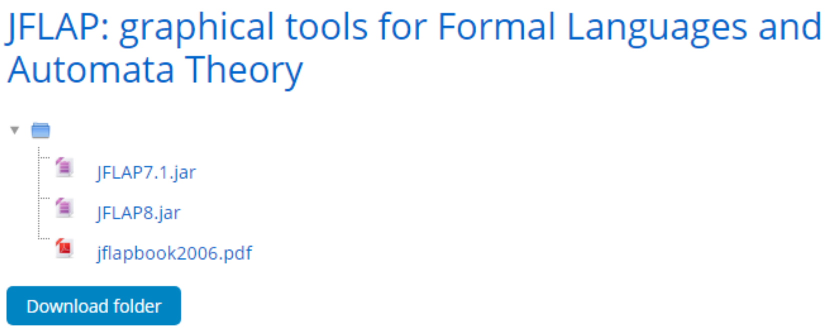
(1) Download JFLAP from the folder: “JFLAP: graphical tools for Formal Languages and Automata Theory” at the top of the Labs page on Moodle.


JFLAP: graphical tools for Formal Languages and Automata Theory

JFLAP7.1.jar

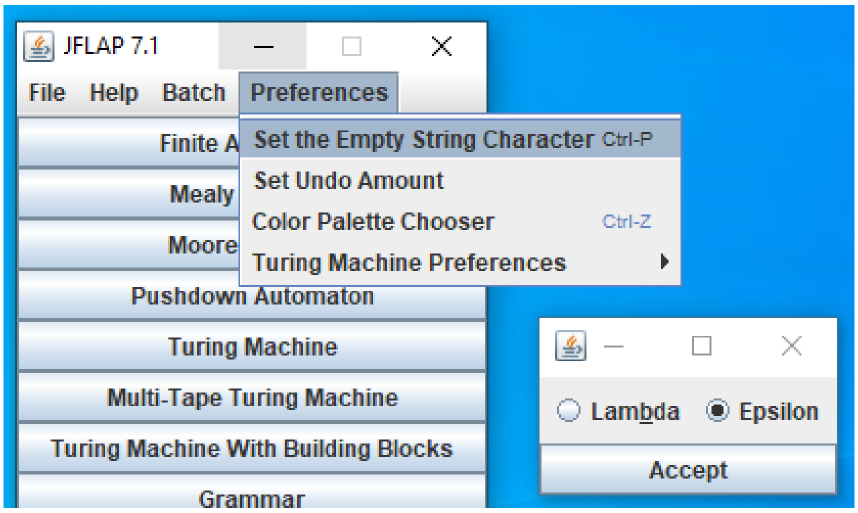
JFLAP8.jar

jflapbook2006.pdf

Follow the JFLAP tutorial at http://www.jflap.org/tutorial/fa/createfa/fa.html. Then use JFLAP to draw and simulate some of the DFAs/NFAs discussed in the lecture.

Please note the following minor differences:

- An **accept** states is called a **final** state in JFLAP.
- By default, the empty string ε is called λ (lambda) in JFLAP. To change this use: `Preferences > Set the Empty String Character > Epsilon`

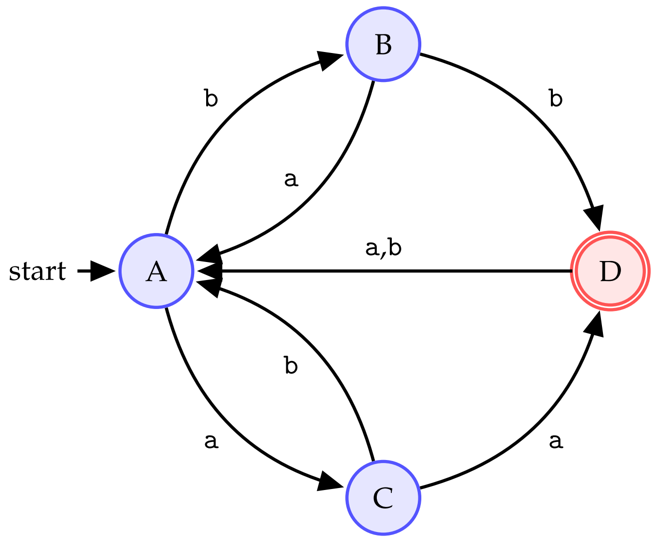

JFLAP 7.1
File
Help
Batch
Preferences
Finite A
Mealy
Moore
Pushdown Automaton
Turing Machine
Multi-Tape Turing Machine
Turing Machine With Building Blocks
Grammar
Set the Empty String Character Ctrl-P
Set Undo Amount
Color Palette Chooser Ctrl-Z
Turing Machine Preferences
Lambda
Epsilon
Accept

