(Let us admit Eq. (5.34) to be true.)

Proof by contradiction: Assume one of the Hamiltonian's ground state |GS> has $M_{z,tot}=M_0>0$. ($M_0$ must be also an integer (why?)). [If $M_0<0$, we can always do a π-rotation around x-axis to flip $M_0$'s sign while keeping in the ground-state energy sector.]

By (5.34), we know that |GS> has $S_{tot}=M_0$. We act on the following ladder operator

$$S^-_{\mathrm{tot}} \equiv \sum_i S^-_i$$

on |GS>, so we obtain a state |**O**> in $M_{z,tot}=M_0-1$ sector, which must be the lowest-energy state because its energy is the same as that of |GS>. Then |**O**> has $S_{tot}=|M_0-1|=M_0-1$ (here we use the fact that $M_0$ is an integer.)$\neq M_0$. But |**O**> and |GS> must have the same $S_{tot}$ because the above ladder operator does not change $S_{tot}$. Thus, we have a contradiction, and it should have been $M_0=0$ at the beginning, and by (5.34), $S_{tot}=0$ and all the ground states are in $M_0=0$ sector. Corollary 5.4 further means there is actually only one unique ground state.

**Remark**: if we really want to use $M_0<0$, we just switch to $S^+_{\mathrm{tot}} \equiv \sum_i S^+_i$