POLITECNICO
MILANO 1863

DIPARTIMENTO DI ELETTRONICA
INFORMAZIONE E BIOINGEGNERIA

DEIB

LOCALIZATION, NAVIGATION AND SMART MOBILITY
Project presentation A.Y. 2023/2024

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Tugrul KOK – 10896465
Zeynep Gul PEHLIVANLI – 10886151

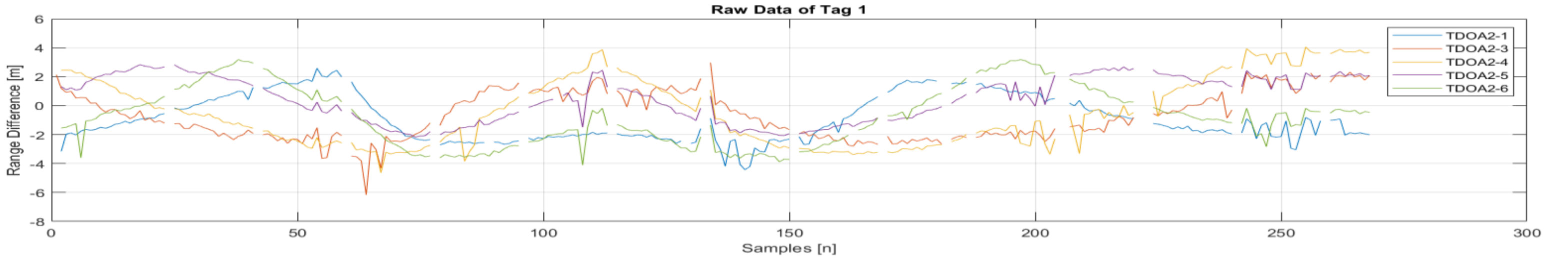
OUTLINES

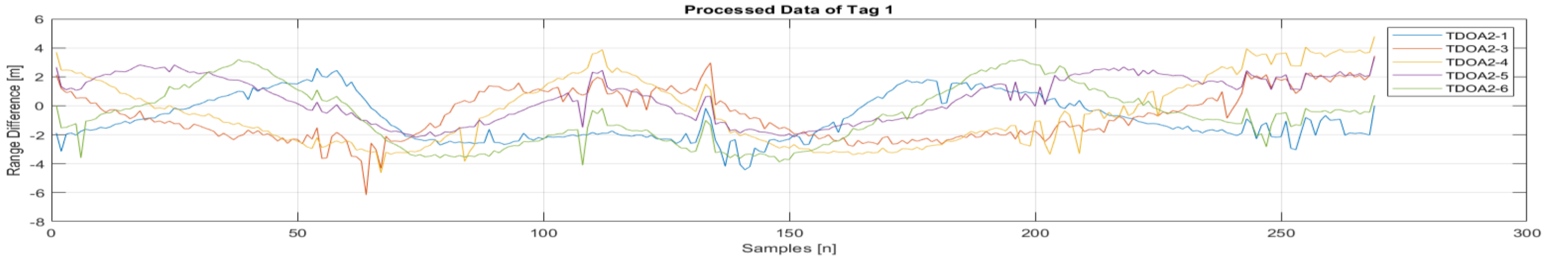
- **Experiment**
- **Preprocessing**
- **Theory Explanation**
- **Simulation Outcomes & Perfomance Analysis**

EXPERIMENT

- Measurements are taken from an AGV that is moving around the area
- 5 TDOA Measurements
- 6 APs at known positions
- 4 UWB Tags
- Sampling Frequency = 10 Hz
- Master AP: #2 Measurements

PREPROCESS


Raw Data of Tag 1
Range Difference [m]
Samples [n]
TDOA2-1
TDOA2-3
TDOA2-4
TDOA2-5
TDOA2-6
6
4
2
0
-2
-4
-6
-8
0
50
100
150
200
250
300


