EEEN 322 PS 1 QUESTIONS

Q1

2.1-8 Determine the power and the rms value for each of the following signals:

(a) $10\cos\left(100t+\frac{\pi}{3}\right)$ **(b)** $10\cos\left(100t+\frac{\pi}{3}\right)+16\sin\left(150t+\frac{\pi}{5}\right)$

(c) $(10+2\sin 3t)\cos 10t$ **(d)** $10\cos 5t\cos 10t$

(e) $10\sin 5t\cos 10t$ **(f)** $e^{j\alpha t}\cos\omega_0 t$

* *

Q2

EXAMPLE 3.13 Find the Fourier transform of a general periodic signal $g(t)$ of period T_0, and hence, determine the Fourier transform of the periodic impulse train $\delta_{T_0}(t)$ shown in Fig. 3.24a.

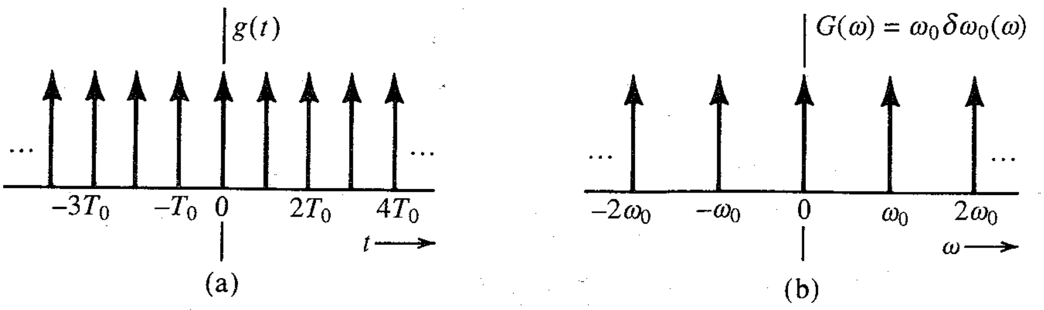

Figure 3.24 Impulse train and its spectrum.

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Q3

EXAMPLE 3.15 Using the time differentiation property, find the Fourier transform of the triangle pulse $\Delta(t/\tau)$ shown in Fig. 3.25a.

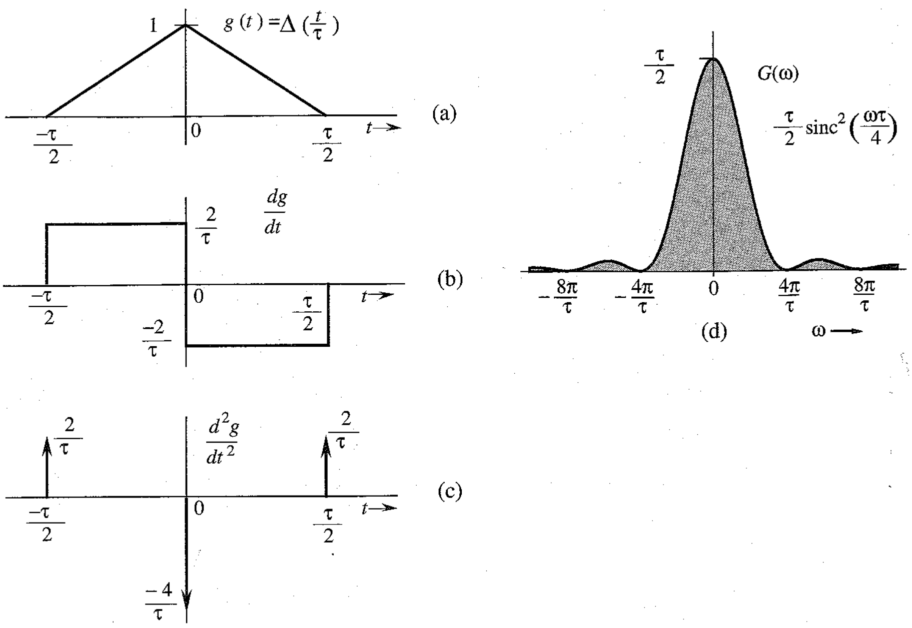

Figure 3.25 Finding the Fourier transform of a piecewise-linear signal using the time differentiation property.

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Q4

3.3-6 The signals in Fig. P3.3-6 are modulated signals with carrier cos $10t$. Find the Fourier transforms of these signals using the appropriate properties of the Fourier transform and Table 3.1. Sketch the amplitude and phase spectra for parts **(a)** and **(b)**. *Hint*: These functions can be expressed in the form $g(t)\cos\,\omega_0 t$.

