

Preorder: The first element given is the root. We can then search for this element in the inorder traversal. The elements of the tree are now uniquely positioned to left & right subtrees by this element. The sequence

{
① { preceding the root in the inorder sequence is the inorder sequence for the left subtree
} }
{
② { succeeding the root in the inorder sequence is the inorder sequence for the right subtree

We can recursively reconstruct left and right subtrees from the preorder and inorder traversals.

Example: Given: preorder and postorder ?

If we have left only or right only subtrees in the tree then the preorder and postorder traversals are not necessarily unique.

Example: Preorder (1 2 3)

Postorder (3 2 1)



Some pre-order and post-order
we cannot reconstruct

Red Black Trees

- ① Every node is either red or black.
- ② The root is black.
- ③ Every leaf (NIL) is black.
- ④ If a node is red, then both of its children are black and its opposite.
- ⑤ For each node, all the paths from the node to descendant leaves contain the same number of black nodes.

Black Height

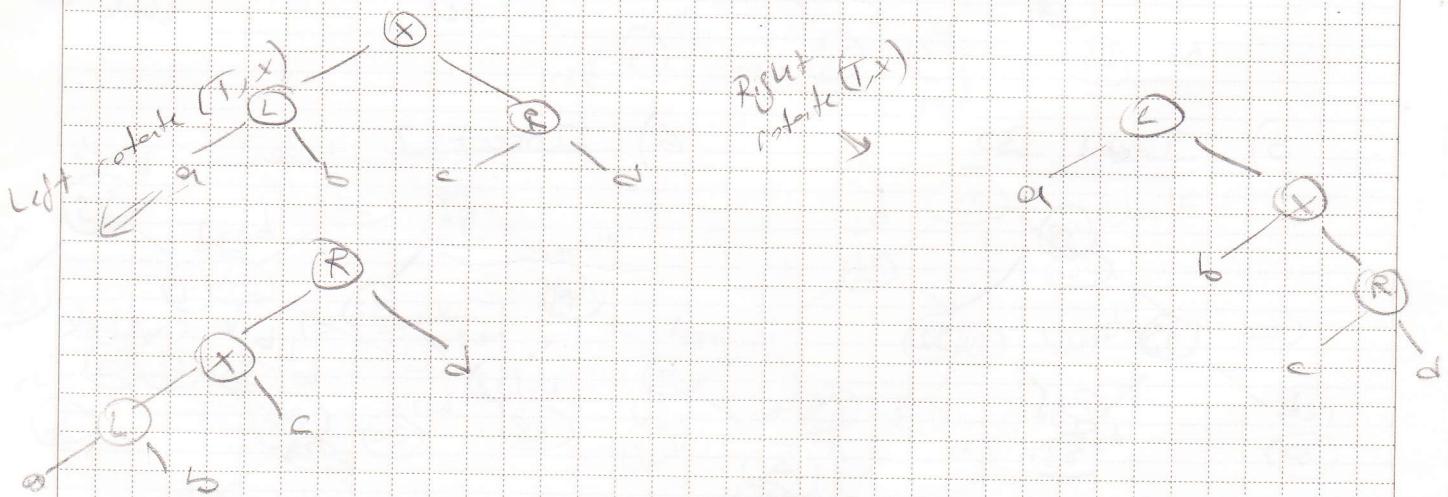
of black nodes on any path from (but not including) a node x down to a leaf denoted by $lh(x)$.

The height of RB tree is at most $2 \log(n+1)$ # of internal nodes

→ search
→ min
→ max
→ successor
→ predecessor

$$O(h) = O(\log n)$$

Rotation



Example: 13, 32, 83, 83, 30, 2, 63, 45, 55, 32 \Rightarrow Red-Black tree

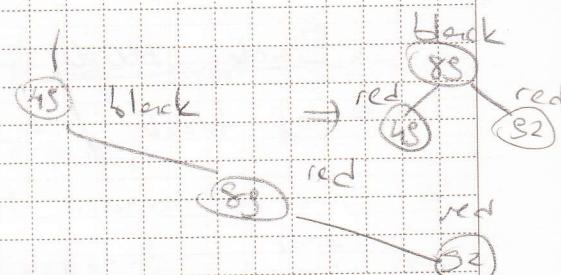
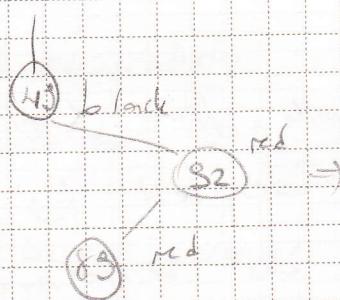
1) Insert 43