Processed Data of Tag 1
Range Difference [m]
Samples [n]
TDOA2-1
TDOA2-3
TDOA2-4
TDOA2-5
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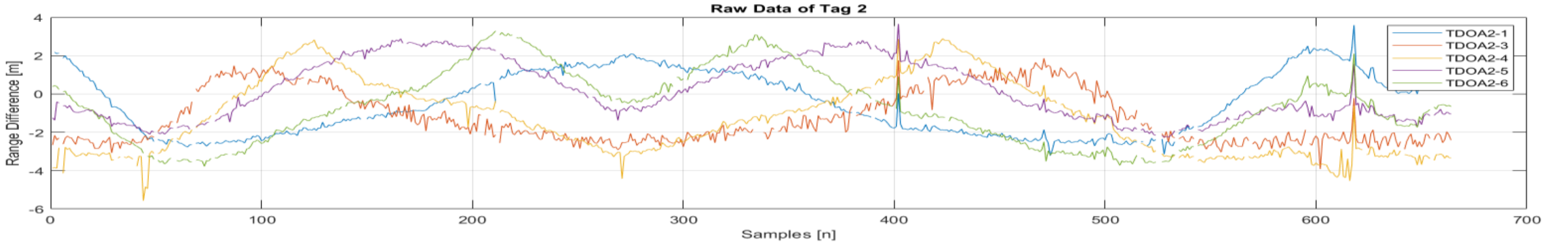
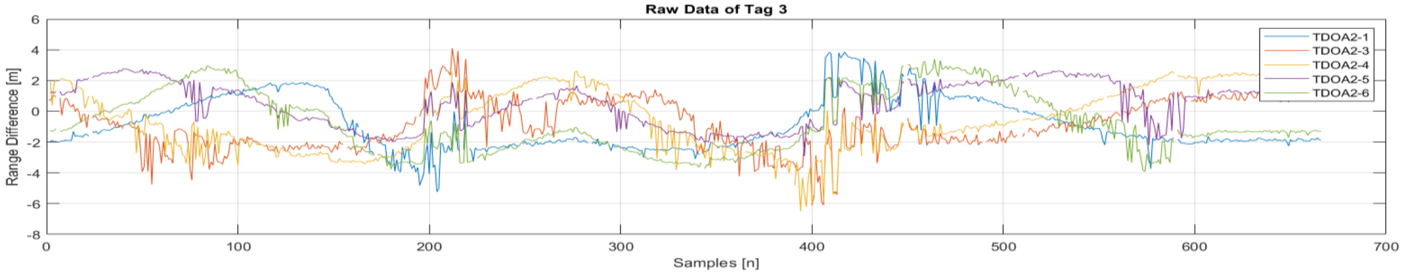
6
4
2
0
-2
-4
-6
-8
0
50
100
150
200
250
300

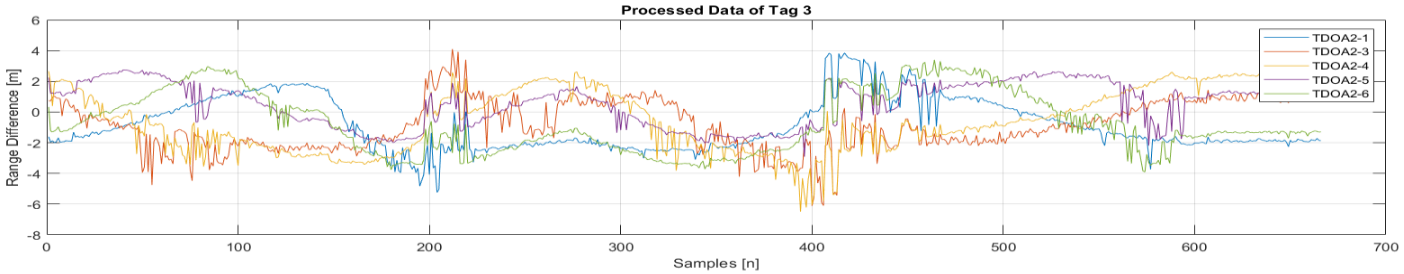

Raw Data of Tag 2
Range Difference [m]
Samples [n]
TDOA2-1
TDOA2-3
TDOA2-4
TDOA2-5
TDOA2-6
4
2
0
-2
-4
-6
0
100
200
300
400
500
600
700


Processed Data of Tag 2
Range Difference [m]
Samples [n]
TDOA2-1
TDOA2-3
TDOA2-4
TDOA2-5
TDOA2-6
4
2
0
-2
-4
-6
0
100
200
300
400
500
600
700

