Normal Dağılım

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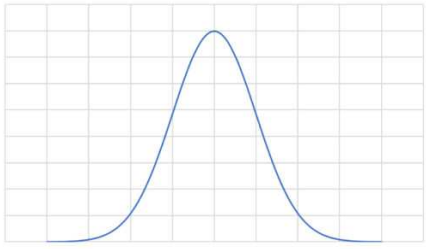
2020-2021 Bahar Dönemi

Normal Dağılım

- İstatistikte en yaygın kullanılan dağılım normal dağılımdır.
- Gauss dağılımı ve çan eğrisi gibi isimlerle de adlandırılmaktadır.



Olasılık Yoğunluk Fonksiyonu (pdf)

Normal dağılımın olasılık yoğunluk fonksiyonu şöyledir:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad , -\infty < x < \infty$$

$\mu = Anakütle\ ortalaması$

$\sigma = Anakütle\ standart\ sapması$

Özellikleri

Bu dağılım μ (ortalama) ve σ (standart sapma) parametrelerine dayanmaktadır.

Dolayısıyla normal bir dağılım $X \sim N(\mu, \sigma^2)$ şeklinde gösterilir.

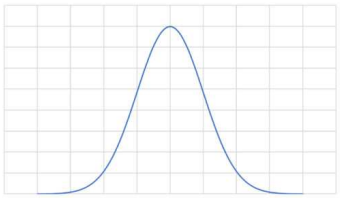
$f(x) \geq 0$

$\int_{-\infty}^{\infty} f(x)dx = 1\,'dir.$

Özellikleri

$x = \mu$ olması durumunda $f_{max} =$??

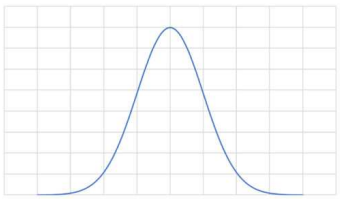
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad , -\infty < x < \infty$$



Özellikleri

$x = \mu$ olması durumunda $f_{max} = \frac{1}{\sigma\sqrt{2\pi}}$

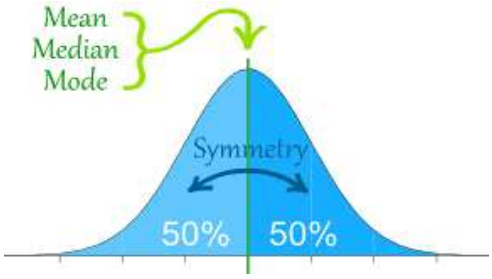
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad , -\infty < x < \infty$$



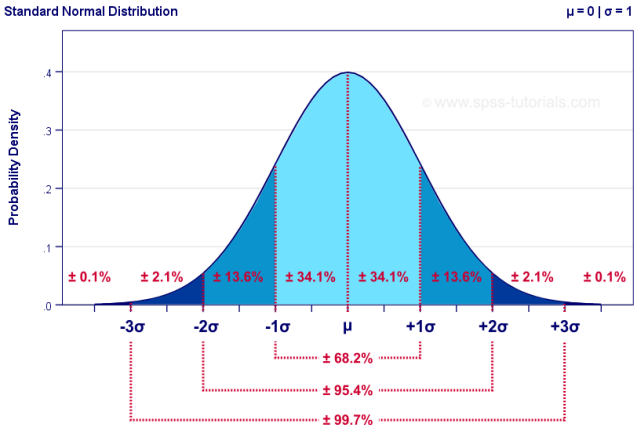
Özellikleri

Normal dağılım eğrisi simetrik olduğundan

ortalama = medyan = mod



Özellikleri

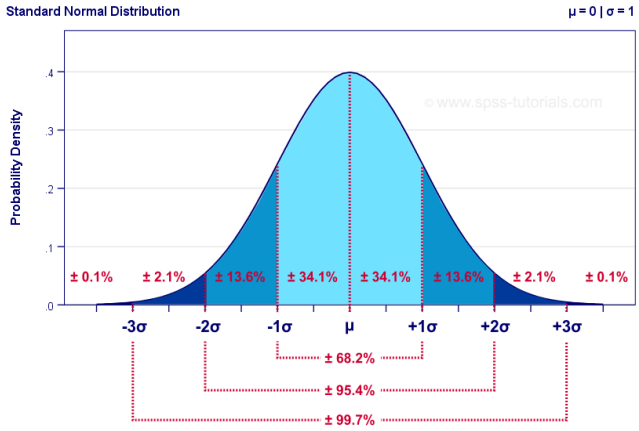


Ortalamanın 1σ sağında ve solunda kalan alan toplam alanın % ….' ine eşittir.

