NUMERICAL METHODS
Week-1
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Introduction

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Why do we need NM?

- **Mathematical models** are an integral part in solving **engineering problems**.
- Many times, these mathematical models are derived from engineering and science principles, while at other times the models may be obtained from experimental data.

Why do we need NM?

- **Mathematical models** generally result in need of using **mathematical procedures** that include but are not limited to
  - Differentiation,
  - Nonlinear equations,
  - Simultaneous linear equations,
  - Curve fitting by interpolation or regression,
  - Integration,
  - Differential equations.
- These mathematical procedures may be suitable to be solved exactly as you must have experienced in the series of calculus
- **BUT IN MOST CASES, the procedures need to be solved approximately using numerical methods.**

Why NM in Computer Engineering?

- To analyze and develop **algorithmic methods** for any **function evaluation**.

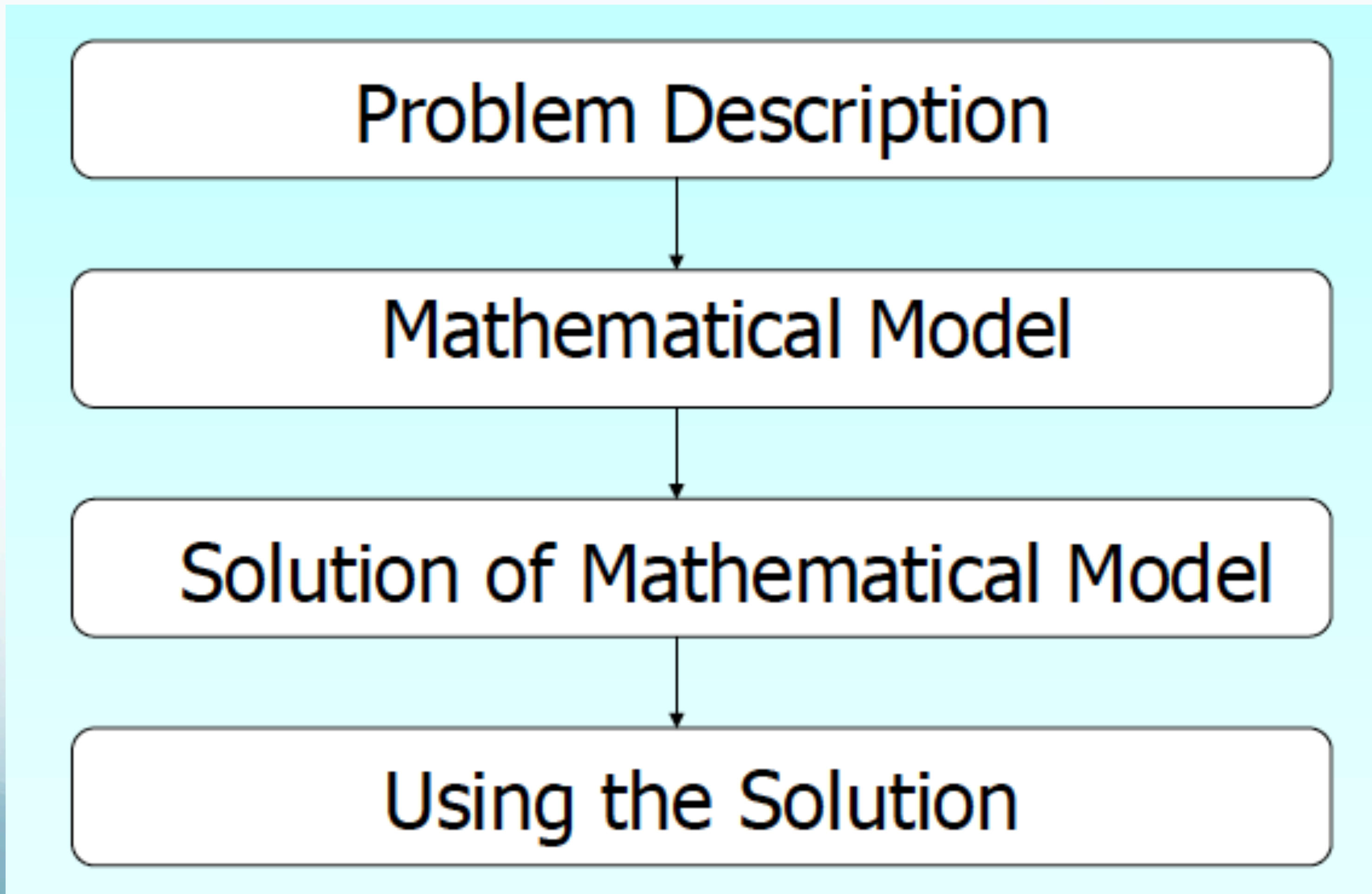
- To explain the role of and the limitations of the computer in solving mathematical and engineering **problems**.

Why NM in Computer Engineering?

