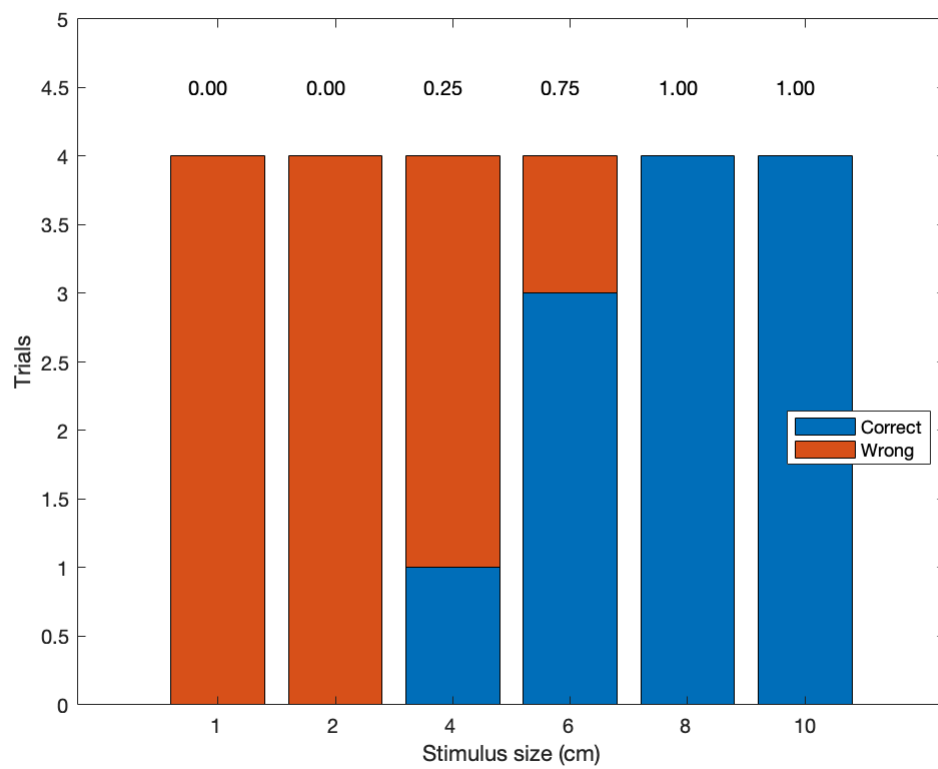




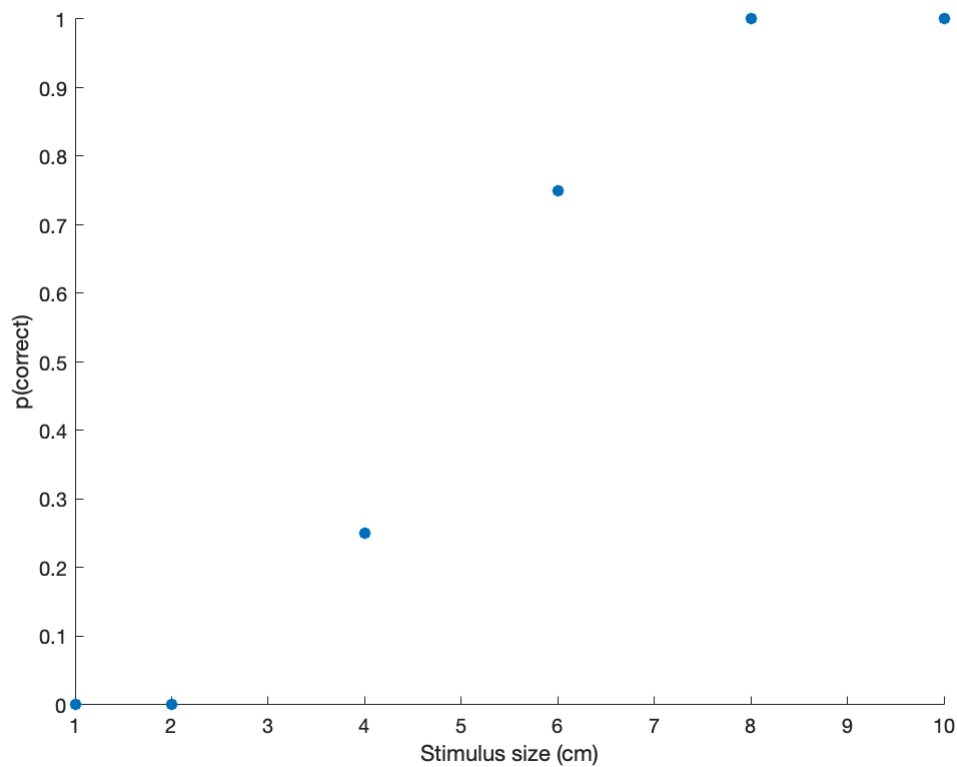
```

        case 2
            Bplt(2,1) = Bplt(2,1) + Given_rnd.Correct(i);
            Bplt(2,2) = Bplt(2,2)+1;
        case 4
            Bplt(3,1) = Bplt(3,1) + Given_rnd.Correct(i);
            Bplt(3,2) = Bplt(3,2)+1;
        case 6
            Bplt(4,1) = Bplt(4,1) + Given_rnd.Correct(i);
            Bplt(4,2) = Bplt(4,2)+1;
        case 8
            Bplt(5,1) = Bplt(5,1) + Given_rnd.Correct(i);
            Bplt(5,2) = Bplt(5,2)+1;
        case 10
            Bplt(6,1) = Bplt(6,1) + Given_rnd.Correct(i);
            Bplt(6,2) = Bplt(6,2)+1;
    end
end
Bplt(:,3) = Bplt(:,1) ./ Bplt(:,2);
% Calculate p(correct) for each stimulus size
figure;
bar([0 4; 0 4; 1 3; 3 1; 4 0; 4 0], 'stacked')
hold on
legend('Correct', 'Wrong', 'location', 'best')
set(gca, 'xticklabels', [1 2 4 6 8 10]);
xlabel('Stimulus size (cm)')
ylabel('Trials')
ylim([0 5])
for i = 1:size(Bplt,1)
    text(i-0.25, 4.5, sprintf('%.2f', Bplt(i,3)))
end