Ortalamanın 2σ sağında ve solunda kalan alan toplam alanın % ….' ine eşittir.

Ortalamanın 3σ sağında ve solunda kalan alan toplam alanın % ….' ine eşittir.

Özellikleri



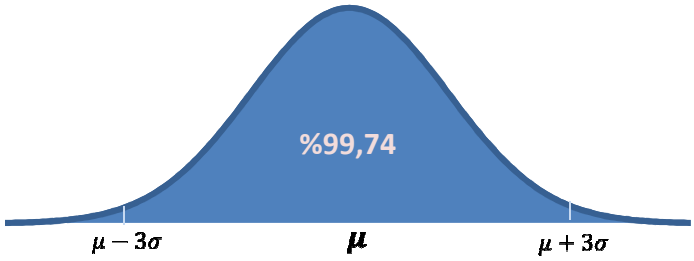
Ortalamanın 1σ sağında ve solunda kalan alan toplam alanın %68,2'sine eşittir.

Ortalamanın 2σ sağında ve solunda kalan alan toplam alanın %95,4'üne eşittir.

Ortalamanın 3σ sağında ve solunda kalan alan toplam alanın %99,7'sine eşittir.

NORMAL DAĞILIM

- Ortalaması μ ve varyansı σ^2 olan normal dağılıma sahip bir X rassal değişkeni için



$$P(\mu - 3\sigma \le x \le \mu + 3\sigma) = 0.9974$$

STANDART NORMAL DAĞILIM

- Olasılık hesaplamasındaki zorluktan dolayı normal dağılım gösteren hesaplamalar için **standart normal dağılım** yaklaşımından yararlanılır.
- Böylece tek bir olasılık tablosu kullanılarak normal dağılımla ilgili olasılık hesaplamaları yapılmış olur.

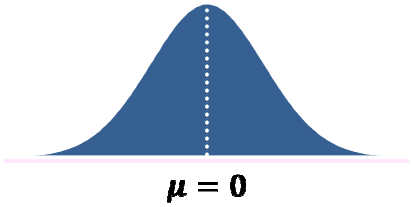
STANDART NORMAL DAĞILIM

- Normal Dağılımın özel bir biçimidir. Normal dağılıma dayalı hesaplamalarda kullanıcılara kolaylık sağlar.
- **Ortalama** $(\mu) = 0$ ve **Varyans** $(\sigma^2) = 1$ dir.
- Standart normal değişken z ile gösterilir.

STANDART NORMAL DAĞILIM

- Eğer bir x değişkeninin normal dağıldığı biliniyorsa
- Yandaki eşitlik ile elde edilen z değerleri ortalaması 0 ve varyansı 1 olan standart normal dağılıma uyar
- Dağılımın grafiği yandaki gibidir.

$$z = \frac{x - \mu}{\sigma}$$



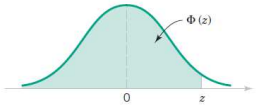
STANDART NORMAL DAĞILIM

- Bu özellik, ortalama ve standart sapmanın değerine bağlı değildir.
- Ortalama ve standart sapma ne olursa olsun X değişkeninin normal dağılması bu özelliğin geçerliği için yeterlidir.
- Çeşitli z değerleri için 0 ile z arasında kalan alanı gösteren **Z** tablosu geliştirilmiştir.
- Bu tablodan yararlanarak normal dağılıma dayalı hesaplamalar yapılabilir.
- Z değeri ile merkez (ortalama) arasında kalan alanı tablo bize verir.
- Z değerlerinin her birine standart skorlar da denir.

