Assignment 5: Convection in stellar models

Convection is one of the major processes affecting the evolution of a star. It is one of the two main transport mechanisms for energy and it causes mixing of chemical composition and angular momentum. While it is hard to treat it correctly in a spherically symmetric framework, the conditions for its emergence can be derived rigorously.

To tell whether a region of the star is subject to convection, we need to check it's temperature gradient. It is common in stellar astrophysics to write it in terms of pressure instead of radius and to use logarithmic values to make it independent of the choice of units,

$$\nabla \equiv \frac{d\ln T}{d\ln P} = \frac{P}{T}\frac{dT}{dP}. \tag{1}$$

Here, ∇ is just a symbol for a quantity and not an operator. The pressure P and temperature T, are values taken from the stratified profile. As discussed on the slides this needs to be compared to the *adiabatic temperature gradient*,

$$\nabla_{\rm ad} \equiv \left(\frac{d\ln T}{d\ln P}\right)_{S,\mu}. \tag{2}$$

This is evaluated at constant entropy S and molecular mass μ. It is a property of the equation of state and does not depend on the stellar profile.

If the stratification is *superadiabatic*, i.e. the actual temperature gradient is higher than the adiabatic one, it is convectively unstable. This is called the *Schwardschild criterion*,

$$\nabla_{\rm ad} - \nabla < 0. \tag{3}$$

Another important quantity is the *radiative temperature gradient*,

$$\nabla_{\rm rad} = \frac{3}{16\pi acG}\frac{\kappa LP}{mT^4}, \tag{4}$$

which is the T gradient needed for radiation to be the sole energy transport mechanism at a given radius. This uses the radiation constant a, speed of light c, gravitational constant G, opacity of the stellar material κ, and luminosity L.

The frequency at which a displaced fluid element in the star will oscillate is called the *Brunt–Väisälä frequency* N. For an ideal gas it is given by,

$$N^2 = \frac{g}{H_P}\left(\nabla_{\rm ad} - \nabla + \nabla_\mu\right), \tag{5}$$

with the pressure scale height $H_P = -dr/d\ln P$ and the composition gradient $\nabla_\mu = d\ln\mu/d\ln P$. It is used in the *Ledoux criterion* for convection, which extends the Schwarzschild criterion by including the effect of the composition gradient,

$$N^2 < 0. \tag{6}$$

Lecture 13 will cover convection in detail.

Tasks

a) Run a model of a $1.2M_\odot$ star to a slightly evolved state with a central hydrogen mass fraction of 0.6.
b) Derive ∇_{ad} for a fully ionized, monatomic, ideal gas. Plot the value stored the MESA profile file (column `grada`) against radius and discuss the difference to your theoretical value.
c) Get the actual temperature gradient ∇ from the profile file (column `gradT`) and add it to the previous plot.
d) Find the regions that are convectively unstable due to the Schwarzschild criterion (Eq. (3)).
e) Compute ∇_{rad} using Eq. (4) and also add it to the plot of the other gradients. To keep the important features visible, you might want to limit the y-axis to the range from 0 to 1.
f) Describe with which of the other gradients ∇ coincides in different regions of the star and give an explanation for that.
g) Repeat these steps for a $1.0M_\odot$ star and qualitatively discuss the difference between the two.
h) *Bonus question:* Compute the square of the Brunt–Väisälä frequency N^2 for the $1.2M_\odot$ star and visualize it in a logarithmic plot showing both positive and negative components. Use it to compare the convective regions according to the Ledoux criterion (Eq. (6)) to the Schwarzschild criterion from Task d).

Report

Prepare a two-page (not including figures) report on this assignment. The bonus question is not needed for a perfect grade, but can be used to make up for a missing task on this or a future assignment.

Every group has to hand in their report at the beginning of the lab class on 11/12/2017.

Resources

- Questions regarding this assignment should be directed to `r.p.ratnasingam2@ncl.ac.uk` or `a.hindle@ncl.ac.uk`.
- The slides and MESA work directory can be found on Blackboard.
- All materials are also available on https://www.mas.ncl.ac.uk/~npe27/PHY3033/
- The MESA work directory is also available via Git.