Unbelievabubble: a 2^{4-1} Study

Katherine Sledjeski, Kate Bitzko, Heer Patel, and Tu Le

Figure1: Fractional Factorial Design

A	B	C	D=ABC
+	+	+	+
-	+	+	-
+	-	+	-
-	-	+	+
+	+	-	-
-	+	-	+
+	-	-	+
-	-	-	-

For our project, we decided to work with soap bubbles. The goal of this experiment was to determine the optimal bubble solution using a mixture of Karo syrup, sugar, water, and dish soap in order to create the most long-lasting bubble. To do so, we created a 2^{4-1} study and set each of the factors to a predetermined high and low value. These values can be seen in Figure 1 of the Relevant Figures portion of this paper. After completing the experiments and conducting a linear regression model on the resulting bubble lifespans, it was determined that in order to create the best bubble, the amount of dish soap, sugar, water, and Karo syrup should all be set to high levels.

Figure 2: Factor Levels

Factor	High	Low
A (Dish Soap)	3 tbsp	1 tbsp
B (Sugar)	1.5 tsp	0.5 tsp
C (Water)	1.5 cups	0.5 cups
D (Karo Syrup)	1.5 tbsp	0.5 tbsp

In order to determine the optimal combination of Karo syrup, water, sugar, and dish soap, we created 8 different bubble solutions each with the high and low values specified in Figure 2 in the relevant figures section. Using these 8 different solutions, we blew twenty bubbles and timed how quickly they popped after being fully formed and off the wand. The person in charge of blowing bubbles remained the same throughout each of the combinations and their replications in order for the bubbles to fall from the same height. The person blowing the bubbles stood in the same place for the entirety of the experiment so the bubbles consistently landed on the same type of surface. We conducted the experiment indoors to avoid having any weather related nuisance factors.

It is important to note before beginning the analysis that, we are looking at a 2^{4-1} design which is a Resolution IV design. Within a resolution IV design, no main effect is aliased with two-way interactions, but the two way interactions may still be aliased with each other. Given that we used generator D=ABC, this gives that AB and CD, BC and AD, as well as AC and BD are aliased with each other. Because of this, we cannot tell which of the interaction effect specifically is causing the change in our data, rather we just know that one of them is having some effect and should set our high and low factors accordingly.

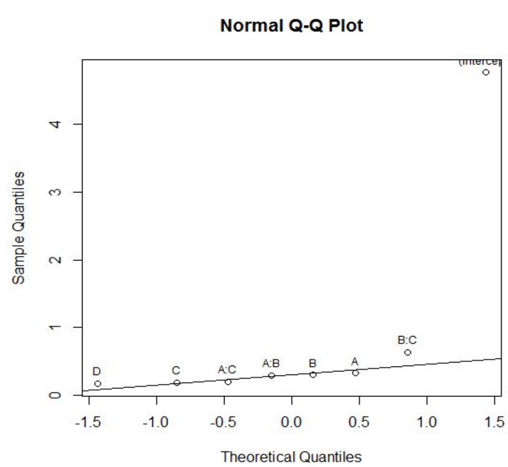


Figure 3: Quantile Plot of Effects

After completing our experiment, using the data obtained and the statistical software, R, we were able to obtain a chart showing the significance of various effects. These effects are plotted and can be seen in Figure 3. From this graph, it appears that the BC interaction and intercept are definitely significant to the model while main effects, A, B, and D may be significant. Next, if we assume a level of significance of 0.05, we can see from the output generated by R that the intercept, main effects A and B, and interaction effects AB and BC are all significant to the model. By the principle of model hierarchy this means that we have to include the effect C within our model. The equation found is as follows:

$$\mathbf{Y = 4.7664 + 0.3256A + 0.3031B + 0.1930C + 0.2916AB + 0.6412BC}$$

From this equation, in order to achieve maximum results, factors A, B, and C should all be set high and the setting of D does not matter because it is not significant in this case. Therefore, to create the best bubble, we should use high levels of water, sugar, and dish soap and either high or low levels of Karo syrup. It makes sense that factor A is significant in our model because dish soap is the basic ingredients in most bubble solutions because soap helps bubbles form. Factor B (sugar) acts as thickener which helps to slow the evaporation of the water that forms the bubbles which causes them to last longer.We can see this simply from looking at the average bubble pop time for solutions with high sugar and dish soap in comparison to solutions with low sugar and low dish soap. Solutions 1 and 5 both had high amounts of sugar and dish soap and the average bubble lasted for 5.686 seconds while Solutions 4 and 8 which had low amounts of sugar and dish soap had an average bubble time of 4.429 seconds. Those with high amounts of sugar and dish soap lasted over 1 second longer than those without. However, before we finalize any decisions about how our factors should be set, it is important to check that our model obeys the assumptions of linear regression models, mainly that the residuals are normally distributed. In order to check this normality, we can use two tools: a Q-Q plot and a plot of the residuals.

