On the Distributional Effects of Monetary Shocks and Market Incompleteness*

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September 27, 2024

Abstract

I study the transmission of distortionary monetary policy shocks under incomplete markets. Using a heterogeneous agents general equilibrium model, I demonstrate that there is a unique fundamental stationary equilibrium, where the distribution of monetary holdings mirrors productivity, but infinite non-fundamental stationary equilibria for a given monetary base in the presence of a frictionless bonds market. Only financially constrained economies return to the fundamental stationary equilibrium after an unforeseeable monetary shock that redistributes monetary holdings, with aggregate effects on output and endogenous price stickiness along the transition. In financially developed economies, distortions are smaller, and effects on aggregate variables are negligible, but monetary shocks create hysteresis by making the consequences of idiosyncratic shocks permanent. While partial market completion enhances welfare by enabling nearly perfect risk sharing, this improvement is limited by the irreversibility of the idiosyncratic shocks. Ultimately, distributional effects are irrelevant for monetary policy transmission to aggregate variables in developed economies but critical in poorer countries.

Keywords: Monetary Shocks, Distributional Effects, Market Incompleteness, Credit, Financial Development

JEL Codes: E41, E44, D52, E31, D31

*I thank Burak Uras, Has van Vlokhoven, Jeffrey R. Campbell, Fabio Braggion, Lucas Chang, Hierro Castro, Sebastião Oliveira, Aleksei Nemtyrev, Aleksandr Minchenko, and the seminar participants at the Macro Study Group (Tilburg University) for their helpful comments.

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1 Introduction

There is growing evidence that monetary policy shocks produce distributional effects along several channels. These shocks can affect relative income, benefit debtors to the detriment of creditors, and, more immediately, change asset prices, redistributing wealth between agents who own financial assets and those who do not. This is important given that, even in the U.S., only 21% of households held publicly traded stocks directly, and only 1.1% held bonds in 2022 according to the Survey of Consumer Finances (SCF). Moreover, households with stocks and bonds,[1] directly or indirectly, are nearly twice as rich as the average. Although well-functioning credit markets could protect agents from wealth shocks through risk-sharing, access to credit markets is limited, as roughly 20% of U.S. families lacked a credit card and 10% had their credit request turned down at least once in the twelve months before the 2022 SCF survey.

These distributional effects can affect the transmission of monetary shocks. As Friedman (1969) noted, the random re-shuffling of monetary holdings across agents is incompatible with an immediate return of the economy to the initial equilibrium, even in a fairly frictionless economy. Agents made richer by the money injection will likely smooth consumption, and incomes would adjust to make the economy converge to the long-run equilibrium. As a result, the wealth redistribution led by monetary shocks should produce a sluggish convergence path and transient redistribution of consumption and income especially in the absence of financial tools that allow for risk sharing. I investigate this claim and study how financial development - in the form of access to credit - affects post-shock dynamics.

I develop a fairly tractable general equilibrium model featuring a cash-in-advance friction and a monopolistically competitive goods market. There is perfect foresight, and prices can be changed at the beginning of each period at no cost. Furthermore, for a richer analysis of how the redistribution of monetary holdings affects inequality, I allow agents to differ with respect to productivity, which generates heterogeneity in monetary holdings, consumption, output, and prices. This model allows me to show that the introduction of one-period bonds is enough to significantly alter the post-shock dynamics. Risk-sharing, allowed by these bonds, nearly neutralizes the short-term distributional effects in consumption and, as a result, the aggregate non-neutrality of money, leading to an immediate convergence.

First, I show that there is a unique fundamental stationary equilibrium, that is, a stationary equilibrium in which, given the market structure and consumer preferences, differences in wealth mirror differences in productivity. However, for a given

[1] Excluded retirement funds, savings, and foreign bonds.

monetary base, any redistribution of monetary holdings across agents is compatible with a stationary equilibrium with borrowing from a frictionless bonds market. This is because agents whose assets are too low given their productivity become indebted and find it optimal to roll their debt indefinitely. Likewise, entrepreneurs who own more assets than their income can sustain decide to permanently maintain savings and receive interest payments. This way, one-period bonds work as perpetuities.

Then, I investigate the dynamics after a one-time unforeseeable monetary shock that happens to an economy at the fundamental stationary equilibrium. I show that the distributional effects of monetary shocks indeed induce a more sluggish and distorted return to the fundamental stationary equilibrium in the absence of access to credit markets. On the other hand, and contrary to Friedman's hypothesis, if there is full enforcement of debt repayments, the economy does not return to the initial allocations, and the induced differences in monetary holdings become permanent. This implies that the effect of these shocks over wealth inequality can be persistent.

The model is closely related to the financial segmentation channel, proposed by Williamson (2008). In this paper, he assumes that households are either *connected* or *unconnected* to financial markets, with no possibility of moving between groups. By *connected*, he means that these households operate frequently in financial markets and, hence, are the first to be affected by monetary shocks. This heterogeneity is well-illustrated by the aforementioned low levels of financial asset ownership observed in the U.S. Throughout, I will use the same classification adopted by Williamson (2008).

For a big enough positive (negative) monetary shock, the consumption of connected households increases (decreases) relative to that of the unconnected, but the revenues of the latter are higher (lower). This mechanism produces endogenous convergence to the long-run equilibrium in economies in the absence of borrowing, as unconnected (connected) households gradually appropriate money that was initially idle, linking portfolio-related distributional effects of monetary shocks to the more indirect, general equilibrium, income-related ones. As a result, consumption and wealth inequality move in the opposite direction of income inequality after the shock.

I allow the whole productivity distribution between connected and unconnected agents to differ and assume, in line with the data, that connected agents are more productive on average. This allows me to capture a dimension of the disparity between both types that was unexplored in Williamson (2008). If the monetary shock benefits, on average, poorer agents, it reduces inequality. However, the shock also creates a wedge between connected and unconnected agents *with the same productivity*. This wedge is diminished by the existence of a bonds market, which unambiguously improves welfare by allowing for risk sharing. Moreover, I show that allocative efficiency fluctuates if the connectedness status is correlated with productivity, as the beneficiaries from the shock cut their labor due to the wealth effect.

The primary contribution of the paper is to examine how the redistribution of agents' wealth - unrelated to fundamentals - affects the transmission of monetary policy shocks. I study transitional dynamics between stationary equilibria while allowing for consumption smoothing. I demonstrate how the introduction of a simple one-period bond fundamentally alters the equilibrium and improves welfare, by mitigating distortions through risk-sharing and resolving the idle cash balances problem in spite of leading to hysteresis in post-shock monetary holdings. Additionally, the model generates real effects and endogenous price stickiness through two mechanisms: (i) unconnected agents keep their prices below that of the connected to restore real balances, and (ii) connected agents lower production and, thus, marginal costs after a positive shock.

These findings highlight differences between economies with well- and poorly-developed credit markets. In poorer economies, monetary policy induces stronger distributional effects on consumption and more output volatility. By contrast, widespread access to financial markets reduces distortions and stabilizes output, even though this means subjecting more people to monetary policy risk through their assets, as it makes the shocks look more like helicopter drops. As a result, distributional effects are irrelevant for monetary policy transmission to aggregate variables in rich economies. This emphasizes the importance of developing a well-functioning financial market to ease distortions caused by unforeseeable monetary shocks while balancing short-run stabilization with policies to address long-run inequality.

Lastly, I conduct a series of sensitivity exercises by changing certain assumptions and parameters. First, I consider the cases of a negative monetary shock and CRRA utility. I also study how different Frisch elasticities affect the transmission of the shock. Then, I show that (i) distortions generated by a positive shock decrease on the number of connected agents; (ii) the paths for most variables are explained by the ratio between the size of the individual and aggregate shocks; and (iii) the fraction of connected agents affects the speed of convergence of the bondless economy. I also briefly discuss the model's implications for futures markets.

The paper is organized as follows. In section 2, I develop the baseline model and analytically study its stationary equilibria. In section 3, I first study the post-shock dynamics in the absence of a bonds market. Then, I introduce bonds, analyze both the market equilibrium and the case of an interest rate endogenously set to zero, and perform a welfare analysis. Section 4 presents the robustness/sensitivity analysis. Lastly, section 5 concludes. All proofs are presented in Appendix A, while Appendix B contains further graphs, and Appendix C, outstanding tables.

1.1 Related Literature

This paper contributes to two strands of the literature. First, it relates to the literature that studies the distributional effects of monetary policy shocks. These shocks have been shown to differentially affect agents based on their income compositions, portfolios, financial market participation and skill level (Hohberger et al.,2020; Coibion et al., 2017; Dolado et al., 2021). Distributional effects may also arise from the heterogeneity of price adjustment (Cravino et al., 2020; Baqaee et al., 2022), risk sharing mechanisms (Berentsen et al., 2007; Rocheteau et al., 2018), and the regressive interaction between inflation tax and economies of scale in credit transactions (Erosa and Ventura, 2002).

The sign of these channels' net effect is an empirical question. Some studies show that consumption and income inequality historically followed unexpected contractionary monetary shocks (Coibion et al., 2017; Furceri et al., 2018), while others find the opposite both in the case of conventional (Davtyan, 2016) and unconventional (Montecino and Epstein, 2015) monetary policy.

Ampudia et al. (2018) distinguish between direct and indirect, general-equilibrium, channels of monetary policy's distributional effects. In my framework, the direct channels are modeled in reduced form through the financial segmentation channel proposed by Williamson (2008), which can be generalized to include wealth effects due to portfolio revaluations driven by monetary policy. These channels have been shown to produce significant distributional effects in several empirical studies (*e.g.*, Doepke and Schneider, 2006; Saiki and Frost, 2014; Ampudia et al., 2018; Auclert, 2019).

The literature on indirect channels has primarily focused on income-related channels, including either wages, capital returns, skill premium, or employment fluctuations (*e.g.*, Gornemann et al., 2016; Coibion et al., 2017; Dolado et al., 2021; Casiraghi et al., 2018). My contribution lies in integrating these direct and indirect channels by modeling the latter as an endogenous response to the former in a way that restores the original equilibrium, as in Friedman (1969)

Two papers in this literature closely related to mine are Williamson (2008) and Grossman and Weiss (1983). My model differs from that of the former by assuming away market segmentation[2] and perfect competition, and allowing for consumption

[2] In his paper, not only financial markets are segmented, but, moreover, connected agents trade *mostly* among themselves in a competitive goods market. The same goes for the unconnected, who also have a partially separate market of their own.

smoothing and borrowing between connected and unconnected,[3] which is central to my analysis. Grossman and Weiss (1983) model open market operations by assuming that agents withdraw money from the bank in alternate periods, which benefits those who can withdraw at a low interest rate in the period when the shock occurs. Their model displays an oscillatory and dampening path for prices and the interest rate toward the long-run equilibrium, with money flowing from shock beneficiaries to the others. Relative to theirs, my framework endogenizes aggregate output response and the mechanisms that lead to long-run convergence, and creates a role for a credit market in channeling idle cash to cash-constrained agents, offering a more general view on monetary distributional effects.