Z tablosu

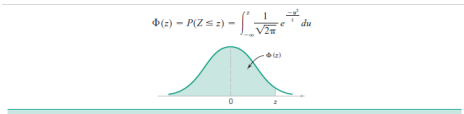
Table I Cumulative Standard Normal Distribution

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du$$



z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00	z
−3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048	−3.9
−3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072	−3.8
−3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108	−3.7
−3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159	−3.6
−3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233	−3.5
−3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337	−3.4
−3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483	−3.3
−3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687	−3.2
−3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968	−3.1
−3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350	−3.0
−2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866	−2.9
−2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555	−2.8
−2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467	−2.7
−2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661	−2.6
−2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210	−2.5
−2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198	−2.4

Kümülatif Z tablosu (+)



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−3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233	−3.5
−3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337	−3.4
−3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483	−3.3
−3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687	−3.2
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−3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350	−3.0
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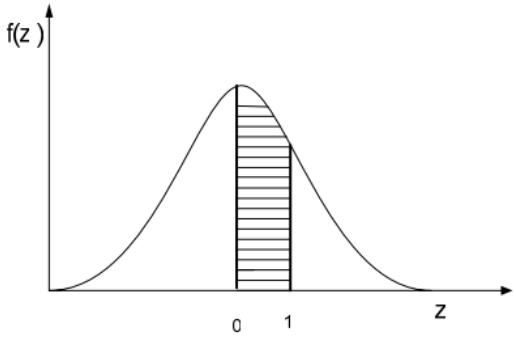
−2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661	−2.6
−2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210	−2.5
−2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198	−2.4
−2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724	−2.3
−2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903	−2.2
−2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864	−2.1
−2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750	−2.0
−1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717	−1.9
−1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930	−1.8
−1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565	−1.7
−1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799	−1.6
−1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807	−1.5
−1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757	−1.4
−1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801	−1.3
−1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070	−1.2
−1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666	−1.1
−1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655	−1.0
−0.9	0.161087	0.163543	0.166023	0.168528	0.171056	0.173609	0.176185	0.178786	0.181411	0.184060	−0.9
−0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855	−0.8
−0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964	−0.7
−0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253	−0.6
−0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538	−0.5
−0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578	−0.4
−0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089	−0.3
−0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740	−0.2
−0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	0.452242	0.456205	0.460172	−0.1
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000	0.0

Kümülatif Z tablosu (-)

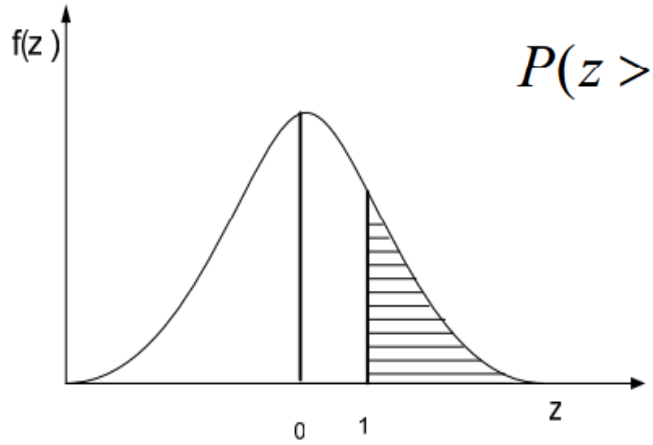
$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856	0.0
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345	0.1
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092	0.2
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732	0.3
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933	0.4
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405	0.5
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903	0.6
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236	0.7
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267	0.8
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913	0.9
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143	1.0
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977	1.1
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475	1.2
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736	1.3
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888	1.4
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083	1.5
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486	1.6
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273	1.7
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621	1.8
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705	1.9
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691	2.0
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738	2.1
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989	2.2
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576	2.3
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613	2.4
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201	2.5
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427	2.6
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365	2.7
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074	2.8
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605	2.9
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999	3.0
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289	3.1
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499	3.2
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650	3.3
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758	3.4
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835	3.5
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888	3.6
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925	3.7
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950	3.8
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967	3.9

