21 Lecture 21: March 15

Last time

- Unusual and influential data (JF chapter 11)

Today

- Anonymous internal midterm evaluations on canvas
- Influence (JF chapter 11)
- Added-variable plots
- Should unusual data be discarded
- Diagnosing non-normality, non-constant error variance, and nonlinearity (JF chapter 12)

Measuring influence

Influence on the regression coefficients combines leverage and discrepancy. The most direct measure of influence simply expresses the impact on each coefficient of deleting each observation in turn:

$$D_{ij} = \hat{\beta}_j - \tilde{\beta}_{j(-i)} \quad \text{for } i = 1, \ldots, n \text{ and } j = 0, 1, \ldots, p$$

where $\hat{\beta}_j$ are the least-squares coefficients calculated for all the data, and the $\tilde{\beta}_{j(-i)}$ are the least-squares coefficients calculated with the ith observation omitted. To assist in interpretation, it is useful to scale the D_{ij} by (deleted) coefficient standard errors:

$$D^*_{ij} = \frac{D_{ij}}{\widehat{SE}_{(-i)}(\tilde{\beta}_{j(-i)})}$$

Following Belsley, Kuh, and Welsh (1980), the D_{ij} are often termed DFBETA_{ij}, and D^*_{ij} are called DFBETAS_{ij}. One problem associated with using D_{ij} or D^*_{ij} is their large number: $n(p+1)$ of each.

Cook's distance calculated as

$$D_i = \frac{\sum_{j=1}^{n} (\tilde{y}_{j(-i)} - \hat{y}_j)^2}{(p+1)\hat{\sigma}^2} = \frac{\hat{\epsilon}_i'^2}{p+1} \times \frac{h_i}{1-h_i}$$

In effect, the first term in the formula for Cook's D is a measure of discrepancy, and the second is a measure of leverage. We look for values of D_i that stand out from the rest.

A similar measure suggested by Belsley et al. (1980)

$$\text{DFFITS}_i = \hat{\epsilon}^*_i \frac{h_i}{1-h_i}$$

Except for unusual data configurations, Cook's $D_i \approx \text{DFFITS}_i^2/(p+1)$.

Numerical cutoffs (suggested)

Diagnostic statistic	Cutoff value
h_i	$2\bar{h} = \frac{2(p+1)}{n}$, ($3\bar{h}$ for small sample)
D^*_{ij}	$\lvert D^*_{ij}\rvert > 1$ or 2 ($2/\sqrt{n}$ for large samples)
Cook's D_i	$D_i > \frac{4}{n-p-1}$
DFFITS	$\lvert\text{DFFITS}_i\rvert > 2\sqrt{\frac{p+1}{n-p-1}}$

Added-variable plots

Unlike the case of SLR, the scatterplot with the response variable and one predictor gives only the marginal effect in MLR. Instead, the added-variable plot (also called a partial-regression plot or a partial-regression leverage plot) gives a graphical inspection over each dimension.

Let $\tilde{Y}_i^{(1)}$ represent the residuals from the least-squares regression of Y on all the Xs except X_1, in other words, the residuals from the following fitted regression equation:

$$Y_i = \tilde{\beta}_0^{(1)} + \tilde{\beta}_2^{(1)} X_{i2} + \cdots + \tilde{\beta}_p^{(1)} X_{ip} + \tilde{Y}_i^{(1)}$$

where the parenthetical superscript (1) indicates the omission of X_1 from the right-hand side of the regression equation. Likewise, $\check{X}_i^{(1)}$ is the residual from the least-squares regression of X_1 on all the other Xs:

$$X_{i1} = \check{\beta}_0^{(1)} + \check{\beta}_2^{(1)} X_{i2} + \cdots + \check{\beta}_p^{(1)} X_{ip} + \check{X}_i^{(1)}$$

Then, the residuals $\tilde{Y}_i^{(1)}$ and $\check{X}_i^{(1)}$ have the following interesting properties:

1. The slope from the least-squares regression of $\tilde{Y}_i^{(1)}$ on $\check{X}_i^{(1)}$ is simply the least-squares slope $\hat{\beta}_1$ from the *full* multiple regression.
2. The residuals from the simple regression of $\tilde{Y}_i^{(1)}$ on $\check{X}_i^{(1)}$ are the same as those from the full regression, that is
$$\tilde{Y}_i^{(1)} = \hat{\beta}_1 \check{X}_i^{(1)} + \hat{\epsilon}_i$$
3. The variation of $\check{X}_i^{(1)}$ is the *conditional variation* of X_1 holding the other Xs constant.