Finally, I also contribute to the literature on incomplete markets, particularly on the role of debt. In the New-Monetarist tradition (Kiyotaki and Wright, 1993; Lagos and Wright, 2005), imperfect credit markets make money essential for transactions. Coupled with idiosyncratic shocks, this leads to idle balances (Rocheteau et al.,2018), which, in the presence of a banking sector, can be reallocated from agents with low marginal utility to those with a higher one (Berentsen et al., 2007). In my framework, as in Berentsen et al. (2007), interest payments to creditors improve allocations, but here, the liquidity provided to cash-constrained households further enhances welfare.

My framework also relates to Eggertsson and Krugman (2012), who show that a rapid tightening of borrowing requirements generates distributional effects and depresses aggregate demand, creating a role for fiscal policy to alleviate debt burdens. In both their model and mine, financial imperfections amplify distributional effects and slow down convergence to the long-run equilibrium. However, while their shocks originate in the credit market, mine are external. This allows me to focus on the role of credit markets in allowing for risk sharing, which generates non-trivial implications.

Access to liquidity through a frictionless credit market goes a long way in completing the market as in Telmer (1993), practically undoing the heterogeneity in consumption. I extend this literature by developing a tractable model that allows for the analytical characterization of the post-shock dynamics with and without a well-functioning credit market. Furthermore, I show that the presence of such a market can introduce hysteresis in post-shock monetary holdings, which might significantly dampen the welfare improvement brought about by financial development. This effect is, to the best of my knowledge, novel to the literature.

[3] In his framework there is a credit market, but only connected agents have access to it.

2 The Model

Consider an economy with a continuum of entrepreneurs with unit mass, who differ in their time-invariant productivity, z. The productivity follows a cumulative distribution function $\mathbb{F}(\cdot)$. Every entrepreneur produces an intermediate good through her own work and derives utility from the consumption of the final good. There is also a final good firm, which operates in a competitive market and produces a composite final good out of the intermediate goods produced by the entrepreneurs. Moreover, I assume that there is a market for riskless one-period pure-discount bonds.

I assume that entrepreneurs have identical preferences. Moreover, as in Williamson (2008), I define *connected agents* as those who frequently trade in financial markets, being, therefore, directly affected by monetary shocks. They correspond to a fraction $\eta \in (0, 1)$ of the population, and their productivity is distributed according to the c.d.f. $F_c(\cdot)$. *Unconnected agents* are, naturally, affected indirectly by monetary shocks and correspond to a fraction $1 - \eta$ of the population. Their productivity is distributed according to the c.d.f. $F_u(\cdot)$. Naturally, $\mathbb{F}(z) = \eta F_c(z) + (1 - \eta)F_u(z)$. Finally, I denote $\eta_i = \eta$ for $i = c$ and $\eta_i = 1 - \eta$ for $i = u$. For simplicity, I assume a common support for these distributions, and that connectedness status is fixed for each agent.

The timing of the model goes as follows:

1. All bonds, $b_{it}(z)$, purchased in the previous period reach maturity;

2. The entrepreneur with productivity z and connectedness status $i \in \{c, u\}$ starts with $m^-_{it}(z)$ units of money, and connected agents may receive an unanticipated and unforeseeable transfer (tax) $\tau m^-_{ct}(z)$ from the government, financed through money creation (destruction). Thus, $m_{it}(z) = (1 + \mathbb{1}_{i=c}\tau)(m^-_{it}(z) + b_{it}(z))$, where $\mathbb{1}_{i=c} = 1$ when the agent is connected, and $\mathbb{1}_{i=c} = 0$ otherwise;

3. The entrepreneur sets a price $p_{it}(z)$ for the good she produces. Given these prices, the final goods firm buys on credit the output of each entrepreneur, $y_{it}(z)$, and produces thereby a composite good Y_t;

4. Each entrepreneur then decides, given P_t, how much of $m_{it}(z)$ to spend on consumption, $C_{it}(z)$, how much to save as idle cash balances, $s_{it}(z) \geq 0$, and how much to save in terms of bonds. Agents have access to a bonds market where they can buy/sell bonds at a price q_t.

5. The final goods firm pays the entrepreneurs for the purchases made in the same period with the money obtained through sales.

6. Sales revenues and the money unspent in the period will sum up to $m^{-}_{i,t+1}(z)$.

As usual, if the bond is bought, $b_{it}(z) > 0$; if it is sold, $b_{it}(z) < 0$. Moreover, monetary shocks are assumed to become immediately known by everyone whenever they take place, that is before agents make any pricing and production decision for that period. Importantly, this timing implies that *entrepreneurs cannot benefit from current sales* because purchases are only paid for at the end of the period. Thus, there is a cash-in-advance (CIA) friction. Moreover, there is imperfect competition in the intermediate goods market, which allows for pricing decisions. I also assume that the monetary shock each entrepreneur receives is proportional to their current monetary holdings.[4] Importantly, as long as $\eta < 1$, we do not have helicopter drops of money.

Since $q_t > 1$ is not possible, $s_{i,t}(z) > 0$ can only happen if: 1) $q_t = 1$ or 2) the bonds market is shut down. I will also assume, henceforth, that, if the nominal interest rate is equal to zero, that is, $q_t = 1$, all the savings will still take place through the bonds market. Hence, savings will never take the form of idle money if agents can buy and sell bonds. This can be rationalized as being the only choice that is robust to small upward trembles in the real interest rate, which would always produce $s_{it}(z) = 0$ for any arbitrary $z \in [\underline{z}, \overline{z}]$.

The problem of an entrepreneur with productivity z and connectedness status $i \in \{c, u\}$ is:

$$\max_{\{C_{it}(z), m^{-}_{i,t+1}(z), h_{it}(z), p_{it}(z), b_{i,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(C_{it}(z)) - \gamma \frac{h_{it}(z)^{1+\zeta}}{1+\zeta} \right] \tag{1}$$

$$\text{subject to} \quad P_t C_{it}(z) + q_t b_{i,t+1} \leq m_{it}(z) \tag{2}$$

$$m^{-}_{i,t+1}(z) + P_t C_{it}(z) + q_t b_{i,t+1}(z) \leq m_{it}(z) + p_{it}(z) y_{it}(z) \tag{3}$$

$$m_{i,t+1}(z) = m^{-}_{i,t+1}(z) + b_{i,t+1}(z) \tag{4}$$

$$y_{it}(z) = z h_{it}(z) \leq D(p_{it}(z), P_t, Y_t) \tag{5}$$

$$b_{i,t+1}(z) \geq -l_t(z, m_{it}(z)) \tag{6}$$

$$C_{it}(z) \geq 0 \tag{7}$$

where $h_{it}(z)$ is labor; $D(p_{it}(z), P_t, Y_t)$ is the demand faced given the chosen price, $p_{it}(z)$; and $l_t(z, m_{it}(z)) \geq 0$ is the borrowing limit. Moreover, let $R_{it}(z) := p_{it}(z) y_{it}(z)$ be the entrepreneur's revenue. As usual, I assume that the utility function satisfies $u \in \mathcal{C}^2$, $u'(\cdot) > 0$, and $u''(\cdot) < 0$. I assume isoelastic labor disutility for the

[4] I make this assumption for two reasons. First, it is fairly tractable given the pre-existing heterogeneity in cash holdings. Second, it seems more plausible to assume that monetary shock affects agents proportionally. For example, if a fall in the interest rate increases the price of a connected agent's assets, this valuation shock should be proportional to their asset holdings.

sake of tractability and that $\zeta \geq 0$. Also for simplicity, I assume linear technology, i.e. $y_{it}(z) = zh_{it}(z)$. Moreover, notice that transfers enter implicitly in the CIA constraint (2), since, if an agent receives the transfers at the beginning of time t, $m_{ct}(z) = (1+\tau)m^{-}_{ct}(z)$. Lastly, given that monetary shocks are unforeseeable, agents assume that $m^{-}_{i,t+1}(z) = m_{i,t+1}(z)$. The final good's firm faces the static problem:

$$\max_{\{y^D_{it}(z)\}_{z\in[\underline{z},\overline{z}],i\in\{c,u\}}} \quad Y_t = \left(\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} y^D_{it}(z)^{\frac{\epsilon-1}{\epsilon}} dF_i(z) \right)^{\frac{\epsilon}{\epsilon-1}} \tag{8}$$

$$\text{subject to} \quad P_t Y_t = \sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} p_{it}(z) y^D_{it}(z) dF_i(z) \tag{9}$$

$$y_t(z) \geq 0 \quad \forall z \in [\underline{z}, \overline{z}] \tag{10}$$

where $y^D_{it}(z)$ is the demand for the intermediate good produced by the entrepreneur with productivity z and connectedness status $i \in \{c,u\}$. Moreover, I assume that $\epsilon > 1$. Finally, I define $\mathbb{T}$ as the set of periods at which the economy finds itself in a particular equilibrium path, and $t_0 := \min \mathbb{T}$. Then, the equilibrium path of this economy is defined as follows:

Definition 1 (Equilibrium path). *An equilibrium path is a series of intermediate goods, final good and bond prices $\{\{p_{it}(z)\}_{z\in[\underline{z},\overline{z}]}, P_t, q_t\}_{i\in\{c,u\},t\in\mathbb{T}}$, individual consumption bundles $\{C_{it}(z)\}_{z\in[\underline{z},\overline{z}],i\in\{c,u\},t\in\mathbb{T}}$, intermediate and final good outputs $\{\{y_{it}(z)\}_{z\in[\underline{z},\overline{z}]}, Y_t\}_{i\in\{c,u\},t\in\mathbb{T}}$, individual labor $\{h_{it}(z)\}_{z\in[\underline{z},\overline{z}],i\in\{c,u\},t\in\mathbb{T}}$, individual monetary holdings and monetary base $\{\{m_{it}(z)\}_{z\in[\underline{z},\overline{z}]}, M_t\}_{i\in\{c,u\},t\in\mathbb{T}}$, and net bonds holdings $\{b_{i,t+1}(z)\}_{z\in[\underline{z},\overline{z}],i\in\{c,u\},t\in\mathbb{T}}$ such that, for every $t \in \mathbb{T}$:*