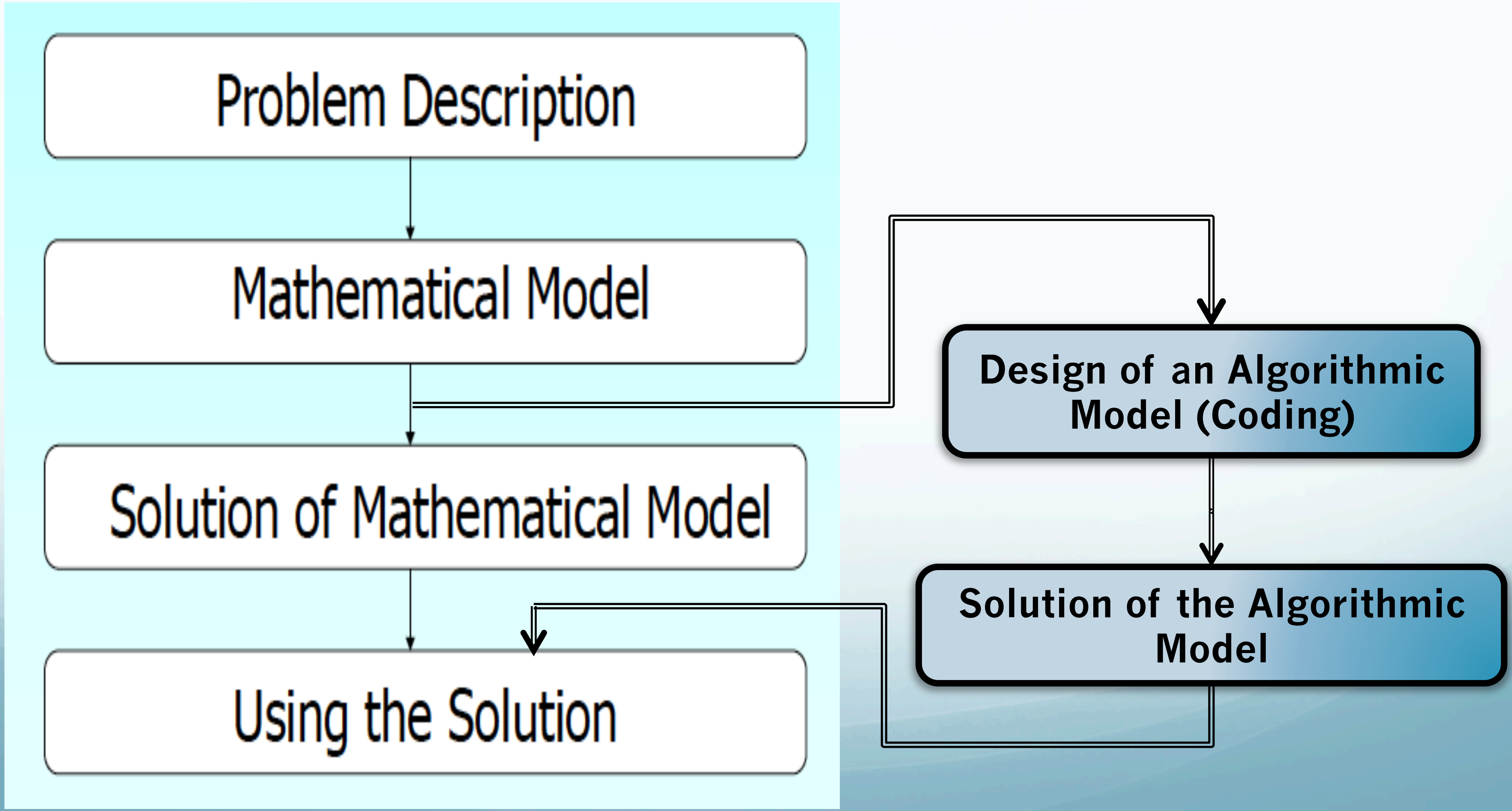
- **Scientific exploration** is often conducted on **computers** rather than laboratory equipment. AT LEAST AT FIRST AS SIMULATION..

- While it is rarely meant to completely replace work in the scientific laboratory, **computer simulation** often complements this work.

