# 11  Lecture 11: Feb 10

## Last time

- Introduction of simple linear regression

## Today

- HW2 posted
- The statistical model of the SLR (JF chapter 6)
- Properties of the Least-Squares estimator
- Inference of SLR model

## Properties of the Least-Squares estimator

Under the strong assumptions of the simple regression model, the sample least squares coefficients $\hat{\beta}_{ls}$ have several desirable properties as estimators of the population regression coefficients $\beta_0$ and $\beta_1$:

- The least-squares intercept and slope are *linear estimators*, in the sense that they are linear functions of the observations $y_i$.
  *Proof:*

- The sample least-squares coefficients are *unbiased estimators* of the population regression coefficients:
$$\mathbf{E}\left(\hat{\beta}_0\right) = \beta_0$$
$$\mathbf{E}\left(\hat{\beta}_1\right) = \beta_1$$

  *Proof:*

- Both $\hat{\beta}_0$ and $\hat{\beta}_1$ have simple sampling variances:

$$\text{Var}(\hat{\beta}_0) = \frac{\sigma_\epsilon^2 \sum x_i^2}{n \sum (x_i - \bar{x})^2}$$
$$\text{Var}(\hat{\beta}_1) = \frac{\sigma_\epsilon^2}{\sum (x_i - \bar{x})^2}$$

  *Proof:*

- Rewrite the formula for $\text{Var}(\hat{\beta}_1) = \frac{\sigma_\epsilon^2}{(n-1)S_X^2}$, we see that the sampling variance of the slope estimate will be small when

- The error variance $\sigma_\epsilon^2$ is small

- The sample size $n$ is large

- The explanatory-variable values are spread out (i.e. have a large variance, $S_X^2$)

- (Gauss-Markov theorem) Under the assumptions of linearity, constant variance, and independence, the least-squares estimators are BLUE (Best Linear Unbiased Estimator), that is they have the smallest sampling variance and are unbiased. (show this)
  *Proof:*

- Under the full suite of assumptions, the least-squares coefficients $\hat{\beta}_0$ and $\hat{\beta}_1$ are the maximum-likelihood estimators of $\beta_0$ and $\beta_1$. (show this)
  *Proof:*

- Under the assumption of normality, the least-squares coefficients are themselves normally distributed. Summing up,

$$\hat{\beta}_0 \sim N(\beta_0, \frac{\sigma_\epsilon^2 \sum x_i^2}{n \sum (x_i - \bar{x})^2})$$

$$\hat{\beta}_1 \sim N(\beta_1, \frac{\sigma_\epsilon^2}{\sum (x_i - \bar{x})^2})$$