1. *Given $\{q_t, P_t\}_{t\in\mathbb{T}}$ and the initial $m_{t_0}(z)$, $\{C_{it}(z), p_{it}(z), y_{it}(z), h_{it}(z), b_{i,t+1}(z)\}_{t\in\mathbb{T}}$ solve the problem (1) of the entrepreneur with $z \in [\underline{z}, \overline{z}]$ and $i \in \{c,u\}$;*

2. *Given prices for the intermediate and final goods, $\{\{p_{it}(z)\}_{z\in[\underline{z},\overline{z}]}, P_t\}_{i\in\{c,u\},t\in\mathbb{T}}$, intermediate goods demand and final good output $\{\{y^D_{it}(z)\}_{z\in[\underline{z},\overline{z}]}, Y_t\}_{i\in\{c,u\},t\in\mathbb{T}}$ solve the problem (8) of the final good firm;*

3. *The intermediate goods markets clear, i.e. $y^D_{it}(z) = y_{it}(z)$ for $z \in [\underline{z}, \overline{z}]$, $i \in \{c,u\}$;*

4. *The final good's market clears, i.e. $\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} C_{it}(z) dF_i(z) = Y_t$;*

5. *The bonds market clears, i.e. $\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} b_{i,t+1}(z) dF_i(z) = 0$;*

6. *The monetary base is owned by entrepreneurs, i.e. $\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} m_{it}(z) dF_i(z) = M_t$.*

2.1 Solution

The solution to the problem of the final good firm, (8), takes the standard form:

$$D(p_{it}(z), P_t, Y_t) = \left(\frac{p_{it}(z)}{P_t}\right)^{-\epsilon} Y_t \qquad \forall z \in [\underline{z}, \overline{z}] \tag{11}$$

and the final good price is given by:

$$P_t = \left(\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} p_{it}(z)^{1-\epsilon} dF_i(z)\right)^{\frac{1}{1-\epsilon}}, \tag{12}$$

which I will, henceforth, refer to as aggregate price. The problem of the entrepreneur with productivity $z \in [\underline{z}, \overline{z}]$ and connectedness status $i \in \{c, u\}$, (1), yields the consumption schedule:

$$C_{it}(z) \begin{cases} = (u')^{-1}\left(\beta \frac{P_t}{P_{t+1}} u'(C_{i,t+1}(z))\right) & \text{if } s_{i,t}(z) > 0 \\ = \frac{m_{it}(z)}{P_t} & \text{if } s_{i,t} = 0 \text{ and } b_{i,t+1}(z) = 0 \\ \leq (u')^{-1}\left(\frac{\beta}{q_t} \frac{P_t}{P_{t+1}} u'(C_{i,t+1}(z))\right) & \text{otherwise} \end{cases}$$

where $(u')^{-1}(\cdot)$ is the inverse of the marginal utility function. This function is well defined because $u'(\cdot)$ is injective and continuous. The cases above correspond, respectively, to 1) partial depletion (that is, the monetary holdings are not fully spent) with idle cash, 2) full depletion (the consumer spends all her money currently), and 3) partial depletion with non-zero bond holdings. Notice that the first case can only happen if the bonds market is shut down, which corresponds to a fully imperfect financial system. Moreover, the strict inequality in the third case is satisfied with equality if, and only if, (6) binds. Furthermore, the price chosen by the entrepreneur is given by:

$$p_{it}(z) = \underbrace{\left(\frac{\epsilon}{\epsilon - 1}\right)}_{\text{(i)}} \underbrace{\frac{\gamma h_{it}(z)^{\zeta}}{z}}_{\text{(ii)}} \underbrace{\frac{P_{t+1}}{\beta u'(C_{i,t+1}(z))}}_{\text{(iii)}} \tag{13}$$

This equation implies that there is a markup, (i), over marginal costs, (ii), and a forward-looking component, (iii), to pricing decisions. To understand the intuition behind this equation, notice that revenues affect how much money the entrepreneur carries to the next period. The value of this money is the value of relaxing the future budget constraint, which is directly related to the marginal utility and the aggregate price in the next period. Whenever consumption will be large in the future, the value of relaxing the next period's budget constraint by carrying more money to the future is lower. Hence, the agent will choose a relatively high price to balance the intertemporal trade-off towards lower current labor disutility. Furthermore, the value of relaxing the budget constraint in the future is decreasing on future aggregate prices. Now, I define the revenue of an arbitrary entrepreneur relative to the average revenue as:

$$\theta_{it(z)} := \frac{p_{it}(z)D(p_{it}(z), P_t, Y_t)}{P_t Y_t} = \left(\frac{p_{it}(z)}{P_t}\right)^{1-\epsilon}, \tag{14}$$

which can be interpreted as the equivalent of a market share in the continuous case, since $R_{it}(z) = \theta_{it}(z)M_t$. It takes values $\theta_{it}(z) \in (0,1)$ if the revenue of the entrepreneur with productivity $z \in [\underline{z}, \overline{z}]$ and connectedness status $i \in \{c,u\}$ is below the average revenue, $\theta_{it}(z) = 1$ if it is equal to average, and $\theta_{it}(z) > 1$ if it is larger. I study now the stationary equilibria of this economy.

2.2 The Stationary Equilibrium

I now define a stable price stationary equilibrium. The term "stable price" aims to restrict attention to stationary equilibria where the monetary base is constant. In these monetary equilibria, $m_{it}(z) = m_{i,t+1}(z)$ and $p_{it}(z) = p_{i,t+1}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{c,u\}$, $M_t = M_{t+1}$ and $P_t = P_{t+1}$. Let $\mathbb{T}^S$ be the set of periods in which the economy is at this kind of equilibrium. I define it as follows:

Definition 2 (Stable price stationary equilibrium). *A stable price stationary equilibrium for this economy is a series of prices* $\{\{p_{it}(z)\}_{z\in[\underline{z},\overline{z}]}, P_t, q_t\}_{i\in\{c,u\},t\in\mathbb{T}^S}$*, consumption, labor and output allocations* $\{\{C_{it}(z), h_{it}(z), y_{it}(z)\}_{z\in[\underline{z},\overline{z}]}\}, Y_t\}_{i\in\{c,u\},t\in\mathbb{T}^S}$*, and bond holdings* $\{b_{i,t+1}(z)\}_{z\in[\underline{z},\overline{z}],i\in\{c,u\},t\in\mathbb{T}^S}$ *which, given* $\tau = 0$ *for and* $t \in \mathbb{T}^S$*, solve* (1) *and* (8) *and make* $m_{it}(z) = m_{i,t+1}(z)$ *for every* $z \in [\underline{z}, \overline{z}]$ *and* $i \in \{c,u\}$ *and, thus,* $M_t = M_{t+1}$*.*

Notice that the above definition implies that real allocations and relative prices are constant in all periods in stable price stationary equilibria. Moreover, since all the stationary equilibria studied throughout the paper are stable price, I will, henceforth, call them simply "stationary equilibria". As will be shown in Proposition 1, there are infinite such equilibria that are compatible with a given monetary base, M_t. I will, therefore, refine this concept further by defining *fundamental stationary equilibria* as:

Definition 3 (Fundamental stationary equilibrium). *A fundamental stationary equilibrium for this economy is a stable price stationary equilibrium where* $m_{it}(z) = R_{it}(z)$ *for every* $z \in [\underline{z}, \overline{z}]$*,* $i \in \{c,u\}$ *and* $t \in \mathbb{T}^S$*.*

A fundamental stationary equilibrium is a stationary equilibrium where differences in monetary holdings across agents reflect differences in their fundamentals, which boil down to differences in productivity given our assumptions. Intuitively, agents are just as rich as their capacity to make money allows. As a result, for every $z \in [\underline{z}, \bar{z}]$ and $t \in \mathbb{T}^S$, $m_{ct}(z) = m_{ut}(z)$, that is, connected and unconnected agents with the same productivity have the same monetary holdings. The proposition below confirms that such a fundamental stationary equilibrium exists and is unique.

Proposition 1. *There is a unique fundamental stationary equilibrium, which requires that $b_{i,t+1}(z) = 0$ for all $z \in [\underline{z}, \bar{z}]$, $i \in \{c, u\}$ and $t \in \mathbb{T}^S$. Moreover, given the fundamental stationary equilibrium distribution of monetary holdings, it is the only possible equilibrium. Lastly, for any function $m_{ct}(\cdot) > 0$ and $m_{lt}(\cdot) > 0$ defined over the domain $[\underline{z}, \bar{z}]$ and satisfying $\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\bar{z}} m_{it}(z) dF_i(z) = M_t$, if $l_t(z, m_{it}(z)) = R_{it}(z)$ for all $z \in [\underline{z}, \bar{z}]$ and $i \in \{c, u\}$, there is a unique (non-fundamental) stationary equilibrium with borrowing compatible with it. This equilibrium requires $q_t = \beta$ for every $t \in \mathbb{T}^S$.*

Apart from the uniqueness of the fundamental stationary equilibrium, Proposition 1 implies that infinite non-fundamental stationary equilibria exist for any given monetary base, as long as the borrowing limit requires only that agents can repay their debt at the beginning of the next period. Any distribution of money is made permanent in such a stationary equilibrium through borrowing. However, the equilibrium is unique for any given distribution. Thus, without financial frictions, no mechanism ensures convergence to a stationary equilibrium where differences in monetary holdings, allocations, and prices fully reflect fundamentals.

The intuition is that indebted agents indefinitely roll over their debt and pay the interest with their sales revenue. Hence, these one-time bonds work just like perpetuities. To put it simply, under the natural borrowing limit $l_t(z, m_{it}(z)) = R_{it}(z)$, which ensures the capacity debt repayment, consumption should either decrease, increase or stay constant for *all* agents according to their Euler equation. Since the monetary base is constant after the shock, this requires constant consumption. Additionally, the amount of cash borrowed is always strictly below the amount of cash that the indebted entrepreneur will earn from sales. The next section investigates the case of an MIT monetary shock to an economy that starts at the fundamental stationary equilibrium.

3 Transition Dynamics

In what follows, I consider an economy that starts at the fundamental stationary equilibrium at $t = 0$ and receives an MIT monetary shock at $t = 1$ in the fashion described at the beginning of section 2. Whenever I refer to the stationary equilibrium, I use the notation X_0 for $X \in \{Y, P, M\}$ and $x_0(z)$ for $x \in \{m, b, C, p, y, h, R, \theta\}$, which does not depend on connectedness status by the definition of fundamental stationary equilibrium. I start by studying a baseline economy with perfect financial frictions and, only after, allow the bonds market to become operational to study how the transmission of the shock is affected by financial development.