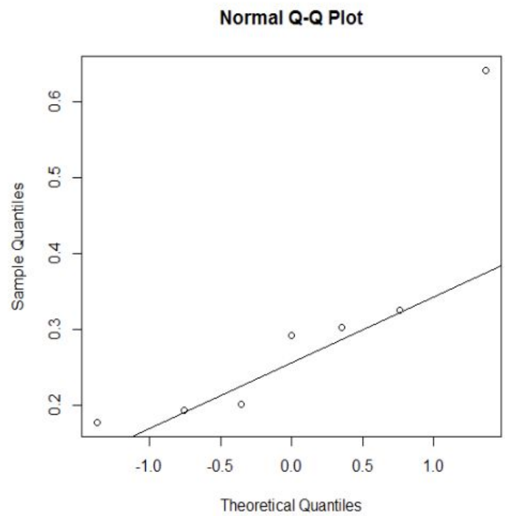
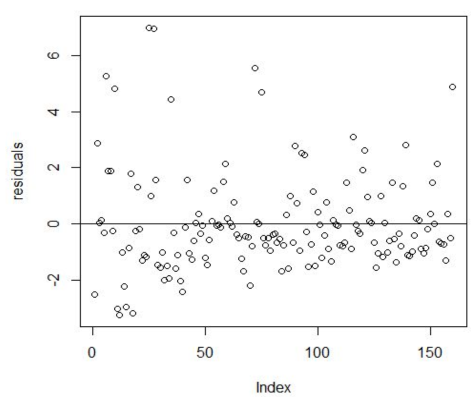


Figure 4: Q-Q Plot Normality Check

For the Q-Q plot we want all of the points to be close to the line. For the plot of residuals, we want the points to look random and unbiased in order to show that the residuals are uncorrelated with mean 0 and constant variance. For the regression equation calculated above, the Q-Q plot can be seen in Figure 4 and the plot of residuals can be seen in Figure 5. Looking at Figure 4, it seems that our residuals were not normally distributed. Several of the quantiles lie extremely far from the normal line and the others are not as tight to the line as we would like. This means that there was most likely a large error in our data set and the model might not be completely accurate. It could also indicate the presence of outliers.



Looking at Figure 5 we can see that for the most part the residuals do look to be independent and identically distributed, there are only a couple of points that would be cause for concern but overall the data looks unbiased.

