

Homework #1

MATH 7360 – Fall 2020

Due: Friday, Sep 11, 2020

Some R exercises

1. Let $a = 0.7$, $b = 0.2$, and $c = 0.1$.
 - (a) Write out 0.7, 0.2, and 0.1 in binary.
 - (b) In R, test whether $(a + b) + c$ equals 1.
 - (c) In R, test whether $a + (b + c)$ equals 1.
 - (d) In R, test whether $(a + c) + b$ equals 1.
 - (e) Explain what you found. Hint: find out how addition is performed on numerics (double).
2. Create the vector $\mathbf{x} = (0.988, 0.989, 0.990, \dots, 1.010, 1.011, 1.012)$.
 - (a) Plot the polynomial $y = x^7 - 7x^6 + 21x^5 - 35x^4 + 35x^3 - 21x^2 + 7x - 1$ at points x_i in \mathbf{x} .
 - (b) Plot the polynomial $y = (x - 1)^7$ at points x_i in \mathbf{x} .
 - (c) Explain what you found.
3. Let $\mathbf{u} = (1, 2, 3, 3, 2, 1)^\top$.
 - a Compute $\mathbf{U} = \mathbf{I} - (2/d)\mathbf{u}\mathbf{u}^\top$ where $d = \mathbf{u}^\top\mathbf{u}$. (This type of matrix is known as an ‘elementary reflector’ or a ‘Householder transformation’.)
 - b Let $\mathbf{C} = \mathbf{U}\mathbf{U}$, the matrix product of \mathbf{U} and itself. Find the largest and smallest off-diagonal elements of \mathbf{C} .
 - c Find the largest and smallest diagonal elements of \mathbf{C} .
 - d Compute $\mathbf{U}\mathbf{u}$. (matrix times vector).
 - e Compute the scalar $\max_i \sum_j |U(i, j)|$.
 - f Print the third row of \mathbf{U} .
 - g Print the elements of the second column below the diagonal.
 - h Let \mathbf{A} be the first three columns of \mathbf{U} . Compute $\mathbf{P} = \mathbf{A}\mathbf{A}^\top$.
 - i Show that \mathbf{P} is idempotent (in other words $\mathbf{P} = \mathbf{P}\mathbf{P}$) by recomputing (e) with $\mathbf{P}\mathbf{P} - \mathbf{P}$.
 - j Let \mathbf{B} be the last three columns of \mathbf{U} . Compute $\mathbf{Q} = \mathbf{B}\mathbf{B}^\top$.
 - k Show that \mathbf{Q} is idempotent by recomputing (e) with $\mathbf{Q}\mathbf{Q} - \mathbf{Q}$.

- 1 Compute $\mathbf{P} + \mathbf{Q}$.
4. Read in the matrix in the file 'oringp.dat' on the failure of O-rings leading to the Challenger disaster. The columns are flight number, date, number of O-rings, number failed, and temperature at launch. Compute the correlation between number of failures and temperature at launch, deleting the last, missing observation (the disaster).
5. Functions

a What are the three components of a function?

b What does the following code return?

```

1 x <- 10
2 f1 <- function(x) {
3   function() {
4     x + 10
5   }
6 }
7 f1(1)()
```

c How could you make this call easier to read?

```

1 mean(, TRUE, x = c(1:10, NA))
```

d Does the following function throw an error when called? Why/why not?

```

1 f2 <- function(a, b) {
2   return(a * 10)
3 }
4 f2(10, stop("This is an error!"))
```

6. Let the $n \times n$ matrix \mathbf{A} have elements $A(i, j) = 1/(|i - j| + 1)$.
 - a Create a function that takes input argument n and output matrix \mathbf{A} .
 - b Compute and print \mathbf{A} for $n = 10$.
 - c Compute and print the Cholesky factorization for \mathbf{A} for $n = 10$. Hint: try `chol()` function.
 - d Find the determinant of \mathbf{A} .