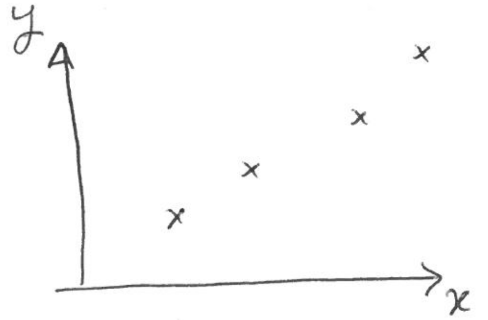
Kovaryans Hesabı

* İki Rassal Değişkenin covaryons/correlation hesabı:

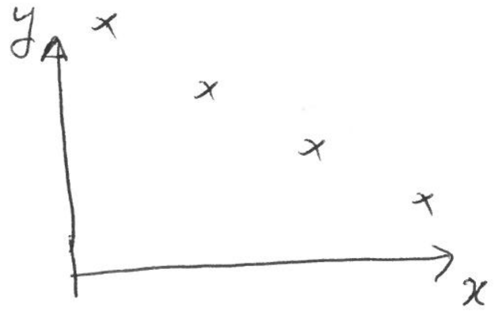
X, Y iki rassal değişken olsun (bağımsız mı?)

X: Rassal olarak seçilmiş bir kişinin boyu
Y: Aynı kişinin kilosu olabilir.

$$Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] = \dots = E[XY] - \mu_X \mu_Y$$



$cov(X,Y) > 0$



$cov(X,Y) < 0$

$cov(X,Y) = 0 \Rightarrow$ birşey söylenemez (uncorrelated)

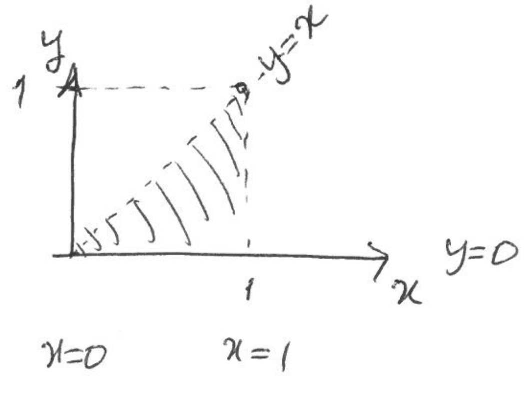
Correlation between X and Y.

$$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \, \sigma_Y} \quad \text{ve} \quad -1 \leq \rho_{XY} \leq +1.$$

Örnek: X, Y sürekli rassal değişkenler olsun.

$f(x,y)$ X ve Y'nin ortak olasılık yoğunluk fonk. olsun.

$$f(x,y) = \begin{cases} \beta x & 0 \leq y \leq x \leq 1 \\ 0 & \text{dd.} \end{cases}$$



$$E(XY) = \int_0^1 \int_0^x xy\, f(x,y)\, dy\, dx = \int_0^1 \int_0^x xy\, 3x\, dy\, dx$$

$$= \int_0^1 3x^2 \underbrace{\int_0^x y\,dy}_{= \frac{y^2}{2}\Big|_0^x} dx = \int_0^1 3x^2 \frac{x^2}{2}\, dx = \frac{3}{2} \int x^4 dx = \frac{3}{2} \cdot \frac{1}{5} = \frac{3}{10}$$

$$E(X) = \int_0^1 \int_0^x x\, f(x,y)\, dy\, dx = \int\int x\, 3x\, dy\, dx = \int_0^1 3x^2 \cdot x\, dx = \frac{3}{4}.$$

$$3\frac{x^4}{4}$$

$$E[Y] = \int_0^1 \int_0^x y \cdot 3x\, dy\, dx = \int_0^1 3x \int_0^x y\, dy\, dx = \int_0^1 3x \frac{x^2}{2}\, dx = \frac{3}{2}\,\frac{1}{4} = \frac{3}{8}$$

$$\mathrm{Cov}(X,Y) = E(X,Y) - \mu_x \mu_y = \underset{(16)}{\frac{3}{10}} - \frac{3}{4} \cdot \underset{(5)}{\frac{3}{8}} = \frac{48-45}{160} = \frac{3}{160}.$$

Çalisma Sorusu:

$$f(x,y) = \begin{cases} kx & 0<x<1,\ 0<y<1 \\ 0 & dd \end{cases}$$

$\mathrm{cov}(X,Y) = ?$

ipucu: önce k sabitini ~~n~~ bulunuz.

$\int\int_{-\infty}^{+\infty} f(x,y)\, dy\, dx = 1$ olduğunu hatırlayınız.

X, Y Kesikli ise,

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

$$E(X) = \sum_{k} k\, P(X=k) = \sum_{k} k\, p_X(k)$$

$$E(Y) = \sum_{k} k\, p_Y(k)$$

$$E(XY) = \sum_{x}\sum_{y} xy\, P(X=x, Y=y) = \sum_{x}\sum_{y} xy\, \underbrace{p(x,y)}_{\text{ortak o.f.}}$$

Örnek:

$p(x,y)$	X = 1	X = 2	X = 3		
Y = 2	0.1	0.1	0.2	0.4	$= P(Y=2)$
Y = 3	0.2	0.3	0.1	0.6	$P(Y=3)$
	0.3	0.4	0.3	1.0	
	$P(X=1)$	$P(X=2)$	$P(X=3)$		

$$E(X) = \sum_{k=1}^{3} k\, p_X(k) = 1.\, p_X(1) + 2\, p_X(2) + 3\, p_X(3) = 0.3 + 0.8 + 0.9 = 2.0$$

$$E(Y) = \sum_{k=2}^{3} k\, p_Y(k) = 2\, p_Y(2) + 3\, p_Y(3) = 2.(0.4) + 3(0.6) = 2.6$$

$$E(XY) = \sum_{x=1}^{3}\sum_{y=2}^{3} xy\, p(x,y) = 1.2.(0.1) + 1.3(0.2) + 2.2(0.1) + 2.3(0.3) + 3.2(0.2) + 3.3(0.1) = 5.1$$

$$Cov(X,Y) = 5.1 - (2)(2.6) = -0.1.$$

$$\rho =$$

$$E(X^2)=\sum_{x=1}^{3} x^2 P_X(x) = 1^2(0.3)+2^2(0.4)+3^2(0.3)$$

$$= 0.3+1.6+2.7 = 4.6.$$

$$\sigma_X^2 = Var(X) = E(X^2)-\mu_X^2 = 4.6-2^2 = 0.6$$

$$E(Y^2)=\sum_{y=2}^{3} y^2 P_Y(y) = 2^2(0.4)+3^2(0.6) = 1.6+5.4 = 7.$$

$$\sigma_Y^2 = Var(Y) = E(Y^2)-\mu_Y^2 = 7-(2.6)^2 = 0.24.$$

$$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = \frac{-0.1}{\sqrt{0.6}\sqrt{0.24}} \cong -0.26$$