CS/CNS/EE 156a
Homework 3

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1. Answer: [b]
Since $\epsilon = 0.05$ and $\mathrm{M} = 1$,

$$2Me^{-2\epsilon^2 N} \leq 0.03$$

$$2 * 1 * e^{-2*0.05^2*N} \leq 0.03$$

$$e^{-2*0.05^2*N} \leq 0.015$$

$$-2 * 0.05^2 * N \leq \ln(0.015)$$

$$\mathrm{N} \geq 839.94 \approx 840$$

Therefore, the answer is [b].

2. Answer: [c]
We go through the same process as what we did for problem 1, now with $\mathrm{M} = 10$.

$$2Me^{-2\epsilon^2 N} \leq 0.03$$

$$2 * 10 * e^{-2*0.05^2*N} \leq 0.03$$

$$e^{-2*0.05^2*N} \leq 0.0015$$

$$-2 * 0.05^2 * N \leq \ln(0.0015)$$

$$\mathrm{N} \geq 1300.46 \approx 1300$$

Therefore, the answer is [c]

3. Answer: [d]
We go through the same process as what we did for problem 1, now with $\mathrm{M} = 100$.

$$2Me^{-2\epsilon^2 N} \leq 0.03$$

$$2 * 100 * e^{-2*0.05^2*N} \leq 0.03$$

$$e^{-2*0.05^2*N} \leq 0.00015$$

$$-2 * 0.05^2 * N \leq \ln(0.00015)$$

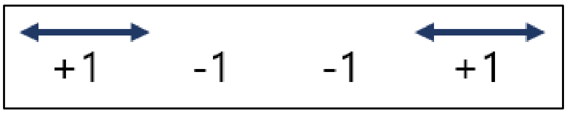
$$\mathrm{N} \geq 1760.9 \approx 1761$$

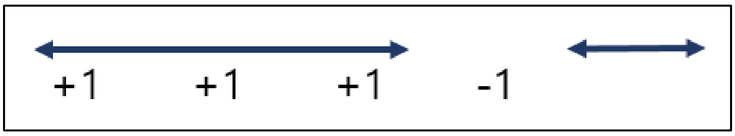
Therefore, the answer is [d].

4. Answer: [b]
Until 4 points, it is easy to shatter them in 3D space with a plane. However, as we add 1 extra point, we can no longer shatter them. For example, when 4 points are on a same plane and the remaining 1 point is not on the plane, there happen many cases that we cannot separate the entire 5 points by using a plane. Therefore, we cannot shatter 5 points in 3-dimension.

5. Answer: [b]
By the lecture, we know that growth functions with break points are polynomials. If we don't have a break point, the growth function is 2^N. Since i) and ii) are polynomials and v) is 2^N (the case when there is no break point), the answer is [b]. Note that iii) and iv) are neither polynomial nor 2^N.

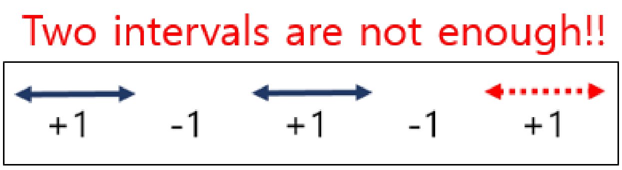
6. Answer: [c]
It is obvious that since we have two arbitrary intervals, we can shatter up to 4 points.
This is because up to 4 points, we can separate all points by using two intervals.
However, when it becomes 5 points, there can be three, distant, +1 groups.
For a more intuitive explanation, see the examples for 4-point case below.
Examples for separating 4 points (*Does not include all cases. These examples are just to help explaining*)





Etc…(Did not draw all cases, but as we can see, 4 points can be separated by using just two intervals)

However, when there are 5 points, we cannot separate them by using just two intervals, as following:



Therefore, the break point is 5.

7. Answer: [c]
Choosing two distinct intervals is equivalent to choosing four end points out of N+1 possible slots. Thus, it is
$\binom{N+1}{4}$
Now, for non-distinct intervals, it is equivalent to choosing a single interval. For single interval, we choose two end points out of N+1 possible slots. It is $\binom{N+1}{2}$.
Finally, we should consider the case when the interval does not cover any point. It is 1.

Therefore, the answer is $\binom{N+1}{4} + \binom{N+1}{2} + 1$.

8. Answer: [d]
Up to 2M, we can shatter the points because even for the worst case, which is the alternating pattern (+1,-1,+1,-1,+1,-1,…), we can cover all M +1 points by using M intervals. (The worst case here indicates the case when it requires largest number of intervals to cover all +1s.) However, when we add one more point to 2M, which becomes 2M+1, then the last point of +1 cannot be covered since we've already used all M intervals. Therefore, the break point of the given hypothesis is 2M+1.

9. Answer: [d]
It is obvious that we can shatter 1 point with a triangle.
For 3 points, it is also obvious that we can shatter them because in the lecture we saw that we could shatter 3 points by using a single line. Since an arbitrary chosen triangle(which has 3 sides) already has a line segment, we can definitely shatter 3 points.
For 5 points, I drew 5 alternating points(+1,-1,+1,-1,+1) on a circle, which are equi-distant from each other. I could shatter them by drawing a triangle including the three +1 points. When I drew the triangle on the circle, I could see that the two -1s were located in the exterior area of the triangle.
For 7 points, I again drew 7 alternating points(-1,+1,-1,+1,-1,+1,-1) on a circle, which are equi-distant from each other. Since there were still three +1 points, I could draw a triangle that included three +1 points as I did for the case of 5 points. As in the case of 5 points, when I drew the triangle, the exterior area of triangle covered four -1 points.
For 8 points, I drew 8 alternating points(-1,+1,-1,+1,-1,+1,-1,+1) on a circle. Since it is now *8 points*, there were *four +1 points* on the circle. Since there were *four* (which is larger than three) +1 points on the circle, I could no longer draw a triangle that covers all of the four +1 points.
Therefore, 7 is the largest number of points in R^2 that can be shattered by the triangle hypothesis.

10. Answer: [b]
For generalization, we can think of ${x_1}^2 + {x_2}^2$, which is equivalent to the distance from origin, as one variable. Then, the range of $a^2 \le {x_1}^2 + {x_2}^2 \le b^2$ is now just an interval that determines +1 and -1, based on the point's distance from the origin. In other words, it is equivalent to a case of single line and single interval. Therefore, choosing two end-points of N+1 intervals and adding 1 for the case when the interval does not cover any point give the answer of $\binom{N+1}{2} + 1$.