```



```
figure;
scatter([1,2,4,6,8,10], [0 0 .25 0.75 1 1], [], 'filled')
xlabel('Stimulus size (cm)'); ylabel('p(correct)')
```



```
% just_save('pscyh_ex_nofit', 'n')
```

## New equation and fits

Four parameter model:

$$\psi(x; \alpha, \beta, \gamma, \lambda) = \gamma + (1 - \gamma - \lambda)F(x; \alpha, \beta)$$

### Parameters

$\psi$  : proportion of correct responses

$x$  : data

$\alpha$  : midpoint OR threshold

$\beta$  : heat OR slope

$\gamma$  : guess rate OR lower asymptote of  $\psi$

$\lambda$  : lapse rate OR upper asymptote of  $\psi$

$F$  : sigmoid function (Weibull, **logistic**, gumbel, cumulative normal, hyperbolic secand, quick)

```
% values
xdata = unique(Given.Size_cm);
trials = Bplt(:,2);
correct = Bplt(:,1);
p_correct = correct ./ trials;
```

```

% Create function to use (with logistic)
fun = @(x,xdata) x(4) + (1 - x(4) - x(3)) * (1 ./ (1 + exp(-x(2)*(xd

% Initial guess for [alpha, beta, lambda, gamma] parameter
x0 = [5 3 0 0];

% Minimize negative log likelihood
coeffs = fmincon(@(x) nloglik_1(x(1), x(2), x(3), x(4), xdata, trial

```

Local minimum found that satisfies the constraints.

