EEEN 202

HW 1 SOLUTIONS

1) a) $$\frac{V_o - V_{dc}}{R} + \frac{1}{L}\int_0^t V_o \, d\tau + C\frac{dV_o}{dt} = 0$$

$$\Rightarrow V_o + \frac{R}{L}\int_0^t V_o \, d\tau + RC\frac{dV_o}{dt} = V_{dc} \qquad (1)$$

b) Laplace equivalent of Equation (1)

$$\Rightarrow V_o(s) + \frac{R}{L}\frac{V_o(s)}{s} + RCsV_o(s) = \frac{V_{dc}}{s}$$

$$\Rightarrow V_o(s) = \frac{V_{dc}/s}{1 + \frac{R}{L}\frac{1}{s} + RCs} = \frac{(1/RC)\,V_{dc}}{s^2 + (1/RC)s + 1/LC} \qquad (2)$$

c) $$i_o = \frac{1}{L}\int_0^t V_o \, d\tau \Rightarrow I_o(s) = \frac{1}{Ls}V_o(s)$$; use Equation (2)

$$\Rightarrow I_o(s) = \frac{\frac{(1/RC)\,V_{dc}}{s^2 + (1/RC)s + 1/LC}}{Ls} = \frac{1/RLC \; V_{dc}}{s\left(s^2 + (1/RC)s + 1/LC\right)}$$

2) a) $F(s) = \dfrac{10s^2 + 512s + 7186}{s^2 + 48s + 625}$

$$\Rightarrow P(s) = \frac{10s^2 + 512s + 7186}{10s^2 + 480s + 6250} \quad ; \quad \begin{array}{r} 10s^2 + 512s + 7186 \\ - \quad 10s^2 + 480s + 6250 \\ \hline 0s^2 \; 32s + 936 \end{array}$$

$$\Rightarrow F(s) = 10 + \frac{32s + 936}{s^2 + 48s + 625}$$

$$= 10 + \frac{C_1}{s + 24 + j7} + \frac{C_2}{s + 24 + j7}$$

$$\Rightarrow C_1 = \left. \frac{32s + 936}{s^2 + 48s + 625} \right|_{s = -24 - j7} = 20 \angle -36{,}87 \quad \Rightarrow C_2 = C_1^* = 20 \angle 36{,}87$$

$$\Rightarrow F(s) = 10 + \frac{20 \angle -36{,}87}{s + 24 - j7} + \frac{20 \angle 36{,}87}{s + 24 + j7}$$

$$\Rightarrow p(t) = 10\,\delta(t) + 20\,e^{-j\,36{,}87}\, e^{(-24+j7)t} + 20\,e^{j\,36{,}87}\, e^{(-24-j7)}$$

$$= 10\,\delta(t) + \left[40\,e^{-24t} \cos(7t + 36{,}87)\right] u(t)$$

2) b) $f(s) = \dfrac{(s+5)^2}{s(s+1)^2} = \dfrac{C_1}{s} + \dfrac{C_2}{(s+1)^2} + \dfrac{C_3}{s+1}$

$\Rightarrow C_1 = \dfrac{(s+5)^2}{s(s+1)^2}\Big|_{s=0} = 25 \qquad C_2 = f(s)\Big|_{s=-1} = -16$

$\Rightarrow C_3 = \dfrac{d}{ds}\left(\dfrac{(s+5)^2}{s}\right)\Big|_{s=-1} = \dfrac{2(s+5)s - (s+5)^2}{s^2}\Big|_{s=-1} = -24$

$\Rightarrow p(t) = [25 - 16t e^{-t} - 24e^{-t}]u(t)$

2) c) $F(s) = \dfrac{s^3+5s^2-50s-100}{s^2+13s+40}$

$$\Rightarrow \begin{array}{l} s^3+5s^2-50s-100 \\ - \; s\times(s^2+13s+40) \\ \hline -8s^2-90s-100 \\ - \; -8\times(s^2+13s+40) \\ \hline 14s+220 \end{array}$$