Steps for Solving an Engineering Problem from a generic perspective



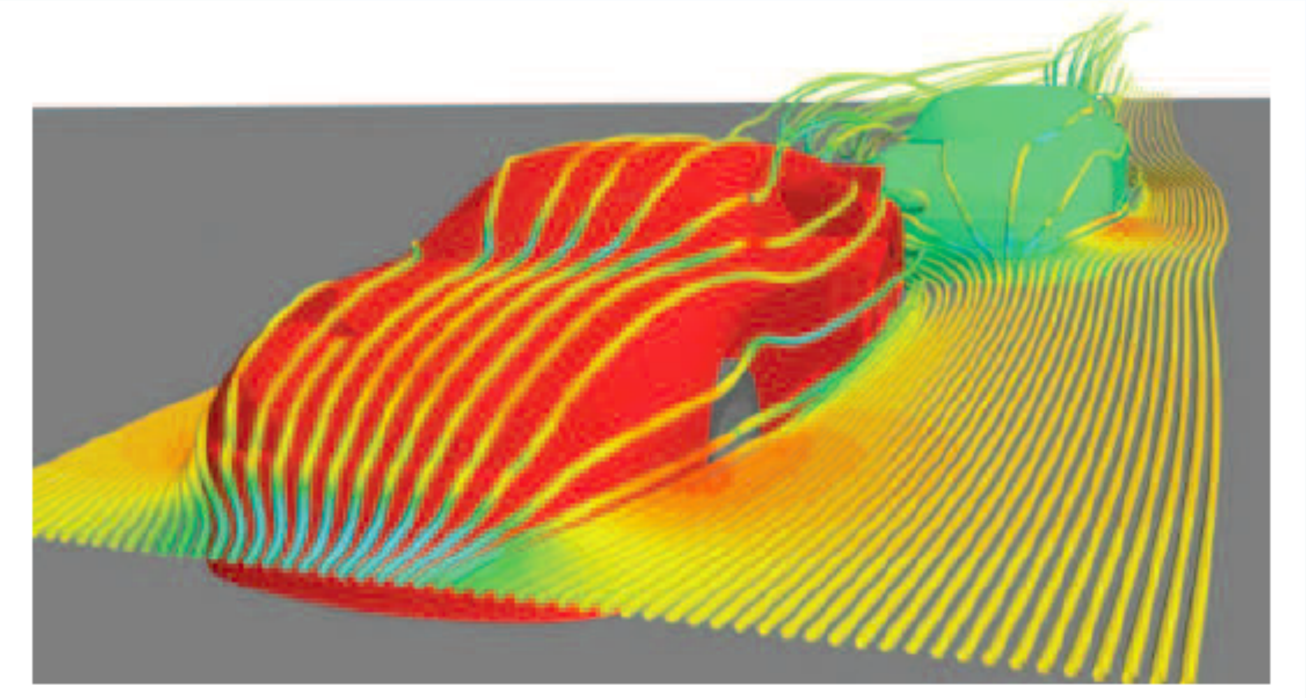
Steps for Solving an Engineering Problem from CE perspetive



Mathematical Procedures

- Linear Equations
- Nonlinear Equations
- Differentiation
- Curve Fitting
  - Interpolation
  - Regression
- Integration
- Differential Equations
- Other Advanced Mathematical Procedures:
  - Partial Differential Equations
  - Optimization
  - Fast Fourier Transforms