Optionally, you may want to read the first chapter from the “JFLAP Book” – available as PDF from the same folder. (Legal copy from https://www2.cs.duke.edu/csed/jflap/jflapbook/).

The aim of this exercise is to get you used to JFLAP *and learn from experimenting with it. Play with it, develop your intuition and understanding of the concepts, experience the mistakes and errors, the failures and successes!*

In particular, if something does not work then ask yourself: what is the source of the problem? How can I fix it?
If it works then ask: can I see the more general pattern? What are the transferable skills/knowledge I can use elsewhere?

(2) Consider the following DFA:



Without using JFLAP, practice *simulating* the behaviour of ths DFA using the following strings. ()For each string, list the sequence of visited states, e.g. $q_0, q_2, q_0, q_1, q_3, \ldots$).

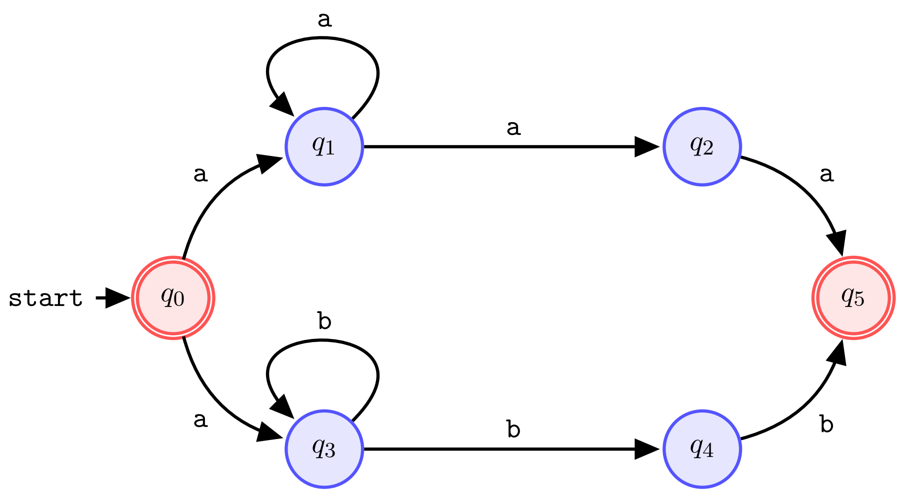
`abba` `babb` `aaa` `bbabba`

You can use the following table to organise your work:

Symbol	State	Symbol	State	Symbol	State	Symbol	State
	A		A		A		A
a		b		a		b	
b		a		a		b	
b		b		a		a	
a		b				b	
						b	
						a	

Produce the formal definition of the above DFA. This should consist of: the alphabet Σ, the set of states Q, the transition function δ, in table form, the start state, and the set of accept states F.

(3) Consider the following NFA:



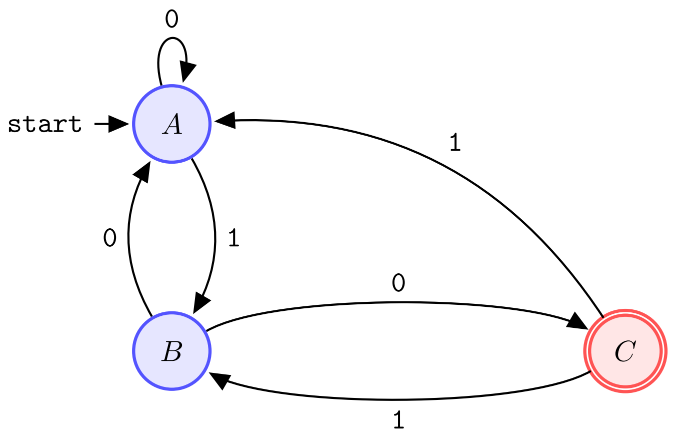
Without using JFLAP, practice *simulating* the behaviour of this NFA using the following strings. (For each string, list the sets of visited states, e.g. $\{q_0\}, \{q_1, q_2\}, \{q_2, q_3\}, \ldots$).