$$\Rightarrow F(s) = s-8+\frac{14s+220}{s^2+13s+40}$$

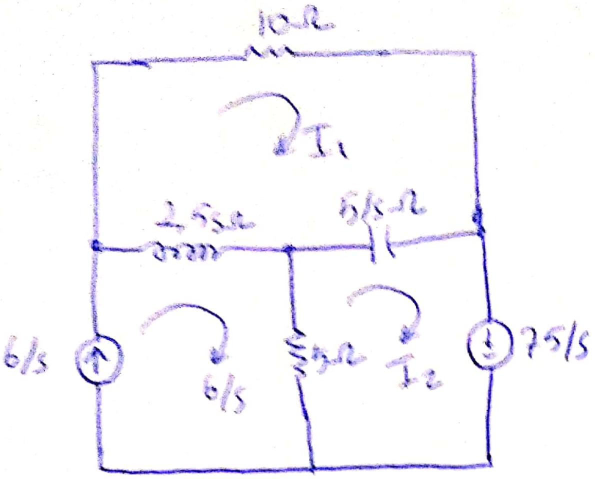
$$= s-8+\frac{14s+220}{(s+5)(s+8)} = \frac{K_1}{s+5}+\frac{K_2}{s+8}+s-8$$

$$\Rightarrow K_1 = \left.\frac{14s+220}{s+8}\right|_{s=-5} = 50$$

$$K_2 = \left.\frac{14s+220}{s+5}\right|_{s=-8} = -36$$

$$\Rightarrow f(t) = \delta'(t) - 8\delta(t) + (50e^{-5t} - 36e^{-8t})u(t)$$

(93)



ⓐ $2{,}5s\left(I_1 - 6/s\right) + 5/s\left(I_1 - I_2\right) + 10 I_1 = 0$

or

$(s^2+4s+2)I_1 - 2I_2 = 6s$
$-I_1 + (s+1)I_2 = -9$

$\Rightarrow \Delta = \begin{vmatrix} (s^2+4s+2) & -2 \\ -1 & (s+1) \end{vmatrix} = s(s+2)(s+3)$

$N_1 = \begin{vmatrix} 6s & -2 \\ -9 & s+1 \end{vmatrix} = 6(s^2+s-3)$

$N_2 = \begin{vmatrix} (s^2+4s+2) & 6s \\ -1 & -9 \end{vmatrix} = -9s^2 - 30s - 18$

$I_1 = \dfrac{N_1}{\Delta} = \dfrac{6(s^2+s-3)}{s(s+2)(s+3)}$

$I_2 = \dfrac{N_2}{\Delta} = \dfrac{-9s^2-30s-18}{s(s+2)(s+3)}$

ⓑ $sI_1 = \dfrac{6(s^2+s-3)}{(s+2)(s+3)}$

$\lim_{s\to\infty} sI_1 = i_1(0^+) = 6A$ $\quad$ $\lim_{s\to 0} sI_1 = i_1(\infty) = -3A$

$sI_2 = \dfrac{-9s^2-30s-18}{(s+2)(s+3)}$

$\lim_{s\to\infty} sI_2 = i_2(0^+) = -9A$ $\quad$ $\lim_{s\to 0} sI_2 = i_2(\infty) = -3A$

ⓒ $$I_1 = \frac{6(s^2+s-3)}{s(s+2)(s+3)} = \frac{C_1}{s} + \frac{C_2}{s+2} + \frac{C_3}{s+3}$$

$$C_1 = \frac{6(-3)}{6} = -3 \qquad C_2 = \frac{6(4-2-3)}{(-2)(1)} = 3 \qquad C_3 = \frac{6(9-3-3)}{(-3)(-1)} = 6$$