Example: NASCAR AUTO

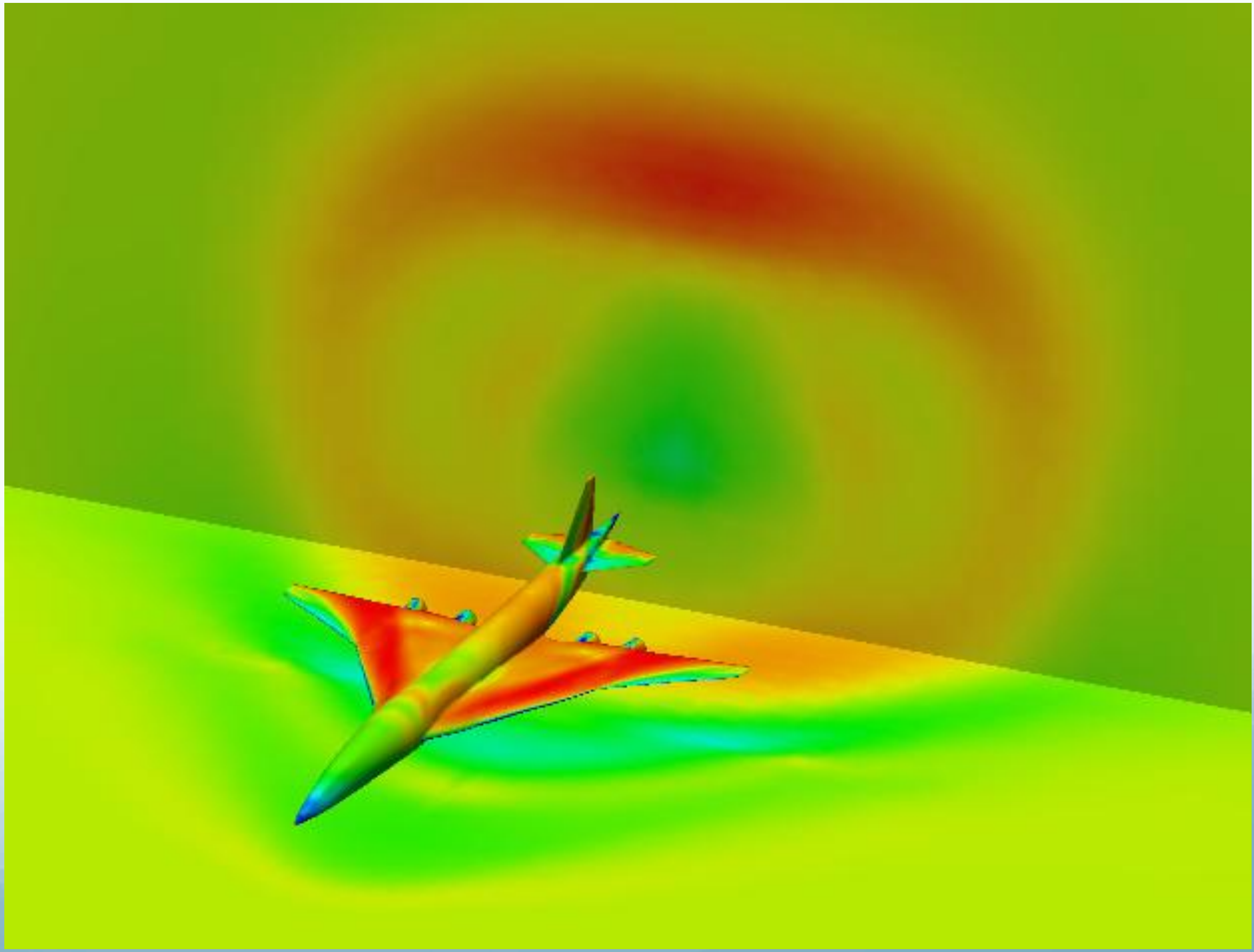


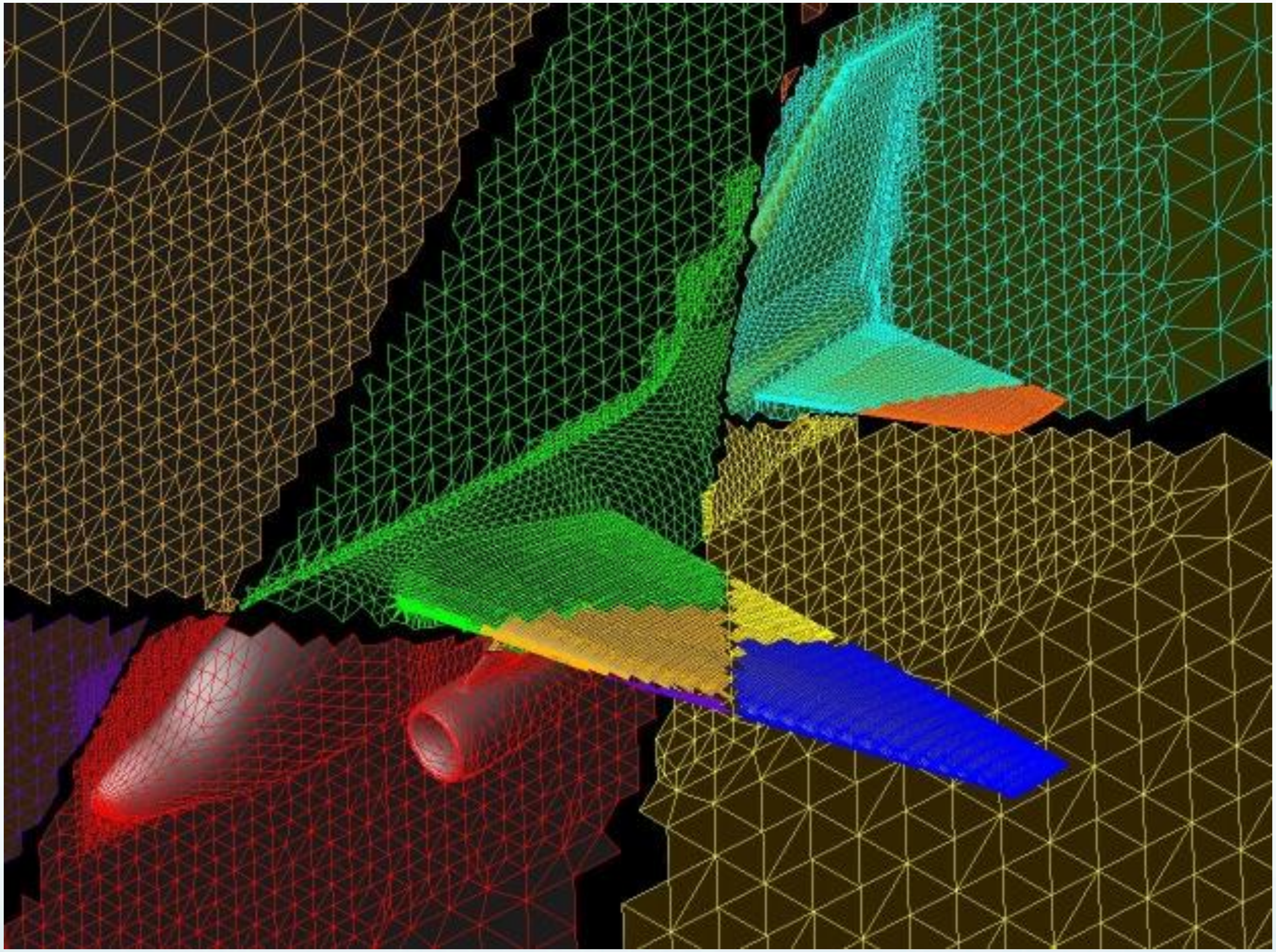
The aerodynamic simulation of two NASCAR autos requires the numerical solution of **partial differential equations (PDEs)** that model the flow of air past the car.

An auto body must be smooth and sleek, so it is often modeled using cubic (or higher order) **splines**.