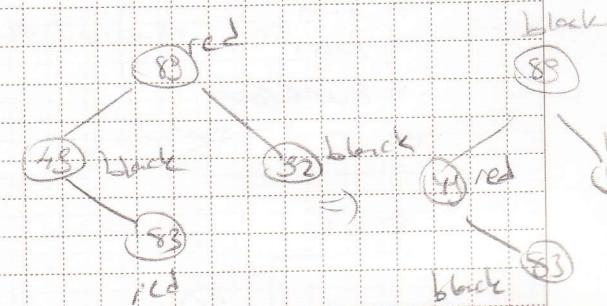
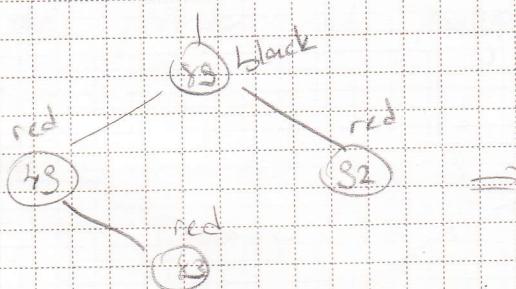
2) Insert 82



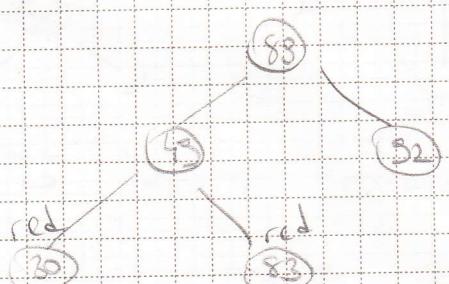
3) Insert 83



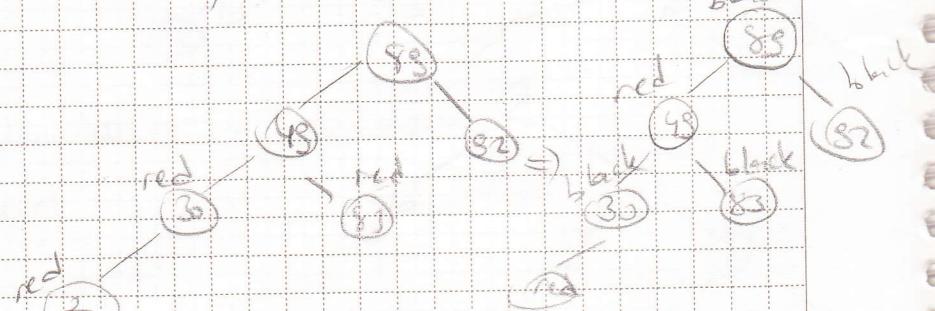
4) Insert 83



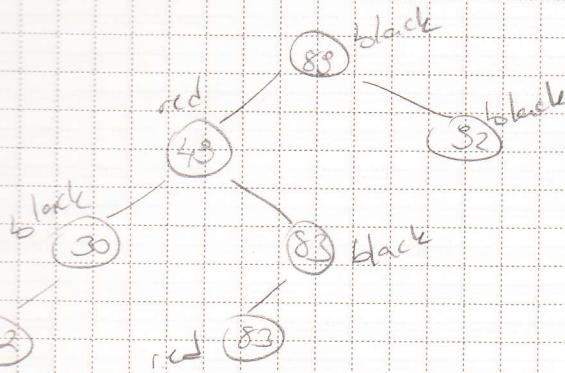