3.1 Baseline Economy

In the baseline economy, I assume that no borrowing can take place, *i.e.*, $l_t(z, m_{it}(z)) = 0$ for $z \in [\underline{z}, \bar{z}]$ and $i \in \{c, u\}$. This amounts to shutting off the bond market completely. I assume that the central bank introduces (withdraws) $\eta\tau M_{c0} > 0$ (< 0) units of money in the economy at $t = 1$, where $M_{c0} := \int_{\underline{z}}^{\bar{z}} m_0(z) dF_c(z)$ is the average monetary holdings of connected agents in the stationary equilibrium. Moreover, let $\tau^A := \eta\tau M_{co}/M_0$ be the proportional aggregate shock. I also define M_t^C as the money in circulation at time t, that is, the amount of money that is demanded in the economy for transaction motive, and $M_t = (1 + \tau^A)M_0$ is the monetary base. I now consider an economy that receives a monetary shock operated through helicopter drops.

3.1.1 Helicopter Drops Of Money

When helicopter drops take place, each agent gets a proportional $\tau_H = \tau^A$ over their monetary holdings. It is easy to see that the only possible equilibrium is one in which all agents fully deplete their money. Essentially, relative monetary holdings are not distorted by the shock since every agent gets the same proportional shock. Thus, the economy goes immediately to the new fundamental stationary equilibrium, in which the monetary base is $M_t = (1 + \tau^A)M_0$ for $t \in \{1, 2, ...\}$. The corollary below formalizes that. The proof can be found in Appendix A.

Corollary 1.1. *After helicopter drops of money, the economy goes immediately to the new fundamental stationary equilibrium.*

Since all agents fully deplete their resources, prices are given by:

$$P_t^H = (1 + \tau^A)P_0 \qquad \text{and} \qquad p_t^H(z) = (1 + \tau^A)p_0(z) \tag{15}$$

for $z \in [\underline{z}, \bar{z}]$, $i \in \{c, u\}$ and $t = \{1, 2, ...\}$, where the H superscript refers to the "helicopter drops equilibrium", and consumption is identical to the consumption level in the initial stationary equilibrium level, that is, $C_{it}^H(z) = C_0(z)$ for $t = \{1, 2, ...\}$. Thus, a monetary shock implemented through helicopter drops is neutral, since prices immediately rise uniformly and enough to put the economy at the new fundamental stationary equilibrium already at $t = 1$.

3.1.2 Uneven Access To The New Money

Now, I assume that the connected agents are the first to have their monetary holdings affected by the monetary shock. With some abuse of notation, I denote the agents who are made richer, in relative terms, by the monetary shock as *high-cash* and the ones that are made relatively poorer as *low-cash*. I denote the former with subscript h and the latter with subscript l. Naturally, if $\tau > 0$, then $h = c$, $m_{h1}(z) = m_{c1}(z) = (1+\tau)m_0(z)$ and $m_{l1}(z) = m_{u1}(z) = m_0(z)$ for any arbitrary $z \in [\underline{z}, \overline{z}]$; whereas $h = u$, $m_{l1}(z) = m_{c1}(z) = (1+\tau)m_0(z)$ and $m_{h1}(z) = m_{u1}(z) = m_0(z)$ when $\tau < 0$. Moreover, I define:

$$\overline{U}_t^{GAP} = \left(\frac{\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\epsilon-1} \frac{u'(C_{it}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}} dF_i(z))}{\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\epsilon-1} \frac{u'(C_{i0}(z))^{\epsilon-1}}{h_{i0}(z)^{\zeta(\epsilon-1)}} dF_i(z)} \right)^{\frac{1}{\epsilon-1}}, \tag{16}$$

which captures the deviation, at a given moment, of a kind of weighted mean of the ratio of marginal utility of consumption over marginal labor disutility relative to the stationary equilibrium value, where the weights are a function of productivity. This expression is well-defined since the utility function is assumed to be equal for all agents, allowing for comparison across them. It will be useful to also define an analogous individual-level measure as:

$$U_{it}^{GAP}(z) = \left(\frac{z^{\epsilon-1} \frac{u'(C_{it}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}}}{z^{\epsilon-1} \frac{u'(C_{i0}(z))^{\epsilon-1}}{h_{i0}(z)^{\zeta(\epsilon-1)}}} \right)^{\frac{1}{\epsilon-1}}. \tag{17}$$

In the following proposition, I characterize the dynamics after the shock.

Proposition 2. *After a monetary shock. there is a certain time $T < \infty$ in which the economy converges to the new fundamental stationary equilibrium. For $t = \{T, T+1, ...\}$, $p_{it}(z) = p^H(z, (1+\tau^A)M_0)$ for $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$, $P_t = P^H((1+\tau^A)M_0)$, and $Y_t = Y_0$. Moreover, for $t = \{T+1, T+2, ...\}$, $C_{it}(z) = C_{i0}(z)$ and $m_{it}(z) = (1+\tau^A)m_0$ for all $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$. Along the transition, that is, for $t = \{1, ..., T-1\}$, we have $C_{lt}(z) \leq C_{ht}(z)$, $\theta_{lt}(z) \geq \theta_{ht}(z)$, $p_{lt}(z) \leq p_{ht}(z)$, $y_{lt}(z) \geq y_{ht}(z)$, $R_{lt}(z) \geq R_{ht}(z)$, $m_{l,t+1}(z) \leq m_{h,t+1}(z)$, and $m_{h,t+1}(z) - m_{ht}(z) \leq m_{l,t+1}(z) - m_{lt}(z)$ for all $z \in [\underline{z}, \overline{z}]$, with strict inequality whenever the high-cash agent does not fully deplete. Moreover, low-cash agents are always more likely to fully deplete than their high-cash counterparts. Finally:*

$$1 + \pi_{t+1} = \overline{U}_{t+1}^{GAP}, \tag{18}$$

and

$$\frac{\theta_{it}(z)}{\theta_{i0}(z)} = \left(\frac{1}{1+\pi_{t+1}}\right)^{\epsilon-1} \frac{\frac{u'(C_{i,t+1}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}}}{\frac{u'(C_{i0}(z))^{\epsilon-1}}{h_{i0}(z)^{\zeta(\epsilon-1)}}} = \left(\frac{U_{i,t+1}^{GAP}(z)}{\overline{U}_{t+1}^{GAP}}\right)^{\epsilon-1}. \tag{19}$$

Proposition 2 implies that the economy eventually reaches the new fundamental stationary equilibrium, where the allocation is identical to the initial one, but prices are different. To understand this, notice that (19) means that the "market share" of a given entrepreneur, $\theta_{it}(z)$, will be higher (lower) whenever her ratio of marginal utility of consumption to labor disutility in the next period will be higher (lower) than the weighted mean, $\overline{U}_{t+1}^{GAP}$. This means that *artificially* poorer agents - *i.e.*, whose real balances fall below its fundamental level - will *tend* to have a higher market share than their fundamentals would suggest[5].

This result follows from the fact that lower future consumption increases the value of holding money, encouraging poorer agents to choose lower prices and work harder than their high-cash counterparts. The resulting increase in the marginal disutility of labor partially offsets the effect of lower consumption. As a result, price and output differences between agents with the same productivity arise due to a wealth effect, which gradually reduces differences in monetary holdings across high- and low-cash agents along the transition path. This is, in essence, the mechanism described in Friedman (1969), and it gradually reduces inequality, as captured by the faster growth of the low-cash agents' monetary holdings compared to their high-cash counterparts, *i.e.* $m_{h,t+1}(z) - m_{ht}(z) \leq m_{l,t+1}(z) - m_{lt}(z)$.

Finally, (18) suggests that inflation (or deflation) persists as long as the weighted mean marginal utility deviates from its fundamental stationary equilibrium level. When distortions are large, inflation is higher. For instance, if $\overline{U}_{t+1}^{GAP} > 1$, the marginal future value of money is relatively high on average, meaning that prices would tend to be *artificially* low - *i.e.*, lower than their final stationary equilibrium level - since many agents accumulate cash to replenish their real balances. As distortions shrink, future prices will be closer to their final level than current prices, resulting in inflation.

For the sake of tractability[6], I assume, henceforth, a logarithmic utility function, *i.e.* $u(\cdot) = \log(\cdot)$. Before I characterize the fundamental stationary equilibrium under the new specification, I define, respectively, the following aggregate and connectedness

[5]What is meant with the word "tend" here is that some artificially poorer agents can still have a lower market share than in the stationary equilibrium if the rise in her future marginal utility is still lower than the rise in the weighted mean, $\overline{U}_{t+1}^{GAP}$, especially if marginal labor goes up enough. As will be shown later, log-utility (and homothetic utility functions in general) rules that possibility out.

[6]The tractability arises mainly due to the elimination of difficulties related to potential non-homothety. However, the property that income and substitution effects cancel out also helps in the analytical proofs but does not seem essential for obtaining the results, as is shown in section 4.

status-specific measures of productivity:

$$\mathcal{Z} := \left(\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\frac{\epsilon-1}{\epsilon}} dF_i(z) \right)^{\frac{\epsilon}{\epsilon-1}} \qquad \text{and} \qquad \mathcal{Z}_i := \left(\int_{\underline{z}_i}^{\bar{z}_i} z^{\frac{\epsilon-1}{\epsilon}} dF_i(z) \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{20}$$

where $i \in \{c, u\}$. The characterization of the log-specific fundamental stationary equilibrium follows as a corollary to Proposition 1.

Corollary 1.2. *In the case of log-utility, for every $t \in \mathbb{T}^S$, aggregate output and final good prices are given by:*

$$Y_t = \mathcal{Z} \left(\frac{\epsilon - 1}{\epsilon} \right) \frac{\beta}{\gamma} \qquad\qquad P_t = \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} \frac{1}{\mathcal{Z}} M_t \tag{21}$$

and individual revenue relative to the average, monetary holdings, consumption, and prices are given by:

$$\theta_{it}(z) = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}^{\frac{\epsilon-1}{\epsilon}}} \qquad\qquad m_{it}(z) = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}^{\frac{\epsilon-1}{\epsilon}}} M_t \tag{22}$$

$$C_{it}(z) = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}^{\frac{\epsilon-1}{\epsilon}}} Y_t \qquad\qquad p_{it}(z) = \left(\frac{z}{\mathcal{Z}} \right)^{-\frac{1}{\epsilon}} P_t \tag{23}$$

for an arbitrary $z \in [\underline{z}, \bar{z}]$

Corollary 1.2 shows that changes in the money supply are fully absorbed into prices, making output is constant across these equilibria[7]. Moreover, notice that $p_{it}(z)$ and P_t at the stationary equilibrium are a function of money supply, which in this case equals M_0. Whenever convenient, I will denote this dependence of prices on the money supply explicitly, by writing $p_i(z, M_t)$ and $P(M_t)$. Furthermore, notice that, for an arbitrary $z \in [\underline{z}, \bar{z}]$, $\theta_t(z), m_{it}(z), C_{it}(z)$ and $p_{it}(z)$ are rescaled versions of their average counterparts, where the rescaling factor depends only on z. Thus, these stationary equilibria reflect fundamentals as more productive entrepreneurs have higher revenues, monetary holdings and consumption, and lower prices than the average.