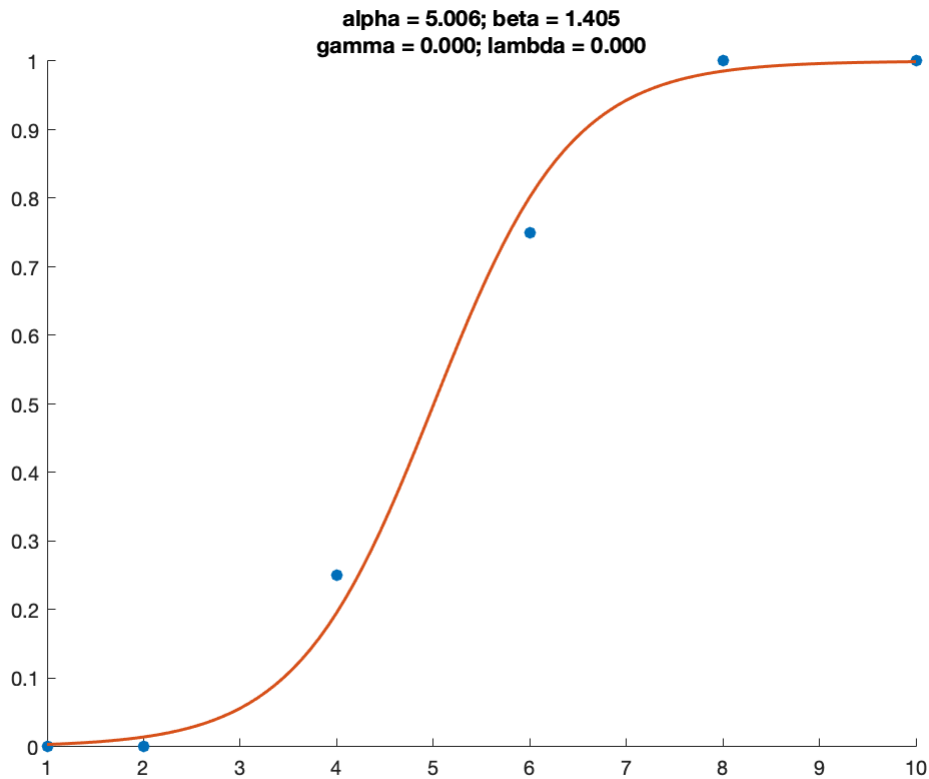
Optimization completed because the objective function is non-decreasing in any feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

```

% plot
figure;
scatter(xdata, p_correct, [], 'filled');
hold on
xL = linspace(xdata(1), xdata(length(xdata)));
yL = fun(coeffs, linspace(xdata(1), xdata(length(xdata))));
plot(xL, yL, 'linewidth', 1.5)
title(sprintf('alpha = %.3f; beta = %.3f\ngamma = %.3f; lambda = %.3

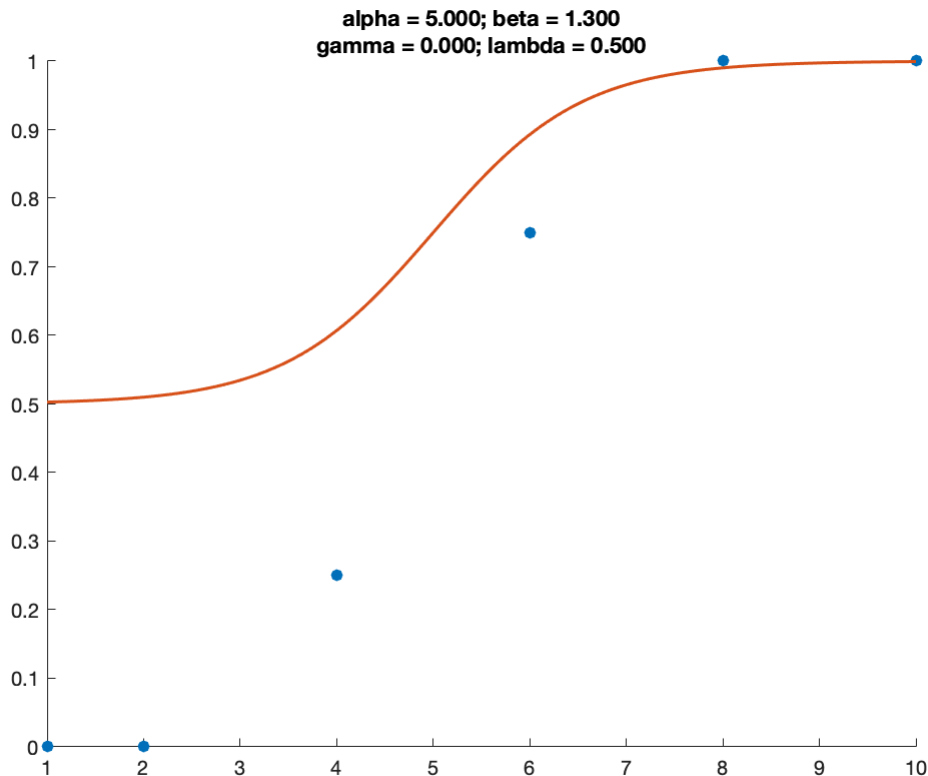
```



## Changing parameters

```
alpha = 5;
beta = 1.3;
lambda = 0;
gamma = 0.5;

% plot
figure;
scatter(xdata, p_correct, [], 'filled');
hold on
yL = fun([alpha, beta, lambda, gamma], linspace(xdata(1), xdata(length(xdata))), 100);
plot(xL, yL, 'linewidth', 1.5)
title(sprintf('alpha = %.3f; beta = %.3f\n gamma = %.3f; lambda = %.3f', alpha, beta, gamma, lambda));
```



What predictions can we make about parameter values for each of the 3 groups of subjects?

## Non-parametric bootstrap

Simulate many experiments using the observers data

```
nboots = 500; % number of bootstraps / simulated experiments
b_coeffs = nan(nboots, 4);
for i = 1:nboots
    % resample with replacement
    for ii = 1:length(xdata)
        true_data = Given_rnd.Correct(Given_rnd.Size_cm == xdata(ii))
        sim_corr = datasample(true_data, length(true_data), 'replace')
        sim_correct(ii,1) = sum(sim_corr);
    end
    % fit a psychometric function
    b_coeffs(i,:) = fmincon(@(x) nloglik_1(x(1), x(2), x(3), x(4), x(5)), ...
    end
```