Figure 21.1 shows that the conditional variation is smaller than its marginal variation – much smaller when X_1 is strongly collinear with other Xs,

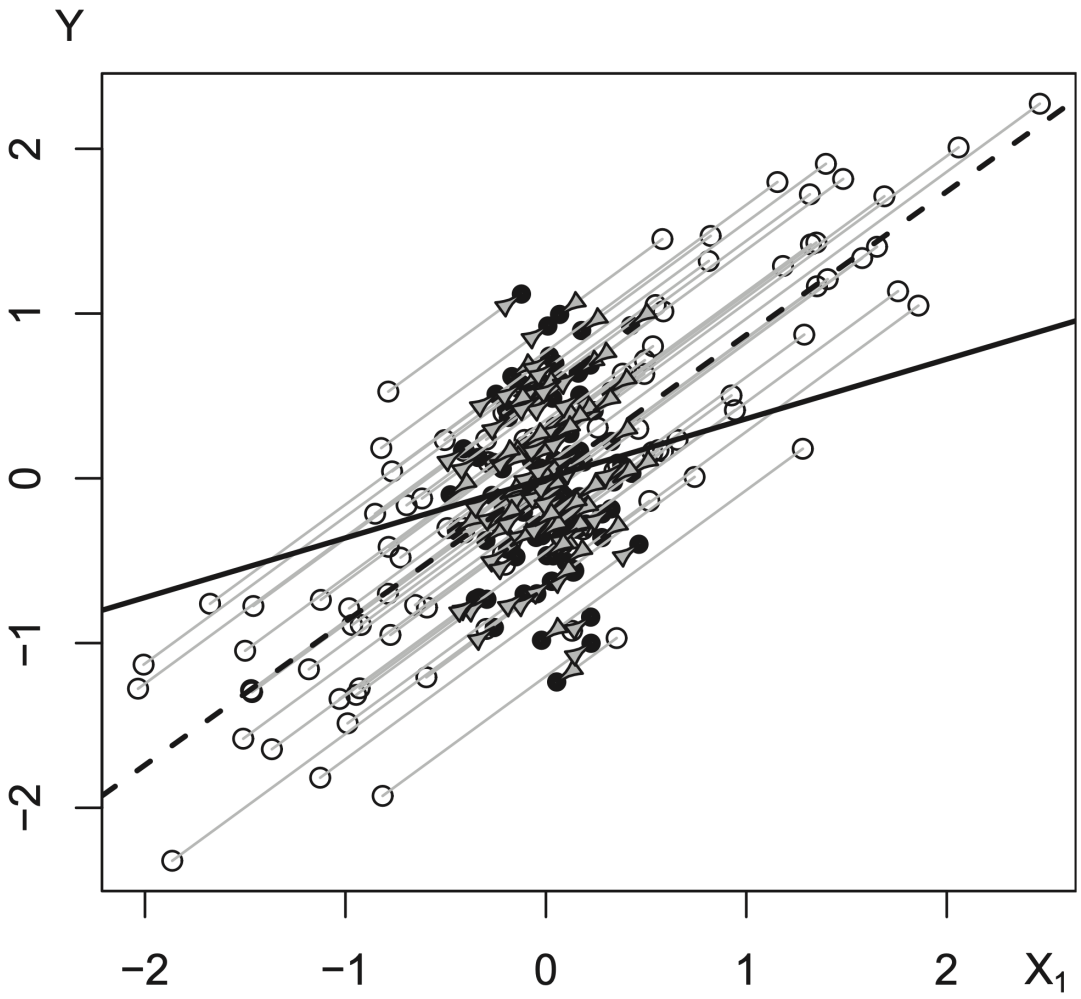

Figure 21.1: The marginal scatterplot (open circles) for Y and X_1 superimposed on the added-variable plot (filled circles) for X_1 in the regression of Y on X_1 and X_2. The variables Y and X_1 are centered at their means to facilitate the comparison of the two sets of points. The arrows show how the points in the marginal scatterplot map into those in the AV plot. In this contrived data set, X_1 and X_2 are highly correlated ($r_{12} = 0.98$), and so the conditional variation in X_1 (represented by the horizontal spread of the filled points) is much less than its marginal variation (represented by the horizontal spread of the open points). The broken line gives the slope of the marginal regression of Y on X_1 alone, while the solid line gives the slope $\hat{\beta}_1$ of X_1 in the MLR of Y on both Xs. JF Figure 11.9.

Figure 21.2 illustrates the added-variable plots using the Duncan's data.