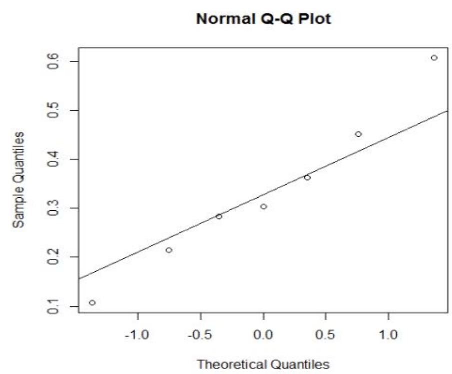


Figure 6: Outlier Points Removed and Redone Q-Q Plot

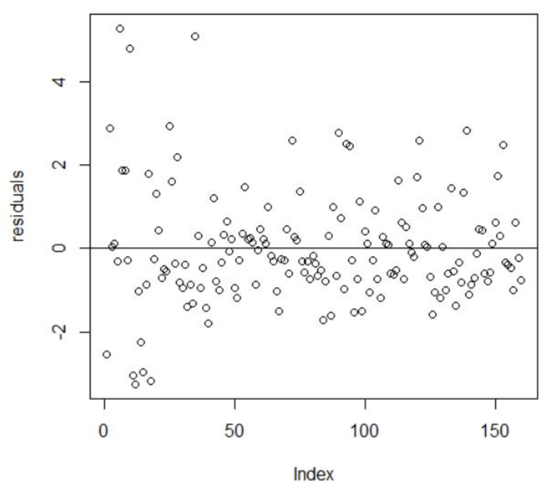


Figure 7: Outliers Removed and Redone Residuals Plot

After analyzing the Q-Q plot of the first regression equation found in Figure 4, it was decided that we needed to further look into potential outliers within each of our data sets. When performing data analysis, you can usually assume that values will cluster around some central data point. But sometimes a few of the values fall too far from the central point. These outlier values can skew the statistical analysis and lead to false or misleading conclusions about the data. In order to identify these outliers, we have to pinpoint the statistical center of the range. To do this, we first find the first and third quartiles of the data. In our case, we used the quartile function in excel in order to do so. Taking these two quartiles, you can calculated the statistical fifty percent of the data set by subtracting the 1st quartile from the third quartile also known as the interquartile range (IQR). Statisticians agree that in order to establish a "upper fence" and "lower fence" for a data set the lower fence is equal to: (1st quartile - IQR*1.5) and the upper fence is equal to: (3rd quartile + IQR*1.5). Any data points lying outside of these upper and lower fences are outliers. However, we cannot simply just remove the outliers from the data set because it will change the orthogonality of the design. Therefore, we took the points that were outliers, removed them, but then re-did the trial in order to obtain a new data point. After the outliers were removed and the new data points were obtained, we re-ran the model in R. This time we got the following regression equation:

$$\mathbf{Y = 4.5691 + 0.4510A + 0.3041B + 0.1075C + 0.2836D + 0.3625AB + 0.6075BC}$$

The effects A,B,D, AB, and BC are significant at the alpha equal to 0.05 level. By model hierarchy, this also means that factor C will be significant. The new Q-Q plots and plot of residuals can be seen in Figures 6 and 7 respectively. By looking at these two figures, it looks like this new regression model is a better fit because the residuals appear to be more normally distributed and uncorrelated.

The results of this study have the potential to be useful to a company that produces and sells bubble mixtures. Customer satisfaction is essential for a company to be successful in the long-run, and customers will want a mixture that is easy to use and creates long-lasting bubbles. We could improve our experiment by using a small fan instead of a person to produce the bubbles. This would provide a steadier stream of air and reduce the opportunities for error in our results. A good way to expand this experiment in the future would be to measure the correlation between the length of time a bubble goes without popping and the length of time it takes to form a successful bubble.

From the fractional factorial design, we know that all four factors A, B, C, D need to be at high setting in order to have high bubble's lasting time, but in order to find the actual optimum of our design, we decide to proceed to response surface methodology. The R output shows that factors A, B and D are significant and if we fit our initial model using only these three main effects, we would have

Y = 4.5080 + 0.5374A + 0.3168B + 0.2890D

The equation shows that from the center point xA= 0, xB= 0, xD= 0, for every 0.5374 units of soap added in the solution, we should add 0.3168 units of sugar and 0.2890 units of syrup. The method of steepest ascent tells us the direction of the hill goes up to the peak point xA= 0, xB = 0, xD = 0;

The center point of the design surface corresponds to the highest setting of factor A which is 1.5 oz of soap. If we choose the basic step size of soap level is 0.5 oz which corresponds to ΔxA = 1, and the slope of xB/ xA = 0.31745/0.52668 = 0.6027. Thus our step size xA = 1 determines that xB 0.6027 oz of sugar. Similarly, the slope of of xz axis is 0.26769/0.52668 = 0.5083, which means the corresponding xC = 0.5083 . Therefore, the path of steepest ascent passess through the point (0,0.0) and have the slope (1, 0.6027, 0.5083). If we sequentially move along the path of steepest ascent, we would approach the true optima of our experiment.

For the central composite design, we convert the measurement units of three factors A, B, D to oz so that it's easier for conversion. The experiment contains 20 solutions, each solution was run five times. We convert the coded values of A, B, D into real measurement by time the corresponding slope of the path of steepest ascent with the low and high value measurements of original experiment to get the ratio.

The result of the central composite design indicates that the second-order model fit is

$$\mathbf{Y = 1.10 + 4.72A + 21.5B + 6.0C + 1.17A^2 - 134B^2 - 12.1C^2 -27.5AB - 13.5\ AD + 148.2BD}$$

However, the output also indicate that only interaction AD and BD are significant. Due to model hierarchy, factor A, B, and D are all significant. Therefore, the final model would be:

$$\mathbf{Y =4.67+ 4.28A - 3.29B - 0.14D - 13.5AD + 148.2BD}$$

The result of the central composite design indicates that we need to set soap at high level because it is the main ingredient of our bubble solution. The negative coefficient of B and D make us think that we need to set sugar and syrup at low setting, but the interaction between B and D are very high, which might mean that the interaction between sugar and syrup can result a very thickener mixture that help to extend to bubble lasting time.