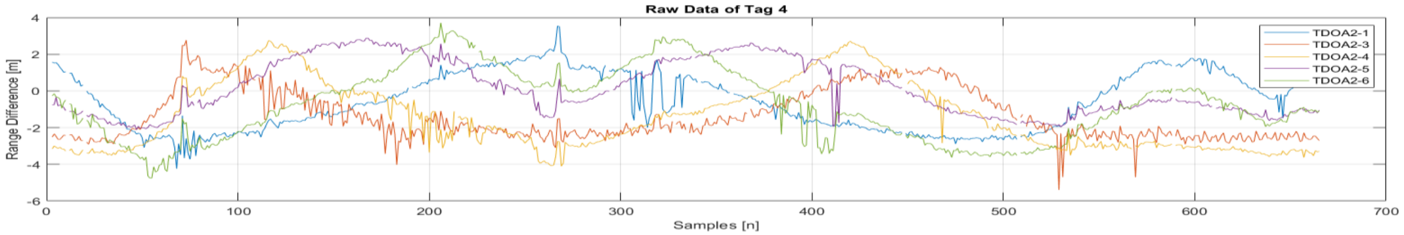
PREPROCESS



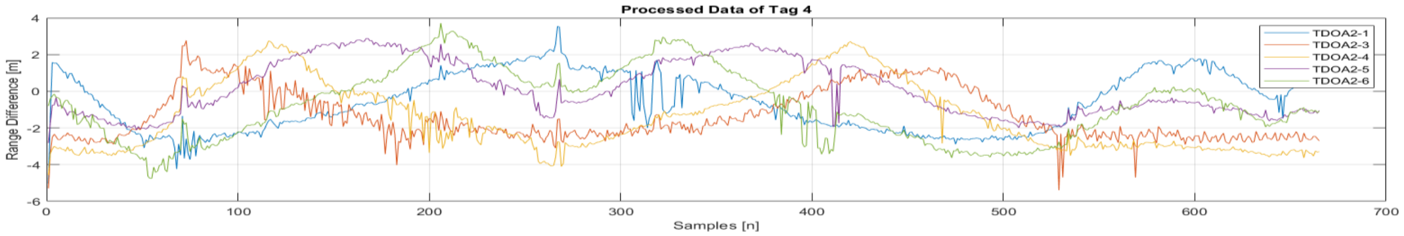
Raw Data of Tag 3
Range Difference [m]
Samples [n]
6
4
2
0
-2
-4
-6
-8
0
100
200
300
400
500
600
700
TDOA2-1
TDOA2-3
TDOA2-4
TDOA2-5
TDOA2-6



Processed Data of Tag 3
Range Difference [m]
Samples [n]
6
4
2
0
-2
-4
-6
-8
0
100
200
300
400
500
600
700
TDOA2-1
TDOA2-3
TDOA2-4
TDOA2-5
TDOA2-6



Raw Data of Tag 4
Range Difference [m]
Samples [n]
4
2
0
-2
-4
-6
0
100
200
300
400
500
600
700
TDOA2-1
TDOA2-3
TDOA2-4
TDOA2-5
TDOA2-6



Processed Data of Tag 4
Range Difference [m]
Samples [n]
4
2
0
-2
-4
-6
0
100
200
300
400
500
600
700
TDOA2-1
TDOA2-3
TDOA2-4
TDOA2-5
TDOA2-6

Measurement Model

General expression: $\rho = h(u) + n$

ρ : Measurements
$h(u)$: Non-linear deterministic measurement function
n : Measurement errors

Since the experiment has 4 tags and 5 measurements for each sample time:

$$\rho_{j,t}^{i} = h_j^i(u_t) + n_{j,t}^{i}$$

i: i_{th} Tag
j: j_{th} measurement
t: sample time
ut: AGV position at sample time

TDOA Measurement

- $h_i(u) = \sqrt{(S_{m,x} - u_x)^2 + (S_{m,y} - u_y)^2} - \sqrt{(S_{i,x} - u_x)^2 + (S_{i,y} - u_y)^2}$
- In our system, measurement matrix for Tag 1 at first sample time,