5) Insert 30



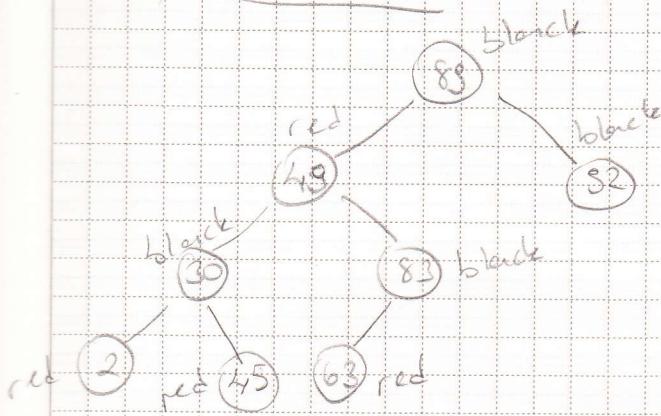
6) Insert 2



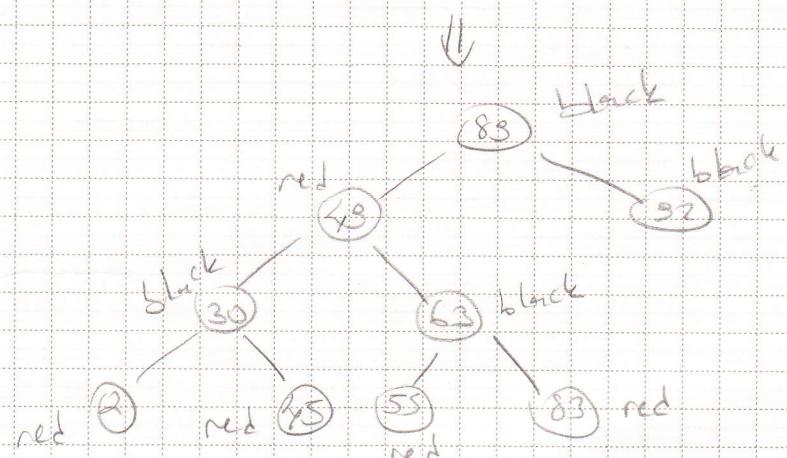
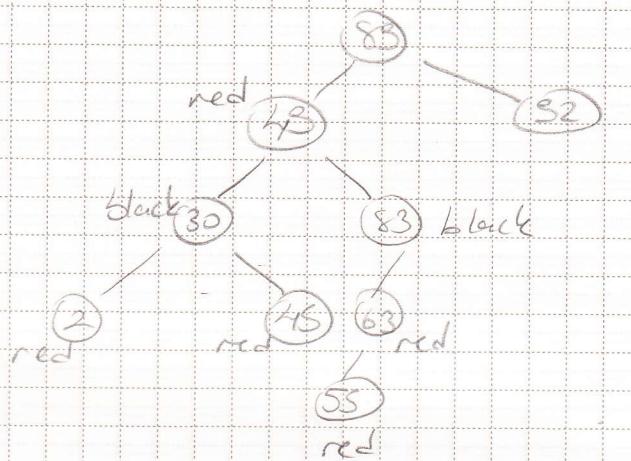
7) Insert 63



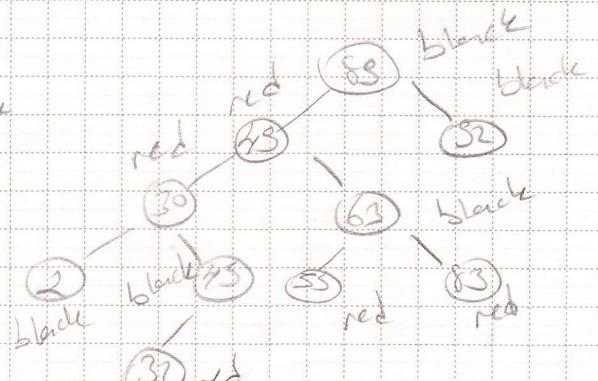
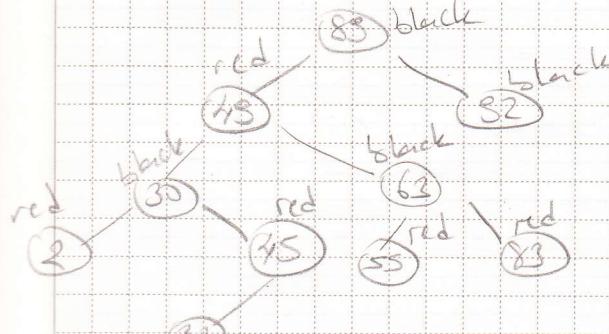
8) Insert 45