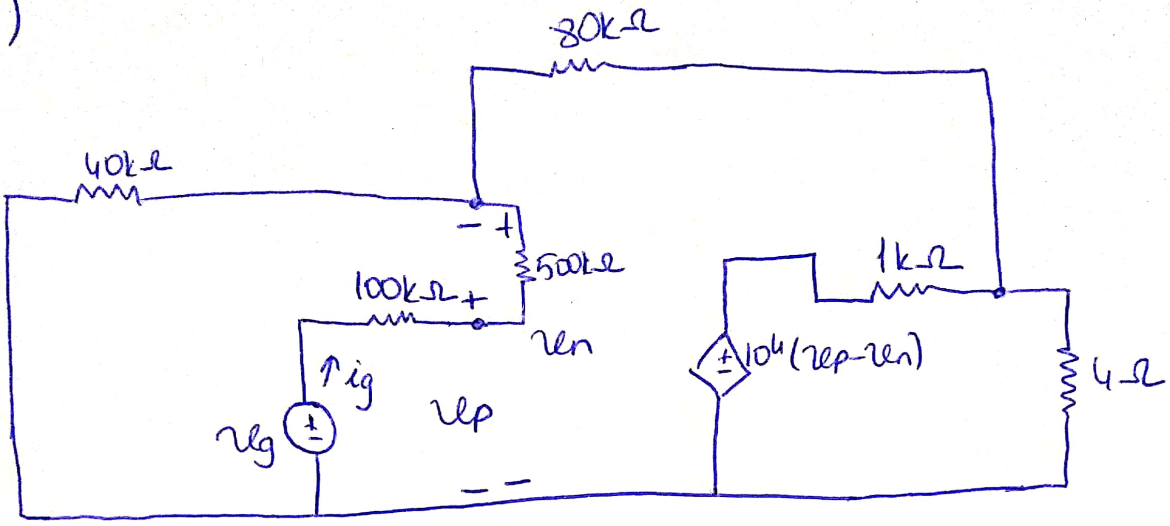
$$i_1(t) = \left[-3+3e^{-2t}+6e^{-3t}\right]u(t)\ A$$

$$I_2 = \frac{-9s^2-30s-18}{s(s+2)(s+3)} = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

$$K_1 = \frac{-18}{6} = -3 \qquad K_2 = \frac{-36+60-18}{(-2)(1)} = -3 \qquad K_3 = \frac{-81+90-18}{(-3)(-1)} = -3$$

$$i_2(t) = \left[-3-3e^{-2t}-3e^{-3t}\right]u(t)\ A$$

Q4)



KCL at non-inverting input;

$$\frac{v_p - v_g}{100k} + \frac{v_p - v_n}{500k} = 0 \quad \Rightarrow 6v_p - 5v_g - v_n = 0$$

KCL at inverting input;

$$\underset{(50)}{\frac{v_n}{40k}} + \underset{(25)}{\frac{v_n - v_o}{80k}} + \underset{(4)}{\frac{v_n - v_p}{500k}} \Rightarrow 79v_n - 25v_o - 4v_p = 0$$

KCL at output;

$$\frac{v_o - v_n}{80k} + \underset{(80)}{\frac{v_o - 10^4(v_p - v_n)}{1k}} + \underset{(20{,}000)}{\frac{v_o}{4}} = 0.$$

$$\Rightarrow 20081v_o - 80 \cdot 10^4 v_p + (80 \cdot 10^4 - 1)v_n = 0$$

$$4/ \quad 6v_p - v_n = 5v_g$$

$$6/ \quad -4v_p + 79v_n = 25v_o$$

$$470v_n = 20v_g + 150v_o \quad \Rightarrow \quad v_n = \frac{2v_g + 15v_o}{47}$$

$$v_p = \frac{237v_g + 15v_o}{6.47}$$

$$20081 v_o - 80.10^4 \left(\frac{237 v_g + 15 v_o}{6.47} - \frac{2 v_g + 15 v_o}{47} \right) - \frac{2 v_g + 15 v_o}{47} = 0$$

$$65662827 v_o = 180000012 v_g$$

$$v_o / v_g = \frac{180000012}{65662827} = 2.7413$$

b) $v_o = 2,7413 \cdot 1 = 2,7413 V$

$$v_p = \frac{237.1 + 15.(2,7413)}{6.47} = 986,2394 mV$$

$$v_n = \frac{2.1 + 15.(2,7413)}{47} = 917,4362 mV$$

c) $v_p - v_n = 986,2394 - 917,4362$
$= 68,8032 mV$
$= 68803,2 \mu V$

d) $i_g = \frac{(1000 - 986,2394).10^{-3}}{100.10^3} = 1,37606 \times 10^7$
$= 137606 pA$

e) on ideal opamp

$v_n = v_p$

$$\frac{2 v_g + 15 v_o}{47} = \frac{237 v_g + 15 v_o}{6.47} \Rightarrow 75 v_o = 225 v_g$$

$$v_o / v_g = 3$$

$v_o = 3.1 = 3V$

$$v_p = \frac{237.1 + 15.3}{6.47} = 1V$$

$$v_n = \frac{2.1 + 15.3}{47} = 1V$$

$\Rightarrow$ $v_p - v_n = 1 - 1 = 0V = 0mV$

$$i_g = \frac{(1000 - 1000).10^{-3}}{100.10^3} = 0A = 0pA$$