Wichmann and Hill 2001 - Fitting, sampling, and goodness of fit

January 4, 2021

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```
clear; clc;
cd('/Users/duncan/OneDrive - University of Delaware - o365/Documents
```

Figure 1

```
% data
sig_int = [0.8, 1.2, 1.4, 1.9, 2.1, 3.5];
p_corr1 = [0.5, 0.62, 0.77, 0.85, 1, 1];
p_corr2 = [0.5, 0.62, 0.77, 0.85, 1, 0.98];
% visualize
figure;
plot(sig_int, p_corr1, 'ko', 'markerfacecolor', 'b')
hold on
plot(sig_int, p_corr2, 'r^', 'markerfacecolor', 'r')
xlim([0 4]); ylim([0.4 1.09])
xlabel('signal intensity'); ylabel('proprotion correct')
% fit
weib = @(x,xdata) 0.5 + (1 - 0.5 - x(3)) * (1 - exp(-(xdata./x(1)).^
fmin_params1 = fmincon(@(x) nloglik(x(1), x(2), x(3), sig_int, p_cor
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreas
feasible directions, to within the value of the optimality tolerance
and constraints are satisfied to within the value of the constraint

<stopping criteria details>
fmin_params1 = 1×3
    1.6346    3.9381    0.0000
```

```
fmin_params2 = fmincon(@(x) nloglik(x(1), x(2), x(3), sig_int, p_cor
```

```
Local minimum found that satisfies the constraints.
```

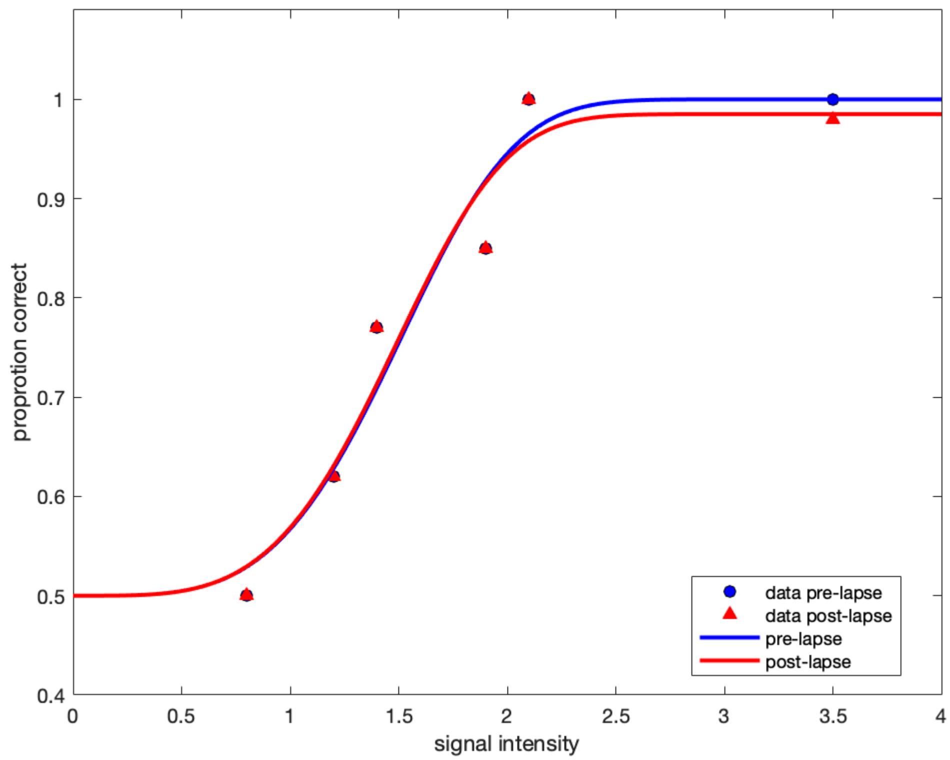
```
Optimization completed because the objective function is non-decreas
feasible directions, to within the value of the optimality tolerance
and constraints are satisfied to within the value of the constraint

<stopping criteria details>
fmin_params2 = 1×3
    1.6062    3.9750    0.0149
```