$$\rho^1_{1,1} = \sqrt{(S_{2,x} - u_{x,1})^2 + (S_{2,y} - u_{y,1})^2} - \sqrt{(S_{1,x} - u_{x,1})^2 + (S_{1,y} - u_{y,1})^2} + n^1_{1,1}$$
$$\rho^1_{2,1} = \sqrt{(S_{2,x} - u_{x,1})^2 + (S_{2,y} - u_{y,1})^2} - \sqrt{(S_{3,x} - u_{x,1})^2 + (S_{3,y} - u_{y,1})^2} + n^1_{3,1}$$
$$\rho^1_{3,1} = \sqrt{(S_{2,x} - u_{x,1})^2 + (S_{2,y} - u_{y,1})^2} - \sqrt{(S_{4,x} - u_{x,1})^2 + (S_{4,y} - u_{y,1})^2} + n^1_{4,1}$$
$$\rho^1_{4,1} = \sqrt{(S_{2,x} - u_{x,1})^2 + (S_{2,y} - u_{y,1})^2} - \sqrt{(S_{5,x} - u_{x,1})^2 + (S_{5,y} - u_{y,1})^2} + n^1_{5,1}$$
$$\rho^1_{5,1} = \sqrt{(S_{2,x} - u_{x,1})^2 + (S_{2,y} - u_{y,1})^2} - \sqrt{(S_{6,x} - u_{x,1})^2 + (S_{6,y} - u_{y,1})^2} + n^1_{6,1}$$

Statistical techniques account for the noise term n making assumptions on its distribution

The ML approach is widely adopted, as it is asymptotically unbiased and efficient:

For known noise distribution $p_n(n)$:

$$F(u) = -lnp(\rho \mid u) = -lnp_n(\rho - h(u) \mid u)$$

If $n \sim N(0, R(u))$

$$F(u) = -ln \mid R(u) \mid + \mid \rho - h(u) \mid^2_{R(u)^{-1}}$$

Bayesian Method: Extended Kalman Filter

Suited for Non-Linear and Gaussian systems

$$\hat{x}_{t|t-1} = F \, * \hat{x}_{t-1|t-1}$$
$$C_{t|t-1} = F \, * C_{t-1|t-1} \, * F^T + Q$$

PREDICTION

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + G_t(\rho_t \, - h_t(\hat{x}_{t|t-1}))$$
$$C_{t|t} = C_{t|t-1} \, - G_t * H_t * C_{t|t-1}$$
$$G_t = C_{t|t-1} \, * H_t^T \left(H_t C_{t|t-1} H_t^T + R_t\right)^{-1}$$

UPDATE

EKF with Random Walk

- Motion Model : $\boldsymbol{x}_t = \boldsymbol{f_t}(\boldsymbol{x_{t-1}}, \boldsymbol{w_{t-1}})$
- Position Estimate
- $\boldsymbol{x}_t = \boldsymbol{u}_t = \begin{bmatrix} u_{x,t} \\ u_{y,t} \end{bmatrix}$, $\begin{cases} u_{x,t} = u_{x,t-1} + T w_{vx,t-1} \\ u_{y,t} = u_{y,t-1} + T w_{vy,t-1} \end{cases}$
- $\begin{bmatrix} u_{x,t} \\ u_{y,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} u_{x,t-1} \\ u_{y,t-1} \end{bmatrix} + T \begin{bmatrix} w_{vx,t-1} \\ w_{vy,t-1} \end{bmatrix}$

$$\left\{ \boldsymbol{w}_{v,t} \sim N(0, \sigma_v^2 \boldsymbol{I}_2) \implies \boldsymbol{x}_t \sim N(\boldsymbol{x}_{t-1}, (T\sigma_v)^2 \boldsymbol{I}_2) \right.$$

- T: Sampling Time
- $\boldsymbol{w}_{v,t-1}$: Driving Process, zero-mean random velocity