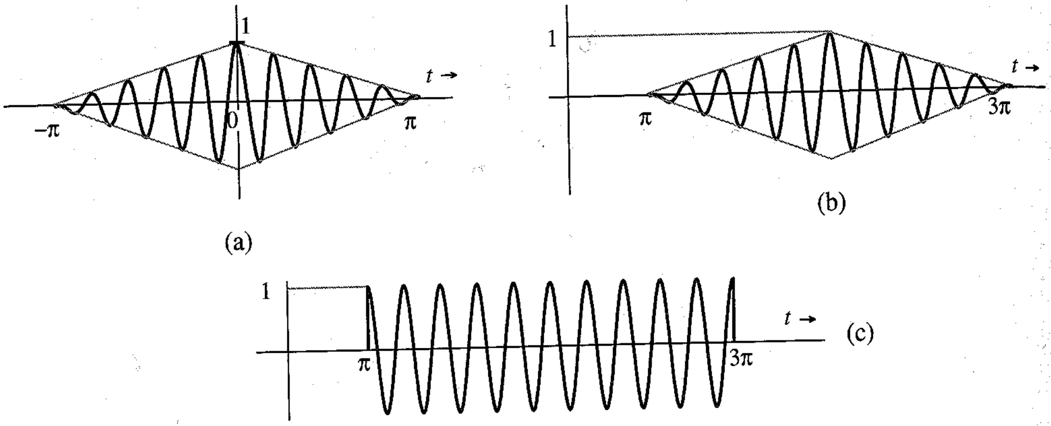

Figure P3.3-6

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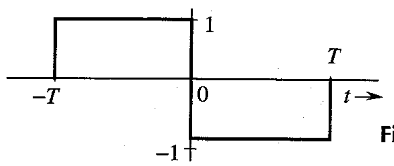
Q5

3.3-9 Find the Fourier transform of the signal in Fig. P3.3-3a by three different methods:

(a) By direct integration using the definition (3.8a).

(b) Using only pair 17 Table 3.1 and the time-shifting property.

(c) Using the time-differentiation and time-shifting properties, along with the fact that $\delta(t) \Longleftrightarrow 1$. Hint: $1 - \cos\,2x = 2\sin^2 x$.



(a)

Figure P3.3-3

EEEN 322 PS 1 SOLUTIONS

2.1-8 (a) Power of a sinusoid of amplitude C is $C^2/2$ [Eq. (2.6a)] regardless of its frequency ($\omega \neq 0$) and phase. Therefore, in this case $P = (10)^2/2 = 50$.

(b) Power of a sum of sinusoids is equal to the sum of the powers of the sinusoids [Eq. (2.6b)]. Therefore, in this case $P = \frac{(10)^2}{2} + \frac{(16)^2}{2} = 178$.

(c) $(10 + 2\sin 3t)\cos 10t = 10\cos 10t + \sin 13t - \sin 3t$. Hence from Eq. (2.6b) $P = \frac{(10)^2}{2} + \frac{1}{2} + \frac{1}{2} = 51$.

(d) $10\cos 5t \cos 10t = 5(\cos 5t + \cos 15t$. Hence from Eq. (2.6b) $P = \frac{(5)^2}{2} + \frac{(5)^2}{2} = 25$.

(e) $10\sin 5t \cos 10t = 5(\sin 15t - \sin 5t$. Hence from Eq. (2.6b) $P = \frac{(5)^2}{2} + \frac{(-5)^2}{2} = 25$.

(f) $e^{j\alpha t}\cos\omega_0 t = \frac{1}{2}\left[e^{j(\alpha+\omega_0)t} + e^{j(\alpha-\omega_0)t}\right]$. Using the result in Prob. 2.1-7, we obtain $P = (1/4) + (1/4) = 1/2$.