3.1.3 Immediate convergence to the stationary equilibrium

In this subsection, I study under what circumstances high-cash agents fully deplete their money already at $t = 1$. Notice that, in this situation, since $s_{it}(z) = 0$ for all $z \in [\underline{z}, \bar{z}]$ and $i \in \{c, u\}$, high-cash agents will have at $t = 2$ only the amount of money

[7]This feature is common to all fundamental stationary equilibria, being, thereby, not specific to the logarithmic functional specification of $u(\cdot)$.

that they manage to obtain through selling their products in the market at $t = 1$, as is the case for their low-cash counterparts. By (13), the price choice does not differ between equally productive high- and low-cash agents, whereby they appropriate, each, $R_1(z) = (1+\tau^A)m_0(z)$. Therefore, individual and aggregate prices are identical to the ones in the helicopter drop case. Output also satisfies $Y_t = Y_0$.

Notice that, by the buyer's first order condition, full depletion happens when:

$$\frac{1}{\beta}\left(1+\tau^A\right)m_0(z) = \frac{1}{\beta}P_2C_{h2} \geq P_1C_{h1} = m_{h1} \tag{24}$$

Condition (24) implies that high-cash agents fully deplete their money if:

$$|\tau| = \tau \leq \frac{1-\beta}{\beta - \eta\frac{M_{c0}}{M_0}} \qquad \text{if } \tau > 0 \tag{25}$$

$$|\tau| = -\tau \leq \frac{1-\beta}{\eta\frac{M_{c0}}{M_0}} \qquad \text{if } \tau > 0 \tag{26}$$

This means that the high-cash agents will fully deplete their money holdings if, and only if, the monetary shock is "low enough". Notice that (25) is only defined for $\eta < \beta\frac{M_0}{M_{co}}$. To understand why, notice that, if $\eta \in \left[\beta\frac{M_0}{M_{c0}}, 1\right]$, then:

$$\frac{1}{\beta}\left(1+\tau^A\right)m_0(z) > (1+\tau)m_0$$

which means that the full depletion condition is satisfied for any value of τ and for all $z \in [\underline{z}, \bar{z}]$. Intuitively, this means that the fall in monetary holdings from one period to the next when the connected agent fully depletes her cash holdings is too small to encourage her to save a positive amount. Thus, henceforth, I shall assume that $\eta \in \left(0, \beta\frac{M_0}{M_{co}}\right)$, which is the most interesting case.

When (24) is satisfied, the economy is at the new equilibrium from period $t = 2$ onwards. However, there are important distributional effects at the period $t = 1$. High-cash and low-cash agents' consumption is given, respectively, by:

$$C_{i1}(z) = C_0(z)\left(\frac{m_{i1}(z)}{(1+\tau^A)m_0(z)}\right) \quad fori \in \{c, u\}.$$

with $C_{h1}(z) > C_0(z)$ and $C_{l1}(z) < C_0(z)$. I now analyze the situation in which (24) is not satisfied.

3.1.4 Gradual convergence to the stationary equilibrium

When the shock is large enough - that is, (24) is not satisfied, - high-cash agents smooth their consumption. Then, prices and output do not go to their equilibrium values immediately anymore. Notice that, for as long as high-cash agents do not fully deplete their resources, we must have $M_t^C < (1+\tau^A)M_0$. Evidently, for any $t \in \{1, 2, ...\}$, $M_t^C > M_0$ for $\tau > 0$ and $M_t^C < M_0$ for $\tau < 0$. In the following proposition, I fully characterize the transition dynamics after the shock. Most of these findings will be shown to generalize to a setup with CRRA utility function in section 4.

Proposition 3. *Let $T^{MAX} < \infty$ be defined as:*

$$T^{MAX} := \arg\max_{T \in \mathbb{N}} \left[\beta^{T-1} > \frac{1+\tau^A}{1+\mathbb{1}_{\tau>0}\tau} \right] \tag{27}$$

Under log-utility, the moment where prices achieve their new stationary equilibrium level satisfies $T \leq T^{MAX}$. Moreover, if we define:

$$\overline{X_{i1}} := X_{i1}(\mathcal{Z}_i) \qquad \text{for } X \in \{C, \theta, m\} \text{ and } i \in \{h, l\} \tag{28}$$

as the average level of consumption, relative revenues, and monetary holdings among either high- or low-cash agents, we have:

$$X_{i1}(z) = \overline{X_{i1}} \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_i^{\frac{\epsilon-1}{\epsilon}}} \qquad \text{for } X \in \{C, \theta, m\} \text{ and } i \in \{h, l\} \tag{29}$$

for all $t = \{0, 1, ...\}$ due to the homothety of log-utility. Moreover, the following results hold for $t = \{1, ..., T-1\}$ and for any arbitrary $z \in [\underline{z}, \overline{z}]$:

$$
\begin{array}{lll}
(a) & \dfrac{m_{ht}(z)}{P_t} > C_{ht}(z) > C_0(z) > C_{lt}(z) = \dfrac{m_{lt}(z)}{P_t} & \\
(b) & C_{h,t+1}(z) < C_{ht}(z) \qquad \textit{and} \qquad m_{h,t+1}(z) < m_{ht}(z) & \\
(c) & C_{l,t+1}(z) > C_{lt}(z) \qquad \textit{and} \qquad m_{l,t+1}(z) > m_{lt}(z) & \\
(d) & p_{ht}(z) > p^H(z, M_t) > p_{lt}(z) > p^H(z, M_t^C) & \\
(e) & P_t > P^H(M_t^C) & \\
(f) & \theta_0(z) > \theta_{h,t+1}(z) > \theta_{ht}(z) & \\
(g) & \theta_0(z) < \theta_{l,t+1}(z) < \theta_{lt}(z) \qquad \textit{and} \qquad p_{l,t+1}(z) > p_{lt}(z) & \\
(h) & h_{l,t+1}(z) = h_{lt}(z) = h_{lt}(z') = h_{l,t+1}(z') \qquad \textit{foreveryz}, z' \in [\underline{z}, \overline{z}] & \\
(i) & Y_0 \geq Y_{t+1} > Y_t \qquad \textit{with strict inequality for } t = \{1, ..., T-2\}. &
\end{array}
$$

Proposition 3 shows that the number of periods needed for reaching the new equilibrium must satisfy condition (27). To build intuition, notice that high-cash agents' monetary holdings fall between $t = 1$, when any arbitrary high-cash agent owns $m_{h1}(z) = \theta_0(z)(1 + \mathbb{1}_{\tau>0}\tau)M_0$, and $t = T+1$, when she owns $m_{h,T+1}(z) = \theta_0(z)(1 + \tau^A)M_0$. For a fixed individual shock τ, this fall depends crucially on how big the aggregate shock, τ^A, is, as it affects the demand that high-cash agents expect to encounter in the market. If the aggregate shock is relatively small, high-cash agents' revenues are lower, encouraging them to keep savings for longer to smooth consumption.

Unlike the case of immediate convergence to the stationary equilibrium, here, when high-cash agents smooth consumption, *money is not neutral in the aggregate*. The proposition above shows that GDP *falls* under log-utility after the shock - for contractionary or expansionist shocks alike - as the price level remains above the helicopter drops level compatible with the amount of money in circulation throughout the transition period, *i.e.*, $P_t > P^H(M_t^C)$ for $t < T$. This means that the aggregate price is higher than the one that ensures money neutrality.

For an expansionist shock, that is, for $\tau > 0$, the price chosen by unconnected agents grows along the transition path because the value of holding money in the next period is given by $\beta/m_{l,t+1}(z)$ for them, that is, it is proportional to the inverse of their future monetary holdings. As they get richer, the value of money decreases, and they allow their prices to approach their final, higher, level. As their "market share" falls over time, the rise in their monetary wealth is due to a higher amount of money in circulation, M_t^C. Finally, notice that even the prices chosen by unconnected agents are such that $p_{ut}(z) > p^H(z, M_t^C)$, meaning that their prices are also *too high* to allow for output to be as large as Y_0.

On the other hand, for a contractionary shock, that is, for $\tau < 0$, the price of the goods chosen by connected agents *undershoots*, since connected agents are made relatively poorer, and they need to decrease their price on impact more than the unconnected to remain competitive. They gradually increase their prices as their monetary holdings grow. They also make a higher production effort than the unconnected, that is $y_{ct}(z) > y_{ut}(z)$ for $t \in \{1, ..., T-1\}$. To study the quantitative implications of the model, I now conduct a simulation.

3.1.5 Simulation

For the simulation, I assume a uniform productivity distribution for simplicity. I also do away with the common support assumption to facilitate calibration[8]. In particular, I fix the lower bound for the productivity of both types of agent at $\underline{z}_c = \underline{z}_c = 0.2$ and calibrate the upper bounds as will be described in more detail below. I normalize the initial monetary base and aggregate prices to $M_0 = 1$ and $P_0 = 1$, which means that the aggregate output is also normalized to $Y_0 = 1$. The parameter values used are summarized in Table 1.

[8]This assumption is useful - though not essential - for the proofs. However, due to the homotheticity of the utility function assumed here, the relaxation of this assumption is inconsequential. Alternatively, we can assume that $F_u(z) = 1$ for $z \in [\overline{z}_u, \overline{z}_c]$ without having to relax it.

Parameter	Definition	Value
ϵ	Elasticity of substitution	11
β	Rate of time discount	0.99
γ	Labor disutility	8.1
η	Fraction of connected agents	0.27
M_0	Initial money supply	1
τ	Individual monetary shock	0.2
$\underline{z}_c$	Minimum productivity among connected	0.2
$\underline{z}_u$	Minimum productivity among unconnected	0.2
$\overline{z}_c$	Maximum productivity among connected	13.311
$\overline{z}_u$	Maximum productivity among unconnected	3.3176

Table 1: Parameter values for the simulation

As usual, I set the elasticity of substitution to $\epsilon = 11$ to get a 10% markup. Moreover, each period is assumed to be a quarter, and, thus, I set the rate of time discount to $\beta = 0.99$ to get a 1% quarterly real interest rate in equilibrium. I also set the shock size to $\tau = 0.2$, which amounts to connected agents becoming 20% richer than the unconnected with the same productivity. In our calibration, this leads to, roughly, a 11% aggregate shock. Although this is fairly large, 1) the shock is not persistent, and 2) as can be seen in Table 4 (see Appendix C), since 1999, shocks to the monetary base of at least 10% occurred in a total of nine quarters, eight of which were positive shocks.