You may want to use a table similar to the one in the previous question.

`aa` `aaaa` `abbaa` `babb` `aaaba` `abbbbbbbaab`

Produce the formal definition $(\Sigma, Q, \delta, q_{\textbf{start}}, F)$ of the above NFA.

(4) Which of the strings listed below does the following NFA accept? (Use pen-and-paper, not JFLAP)



- 10011010
- 01010011
- 00010111
- 0010010

(5) The formal description $(Q, \Sigma, \delta, q_{\text{start}}, F)$ of a DFA is given by

$$(\{q_1, q_2, q_3, q_4, q_5\}, \{\mathtt{d}, \mathtt{u}\}, \delta, q_1, \{q_5\}),$$

where δ is given by the following table

	d	u
$*q_1$	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
$\rightarrow q_5$	q_4	q_5

Give the *state diagram* of this machine.

Hint: It looks like a ladder or a lift going between floors. (u: go up, d: go down.)

(6) Use JFLAP to design simple DFAs which recognize the following languages over the alphabet $\Sigma = \{a, b\}$

1) The language of strings which begin with a.
2) The language of strings which end with b.
3) The language of strings which either begin **or** end with b.
4) The language of strings which begin with a **and** end with b.
5) The language of strings which contain the substring ba.
6) The language of strings with all the a's on the left and b's on the right
7) The language strings consisting of alternating a's and b's.

(7) Use JFLAP to produce NFAs to recognize the following languages over $\Sigma = \{0, 1\}$

1) The language of strings which begin and end with 01.
2) The language of strings which do **not** end with 01.

   Hint: Recall that DFAs are a special case of NFAs. For this problem, it may be easier to first create a DFA that accepts *strings that* ***end*** *with* 01, then we flip the accepting states into non-accepting ones, and vice versa. This will produce a DFA that accepts *strings that* ***do not end*** *with* 01, as required.
   This is a useful technique, but note that **it only works for DFAs – it does not work for NFAs.**

3) The language of strings which begin and end with different symbols.
4) The language of strings of odd length.
5) The language of strings which contain an even number of 0's.
6) The language of binary numbers which are divisible by 4.

(8) If a is a *symbol* from an alphabet Σ then $\mathtt{a}^n$ denotes the string which consists of n successive copies of a.

Similarly, if x is a *string* of symbols then x^n denotes the string which consists of n successive copies of x. For example, $\mathtt{a}^2 = \mathtt{aa}$ and $(\mathtt{ab})^2 = \mathtt{abab}$.

Let $\Sigma = \{0, 1\}$. Write $0^4, 1^4, (10)^3, 10^3$ explicitly as strings in the usual form.

(9) If Σ is an alphabet then Σ^n denotes the set of all strings over Σ which have length exactly n symbols.

1) Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. Find Σ^2.
2) Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Find Σ^3.

(10) If Σ is an alphabet then the set of all finite-length strings over it is denoted by Σ^*.

Let $\Sigma_1 = \{\mathtt{a}\}$ and $\Sigma_2 = \{\mathtt{a}, \mathtt{b}\}$. List the strings of length 0, 1, 2, 3, and 4 over these two alphabets. Write these in the form $\Sigma_1^* = \{\ldots\}$ and $\Sigma_2^* = \{\ldots\}$.

