frame is important

- in graphics, we often keep track of a number of frames
  - each object, the camera, the world ...
  - so we need to be careful how we use matrices.
- given point and matrix is not enough to specify mapping
- for example point $\tilde{p}$ and the matrix

$$\mathbf{S} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- the matrix is non-uniform scaling
- fix a frame $\vec{\mathbf{f}}^{\mathbf{t}}$
- in this frame $\tilde{p} = \vec{\mathbf{f}}^t \mathbf{c}$
- transform with matrix $\vec{\mathbf{f}}^{\mathbf{t}}\mathbf{c} \Rightarrow \vec{\mathbf{f}}^{\mathbf{t}}\mathbf{Sc}$
- the stretches by factor of two in first axis of $\vec{\mathbf{f}}^t$
- see fig

other frame

- pick some other frame $\vec{\mathbf{a}}^t$.
- relationship between bases $\vec{\mathbf{a}}^{\mathbf{t}} = \vec{\mathbf{f}}^{\mathbf{t}}\mathbf{A}$.
- express same point as $\tilde{p} = \vec{\mathbf{f}}^{\mathbf{t}}\mathbf{c} = \vec{\mathbf{a}}^{\mathbf{t}}\mathbf{A}^{-\mathbf{1}}\mathbf{c} = \vec{\mathbf{a}}^{\mathbf{t}}\mathbf{d}$,
- use matrix S we get $\vec{\mathbf{a}}^{\mathbf{t}}\mathbf{d} \Rightarrow \vec{\mathbf{a}}^{\mathbf{t}}\mathbf{Sd}$.
  - (equiv statement $\vec{\mathbf{a}}^{\mathbf{t}}\mathbf{A}^{-\mathbf{1}}\mathbf{c} \Rightarrow \vec{\mathbf{a}}^{\mathbf{t}}\mathbf{SA}^{-\mathbf{1}}\mathbf{c}$.)
- the same point $\tilde{p}$ is stretched about first axis of $\vec{\mathbf{a}}^t$
- see fig
- also rot fig

left-of rule

- point is transformed with respect to the the frame that appears immediately to the left of the transformation matrix in the expression.
- We read

$$\tilde{p} = \vec{\mathbf{f}}^t\mathbf{c} \Rightarrow \vec{\mathbf{f}}^t S\mathbf{c}$$

  - as "$\tilde{p}$ is transformed by S with respect to $\vec{\mathbf{f}}^t$".
- We read

$$\tilde{p} = \vec{\mathbf{a}}^t A^{-1}\mathbf{c} \Rightarrow \vec{\mathbf{a}}^t SA^{-1}\mathbf{c}$$

  - as "$\tilde{p}$ is transformed by S with respect to $\vec{\mathbf{a}}^t$".

more generally

- We read

$$\tilde{p} = \vec{\mathbf{f}}^t AB\mathbf{c} \Rightarrow \vec{\mathbf{f}}^t ASB\mathbf{c}$$

- as "$\tilde{p}$ is transformed by S with respect to $\vec{\mathbf{f}}^t A$".

for frames

- same for transformations of frames
- We read

$$\vec{\mathbf{f}}^t \Rightarrow \vec{\mathbf{f}}^t S$$

  - "$\vec{\mathbf{f}}^t$ is transformed by S with respect to $\vec{\mathbf{f}}^t$".

- We read

$$\vec{\mathbf{f}}^t = \vec{\mathbf{a}}^t A^{-1} \Rightarrow \vec{\mathbf{a}}^t S A^{-1}$$

  - as "$\vec{\mathbf{f}}^t$ is transformed by S with respect to $\vec{\mathbf{a}}^t$".

more generally

- We read

$$\vec{\mathbf{g}}^t = \vec{\mathbf{f}}^t AB \Rightarrow \vec{\mathbf{f}}^t ASB$$

  - as "$\vec{\mathbf{g}}^t$ is transformed by S with respect to $\vec{\mathbf{f}}^t A$".

auxiliary frame

- we may wish to transform a frame $\vec{\mathbf{f}}^t$ in some specific way represented by a matrix M, with respect to some auxiliary frame $\vec{\mathbf{a}}^t$.
  - For example, we may be using some frame to model the planet Earth, and we now wish the Earth to rotate around the Sun's frame.
- let $\vec{\mathbf{a}}^t = \vec{\mathbf{f}}^t A$
- then The transformed frame can then be expressed as

$$\begin{aligned} & \vec{\mathbf{f}}^t && (1) \\ = \; & \vec{\mathbf{a}}^t A^{-1} && (2) \\ \Rightarrow \; & \vec{\mathbf{a}}^t M A^{-1} && (3) \\ = \; & \vec{\mathbf{f}}^t A M A^{-1} && (4) \end{aligned}$$

multiple transformations

- using the "left of" rule
- example:
  - a rotation matrix R rotating a point by θ degrees about origin
  - translation matrix T, translating the point by one unit in the direction of the first frame axis.

interp 1

- given tform

$$\vec{\mathbf{f}}^t \Rightarrow \vec{\mathbf{f}}^t TR$$

- break into 2 steps
- In the first step

$$\vec{\mathbf{f}}^t \Rightarrow \vec{\mathbf{f}}^t T = \vec{\mathbf{f}'}^t$$

  - $\vec{\mathbf{f}}^t$ is transformed by T with respect to $\vec{\mathbf{f}}^t$ and we call the resulting frame $\vec{\mathbf{f}}'^t$.

- In the second step,

$$\begin{array}{rcl} \vec{\mathbf{f}}^t T & \Rightarrow & \vec{\mathbf{f}}^t T R \\ \vec{\mathbf{f}'}^t & \Rightarrow & \vec{\mathbf{f}'}^t R \end{array}$$

  - This is interpreted as: $\vec{\mathbf{f}}'^t$ is transformed by R with respect to $\vec{\mathbf{f}}'^t$.

other way

- In the first step

$$\vec{\mathbf{f}}^t \Rightarrow \vec{\mathbf{f}}^t R = \vec{\mathbf{f}}^{\circ t}$$

  $\vec{\mathbf{f}}^t$ is transformed by R with respect to $\vec{\mathbf{f}}^t$ and we call the resulting frame $\vec{\mathbf{f}}^{\circ t}$.

- In the second step,

$$\vec{\mathbf{f}}^t R \Rightarrow \vec{\mathbf{f}}^t T R$$

  $\vec{\mathbf{f}}^{\circ t}$ is transformed by T with respect to $\vec{\mathbf{f}}^t$.

summary

- both interps can be useful
- left to right, wrt latest (local)
  - right to left, wrt original frame (global)
- ex. 8-11