Example: AIRCRAFT

Sampled **Euler** Calculation





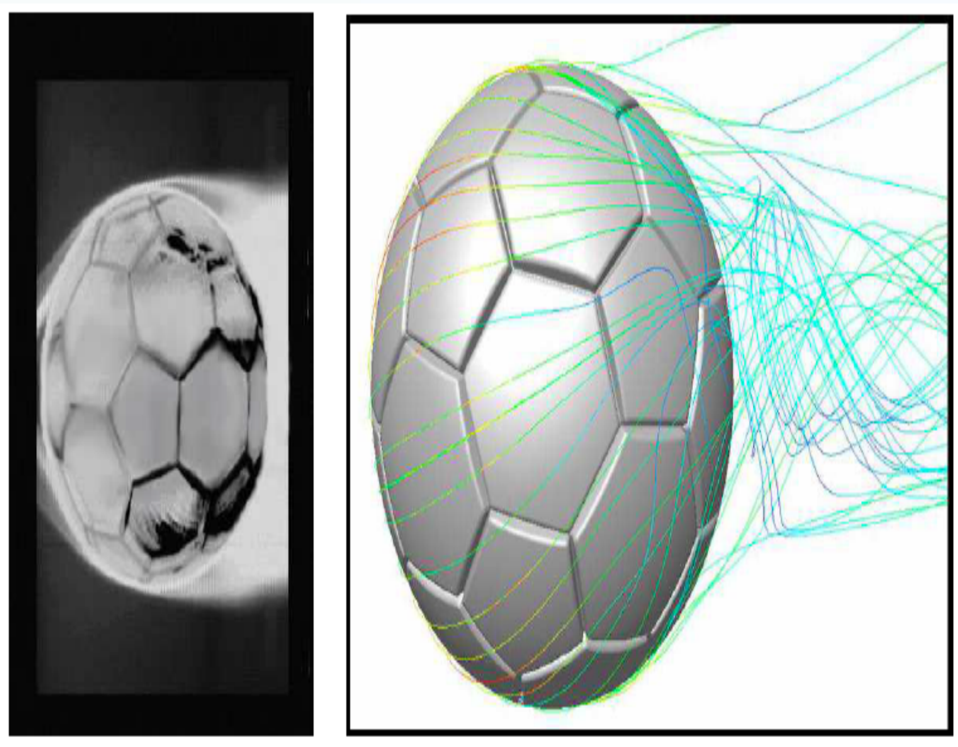
Parallel computation on an **unstructured mesh** showing the domain decomposition of 16 processors of a distributed memory computer.

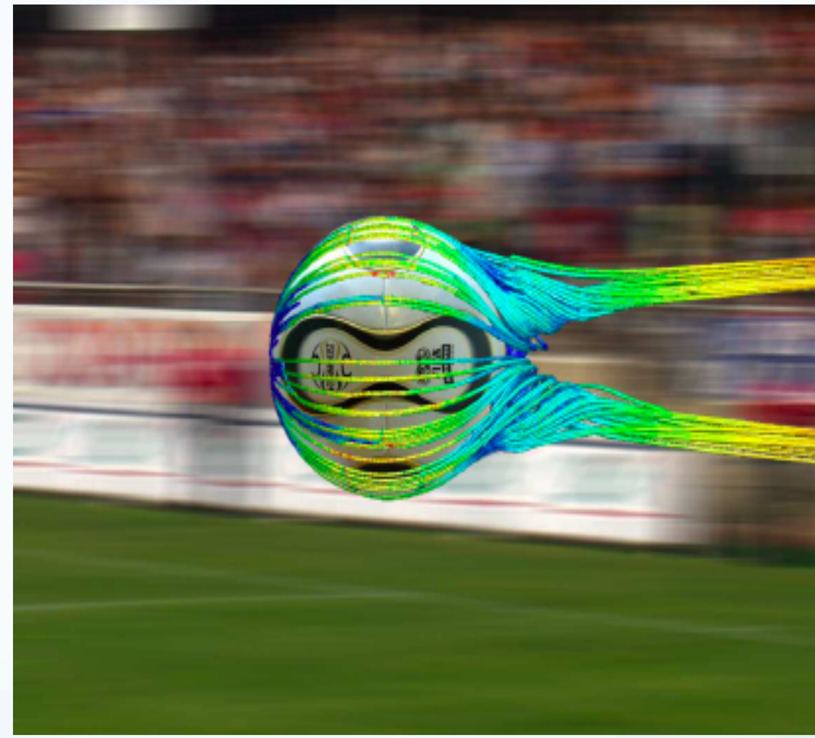
Example: YODA

Yoda's fight scene between Yoda with Count Dooku in Star Wars Episode II.

Two layers of Yoda's clothing, seen in (b) and (c), were computed separately. A process known as **collision detection** ensured that the inner layer of clothing did not intersect the outer robe and become visible.

