V_s(s)$$

05

$$\Rightarrow \quad \frac{(1+s)^2+s}{1+s}V_1(s)=V_s(s)$$

$$\Rightarrow \quad V_1(s)=\frac{s+1}{s^2+3s+1}V_s(s)$$

and we also have from voltage division

$$V_o(s)=\frac{1}{(1/s)+1}V_1(s)$$

03

$$=\frac{s}{s+1}V_1(s)$$

Hence;

$$V_o(s) = \frac{s}{\cancel{s+1}} \, \frac{\cancel{s+1}}{s^2+3s+1} V_s(s)$$

$$\Rightarrow \quad \frac{V_o(s)}{V_s(s)} = \frac{s}{s^2+3s+1} \triangleq H(s)$$ 02

2. We consider

$$H(j\omega) = \frac{j\omega}{(j\omega)^2+3j\omega+1} = \frac{j\omega}{1-\omega^2+j3\omega}$$ 01

and

$$|H(j\omega)| = \frac{\omega}{\sqrt{(1-\omega^2)^2+9\omega^2}}$$

- we find that

$$|H(j\omega)|_{\omega=0} = 0$$

$$\lim_{\omega\to\infty} |H(j\omega)| = \lim_{\omega\to\infty} \frac{1}{\sqrt{9+\left(\frac{1-\omega^2}{\omega}\right)^2}}$$

02

$$= \lim_{\omega\to\infty} \frac{1}{\sqrt{9+\left(\underbrace{\cancel{\tfrac{1}{\omega}}}_{0}-\omega\right)^2}}$$

$$= \lim_{\omega\to\infty} \frac{1}{\sqrt{9+\omega^2}}$$

$$= 0$$

Therefore;

- the type of the filter is justified to be band-pass, because the magnitude of the transfer function for both low and high frequencies approach zero (02)

Moreover;

- we also find that

$$|H(j\omega)| = \frac{1}{3} \frac{3\omega}{\sqrt{(1-\omega^2)^2 + (3\omega)^2}}$$

$$= \frac{1}{3} \frac{1}{\sqrt{1 + \left(\frac{1-\omega^2}{3\omega}\right)^2}}$$

has its maximum value when $\left(\frac{1-\omega^2}{3\omega}\right)^2 = 0$

$\Rightarrow \quad \omega = 1 \text{ rad/sec} \triangleq \omega_0$ (center/corner frequency)

and

$$H_{max} \triangleq H(j\omega_0)$$

$$= H(j)$$

$$= \frac{1}{3}$$

c. We shall consider

$$H(s) = \frac{1}{3} \frac{3s}{s^2 + 3s + 1}$$

$$\triangleq \frac{1}{3} \frac{\beta s}{s^2 + \beta s + \omega_0^2}$$

$\Rightarrow \quad \beta = 3 \text{ rad/sec} \quad \text{(bandwidth)} \quad 02$

$\omega_0^2 = 1 \quad \Rightarrow \quad \omega_0 = 1 \text{ rad/sec} \quad 02$

and

$$|H(j\omega_c)| = \frac{1}{3} \frac{1}{\sqrt{1+\left(\frac{1-\omega_c^2}{3\omega_c}\right)^2}}$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{3}$$

$$\Rightarrow \quad \left(\frac{1-\omega_c^2}{3\omega_c}\right)^2 = 1 \quad \Rightarrow \quad 1-\omega_c^2 = \pm 3\omega_c$$

$$\Rightarrow \quad \omega_c^2 \pm 3\omega_c - 1 = 0$$

$$\omega_{c1} = \frac{-3+\sqrt{9+4\cdot 1}}{2} \quad 02 \quad , \quad \omega_{c2} = \frac{3+\sqrt{9+4\cdot 1}}{2} \quad 02$$

$$= 0.3028 \text{ rad/sec} \qquad\qquad = 3.3028 \text{ rad/sec}$$

and

$$Q = \frac{\omega_0}{\beta} = \frac{1}{3} \quad 02$$

Problem 4) We have

$$Q = \frac{\omega_0}{\beta} = \frac{2}{3}, \quad \omega_0 = 4k \text{ rad/sec}$$

$$\Rightarrow \frac{\cancel{4}^{2} \cdot 10^3}{\beta} = \frac{\cancel{2}}{3} \Rightarrow \beta = 6 \cdot 10^3 \text{ rad/sec}$$

a.

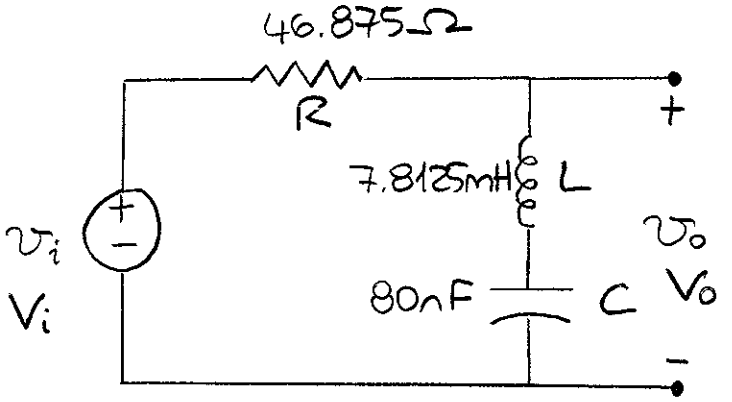
$$\omega_0 = \sqrt{\frac{1}{LC}} \Rightarrow 4 \cdot 10^3 = \sqrt{\frac{1}{L \cdot 80 \cdot 10^{-5}}}$$

$$\Rightarrow 4 \cdot \cancel{10^3} = \frac{10^{\cancel{4}}}{\sqrt{8L}} \Rightarrow 8L = \frac{1}{16}$$

$$\Rightarrow L = 7.8125 \text{ mH}$$ 04

$$\beta = \frac{R}{L} \Rightarrow 6 \cdot 10^3 = \frac{R}{(1/128)}$$

$$\Rightarrow R = 46.8750 \Omega$$ 04



02

b. Considering the s-domain equivalent circuit allows to obtain

$$\frac{V_o(s)}{V_i(s)} = \frac{sL + (1/sC)}{R + sL + (1/sC)}$$

$$= \frac{LCs^2 + 1}{LCs^2 + RCs + 1}$$

$$= \frac{s^2 + (1/LC)}{s^2 + (R/L)s + (1/LC)}$$

$$= \frac{s^2 + \omega_0^2}{s^2 + \beta s + \omega_0^2}$$

$$= \frac{s^2 + 16.10^6}{s^2 + 6.10^3 s + 16.10^6} \qquad 10$$

$$\triangleq H(s)$$

c. $$\beta = \frac{\omega_0}{Q} = \frac{4.10^3}{2/3}$$

$$= 6 \text{ krad/sec} \qquad 01$$

$$\omega_{c1} = \frac{-6.10^3 + \sqrt{(6.10^3)^2 + 4.4.10^3}}{2}$$

$$= \frac{-6.10^3 + \sqrt{52}.10^3}{2}$$

$$= 605.5513 \text{ rad/sec} \qquad 02$$

$$\omega_{c2} = \frac{6.10^3 + \sqrt{52}.10^3}{2}$$

$$= 6605.5513 \text{ rad/sec} \qquad 02$$