Note that
$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \Sigma^4 \cup \cdots .$$

Basic

(1) Intersection, union and difference of DFAs

Given two languages described by two DFAs, the idea is to construct a new DFA that simulates simultaneously-running the given DFAs. To do this, the states of the new DFA will be pairs of states from the original DFAs.
Suppose $\mathcal{M}_1 = (Q_1, \Sigma, \delta_1, q_{\text{start }1}, F_1)$ and $\mathcal{M}_2 = (Q_2, \Sigma, \delta_2, q_{\text{start }2}, F_2)$ are DFAs recognizing L_1 and L_2, respectively.
Let $\mathcal{M}$ be the DFA given by $(Q, \Sigma, \delta, q_0, F)$ where

$$\begin{aligned} Q &= Q_1 \times Q_2 \\ q_0 &= (q_{\text{start }1}, q_{\text{start }2}) \end{aligned}$$

and the transition function δ is defined by

$$\delta\Big((p,q),\sigma\Big) = \Big(\delta_1(p,\sigma), \delta_2(q,\sigma)\Big)$$

for all $(p,q) \in Q_1 \times Q_2$ and $\sigma \in \Sigma$.
Then

- $\mathcal{M}$ recognizes $L_1 \cap L_2$ if

$$F = \{(p,q) \mid p \in F_1 \wedge q \in F_2\} \quad = F_1 \times F_2$$

- $\mathcal{M}$ recognizes $L_1 \cup L_2$ if

$$\begin{aligned} F &= \{(p,q) \mid p \in F_1 \vee q \in F_2\} \\ &= \{(p,q) \mid p \in F_1\} \ \cup \ \{(p,q) \mid q \in F_2\} \\ &= (F_1 \times Q_2) \cup (Q_1 \times F_2) \end{aligned}$$

- $\mathcal{M}$ recognizes $L_1 - L_2$ if

$$F = \{(p,q) \mid p \in F_1 \wedge q \notin F_2\} \quad = F_1 \times (Q_2 - F_2)$$

1) Let the languages L_1 and L_2 be given by:

- $L_1 = \{w \mid w = \mathtt{a}^n\mathtt{b}v$ where n≥ 0 and $v \in \Sigma^*\}$
- $L_2 = \{w \mid w = \mathtt{a}^n\mathtt{b}v$ where $n \geq 0$ and $v \in \Sigma^*\}$

over $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Recall that Σ^* is the set of all strings over Σ of finite length.

First, design DFAs recognising L_1 and L_2, then use the above method to construct a DFA for

$$L_1 \cap L_2 = \{w \mid w \text{ has at least one } \mathtt{a} \textbf{ and } \text{at least one } \mathtt{b}\},$$

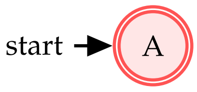
then outline how it can easily be changed for $L_1 \cup L_2$ and $L_1 - L_2$.

2) Use the same method for the language

$$\{w \mid w \text{ has an even number of } \mathtt{a}\text{'s } \textbf{and} \text{ each } \mathtt{a} \text{ is followed by at least one } \mathtt{b}\}.$$

(2) **(Complement of a language)**

In the special case where $L_1 = \Sigma^*$, the language of all possible strings of finite length over the given alphabet Σ, we have the following NFA



i.e. $\mathcal{M}_1 = (Q_1, \Sigma, \delta_1, q_{\text{start }1}, F_1)$ where

$$\begin{array}{rcl} Q_1 & = & \{A\} \\ \Sigma & = & \Sigma \\ \delta_1 & : & (q,\sigma) \mapsto \{A\} \\ q_{\text{start }1} & = & A \\ F_1 & = & \{A\} \end{array}$$

This choice for $\mathcal{M}_1$ provides us with a method to produce the **complement** of L_2 which is defined as $\Sigma^* - L_2 = L_1 - L_2$. Following the result from the previous question, let

$$\begin{array}{rcl} Q & = & Q_1 \times Q_2 = \{q_{\text{start }1}\} \times Q_2 \\ F & = & F_1 \times (Q_2 - F_2) = \{q_{\text{start }1}\} \times (Q_2 - F_2) \end{array}$$

This means that the new DFA is essentially the **DFA for L_2 but with the accepting and non-accepting states "flipped." (swapping roles.)**

Use this observation to produce a DFA for the language

$$\{w \mid w \text{ does } \textbf{not} \text{ contain the substring } \texttt{baba}.\}$$

This does not always work for NFAs – give an example to show that it does not work. (Counter example)

Advanced

In your preferred programming language, implement a class for representing and simulating DFAs and NFAs.