(6) Identifying Black hats & white hats

(a) Assuming white hats are minority while the black hats are the majory in the classroom. A pairwise test between students only reveals either "they are same color hats" or "atleast one is black". Tests therefore divide students into monochromatic, but test outcomes donot reveal which color each component has. Because black hats can lie. They can alway answer accordingly so that they produce the same test pattens so of white hats. When there are more than $n/2$ blacks, this cannot be solved.
$\exists$ atleast two distinct colors of hats with all test results that differ on which particular students are white.
Hence pairwise tests cannot uniquely identify white hats.

6 (b)

When running pairwise in such a scenario:

(i) If both say other is white $\longrightarrow$ keep one student from them

(ii) otherwise $\xrightarrow{\text{Atleast one black}}$ Remove both the students

such that we can conduct $n/2$ interviews leading to have $\leq \frac{n}{2}$ students.

Now we have to show, majority white remaining

Let x = No. of WW pairs, y = No. of BB pairs

z = No. of mixed, WB pairs

$$\therefore W = 2x + z \quad \& \quad B = 2y + z$$

$W - B = 2(x - y)$, since we made the assumption $W > B$

$$x - y = \frac{W - B}{2} > 0 \quad \therefore x > y$$

(vi)

(c) since $W > B$, so $x > y$ after $n/2$ interviews conducted.

Reduction of sizes go on $n, n_1, n_2, \dots, 1$ then we have to have interviews $[n/2] + [n_1/2] + [n_2/2] + \dots$ and so on. This sum is less than n.

$\therefore$ at most $n-1$ interviews need to conduct with each of the other $n-1$ students

$\therefore$ Total $= O(n)$ //