Example: BALL





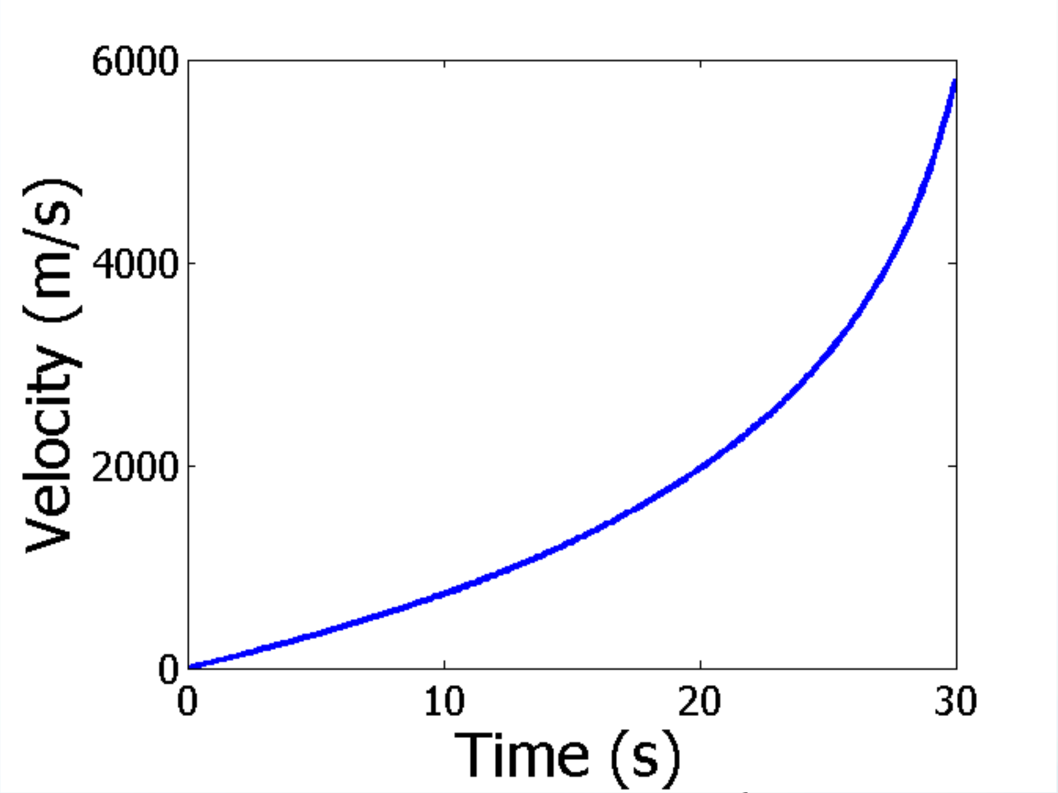
The aerodynamic behavior of a soccer ball can be modeled by NM. High speed airflow path lines colored by local velocity over the soccer ball.→ Designed and solved by **Differential Equations.**

Example: WWW

A representation of the graph of the **World Wide Web** created by David Gleich

Example: Differentiation

What is the acceleration at t=7 seconds?



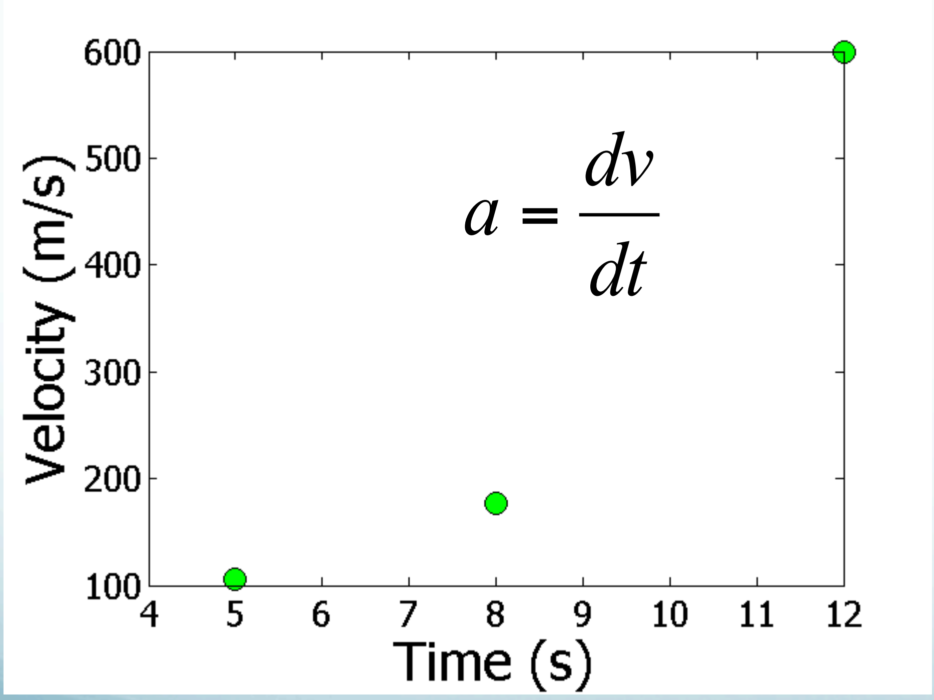
$$v(t) = 2200 \ln\left(\frac{16\times10^4}{16\times10^4 - 5000t}\right) - 9.8t$$

$$a = \frac{dv}{dt}$$

Differentiation

What is the acceleration at t=7 seconds?