EKF with Random Force

- Position and Velocity Estimate

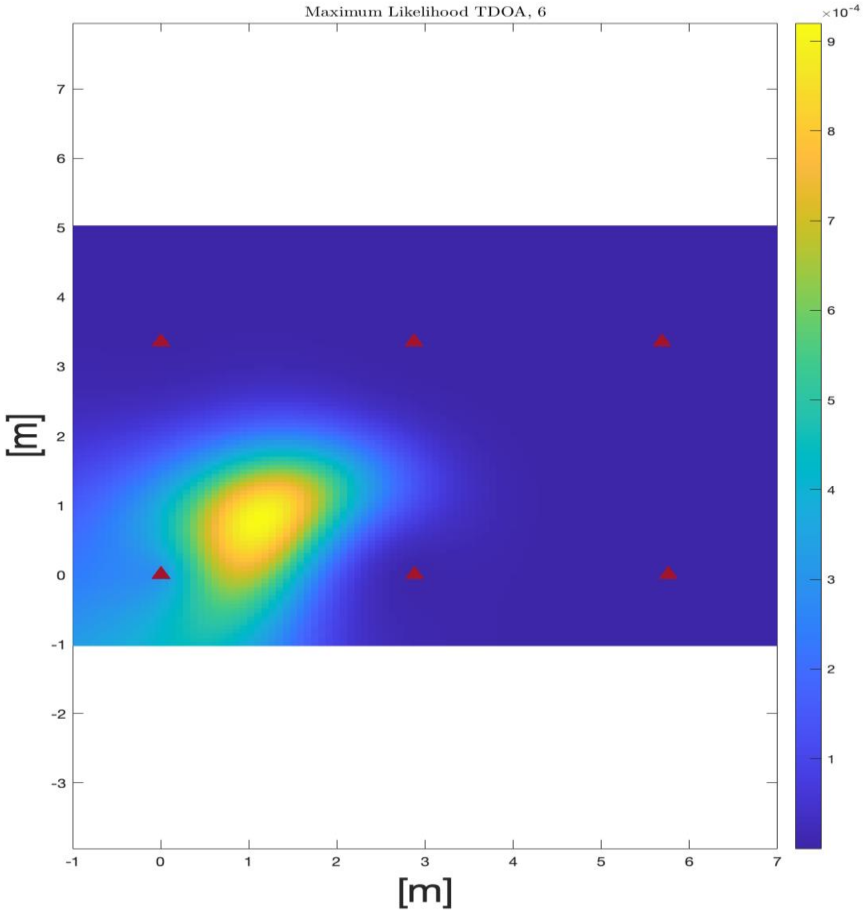
- $\boldsymbol{x}_t = \begin{bmatrix} u_{x,t} \\ u_{y,t} \\ v_{x,t} \\ v_{y,t} \end{bmatrix}$, $\begin{cases} u_{x,t} = u_{x,t-1} + T v_{x,t-1} + \frac{T^2}{2} w_{ax,t-1} \\ u_{y,t} = u_{y,t-1} + T v_{y,t-1} + \frac{T^2}{2} w_{ay,t-1} \\ v_{x,t} = v_{x,t-1} + T w_{ax,t-1} \\ v_{y,t} = v_{y,t-1} + T w_{ay,t-1} \end{cases}$

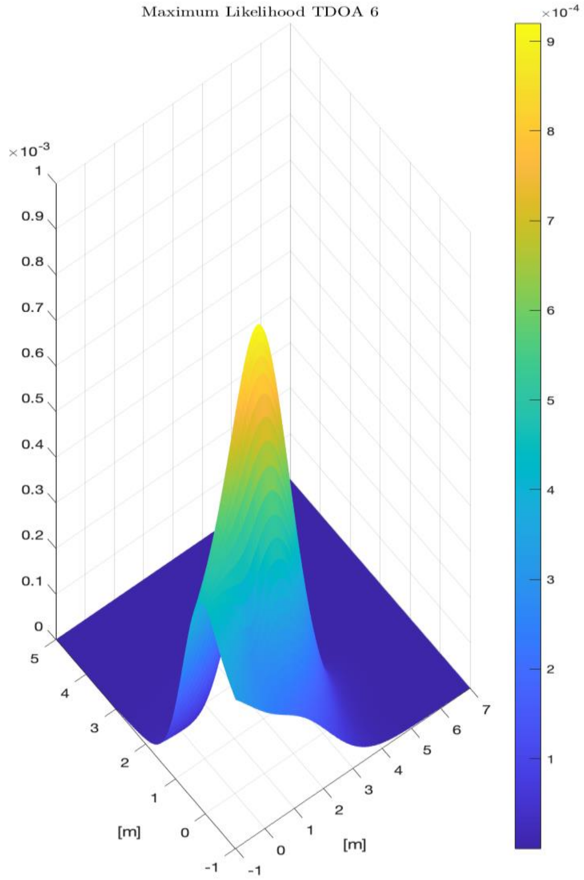
- $\begin{bmatrix} u_{x,t} \\ u_{y,t} \\ v_{x,t} \\ v_{y,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} u_{x,t-1} \\ u_{y,t-1} \\ v_{x,t-1} \\ v_{y,t-1} \end{bmatrix} + \begin{bmatrix} \frac{T^2}{2} & 0 \\ 0 & \frac{T^2}{2} \\ T & 0 \\ 0 & T \end{bmatrix} * \begin{bmatrix} w_{ax,t-1} \\ w_{ay,t-1} \end{bmatrix}$

$$\left\{ \boldsymbol{w}_{a,t} \sim N(0, \sigma_a^2 \boldsymbol{I}_2) \right. \Longrightarrow \boldsymbol{x}_t \sim N(\boldsymbol{F}\boldsymbol{x}_{t-1}, \sigma_a^2 \boldsymbol{L}\boldsymbol{L}^T)$$