I use data from the Survey of Consumer Finances (SCF) to retrieve the fraction of connected agents in the economy. The SCF contains information on the fraction of the U.S. population that owns each type of financial asset (*e.g.* bonds, stocks, investment funds, etc.). As usual in limited participation literature[9], I consider both stock and bond ownership, direct or through mutual funds. I disregard, whenever possible, foreign bonds, since, by our definition, asset owners should get richer with interest rate cuts conducted by the Fed. I also disregard retirement funds and US savings bonds, which are, respectively, highly illiquid and not so relevant.

In 2022, the fraction of American households that fit our definitions, according to the SCF, was 27%, and their income was, on average, $2,0628$ higher than that of the

[9] In fact, Mankiw and Zeldes (1991) and Vissing-Jørgensen (2002) show that the consumption of stock owners is significantly more reactive to fluctuations in the excess return on the stock market than the general population, which is in line with our definition of the connected agents. In particular, in the case of bondholders, according to the latter author, the intertemporal elasticity of substitution (IES) is even larger than for stockholders.

average American household[10]. I therefore, set $\eta = 0.27$ and calibrate $\overline{z}_c$, $\overline{z}_u$ and γ to obtain 1) the normalizations described above, 2) a stationary equilibrium labor supply of $1/3$[11], corresponding to an 8-hour workday, and 3) a ratio of the monetary wealth of connected agents and the whole population of $M_{c0}/M_0 = 2.06$. These targets are achieved exactly by construction. By setting $\underline{z}_c = \underline{z}_u = 0.2$, we also obtain a ratio of the income of the 90^{th} to the 10^{th} of approximately 11.2149, which is close to the actual ratio, according to the SCF, of $10,4762$.

I compute Gini indexes for consumption, revenues, and monetary holdings. The stationary equilibrium income Gini obtained is $Gini_0 \approx 0.4363$. This is quite close to the actual one, which has fluctuated around 0.4 since the 1990s. I also compute the total factor productivity (TFP) in the simulations. The idea behind the TFP measure is to see how productive a representative household would need to be to produce the aggregate output given the average amount of labor in the economy $\overline{h}_t = \sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} l_{it}(z) dF_i(z)$. Since the technology is linear, this means that $TFP_t := Y_t/\overline{h}_t$.

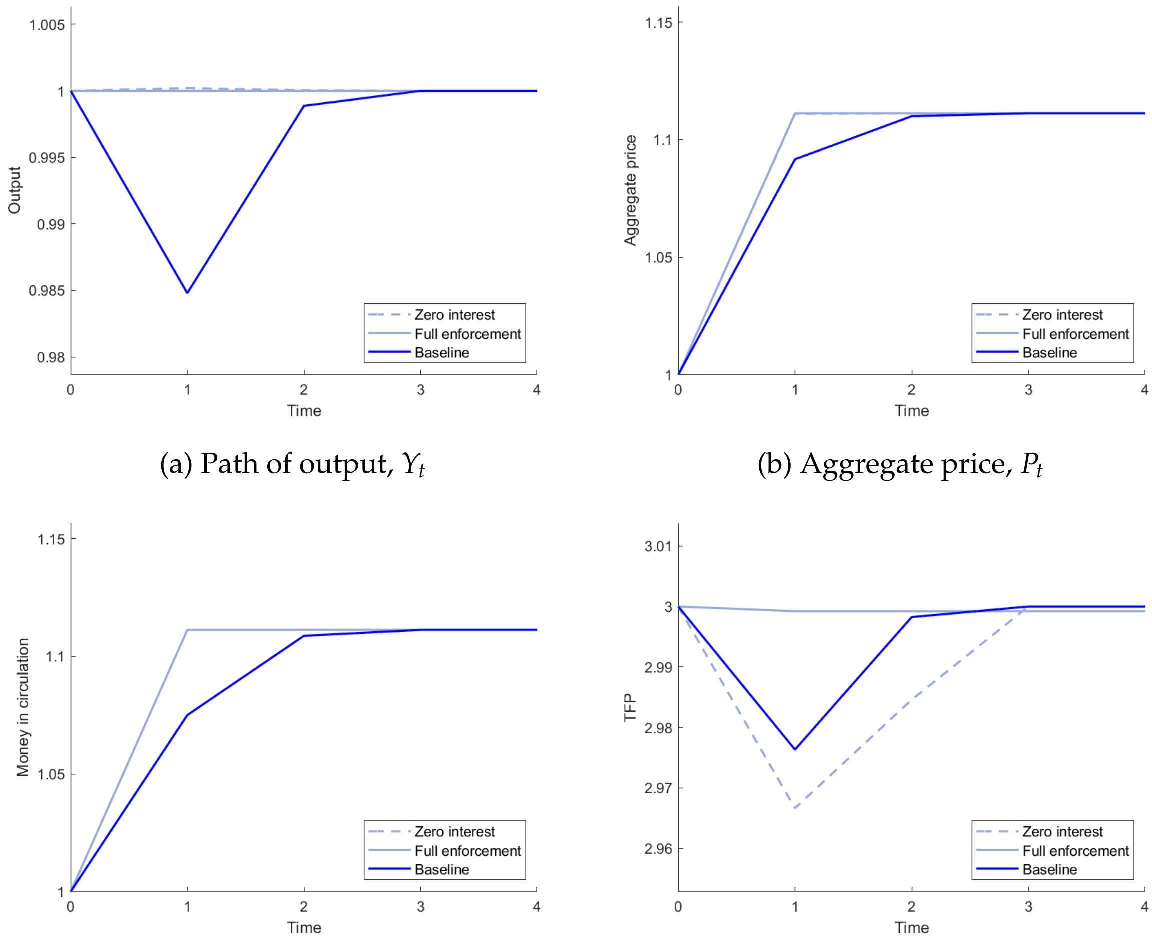
Figure 1 and Figure 2 show, respectively, the paths for aggregate and individual level variables under the baseline economy, the economy with full enforcement of bond contracts - discussed in subsection 3.2, - and the zero interest rate economy - discussed in subsection 3.3. Figure 2 considers connected and unconnected agents with productivity $z_c = z_u = \mathcal{Z}$. Since the logarithmic utility is homothetic, these graphs are identical for other productivity levels up to a re-scaling.

In our calibration, connected agents do not fully deplete their cash only for the first two quarters. Figure 1a indicates that output falls approximately 1.5% on impact, and nearly recovers at $t = 2$. This fall is concentrated on the production of connected agents, as can be seen in Figure 2a. This is because, as shown in Proposition 3, labor - and, hence, output - of unconnected agents is constant, since they set the price to keep the demand they face unchanged. Notice that, since connected agents are more efficient on average, allocative efficiency goes down with the shock - as can be seen in Figure 1d. This means that monetary shocks can generate fluctuations in TFP if they redistribute resources towards more/less productive agents.

Figure 2a shows exactly the dynamics described above for individual consumption. Connected agents' consumption rises by approximately 3.96% on impact. The consumption of the other agents falls by around 8.39%, due to the rise in the price of the final good. Moreover, Figure 1c shows that 67.46% of the injected money is put

[10] I consider households' reported revenues in a normal year.

[11] In the model, for simplicity, I do not assume any upper bound to the labor of the entrepreneur. Nevertheless, for the simulation, I set the labor endowment to 1. This does not affect the solutions for as long as the labor constraint does not bind.



(a) Path of output, Y_t

(b) Aggregate price, P_t

(c) Money in circulation, M_t^C

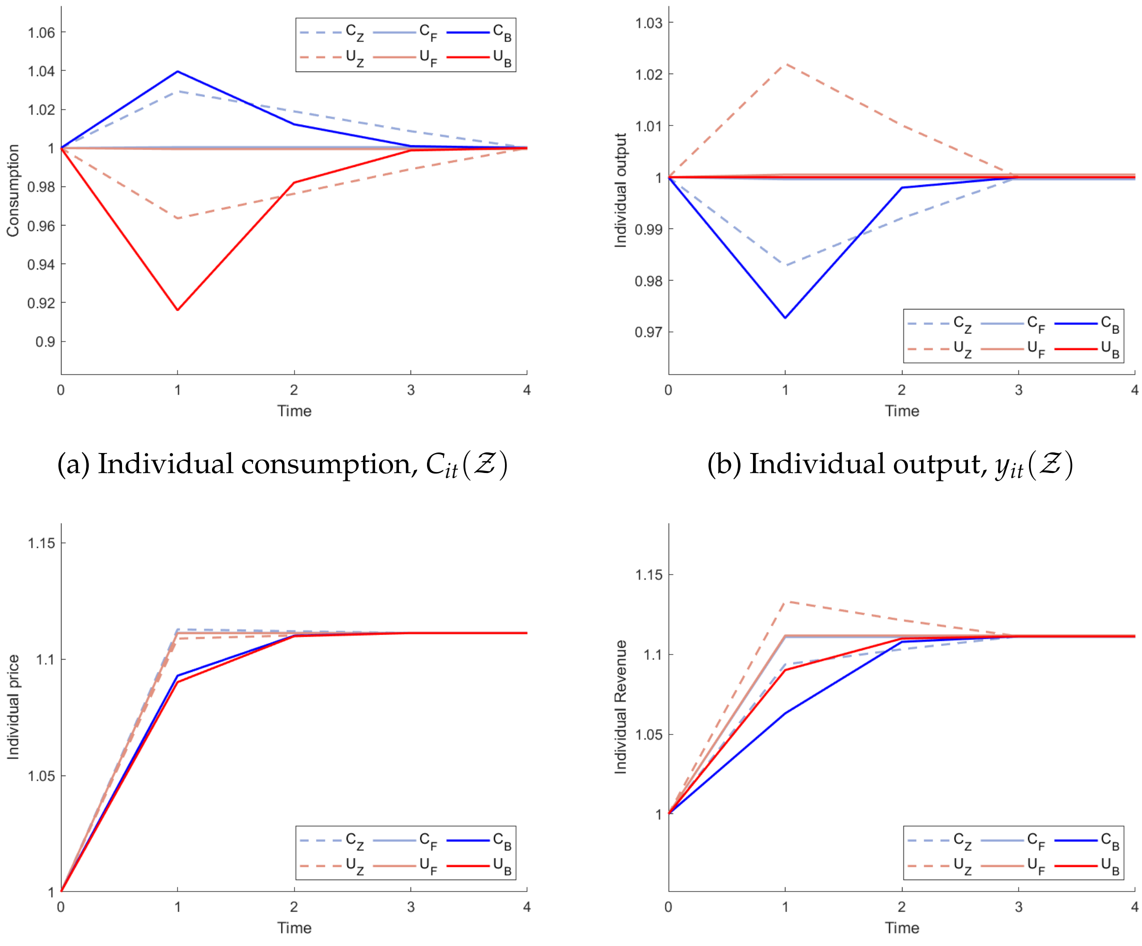
(d) Total Factor Productivity

Figure 1: Paths for aggregate variables in the three economies

in circulation at $t = 1$. The reason behind this is the fact that, as can be seen in Figure 2d, connected agents' revenues grow less than that of the unconnected, due to the reduction in their output. So, even though they fully deplete the extra money in three periods, the fact that they anticipate a fall in their income at $t = 1$ makes them save around $0.3254\tau m_0(\mathcal{Z})$ to afford to consume $P_2 C_{c2}(\mathcal{Z}) = \beta P_1 C_{c1}(\mathcal{Z})$.

