Subdivision Surfaces

- theory was generalized from tensor product B-spline surfaces
- allows for use of a single control mesh to describe entire geometry no patching
- start with a mesh
- use rules to refine the mesh
- surface is the infinite limit of this process, but typically only apply it a small number of times.
- gives us a smooth surface that approximates the control mesh
- special rules can be added in to get creases where desired

Catmull-Clark

- start with watertight mesh M^0
- apply a set of connectivity updates to get a new refined mesh M^1, with its own connectivity and geometry.
- connectivity of M^1:
  - For each vertex v in M^0, we associate a new "vertex-vertex" v_v in M^1.
  - For each edge e in M^0 we associate a new "edge-vertex" v_e in M^1.
  - For each face f in M^0 we associate a new "face-vertex" v_f in M^1.
  - connected up as shown.
- in M^1, all faces are quads.
- In M^1 we call any vertex of valence four "ordinary" and any vertex of valence different from four "extraordinary".

recursive subdivision

- We apply this subdivision process recursively.
- Given M^i, for any $i \geq 1$, we apply the same subdivision rules to obtain a finer mesh M^{i+1}.
- In the new mesh, we have roughly 4 times the number of vertices.
- verify that the number of extraordinary vertices stays fixed
- Thus, during subdivision, more and more of the mesh looks locally rectilinear, with a fixed number of isolated extraordinary points in between.
  - this is nice for the theory

geometry rules

- First, let f be a face in M^i surrounded by the vertices v_j (and let m_f be the number of such vertices).
- We set the geometry of each new face-vertex v_f in M^{i+1} to be

$$v_f = \frac{1}{m_f} \sum_j v_j \tag{1}$$

- the centroid of the vertices in M^i defining that face

edge rules

- Next, let e be an edge in M^i connecting the vertices v_1 and v_2, and separating the faces f_1 and f_2. We set the geometry of the new edge-vertex in M^{i+1} to be

$$v_e = \frac{1}{4}(v_1 + v_2 + v_{f_1} + v_{f_2}) \tag{2}$$

vertex rules

- Finally let v be a vertex in M^i connected to the n_v vertices v_j and surrounded by the n_v faces f_j.
- Then, we set the geometry of the new vertex-vertex in M^{i+1} to be

$$v_v = \frac{n_v - 2}{n_v} v + \frac{1}{n_v^2} \sum_j v_j + \frac{1}{n_v^2} \sum_j v_{f_j} \tag{3}$$

  For an ordinary vertex, with valence $n_v = 4$, this becomes

$$v_v = \frac{1}{2} v + \frac{1}{16} \sum_j v_j + \frac{1}{16} \sum_j v_{f_j} \tag{4}$$

  - nice for theory

theory

- it has been proven that this converges
- it converges to a surface with continouse first derivative

to implement

- need a mesh data structure, which we give you