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance



and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance

and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance



and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance

and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

<stopping criteria details>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in all feasible directions, to within the value of the optimality tolerance and constraints are satisfied to within the value of the constraint

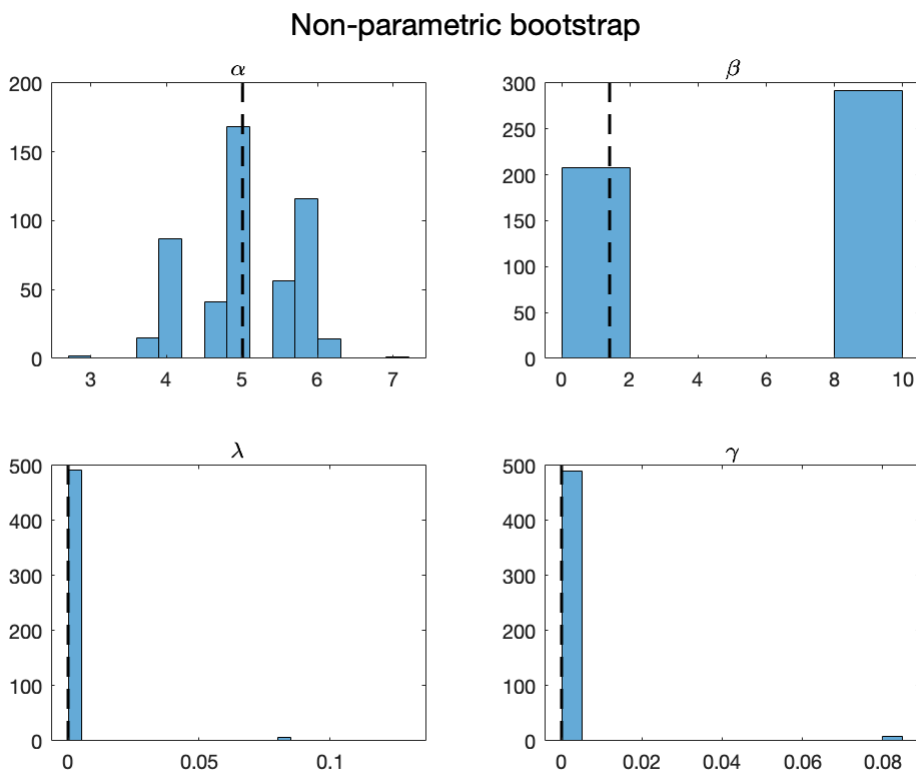
<<a href = "matlab: createExitMsg({'optimlib:sqpLineSearch:Exit1base

```
% Plot distribution for all parameters
figure;
for i = 1:size(b_coeffs,2)
    subplot(2,2,i);
    histogram(b_coeffs(:,i))
    hold on
```

```

if i == 1
    vline(coeffs(1), 'k', '--', 1.5);
    title('\alpha')
elseif i == 2
    vline(coeffs(2), 'k', '--', 1.5);
    title('\beta')
elseif i == 3
    vline(coeffs(3), 'k', '--', 1.5);
    title('\lambda')
else
    vline(coeffs(4), 'k', '--', 1.5);
    title('\gamma')
    sgtitle('Non-parametric bootstrap')
end
end

```



## Parametric Bootstrap

Simulate many experiments using the observers parameters

```
function nll = nloglik_1(alpha, beta, lambda, gamma, signal_intensity)
% INPUTS:
% alpha: threshold / PSE (point of subjective equality)
% beta: slope / rate of change / heat
% lambda: lapse rate (probability of an incorrect response, which is
% independent of stimulus intensity)
% gamma: guess rate (probability of a correct response when the stimulus
% not detected by the underlying sensory mechanism)

% OUTPUT:
% nll: negative log likelihood
```

Deriving negative log likelihood from likelihood of a binomial distribution.

Write out likelihood of a binomial distribution, where  $n$  denotes total number of trials sampled at each stimuli,  $x$  denotes number of correct trials, and  $p$  denotes probabilities from psychometric function.

$$L(p; x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \quad (1)$$

Take the negative log of equation 1.

$$= -\log \left[ \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \right] \quad (2)$$

Distribute the log.

$$= -\log \left( \frac{n!}{x!(n-x)!} \right) + \log(p^x) + \log((1-p)^{n-x}) \quad (3)$$

Note: Log rule:  $\log(a*b) = \log(a) + \log(b)$

Remove constant, distribute exponents and negative sign.

$$= -x \log(p) - (n-x) \log(1-p) \quad (4)$$

Note: Log exponent rule:  $\log(a^b) = b \log(a)$

```
% Create psychometric function
I = gamma + (1 - gamma - lambda) * (1 ./ (1 + exp(-beta*(signal_intensity - alpha))))

% find nll
nll = -sum(correct .* log(I) + (trials - correct) .* log(1 - I));
```

end