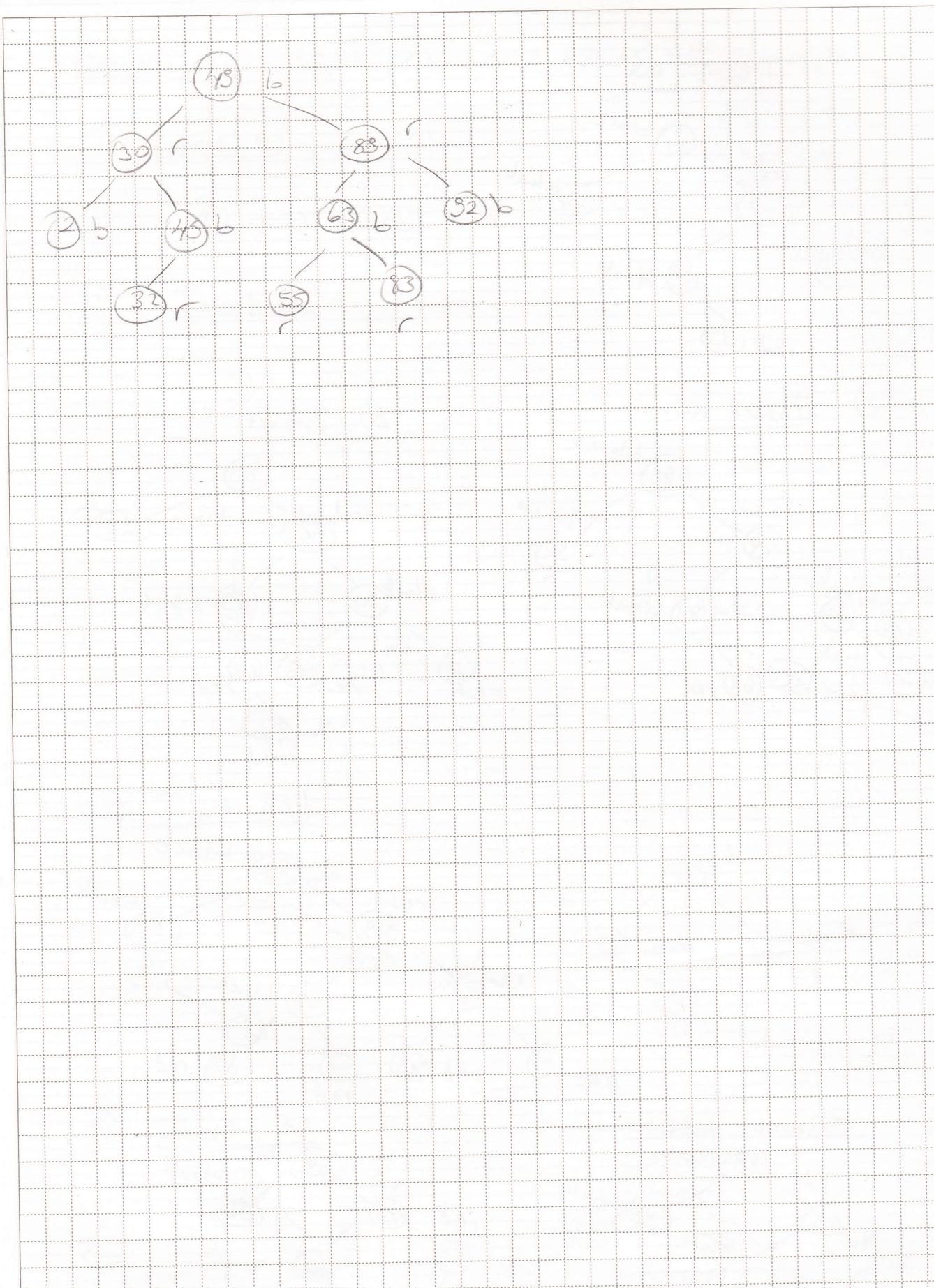
9) Insert 55



Insert 32



			/	/
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HASHING

- How to make searching even

Q1) With hashing

Hashing: Given a key, use a hash function to compute an index of a hash table.

- Finds, insertions, deletions.