```
% fmin_params1 = fminsearch(@(x) nloglik(x(1), x(2), x(3), sig_int,
% fmin_params2 = fminsearch(@(x) nloglik(x(1), x(2), x(3), sig_int,
% psi_fun_free = @(x,xdata) 0.5 + (1 - 0.5 - x(3)) * (1 - exp(-(xdat
% psi_fun_fixed = @(x,xdata) 0.5 + (1 - 0.5 - x(3)) * (1 - exp(-(xda
% fit_free = lsqcurvefit(psi_fun_free, [2 4 0], sig_int, p_corr1)
% fit_fixed = lsqcurvefit(psi_fun_fixed, [2 4 0], sig_int, p_corr1)

xline = linspace(0, 4);
plot(xline, weib(fmin_params1, xline), 'b', 'linewidth', 2)
plot(xline, weib(fmin_params2, xline), 'r', 'linewidth', 2)
% plot(xline, psi_fun_free(fit_free, xline), 'k-', 'linewidth', 2)
% plot(xline, psi_fun_fixed(fit_fixed, xline), 'k--', 'linewidth', 2
% title(sprintf('Parameters1 = alpha = %.3f; beta = %.3f; gamma = %.
%     fit_free(4), fit_free(3), fit_free(1), fit_free(2), fit_fixed(

legend('data pre-lapse', 'data post-lapse', 'pre-lapse', 'post-lapse
```




```
% Again with 4 free parameters
% visualize
figure;
plot(sig_int, p_corr1, 'ko', 'markerfacecolor', 'b')
hold on
plot(sig_int, p_corr2, 'r^', 'markerfacecolor', 'r')
xlim([0 4]); ylim([0.4 1.09])
xlabel('signal intensity'); ylabel('proprotion correct')
% fit
weib = @(x,xdata) x(4) + (1 - x(4) - x(3)) * (1 - exp(-(xdata./x(1))
fmin_params1 = fmincon(@(x) nloglik_1(x(1), x(2), x(3), x(4), sig_in
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreas
feasible directions, to within the value of the optimality tolerance
and constraints are satisfied to within the value of the constraint

<stopping criteria details>
fmin_params1 = 1×4
    1.6346    3.9381    0.0000    0.5000
```

```
fmin_params2 = fmincon(@(x) nloglik_1(x(1), x(2), x(3), x(4), sig_in
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreas
feasible directions, to within the value of the optimality tolerance
and constraints are satisfied to within the value of the constraint

<stopping criteria details>
fmin_params2 = 1×4
    1.6062    3.9750    0.0149    0.5000
```

```
fmin_params3 = fmincon(@(x) nloglik_1(x(1), x(2), x(3), x(4), sig_in
```

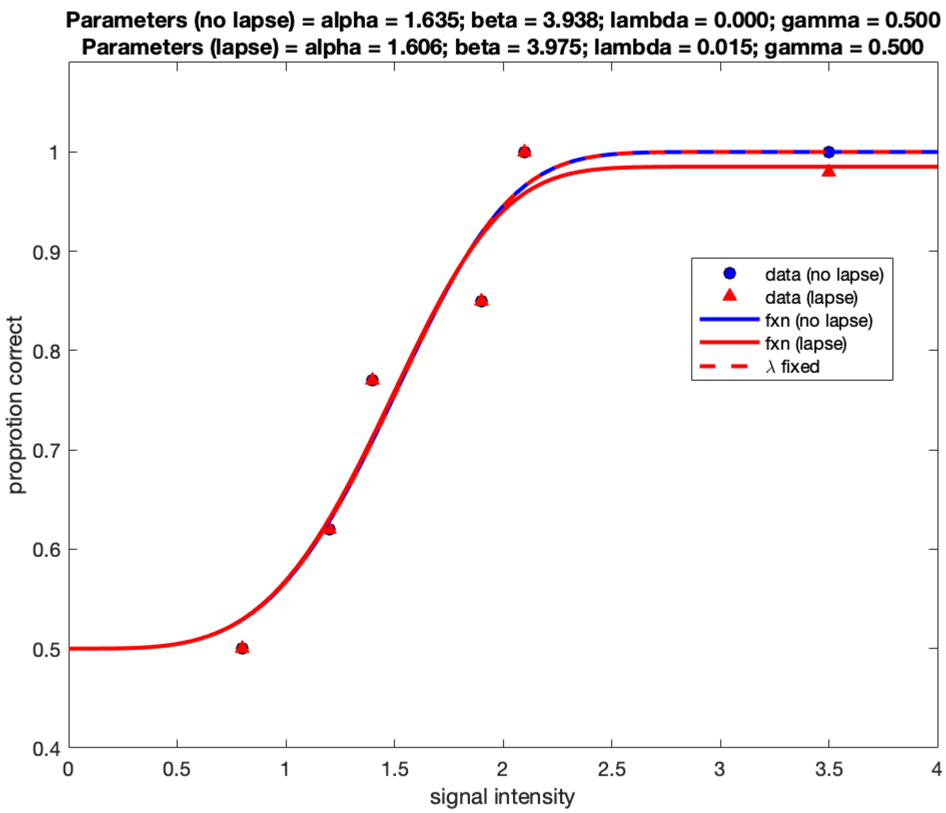
```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreas
feasible directions, to within the value of the optimality tolerance
and constraints are satisfied to within the value of the constraint

