Assignment 4: Simple stellar models using polytropes

Before the age of digital computing scientists were already constructing stellar models analytically by making some simplifying assumptions. A very rewarding approach is to assume that pressure only depends on density and not on temperature. This is called a *polytropic* equation of state, characterised by the *polytropic index* n,

$$P = K_P \rho^{(n+1)/n}. \tag{1}$$

This assumption and the technique of non-dimensionalisation yields the *Lane–Emden equation*,

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d\Theta}{d\xi}\right) = -\Theta^n. \tag{2}$$

This uses the non-dimensional density and radius variables,

$$\theta(\xi)^n = \frac{\rho}{\rho_c}, \quad \xi = \frac{r}{\alpha}, \tag{3}$$

with the central density of the star ρ_c and

$$\alpha^2 = \frac{(n+1)K_P}{4\pi G \rho_c^{(n-1)/n}}. \tag{4}$$

In this assignment we will obtain polytropic profiles for different values of n and compare them to more realistic stellar models computed with MESA.

Lecture 9 will cover polytropes in detail.

Tasks

a) Use the Python script `lane_emden.py` and create a plot of non-dimensional density Θ against non-dimensional radius ξ for values of n from 0 to 4.5 in steps of 0.5.

b) Use the provided inlists to calculate three stellar models, one of a pre-main sequence star, one of a main sequence star, and one of a white dwarf. You do this in the same work directory for all by just changing the filenames in the file `inlist`. The outputs are stored with distinct filenames.

c) Compare each of the MESA models to polytropic profiles of different n, following these steps:
   1. Compute the constant K_P for each MESA model using Eq. (1) and the values at the centre of the star.
   2. Plot ρ/ρ_c of the MESA model vs. radius. Add plots of Θ^n for different polytropic profiles. To convert $\Theta(\xi)$ to $\Theta(r)$ you have to compute the corresponding value of α using Eq. (4).
   3. Find the value of n that matches your model most closely.
   4. Compare the total radius of the polytrope to the radius of the MESA model.
   5. *Bonus question:* Compute the total mass of the polytrope using $m(r) = 4\pi\rho_c r^3(-1/\xi)d\Theta/d\xi$ and compare it to the mass of the MESA model.
   6. *Bonus question:* Give a physical explanation why each MESA model is expected to match or not to match a polytropic profile, at least in parts.

Report

Prepare a two-page (not including figures) report on this assignment. The bonus questions are not needed for a perfect grade, but can be used to make up for a missing task on this or a future assignment.

Every group has to hand in their report at the beginning of the lab class on 27/11/2017.

Resources

- Questions regarding this assignment should be directed to `r.p.ratnasingam2@ncl.ac.uk` or `a.hindle@ncl.ac.uk`.
- The slides and MESA work directory and Lane–Emden solver can be found on Blackboard.
- All materials are also available on https://www.mas.ncl.ac.uk/~npe27/PHY3033/
- The MESA work directory and the Lane–Emden solver is also available via Git.