Index = hash-function(key)

Limitations:

- table size is limited
- critical issues for implementation

Two steps:

1) Design a good hash function

a) Fast to compute

b) Minimizing the number of collisions

2) Design a method to resolve collisions when they occur

Separate chaining

open addressing

① Separate Chaining

- + Keep a linked list of keys that hash to the same value.
- Advantage: deletion is easy
- Disadvantage = memory allocation in linked list manipulation will slow down the program

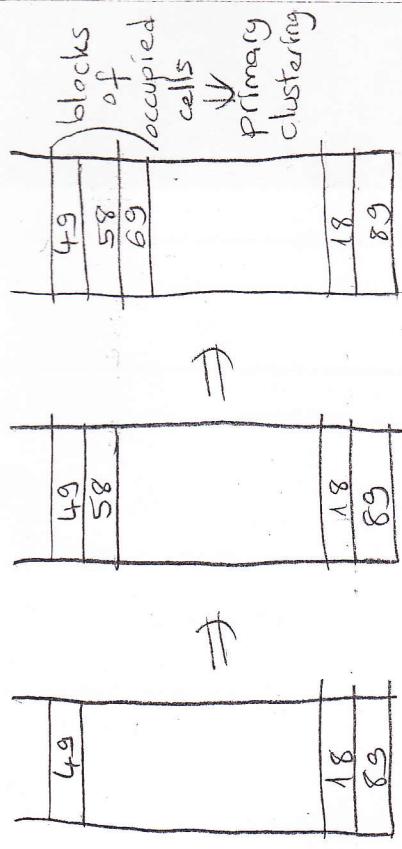
② Open Addressing

◦ Linear probing
 $h_1(k) = (\text{hash}(k) + i) \bmod m$ → primary clustering
 probed sequentially

◦ Quadratic probing:

$h_2(k) = (\text{hash}(k) + i^2) \bmod m$ → secondary clustering
 Double hashing: $h_3(k) = [\text{hash}(k)] \bmod m$

Cluster = a block of contiguous table entries.
Example = Insert 49, 58, 69 with
 $\text{hash}(k) = k \bmod 10$ and linear probing
 $49 \bmod 10$

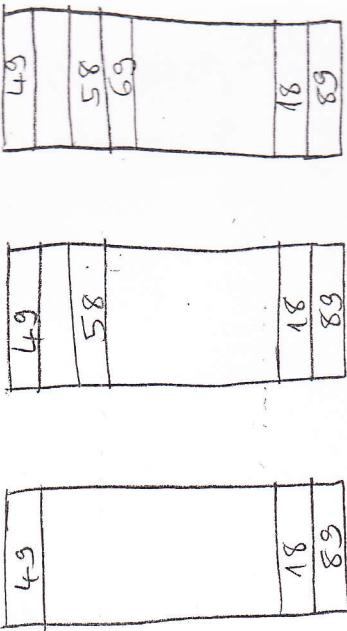


Any key that hashed into the cluster will require several attempts to resolve the collision.

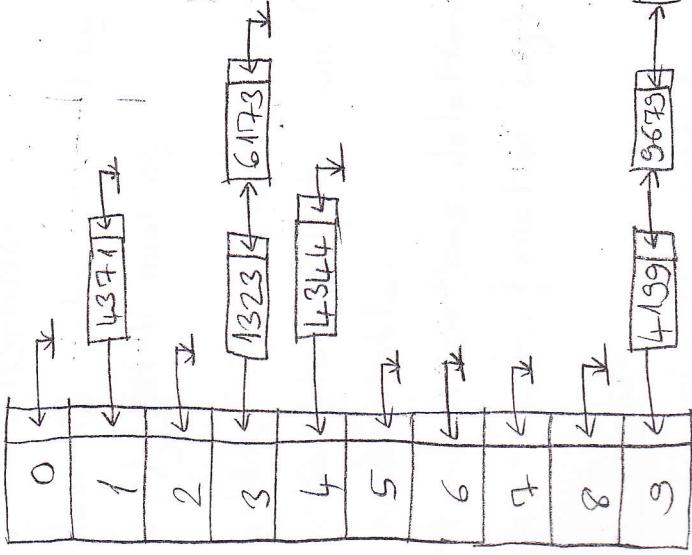
Quadratic Probing

→ helps to eliminate primary clustering
 → two keys with different positions will have different probe sequences.

(a)



→ Keys that hash to the same position will probe the same alternative calls → secondary clustering



Given 4371, 1323, 6173, 4199, 4344, 9679, 1989 hash function $h(x) = x \bmod 10$

Q: What are the results of the following?

- Separate chaining hash table.
- Open addressing hash table using linear probing.
- Open addressing hash table with second hash function $h2(x) = 7 - (x \bmod 7)$