<stopping criteria details>
fmin_params3 = 1×4
    1.6346    3.9381         0    0.5000
```

```
xline = linspace(0, 4);
plot(xline, weib(fmin_params1, xline), 'b', 'linewidth', 2)
plot(xline, weib(fmin_params2, xline), 'r', 'linewidth', 2)
plot(xline, weib(fmin_params3, xline), 'r--', 'linewidth', 2)
title(sprintf('Parameters (no lapse) = alpha = %.3f; beta = %.3f; la
    fmin_params1(1), fmin_params1(2), fmin_params1(3), fmin_params1(

legend('data (no lapse)', 'data (lapse)', 'fxn (no lapse)', 'fxn (la
```



Deviance

First derive equation 5 from 4.

$$D = 2\log\left[\frac{L(\theta_{\max}; y)}{L\left(\hat{\theta}; y\right)}\right] = 2\left[l(\theta_{\max}; y) - l\left(\hat{\theta}; y\right)\right] \qquad (4)$$

Note: Log rule: log(a - b) = log(a / b).

Let *k* denote # of x values (i.e. stimulus intensities), *n* denote # of trials at each x-value/ stimulus intensiities, *y* denote probability from model fit (therefore $\psi(x; \theta_{\max})$ = *y)*, *p* denote $\psi\left(x; \hat{\theta}\right)$.

$\psi(x; \theta_{\max})$ refers to a function where the number of parameters equals the number of data (i.e. a "perfect fit" where function goes through each data point). Therefore, we can just treat the data as this function.

$$D = 2\left[\sum_{i=1}^{k} \log\left(\frac{n_i}{y_i n_i}\right) + y_i n_i \log \psi(x_i;\theta_{\max}) + (1-y_i)n_i \log(1-\psi(x;\theta_{\max})) - \log\left(\frac{n_i}{y_i n_i}\right) + y_i n_i \log \psi\left(x\right.\right.$$

Simplify terms.

$$D = 2\left[\sum_{i=1}^{k} y_i n_i \log(\psi(x;\theta_{\max})) - -y_i n_i \log\left(\psi\left(x_i;\hat{\theta}\right)\right) + (1-y_i)n_i \log(1-\psi(x_i;\theta_{\max})) - (1-y_i)n_i \log\left(\right.\right.$$

Simplify further.

$$D = 2\sum_{i=1}^{k} y_i n_i \log\left(\frac{y_i}{p_i}\right) + (1-y_i)n_i \log\left(\frac{1-y_i}{1-p_i}\right)$$

```
n = 100;
for i = 1:length(sig_int)
    A = n * p_corr1(i) * log(p_corr1(i)/psi_fun_free(fit_free,sig_in
    B = n * (1-p_corr1(i))
    C = log((1 - p_corr1(i)) / (1 - psi_fun_free(fit_free,sig_int(i)
    d(i) = A + B * C;
end
```