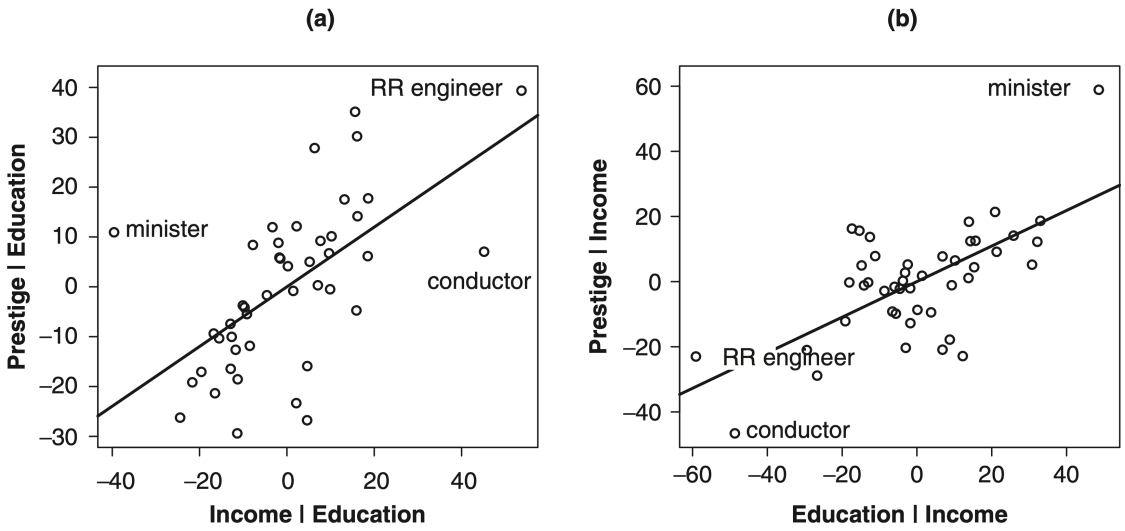

Figure 21.2: Added-variable plots for Duncan's regression of occupational prestige on the (a) income and (b) education levels of 45 US occupations in 1950. Three unusal observations, *miniters*, *conductors*, and *railroadengineers*, are identified on the plots. The added-variable plot for the intercept $\hat{\beta}_0$ is not shown. JF Figure 11.10.

The added-variable plot for income in Figure 21.2(a) reveals three observations that exert substantial leverage on the income coefficient:

- *minister*, whose income is unusually low given the educational level of the occupation
- *conductor*, whose income is unusally high given education
- *railroad engineer*, whose income is relatively high given education.

Remember that the horizontal variable in this added-variable plot is the residual from the regression of income on education, and thus values far from 0 in this direction are for occupations with incomes that are unusually high or low given their levels of education.

Should unusual data be discarded?

In practice, although problematic data should not be ignored, they also should not be deleted automatically and without reflection:

- It is important to investigate *why* an observation is unusual. Truly "bad" data (e.g., an error in data entry) can often be corrected or, if correction is not possible, thrown away. When a discrepant data point is correct, we may be able to understand why the observation is unusual. For Duncan's data, for example, it makes sense that ministers enjoy prestige not accounted for by the income and educational levels of the occupation and for a reason not shared by other occupations. In a case like this, where an outlying observation has characteristics that render it unique, we may choose to set it aside from the rest of the data.

- Alternatively, outliers, high-leverage points, or influential data may motivate model respecification, and the pattern of unusual data may suggest the introduction of additional explanatory variables. We noticed, for example, that both conductors and railroad engineers had high leverage in Duncan's regression because these occupations combined relatively high income with relatively low education. Perhaps this combination of characteristics is due to a high level of unionization of these occupations in 1950, when the data were collected. If so, and if we can ascertain the levels of unionization of all of the occupations, we could enter this as an explanatory variable, perhaps shedding further light on the process determining occupational prestige.

- Except in clear-cut cases, we are justifiably reluctant to delete observations or to respecify the model to accommodate unusual data. Some researchers reasonably adopt alternative estimation strategies, such as robust regression, which continuously downweights outlying data rather than simply discarding them. Because these methods assign zero or very small weight to highly discrepant data, however, the result is generally not very different from careful application of least squares, and , indeed, robust-regression weights can be used to identify outliers.

- Finally, in large samples, unusual data substantially alter the results only in extreme instances. Identifying unusual observations in a large sample, therefore, should be regarded more as an opportunity to learn something about the data not captured by the model that we have fit, rather than as an occasion to reestimate the model with the unusual observations removed.