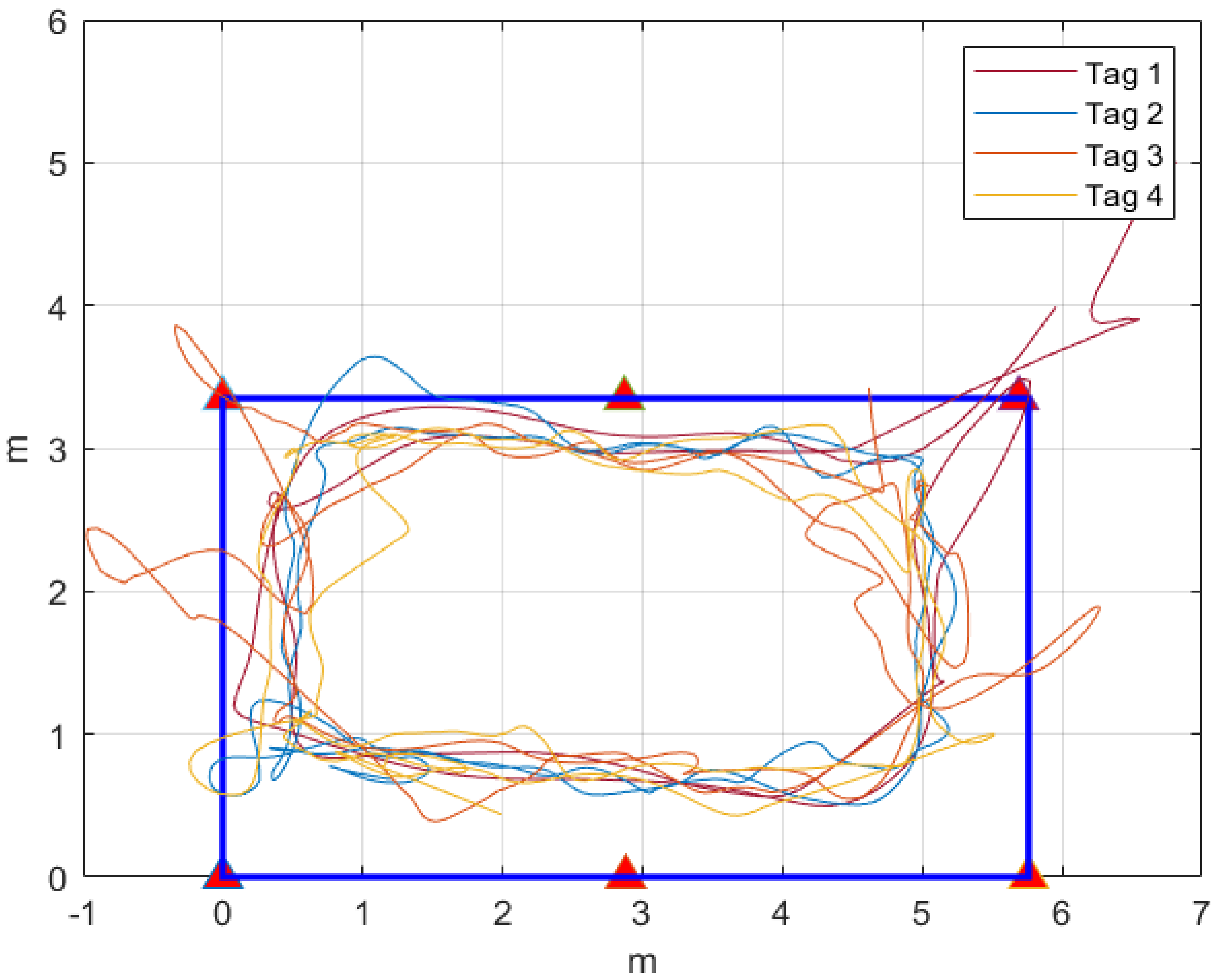
- T: Sampling Time
- $\boldsymbol{w}_{a,t-1}$: Driving Process, zero-mean random acceleration