Time (s)	5	8	12
Vel (m/s)	106	177	600



$$a = \frac{dv}{dt}$$

Example: Simultaneous Linear Equations

Find the velocity profile, given

Time (s)	5	8	12
Vel (m/s)	106	177	600

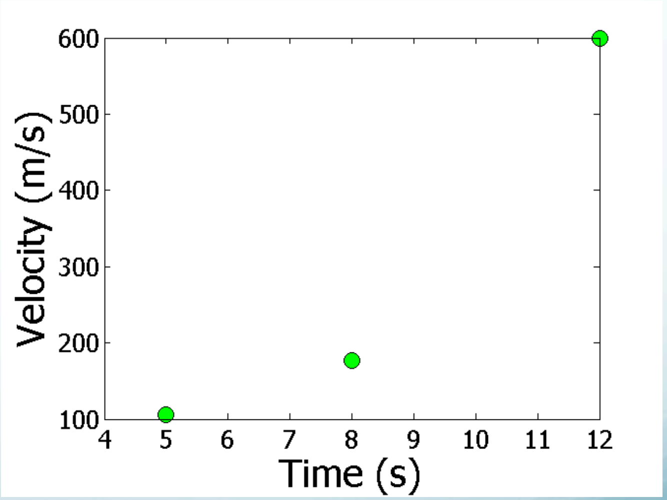
$$v(t) = at^2 + bt + c,\ 5 \le t \le 12$$