All these patterns are reflected in the Gini indexes, as can be seen in Figure 3. In this graph, I use the following notation: I denote each Gini index as G_k, where $G \in \{C, R, M\}$ corresponds, respectively, to the consumption, revenue, and monetary holdings Gini; and $k \in \{B, F, Z\}$ correspond, respectively to the baseline, full enforcement and zero interest rate economies. Notice that the Gini for monetary wealth is the one that goes up the most, indicating a big increment in wealth inequality, as - the relatively rich - connected agents become richer with the shock.

The Gini for income goes down, which reflects the mechanism that re-establishes the equilibrium, since - the relatively poor - unconnected agents choose low prices



(a) Individual consumption, $C_{it}(\mathcal{Z})$

(b) Individual output, $y_{it}(\mathcal{Z})$

(c) Individual prices, $p_{it}(\mathcal{Z})$

(d) Individual revenues, $R_{it}(\mathcal{Z})$

Figure 2: Paths for individual variables in the three economies

to raise their revenues and recover their real monetary balances. As a result, consumption inequality grows, but less than wealth inequality. Concerning the aggregate price, Figure 1b indicates that roughly 82.4% of the increment in the aggregate prices happens already at the first period after the shock. This means the model produces endogenous aggregate price stickiness since money is gradually put into circulation due to the consumption-smoothing behavior of connected agents[12].

As for individual prices, as can be seen in Figure 2c, the differences between connected and unconnected agents are small, and, hence, the bulk of the disparities in revenue is due to the adjustment on the labor margin. To better understand this, I decompose the price series into four components: labor, future consumption, future

[12]Elsewhere, it was shown that the aggregate price grows *more* than what should be the case, given the amount of money put in circulation, that is, $P_t > P^H(M_t^C)$ for the periods $t = 1, ..., T$. Nevertheless, the stickiness comes from the fact that $P_t < P^H((1+\tau^A)M_0)$ for $t = 1, ..., T$, that is, the price is *lower* than the final level it attains when the whole money supply is in circulation.

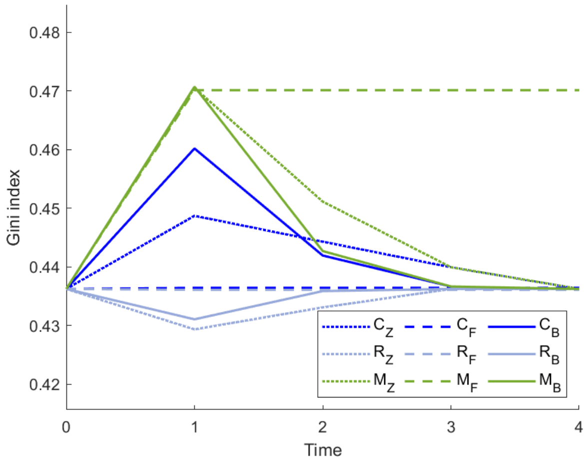
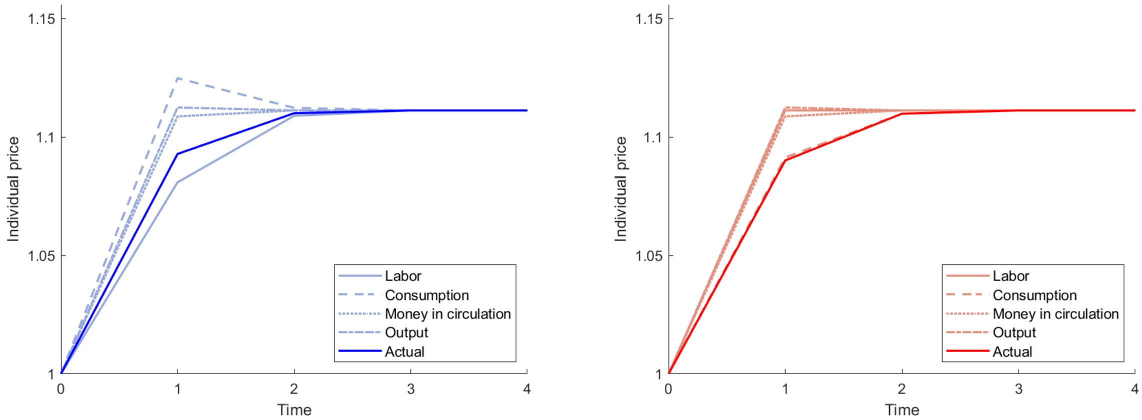


Figure 3: Gini indexes in all economies



(a) Connected agents' prices, $p_{ct}(\mathcal{Z})$

(b) Unconnected agents' prices, $p_{ut}(\mathcal{Z})$

Figure 4: Decomposition of individual prices in the baseline economy

money in circulation, and future output. The idea of this decomposition is that, I plug into the price-setting equation, (13), the actual paths for, respectively, $h_{it}(z)$, $C_{i,t+1}(z)$, M^C_{t+1} and Y_{t+1} - since $P_{t+1} = M^C_{t+1}/Y_{t+1}$, - while keeping the other variables constant at, respectively, $h_0(z)$, $C_0(z)$, M_T and Y_0. I plot the results on Figure 4.

As can be seen in the graph, for connected agents, the fall in labor is the main source of price sluggishness since it implies lower marginal disutility of labor and, hence, lower marginal costs. The gradual introduction of money also contributes to the sluggishness to a lesser extent, since it increases the future value of money relative to the case where the money is injected all at once. Consumption smoothing by connected agents, however, produces an overshooting pattern since a higher future consumption lowers the value of having money in the next period for them.

For the unconnected agents, on the other hand, beyond the money in circulation, the fall in consumption is the main factor pushing their prices downwards, which is needed to remain competitive and replenish their purchasing power through higher revenues. Now, given that part of the dynamics described in this section are, directly or indirectly, due to the gradual introduction of money, the most natural next step is to allow for borrowing to take place, which means that no amount of cash is kept idle. In the next subsection, I present a version of the model with credit.

3.2 Full Enforcement

Now, I consider the case full enforcement of debt contracts. As a result, bond sales are subject to a natural borrowing limit, namely, $l_t(z, m_{it}(z)) = m^-_{i,t+1}(z) = R_{it}(z)$ for all periods, meaning that, since the repayment of one's debt is always enforced, the entrepreneur must only have enough cash at the beginning of $t+1$ to repay her debt. This means that for $i \in \{h, l\}$:

$$q_t = \beta \frac{P_t C_{it}(z)}{P_{t+1} C_{i,t+1}(z)}$$

In the stationary equilibrium, we must have $q_0 = \beta$, since $P_t C_{it}(z) = P_{t+1} C_{i,t+1}(z)$ for all individuals with $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$. However, $b_0(z) = 0$ for all entrepreneurs, since the economy starts at a fundamental stationary equilibrium.

3.2.1 Helicopter drops of money

Since the helicopter drops case does not distort relative monetary balances, and since it brings the economy immediately to the new stationary equilibrium, the dynamics would be identical here as in the case where the bonds were absent. In the case of helicopter drops, equally productive agents remain identical after the shock. Hence, in this case, there would be no role for borrowing. Besides, $q^H = \beta = q_0$. Next, I analyze the situation where there is an uneven monetary injection.

3.2.2 Uneven access to the new money and full enforcement of bond contracts

As before, I assume that connected agents are the first to see their monetary holdings change. Then, the following corollary to Proposition 1 holds.

Corollary 1.3. *For any $\tau \neq 0$, if there is full enforcement of bond contracts, the economy goes immediately to the new stationary equilibrium at $t = 1$. The equilibrium bond price and interest rate are, respectively, $q_t = \beta$ and $i_t = (1-\beta)/\beta$ for $t = 1, 2, ...$ Moreover, for all periods $t = 1, 2, ...$, we have:*

- $C_{it}(z) = C_{i,t+1}(z)$ *and* $m_{it}(z) = m_{i,t+1}(z)$ *for* $i \in \{c, u\}$ *and* $z \in [\underline{z}, \overline{z}]$*;*
- $C_{ht}(z) > C_{lt}(z)$*,* $p_{ht}(z) > p^H(z, (1+\tau^A)M_0) > p_{lt}(z)$ *and* $R_{lt}(z) > \theta_0(z)M_t > R_{ht}(z)$*;*

Moreover, for $t = 2, 3, \ldots$*, we have* $b_{it}(z) = b_{i,t+1}(z)$ *for* $i \in \{c, u\}$ *and* $z \in [\underline{z}, \overline{z}]$*. Expenditures with consumption are a convex combination between* $m_{it}(z)$ *and* $R_{it}(z)$*, that is:*

$$P_t C_{ct}(z) = (1-\beta)(1+\tau)m_0(z) + \beta R_{ct}(z) \tag{30}$$
$$P_t C_{ut}(z) = (1-\beta)m_0(z) + \beta R_{ut}(z) \tag{31}$$

Let $\overline{\mathcal{R}}(z) := (1-\beta)\tau m_0(z)/\beta$*. Then, there is an upper and a lower bound to the difference in revenues between connected and unconnected agents:*

$$\overline{\mathcal{R}}(z) > R_{ut}(z) - R_{ct}(z) > 0 \qquad \text{if } \tau > 0 \tag{32}$$
$$\overline{\mathcal{R}}(z) < R_{ut}(z) - R_{ct}(z) < 0 \qquad \text{if } \tau < 0 \tag{33}$$

and for the difference in consumption expenditures between them:

$$\beta\overline{\mathcal{R}}(z) > P_t C_{ct}(z) - P_t C_{ut}(z) = \beta\left[\overline{\mathcal{R}}(z) - \beta(R_{ut(z)} - R_{ct}(z))\right] > 0 \quad \text{for } \tau > 0, \tag{34}$$
$$\beta\overline{\mathcal{R}}(z) < P_t C_{ct}(z) - P_t C_{ut}(z) = \beta\left[\overline{\mathcal{R}}(z) - \beta(R_{ut(z)} - R_{ct}(z))\right] < 0 \quad \text{for } \tau < 0. \tag{35}$$