$$P(0 < z < 1) = ?$$



$$P(0 < z < 1) = 0{,}3413$$

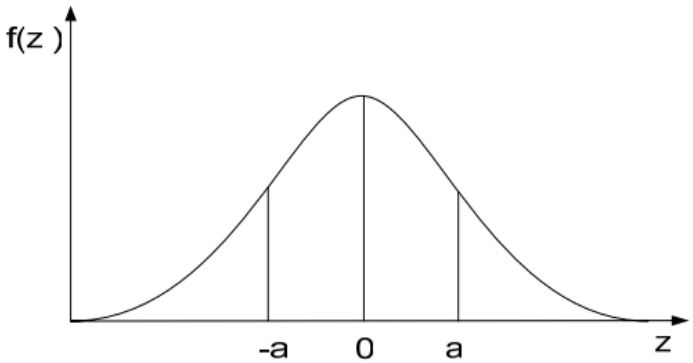


$$P(z > 1) = ?$$

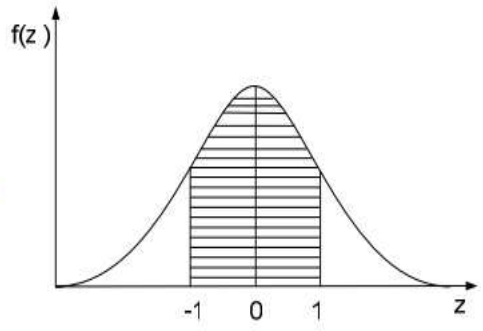
0,5 $- P(0 < z < 1) =$ **0,5** $- 0{,}3413 = 0{,}1587$

Simetrik özelliğinden dolayı 0'da eşit uzaklıktaki Z değerlerinin 0 ile arasında kalan alanların değerleri birbirine eşittir.

$$P(0 < z < a) = P(-a < z < 0)$$



$$P(-1 < z < 1) = ?$$



$$P(-1 < z < 1) = P(-1 < z < 0) + P(0 < z < 1)$$
$$= 2 * P(0 < z < 1) = 2(0{,}3413) = 0{,}6826$$

$$P(-1{,}56 < z < -0{,}95) = ?$$

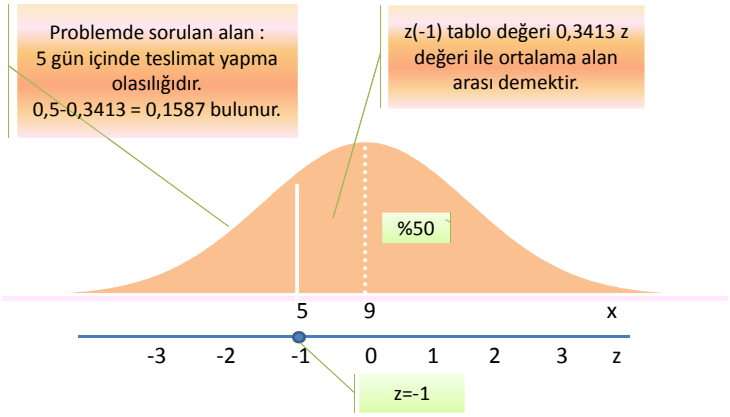
$$P(-1{,}56 < z < -0{,}95) = P(-1{,}56 < z < 0) - P(-0{,}95 < z < 0)$$
$$= 0{,}4406 - 0{,}3289 = 0{,}1117$$

Örnek 1

- Bir tedarikçinin gelen siparişler için hammadde teslimat süresi, ortalaması μ=9 gün ve standart sapması σ=4 gün olan normal dağılıma sahiptir. Rassal seçilen bir hammadde siparişinin 5 gün içinde teslim edilmesi olasılığı nedir?

Örnek 1

- $P(X \leq 5) = ?$
- $P(Z \leq -1) = ?$
- z= (5-9) / 4 = -1



Örnek 3

- Bir kliniğe belli bir şikayetler gelen hastaların yaşlarının normal dağılıma sahip olduğu, ortalamasının 37,5 ve standart sapmasının ise 7,6 olduğu bilinmektedir.

a) Rassal olarak seçilen bir hastanın 44 yaşından genç olması olasılığı nedir?

Örnek 3a)

$$P(X < 44) = ?$$

$$P\left(\frac{X-\mu}{\sigma} < \frac{44-37,5}{7,6}\right) =$$

$$P(z < 0,86) = 0,805106$$

Rassal seçilen bir hastanın 44 yaşından genç olma olasılığı %80,51'dir.

Örnek 3b)

- Rassal seçilen bir hastanın yaşının 46 ile 54 arasında olması olasılığı nedir?

Örnek 3b)

$$P(46 < X < 54) = ?$$

$$P\left(\frac{46-37,5}{7,6} < \frac{X-\mu}{\sigma} < \frac{54-37,5}{7,6}\right) =$$

$$P(1,12 < z < 2,17) = 0,984997 - 0,868643 = 0,116354$$

Rassal seçilen bir hastanın 46 ile 54 yaş arasında olması olasılığı % 11,64'tür.