CMPS 6610 Extra Credit

In this extra credit assignment, we will test and review concepts you have learned since the midterm exam. Please add your written answers to `answers.md` which you can convert to a PDF using `convert.sh`. Alternatively, you may scan and upload written answers to a file named `answers.pdf`.

1. **Algorithmic Paradigms**

- 1a. In learning about algorithmic paradigms, you encountered the unit weight scheduling problem. Given n tasks, what is the work and span of solving the unit-weight task scheduling problem by brute force?
- 1b. Now suppose we generalize this task scheduling problem by giving each task a weight. The goal is now to schedule a set of non-overlapping tasks with maximum weight. Does the greedy algorithm still work? If so, provide a new proof. If not, provide a counterexample.
- 1c. State the optimal substructure property for the weighted task scheduling problem, and give the work and span of the resulting dynamic programming algorithm.

2. **Graphs**

- 2a. We learned that we could use breadth-first search to solve the shortest path problem in an unweighted graph. Why doesn't it work for weighted graphs? Give a high-level reason as well as a counterexample.
- 2b. For the shortest path problem, is the Bellman-Ford algorithm superior to Dijkstra's algorithm? Compare and contrast these two approaches.
- 2c. You learned the cut property for minimum spanning trees. There is another useful fact called the *cycle property* for minimum spanning trees which states the following:

  Given a graph G that contains a cycle, then the largest weight cycle in that graph **cannot** be contained in any minimum spanning tree.

  Prove the cycle property.

3. **Randomization**

- 5a. In the last problem set you were asked to use Markov's inequality in Problem 1. Prove Markov's inequality by using the definition of the expected value of a random variable.