Assignment 1: Hydrostatic Equilibrium

Stars spend a large fraction of their lives in almost perfect *hydrostatic equilibrium* (HSE). This is the state when the force exerted outwards by the pressure gradient is exactly balanced by gravity. Stars only depart significantly from HSE in very late stages of their life when the timescale of nuclear burning becomes comparable to the dynamical timescale.

The equation for HSE in Lagrangian (mass) coordinates is

$$\frac{\partial p}{\partial m} = -\frac{Gm}{4\pi r^4}. \tag{1}$$

This is one of the equations of stellar structure being solved in MESA, so we expect to be satisfied everywhere on the main sequence. But in the reality of numerical simulations no equation is ever fulfilled exactly.

Tasks

a) Run a simulation of a $NM_\odot$ star until it is in the core helium burning phase, where N is your group number. Stop the simulation when the central mass fraction of helium drops below 0.25. The `inlist` files in the provided work directory have been set up accordingly already, except for the mass of the star.

b) Load the last profile file into a data analysis/plotting software (e.g., Python). Compute the left-hand (LHS) and right-hand side (RHS) of Eq. (1) at every cell boundary. Review the slides on intensive vs. extensive quantities to find out how to translate a cell-centred value to a boundary value. Approximate the derivative on the LHS using a finite difference (see slides for more details).

c) The absolute difference of LHS and RHS is not very meaningful because it is cannot be compared between different stars or even the same star at different times. Compute the *relative difference* instead. It is defined as

$$\Delta = \frac{|\mathrm{LHS} - \mathrm{RHS}|}{\max(|\mathrm{LHS}|, |\mathrm{RHS}|)}.$$

Plot the logarithm of Δ against the mass coordinate.

d) What is the theoretical maximum precision achievable using *double-precision* floating-point arithmetic?

e) Bonus question: There are a few single data points with much higher errors than the others. Can you find a feature in the stellar profile that correlates with these points?

Report

Prepare a one-page report on this assignment. Your report should give a very brief summary of HSE and answer all questions from the *Tasks* section. Also include the plot from Task c (it does not count towards the page limit).

Every group has to hand in their report at the beginning of the lab class on 16/10/2017. As this is the first report, it will be marked, but will not affect your grade.

Resources

- Questions regarding this assignment should be directed to `r.p.ratnasingam2@ncl.ac.uk` or `a.hindle@ncl.ac.uk`.
- The slides shown in the introduction and the template work directory can be found on Blackboard.
- The template work directory is also available via Git.