```
A = 0.2717
B = 50
C = -0.0054
A = -2.4764
B = 38
C = 0.0688
A = 4.0209
B = 23
C = -0.1573
A = -5.6441
B = 15.0000
C = 0.4927
A = 4.9744
```

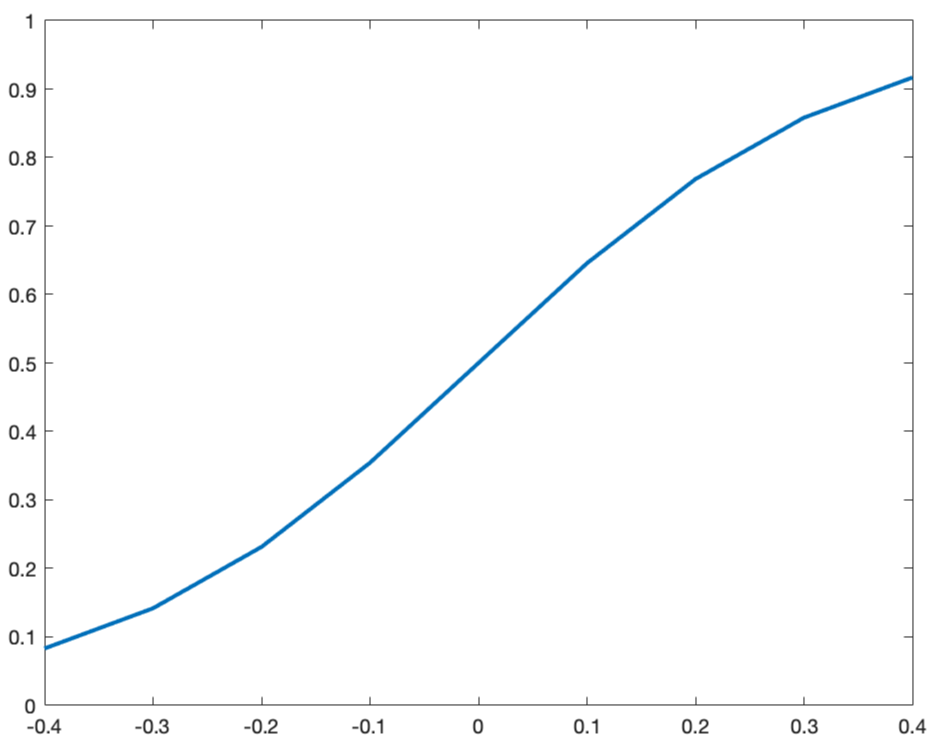
```
B = 0
C = -Inf
A = -0.6770
B = 0
C = -Inf
```

```
D = 2 * sum(d)
```

```
D = NaN
```

Testing

```
psi = @(x,xdata) x(4) + (1 - x(3) - x(4)) * (1 ./ (1 + exp(-x(1) * (
intensity = -0.4:0.1:0.4;
figure;
plot(intensity, psi([6 0 0 0], intensity), 'linewidth', 2)
```




```
function nll = nloglik(alpha, beta, lambda, signal_intensity, p_corr
% INPUTS:
% alpha:

% OUTPUT:
```

```
% nll: negative log likelihood
```

Deriving negative log likelihood from likelihood of a binomial distribution.

Write out likelihood of a binomial distribution, where *n* denotes upper limit, *x* denotes probabilities from subject, and *p* denotes probabilities from model fit.

$$L(p;x) = \frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x} \qquad (1)$$

Take the negative log of equation 1.

$$= -\log\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right] \qquad (2)$$

Distribute the log.

$$= -\log\left(\frac{n!}{x!(n-x)!}\right) + \log(p^{x}) + \log((1-p)^{n-x}) \qquad (3)$$

Note: Log rule: log(a*b) = log(a) + log(b)

Remove constant, distribute exponents and negative sign.

$$= -\mathrm{x}\log(p) - (n-x)\log(1-p) \qquad (4)$$

Note: Log exponent rule: log(a^b) = b*log(a)

```
% Create psychometric function (Weibull)
I = 0.5 + (1 - 0.5 - lambda) * (1 - exp(-(signal_intensity./alpha).^

% find nll
nll = -sum(p_correct .* log(I) + (1 - p_correct) .* log(1 - I));

end
function nll = nloglik_1(alpha, beta, lambda, gamma, signal_intensit
% INPUTS:
% alpha: threshold / PSE (point of subjective equality)
% beta: slope / rate of change / heat
% lambda: lapse rate (probability of an incorrect response, which is
% independent of stimulus intensity)
% gamma: guess rate (probability of a correct response when the stim
```

```
% not detected by the underlying sensory mechanism)

% OUTPUT:
% nll: negative log likelihood
```

Deriving negative log likelihood from likelihood of a binomial distribution.

Write out likelihood of a binomial distribution, where *n* denotes upper limit, *x* denotes probabilities from subject, and *p* denotes probabilities from model fit.

$$L(p;x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x} \qquad (1)$$

Take the negative log of equation 1.

$$= -\log\left[\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}\right] \qquad (2)$$

Distribute the log.

$$= -\log\left(\frac{n!}{x!(n-x)!}\right) + \log(p^x) + \log((1-p)^{n-x}) \qquad (3)$$

Note: Log rule: log(a*b) = log(a) + log(b)

Remove constant, distribute exponents and negative sign.

$$= -\mathrm{x}\log(p) - (n-x)\log(1-p) \qquad (4)$$

Note: Log exponent rule: log(a^b) = b*log(a)

```
% Create psychometric function (Weibull)
I = gamma + (1 - gamma - lambda) * (1 - exp(-(signal_intensity./alph

% find nll
nll = -sum(p_correct .* log(I) + (1 - p_correct) .* log(1 - I));

end
```