* *

EXAMPLE 3.13

A periodic signal $g(t)$ can be expressed as an exponential Fourier series as

$$g(t) = \sum_{n=-\infty}^{\infty} D_n\, e^{jn\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_0}$$

Therefore,

$$g(t) \Longleftrightarrow \sum_{n=-\infty}^{\infty} \mathcal{F}[D_n\, e^{jn\omega_0 t}]$$

Now from Eq. (3.20a), it follows that

$$g(t) \Longleftrightarrow 2\pi \sum_{n=-\infty}^{\infty} D_n\, \delta(\omega - n\omega_0) \qquad (3.41)$$

Equation (2.89) shows that the impulse train $\delta_{T_0}(t)$ can be expressed as an exponential Fourier series as

$$\delta_{T_0}(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} e^{jn\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_0}$$

Here $D_n = 1/T_0$. Therefore, from Eq. (3.41),

$$\delta_{T_0}(t) \Longleftrightarrow \frac{2\pi}{T_0} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

$$= \omega_0 \delta_{\omega_0}(\omega) \qquad \omega_0 = \frac{2\pi}{T_0} \qquad (3.42)$$

Thus, the spectrum of the impulse train also happens to be an impulse train (in the frequency domain), as shown in Fig. 3.24b.

* *

EXAMPLE 3.15

To find the Fourier transform of this pulse we differentiate it successively, as shown in Fig. 3.25b and c. The second derivative consists of a sequence of impulses, as shown in Fig. 3.25c. Recall that the derivative of a signal at a jump discontinuity is an impulse of strength equal to the amount of jump. The function dg/dt has a positive jump of $2/\tau$ at $t = \pm\tau/2$, and a negative jump of $4/\tau$ at $t = 0$. Therefore,

$$\frac{d^2g}{dt^2} = \frac{2}{\tau}\left[\delta\left(t+\frac{\tau}{2}\right) - 2\delta(t) + \delta\left(t-\frac{\tau}{2}\right)\right] \tag{3.49}$$

From the time differentiation property (3.48),

$$\frac{d^2g}{dt^2} \Longleftrightarrow (j\omega)^2G(\omega) = -\omega^2G(\omega) \tag{3.50a}$$

Also, from the time-shifting property (3.30),

$$\delta(t-t_0) \Longleftrightarrow e^{-j\omega t_0} \tag{3.50b}$$

Taking the Fourier transform of Eq. (3.49) and using the results in Eqs. (3.50), we obtain

$$-\omega^2G(\omega) = \frac{2}{\tau}\left(e^{j\frac{\omega\tau}{2}} - 2 + e^{-j\frac{\omega\tau}{2}}\right) = \frac{4}{\tau}\left(\cos\frac{\omega\tau}{2} - 1\right) = -\frac{8}{\tau}\sin^2\left(\frac{\omega\tau}{4}\right)$$

and

$$G(\omega) = \frac{8}{\omega^2\tau}\sin^2\left(\frac{\omega\tau}{4}\right) = \frac{\tau}{2}\left[\frac{\sin(\omega\tau/4)}{\omega\tau/4}\right]^2 = \frac{\tau}{2}\,\text{sinc}^2\left(\frac{\omega\tau}{4}\right) \tag{3.51}$$

The spectrum $G(\omega)$ is shown in Fig. 3.25d. This procedure of finding the Fourier transform can be applied to any function $g(t)$ made up of straight-line segments with $g(t) \to 0$ as $|t| \to \infty$. The second derivative of such a signal yields a sequence of impulses whose Fourier transform can be found by inspection. This example suggests a numerical method of finding the Fourier transform of an arbitrary signal $g(t)$ by approximating the signal by straight-line segments.

*

3.3-6 **Fig. (a)** The signal $g(t)$ in this case is a triangle pulse $\Delta(\frac{t}{2\pi})$ (Fig. S3.3-6) multiplied by $\cos 10t$.

$$g(t) = \Delta\left(\frac{t}{2\pi}\right)\cos 10t$$

Also from Table 3.1 (pair 19) $\Delta(\frac{t}{2\pi}) \Longleftrightarrow \pi\,\text{sinc}^2(\frac{\pi\omega}{2})$ From the modulation property (3.35), it follows that

$$g(t) = \Delta\left(\frac{t}{2\pi}\right)\cos 10t \Longleftrightarrow \frac{\pi}{2}\left\{\text{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \text{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\}$$

The Fourier transform in this case is a real function and we need only the amplitude spectrum in this case as shown in Fig. S3.3-6a.

Fig. (b) The signal $g(t)$ here is the same as the signal in Fig. (a) delayed by 2π. From time shifting property, its Fourier transform is the same as in part (a) multiplied by $e^{-j\omega(2\pi)}$. Therefore

$$G(\omega) = \frac{\pi}{2}\left\{\text{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \text{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\}e^{-j2\pi\omega}$$

The Fourier transform in this case is the same as that in part (a) multiplied by $e^{-j2\pi\omega}$. This multiplying factor represents a linear phase spectrum $-2\pi\omega$. Thus we have an amplitude spectrum [same as in part (a)] as well as a linear phase spectrum $\angle G(\omega) = -2\pi\omega$ as shown in Fig. S3.3-6b. the amplitude spectrum in this case as shown in Fig. S3.3-6b.