<u>Relevant figures</u>

Figure1: Fractional Factorial Design

A	B	C	D=ABC
+	+	+	+
-	+	+	-
+	-	+	-
-	-	+	+
+	+	-	-
-	+	-	+
+	-	-	+
-	-	-	-

Figure 2: Factor Levels

Factor	High	Low
A (Dish Soap)	3 tbsp	1 tbsp
B (Sugar)	1.5 tsp	0.5 tsp
C (Water)	1.5 cups	0.5 cups

D (Karo Syrup)	1.5 tbsp	0.5 tbsp

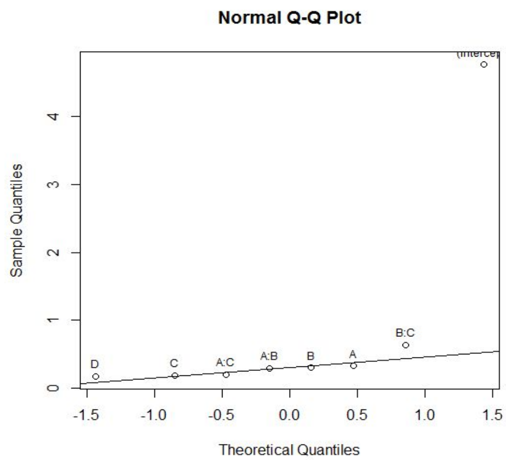


Figure 3: Quantile Plot of Effects

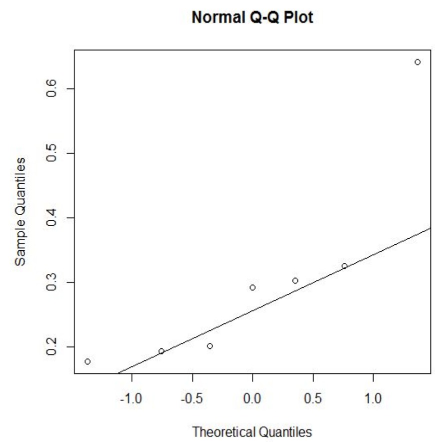


Figure 4: Q-Q Plot Normality Check

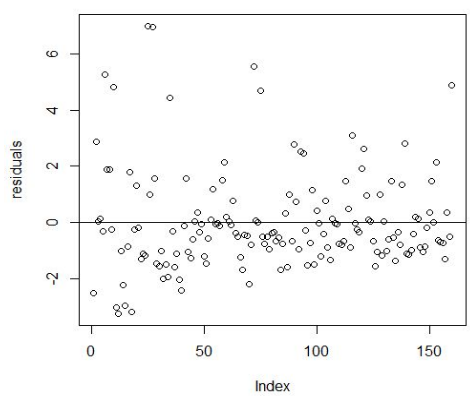


Figure 5: Plot of Residuals Normality Check

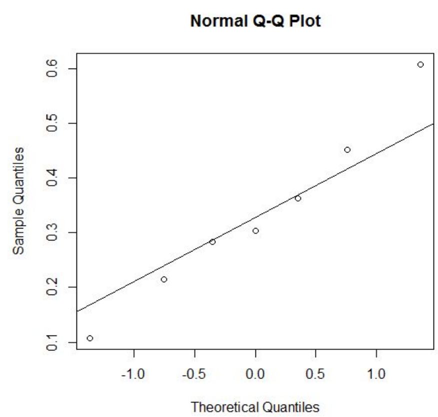


Figure 6: Outlier Points Removed and Redone Q-Q Plot

Figure 7: Outliers Removed and Redone Residuals Plot

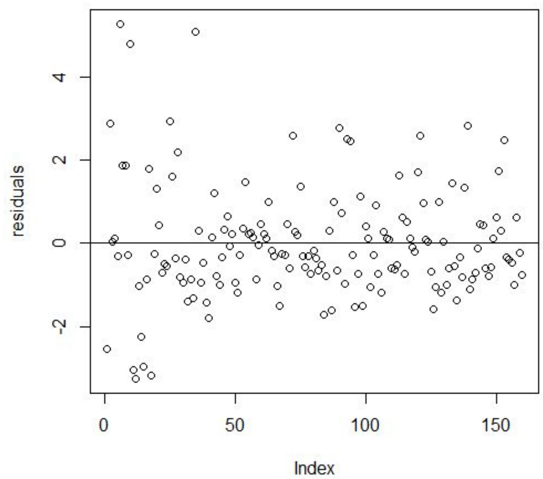


Figure 7: Outliers Removed and Redone Residuals Plot