SIMULATION OUTCOMES & PERFOMANCE ANALYSIS

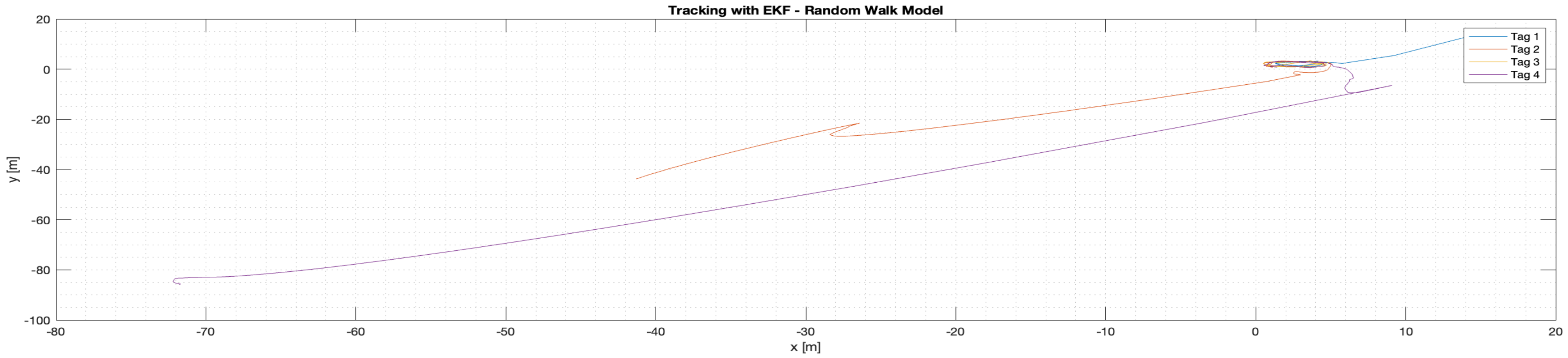


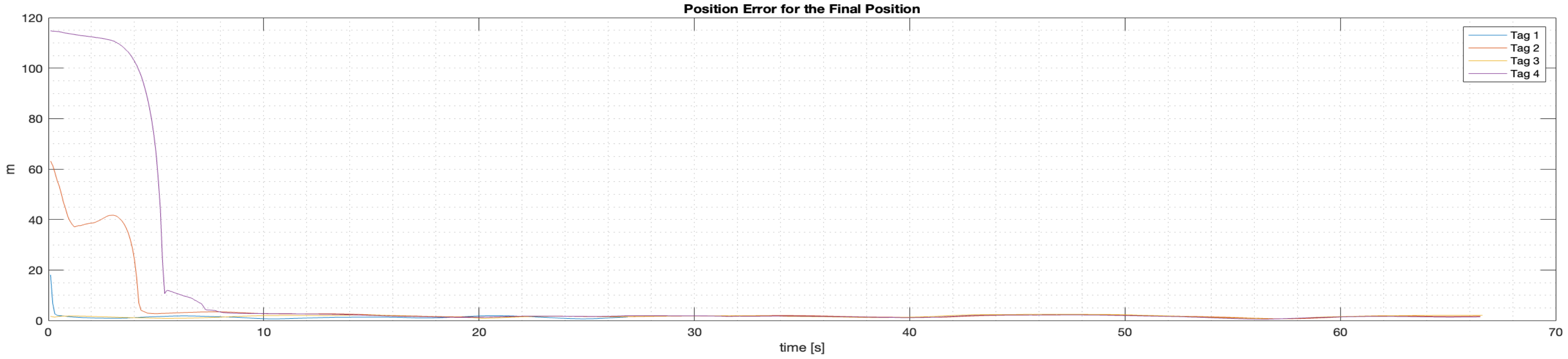


SIMULATION OUTCOMES & PERFOMANCE ANALYSIS



SIMULATION OUTCOMES & PERFOMANCE ANALYSIS





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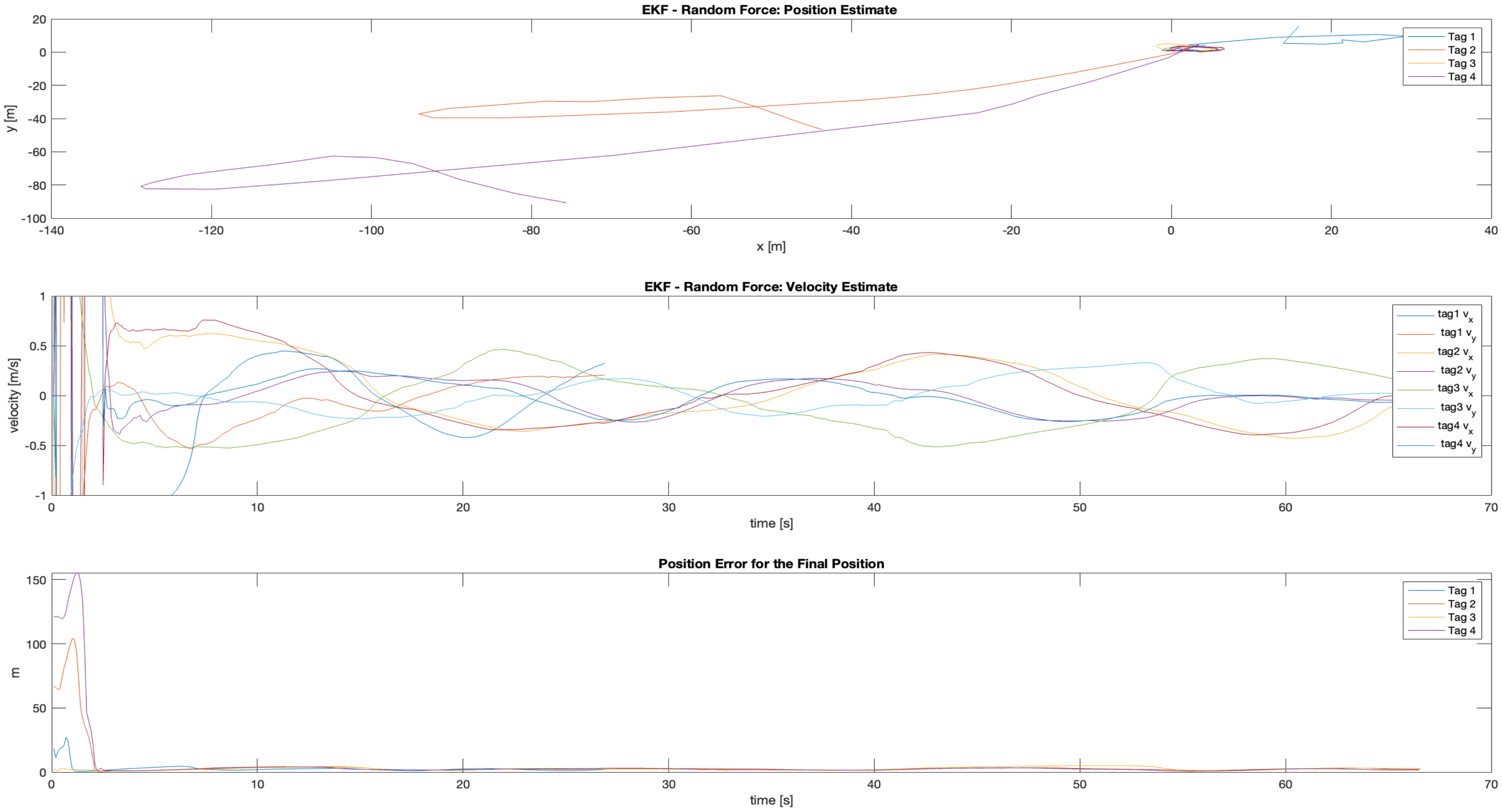

SIMULATION OUTCOMES & PERFOMANCE ANALYSIS

Criteria	Extended Kalman Filter (EKF)	Maximum Likelihood Estimation (MLE)
Modeling	Adaptable to dynamic systems	Linear and nonlinear
Computational Complexity	High (due to Jacobian computations)	Low (simple likelihood calculation)
Parameter Estimation	Model-based predictions	Data-driven predictions
Prediction Accuracy	Good, dependent on system model	Dependent on data and modeling
Applicability to Fast Changing Dynamics	Weak	Strong
Sensitivity to Measurement and Model Errors	Sensitive	Less sensitive