Note: In the above solution. we first multiplied the triangle pulse $\Delta(\frac{t}{2\pi})$ by cos $10t$ and then delayed the result by 2π. This means the signal in Fig. (b) is expressed as $\Delta(\frac{t-2\pi}{2\pi})\cos 10(t-2\pi)$.

We could have interchanged the operation in this particular case. that is. the triangle pulse $\Delta(\frac{t}{2\pi})$ is first delayed by 2π and then the result is multiplied by cos $10t$. In this alternate procedure. the signal in Fig. (b) is expressed as $\Delta(\frac{t-2\pi}{2\pi})\cos 10t$.

This interchange of operation is permissible here only because the sinusoid cos $10t$ executes integral number of cycles in the interval 2π. Because of this both the expressions are equivalent since $\cos 10(t-2\pi) = \cos 10t$.

Fig. (c) In this case the signal is identical to that in Fig. b. except that the basic pulse is $\text{rect}(\frac{t}{2\pi})$ instead of a triangle pulse $\Delta(\frac{t}{2\pi})$. Now

$$\text{rect}\left(\frac{t}{2\pi}\right) \Longleftrightarrow 2\pi\,\text{sinc}(\pi\omega)$$

Using the same argument as for part (b). we obtain

$$G(\omega) = \pi\{\text{sinc}[\pi(\omega+10)] + \text{sinc}[\pi(\omega-10)]\}e^{-j2\pi\omega}$$

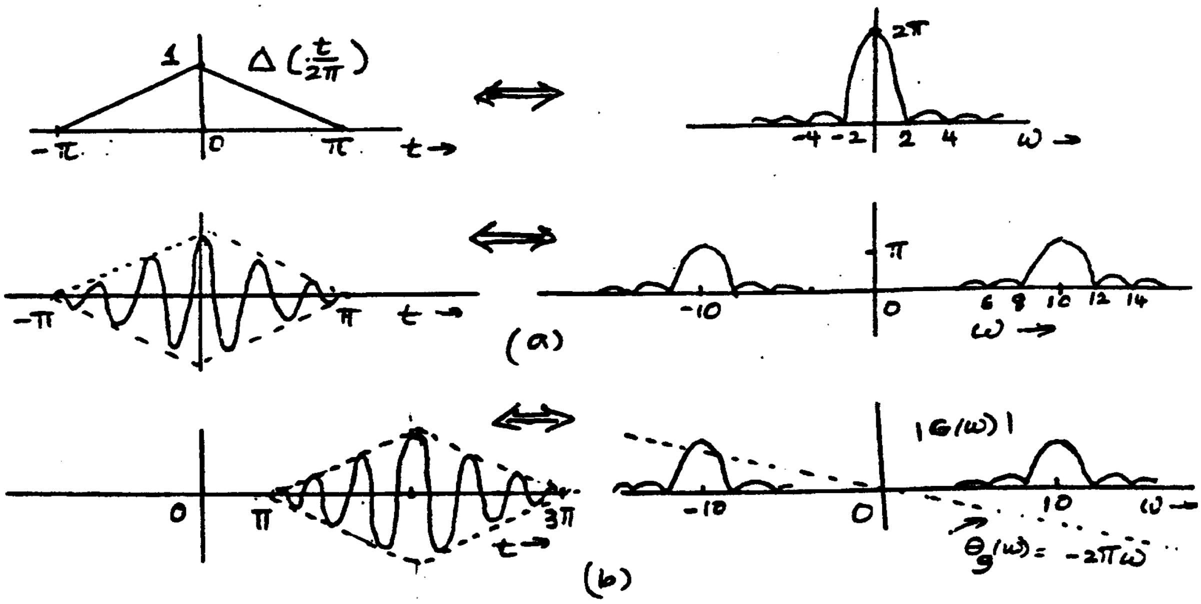


Fig. S3.3-6

* *

3.3-9 **(a)**

$$G(\omega) = \int_{-T}^{0} e^{-j\omega t}\,dt - \int_{0}^{T} e^{-j\omega t}\,dt = -\frac{2}{j\omega}[1 - \cos\omega T] = \frac{j4}{\omega}\sin^2\left(\frac{\omega T}{2}\right)$$