Three simultaneous linear equations

$$25a + 5b + c = 106$$

$$64a + 8b + c = 177$$

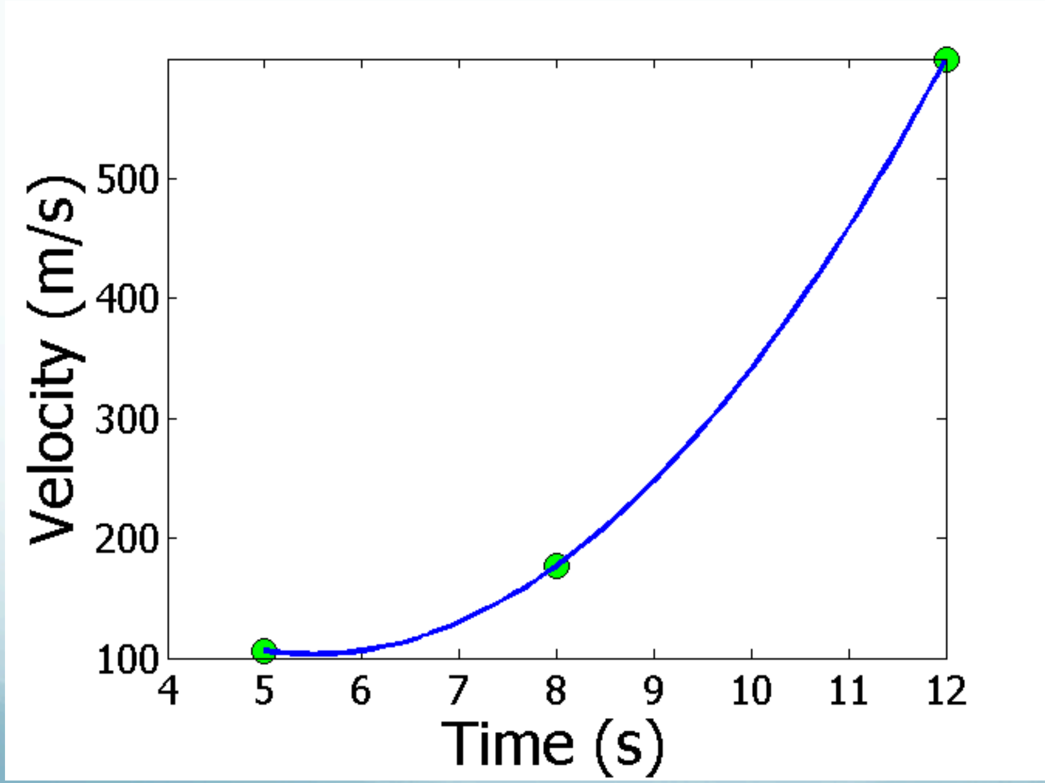
$$144a + 12b + c = 600$$



Example: Interpolation

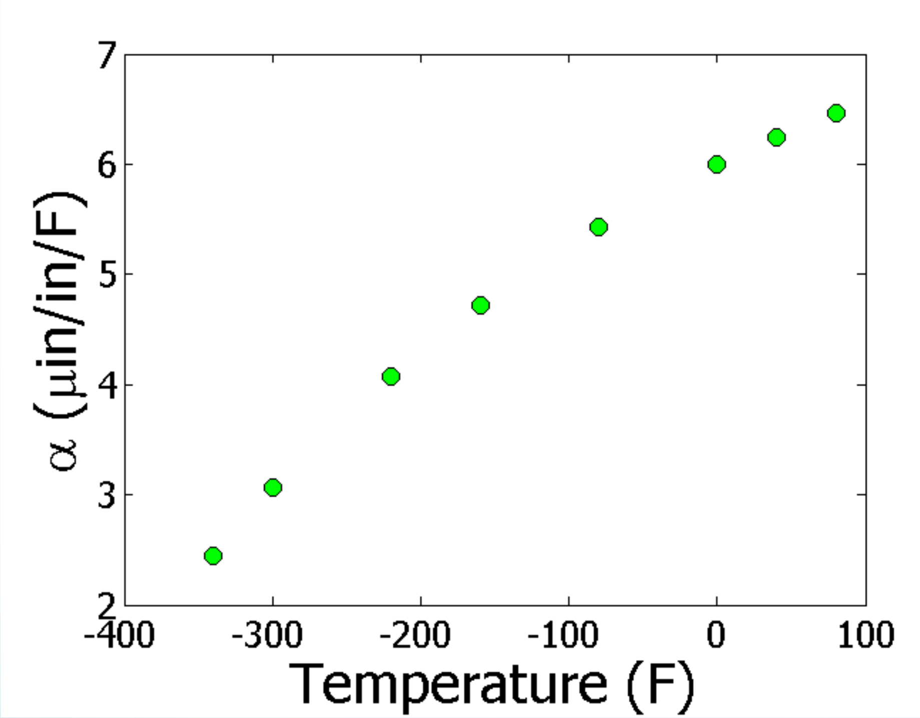
What is the velocity of the rocket at t=7 seconds?

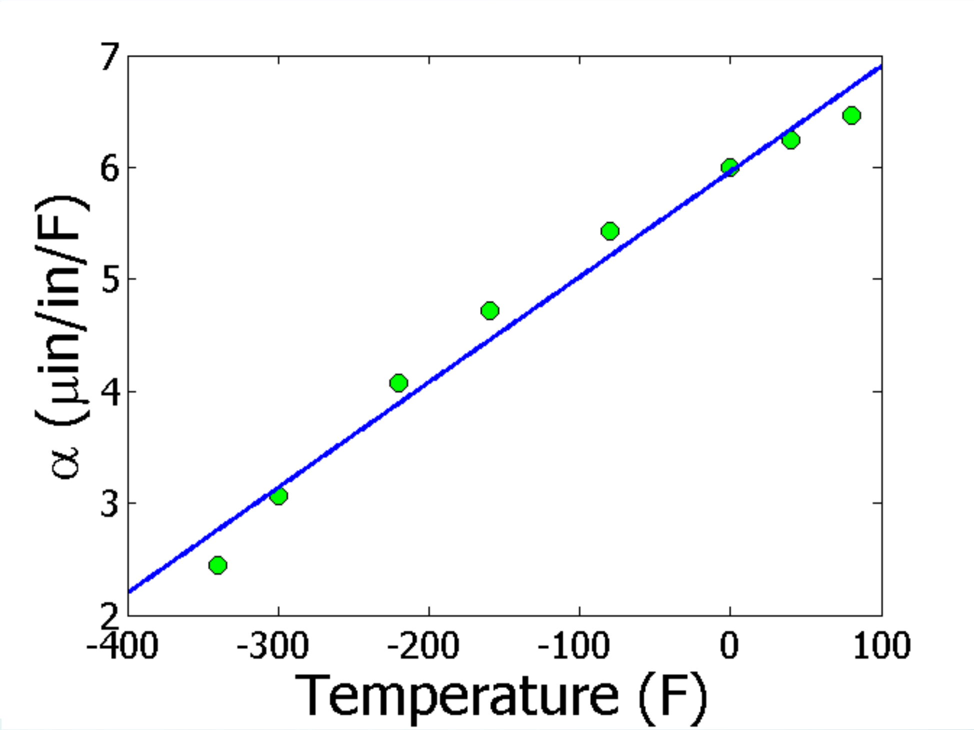
Time (s)	5	8	12
Vel (m/s)	106	177	600



Example: Regression

Thermal expansion coefficient data for cast steel





NM and Algorithms..

Evaluation of a Polynomial Function

$$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n$$

we group the terms in a **nested multiplication**:

$$p(x) = a_0 + x(a_1 + x(a_2 + \cdots + x(a_{n-1} + x(a_n)) \cdots))$$

integer i, n; **real** p, x; **real array** $(a_i)_{0:n}$
$p \leftarrow a_n$
for $i = n - 1$ **to** 0 **do**
    $p \leftarrow a_i + xp$
end for

Taking the derivative of a function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

A computer has the capacity of imitating the limit operation by using a sequence of numbers h such as

$$h = 4^{-1}, 4^{-2}, 4^{-3}, \ldots, 4^{-n}, \ldots$$

The sequence 1/4n consists of machine numbers in a binary computer and, for this experiment on a 32-bit computer.

The following is pseudocode to compute f(x) at the point x = 0.5, with f (x) = sin x:

program *First*
integer i, $i\max$, $n \leftarrow 30$
real *error*, y, $x \leftarrow 0.5$, $h \leftarrow 1$, $e\max \leftarrow 0$
for $i = 1$ **to** n **do**
    $h \leftarrow 0.25h$
    $y \leftarrow [\sin(x + h) - \sin(x)]/h$
    $error \leftarrow |\cos(x) - y|$;   **output** i, h, y, *error*
    if $error > e\max$ **then** $e\max \leftarrow error$; $i\max \leftarrow i$ **end if**
end for
    output $i\max$, $e\max$
end program *First*