(b)

$$g(t) = \text{rect}\left(\frac{t + T/2}{T}\right) - \text{rect}\left(\frac{t - T/2}{T}\right)$$

$$\text{rect}\left(\frac{t}{T}\right) \Longleftrightarrow T\text{sinc}\left(\frac{\omega T}{2}\right)$$
$$\text{rect}\left(\frac{t \pm T/2}{T}\right) \Longleftrightarrow T\text{sinc}\left(\frac{\omega T}{2}\right) e^{\pm j\omega T/2}$$

and

$$\begin{aligned} G(\omega) &= T\text{sinc}\left(\frac{\omega T}{2}\right)[e^{j\omega T/2} - e^{-j\omega T/2}] \\ &= 2jT\text{sinc}\left(\frac{\omega T}{2}\right)\sin\frac{\omega T}{2} \\ &= \frac{j4}{\omega}\sin^2\left(\frac{\omega T}{2}\right) \end{aligned}$$

(c)

$$\frac{df}{dt} = \delta(t + T) - 2\delta(t) + \delta(t - T)$$

The Fourier transform of this equation yields

$$j\omega G(\omega) = e^{j\omega T} - 2 + e^{-j\omega T} = -2[1 - \cos\omega T] = -4\sin^2\left(\frac{\omega T}{2}\right)$$

Therefore

$$G(\omega) = \frac{j4}{\omega}\sin^2\left(\frac{\omega T}{2}\right)$$

Unit Gate Function

We define a unit gate function rect (x) as a gate pulse of unit height and unit width, centered at the origin, as shown in Fig. 3.7a:

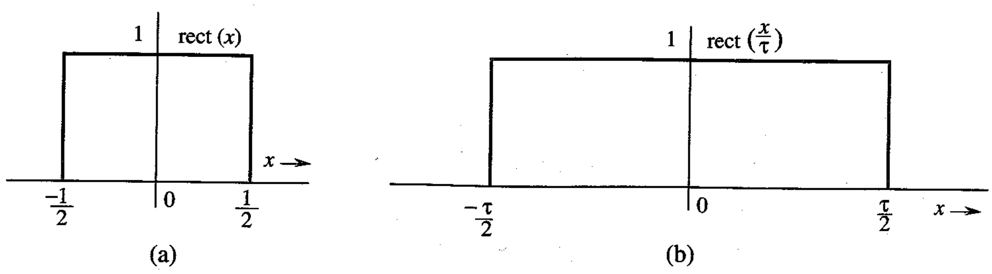


Figure 3.7 Gate pulse.

$$\text{rect}\,(x) = \begin{cases} 0 & |x| > \frac{1}{2} \\ \frac{1}{2} & |x| = \frac{1}{2} \\ 1 & |x| < \frac{1}{2} \end{cases} \tag{3.14}$$

The gate pulse in Fig. 3.7b is the unit gate pulse rect (x) expanded by a factor τ and therefore can be expressed as rect (x/τ) (see Sec. 2.3.2). Observe that τ, the denominator of the argument of rect (x/τ), indicates the width of the pulse.

Unit Triangle Function

We define a unit triangle function $\Delta(x)$ as a triangular pulse of unit height and unit width, centered at the origin, as shown in Fig. 3.8a:

$$\Delta(x) = \begin{cases} 0 & |x| > \frac{1}{2} \\ 1 - 2|x| & |x| < \frac{1}{2} \end{cases} \tag{3.15}$$

The pulse in Fig. 3.8b is $\Delta(x/\tau)$. Observe that here, as for the gate pulse, the denominator τ of the argument of $\Delta(x/\tau)$ indicates the pulse width.

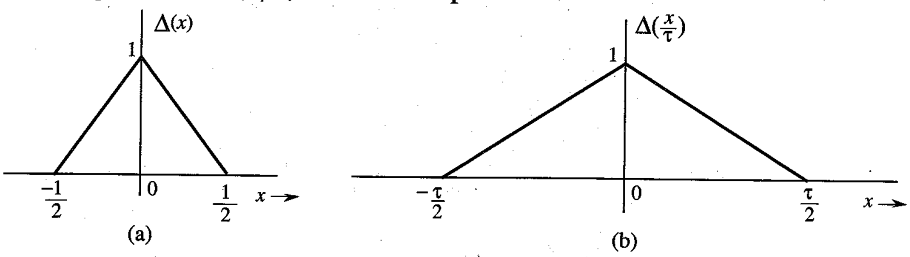


Figure 3.8 Triangle pulse.