According to the corollary above, the economy immediately goes to the new stationary equilibrium, which is characterized by persistent consumption differences between connected and unconnected agents, meaning that the shock creates hysteresis in monetary wealth. Thus, monetary holdings remain forever identical to their post-shock levels. The fact that high-cash agents will receive interest payments in the next period allows them to set a higher price, work less, and receive a lower revenue currently while maintaining a higher consumption standard indefinitely: they essentially have a future real claim on part of other agents' current revenues. Corollary 1.2 also implies that $|R_{ut}(\mathcal{Z}) - R_{ct}(\mathcal{Z})| < 0.002$ and $|P_t C_{ct}(\mathcal{Z}) - P_t C_{ut}(\mathcal{Z})| < 0.002$. For reference, in the bondless version of the model, at $t = 1$, these gaps were $|R_{ut}(\mathcal{Z}) - R_{ct}(\mathcal{Z})| = 0.0271$ and $|P_t C_{ct}(\mathcal{Z}) - P_t C_{ut}(\mathcal{Z})| = 0.1349$ at $t = 1$. Thus, as expected, the presence of bonds reduces the heterogeneity between agents dramatically, but at the expense of making these smaller differences permanent.

As expected, Figure 1a shows that aggregate output is unaffected by the monetary shock, meaning that, if bonds are present and there is perfect enforcement of bond contracts, monetary policy is neutral in the aggregate. Figure 1b shows that the same holds for the aggregate price. Therefore, in this setup, a representative agent model would not be a bad approximation of the aggregate effects of the shock. As can be seen in Figure 2a, the gap in consumption is almost unnoticeable. As a result, individual prices are also not very different, as can be seen in Figure 2c. The differences in revenues and output are, however, more perceptible. In particular, unlike before, the output produced by unconnected agents is now slightly *larger* than in the initial stationary equilibrium, at $t = 0$, in contrast to the flat line obtained in Figure 2b, although this increment is quantitatively insignificant. It is straightforward to show that the output produced by the unconnected agents is:

$$y_{ut}(z) = \underbrace{z\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}}_{y_0(z)} \underbrace{-(1-\beta)\frac{b_{it}(z)}{p_{ut}(z)}}_{>0},$$

The first term in the expression is the output needed to balance the trade-off between current labor and future consumption in the absence of interest payments. The increment is the extra output that needs to be produced and sold for the agent to pay these interests on the bonds sold. In fact, Corollary 1.2 implies that $P_t C_{ut}(z) < R_{ut}(z)$, and the difference between both is precisely given by coupon payments. Hence, the reason for the monetary policy to be neutral in the aggregate in the presence of a market for bonds results from 1) a smaller increment in connected agents' consumption offsetting part of the fall in output by increasing the value of future consumption, and 2) the increment in production by unconnected agents to pay interests.

3.3 Exogenously Set Interest Rate

Now, I exogenously set the interest rate at $\beta < q_t \leq 1$. This is relevant, not only because interest rates fall with an expansionist shock, but also because it gives us a middle ground between the two previous cases. In fact, I will concentrate on the case where $q_t = 1$, meaning that there are no interest payments to connected agents, but, still, their idle cash balances are channeled to the unconnected. For $q_t > \beta$, there would be an excess supply of bonds. Thus, bond sales would be restricted by the demand. I assume:

$$l_t(z, m_{ut}(z)) = \frac{\theta_{ut}(z)}{\theta_{ut}(\mathcal{Z}_u)}\left(\frac{\eta}{1-\eta}\int_{\underline{z}}^{\bar{z}} b_{c,t+1}(z)dF_c(z)\right),$$

meaning that each unconnected agent can obtain a fraction of total bonds that is proportional to their "market share" relative to that of the average unconnected agent. An analogous assumption can be made for the connected agents in the case of $\tau < 0$. Since the nominal interest rate is exogenous, it does not return to its equilibrium value endogenously. Hence, I will, for simplicity, assume that q_t remains constant at the exogenously set level during the whole transition. Proposition 4 ensures that the economy should eventually return to the fundamental stationary equilibrium.

Proposition 4. *If there is a constant $\beta < q_t \leq 1$, then there must be a period $T < \infty$ at which high-cash agents decide to fully deplete their extra money.*

The intuition for this result is the following: since the interest rate is lower than its equilibrium value, connected agents are not compensated enough for giving up on current consumption. Thus, their optimal consumption path features decreasing consumption during the transition and, thus, decreasing monetary holdings. Eventually, these holdings will be so close to their stationary equilibrium level that these agents are better off spending it all at once, as in the bondless economy.

Figure 1b shows that the aggregate price exhibits nearly zero stickiness. As seen in Figure 6 in Appendix B, the labor and consumption components of prices cancel out for both connected and unconnected agents. This occurs because unconnected face higher labor disutility, raising their marginal cost as they need to work harder to pay interests. This offsets the incentives to set lower prices due to lower consumption. Consequently, $Y_t = M_t^C / P_t$ is also nearly unaffected, increasing negligibly.

Figure 2a indicates that the connected agents' consumption grows less than in the baseline economy due to higher prices. Its wedge relative to the consumption of the unconnected is much lower than in the baseline economy but higher than in the full enforcement one. Figure 2b shows that the output and revenues of all agents exceed the baseline levels, with unconnected agents' revenue overshooting. However, in Appendix B, Figure 7 indicates that this overshooting behavior is not sufficient to make the monetary holdings of connected and unconnected converge as fast as in the baseline economy, due to debt amortization.

Overall, the aggregate variables, except for TFP, are compatible with a representative agent model, but the differences between connected and unconnected agents remain significant, which is reflected in the Gini indexes. Still, Figure 3 shows that, although the Gini for monetary wealth and income do not change by very much relative to the baseline, the consumption Gini increases by much less now, which illustrates the role played by bonds in allowing for risk sharing.

3.4 Welfare

I now move on to analyze the welfare consequences of the models here presented. To begin, I adopt a utilitarian specification of the welfare function and give all individuals equal weight. The function is, then, given by:

$$W_t = \sum_{s=t}^{\infty} \beta^{s-1} \sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} \omega_i(z) \left(log(C_{is}(z)) - \gamma \frac{h_{is}(z)^{1+\zeta}}{1+\zeta} \right) dF_i(z)$$

where the individual weight $\omega_i(z) = 1$ for all $z \in [\underline{z}, \bar{z}]$, $i \in \{c, u\}$ and $t = 1, 2, ...$ Now I define the short-run consumption equivalent as follows:

$$W_t = log(\Phi C_0(z)) - \gamma \frac{h_0(z)^{1+\zeta}}{1+\zeta} + \frac{\beta}{1-\beta} W_0,$$

where $\Phi - 1$ is the uniform increment/decrement to the fundamental stationary equilibrium consumption of all agents *in the first period* that would yield the same welfare level as the actual allocation. This measure is used for the sake of readability, since, due to the transitory nature of the shock, it does not matter in the long run. As a result, the usual consumption equivalent measure in terms of lifetime consumption is two orders of magnitude smaller.

Table 2 shows the results. Appendix C contains tables for some counterfactual exercises aimed at better understanding what drives the welfare differences. I have included the cases for $M_{c0}/M_0 \in \{2.06, 1, 0.6\}$ for ease of comparison. The individual level shocks are kept at $\tau = 0.2$ throughout, and the aggregate shock is $\tau^A = 0.1112$. Hence, I let the value of the fraction of connected agents be, respectively, $\eta \in \{0.27, 0.5562, 0.927\}$ to ensure that the aggregate shock is constant[13].

Model	$M_{c0}/M_0 = 2.06$	$M_{c0}/M_0 = 1$	$M_{c0}/M_0 = 0.6$
Baseline	-5.5304%	-0.4016%	6.6595%
Full enforcement	-5.0228%	-0.0021%	6.8995%
Zero interest rate	-5.0894%	-0.1006%	6.755%

Table 2: Welfare analysis

The table implies that the consumers in this economy would be just as well off by accepting a one-period fall of 5.53% in their consumption as in the baseline economy under the calibration adopted above. Notice that welfare falls with the monetary shock for $M_{c0}/M_0 = 2.06$ and $M_{c0}/M_0 = 1$, but it goes up for $M_{c0}/M_0 = 0.6$ relative to the situation where the shock does not occur. The reason is that, in the latter scenario, connected agents are, on average, poorer. As a result, a positive monetary shock reduces inequality and increases the consumption of agents that, on average, derive a higher marginal utility from consumption. The baseline economy is the one with the lowest welfare among the shocked economies.

The full enforcement economy is where welfare is the highest among the shocked economies. The improvement in welfare is not substantial, being approximately 0.5% for the baseline calibration. However, it nearly undoes the fall in welfare caused by the

[13]As will be shown in section 4, changing η and M_{c0}/M_0 leads to the same aggregate and individual level paths.

shock in the case where $M_{c0}/M_0 = 1$. The zero interest rate model generates a middle ground between both limiting cases, although it is closer to the full enforcement scenario. This indicates that: 1) financial frictions drive the welfare after a monetary shock down, and 2) an exogenously low interest rate reduces welfare relative to the equilibrium - *natural* - rate.

Moreover, the table also shows that monetary policy has two kinds of distributional effects: on the one hand, it unequivocally *decreases* welfare by increasing inequality between connected and unconnected agents with the same productivity; and, on the other hand, it may benefit mostly agents that have on average higher/lower productivity, yielding thereby an ambiguous effect over inequality, depending on how well-off connected agents are relative to the unconnected.

Table 5, in Appendix C, shows that the main factor lowering welfare in the baseline economy is the increment in inequality for $M_{c0}/M_0 \in \{2.06, 1\}$, though the fall in output slightly contributes as well. For $M_{c0}/M_{u0} = 0.6$, the shock reduces inequality between connected and unconnected agents, improving welfare. Table 6 presents counterfactuals for the full enforcement economy. Imposing that, after T periods, the full enforcement economy goes to the same equilibrium as the other ones results in minimal welfare loss. This indicates that permanent non-fundamental inequality is the main driver of the welfare decline under full enforcement.

Finally, three exercises regarding the zero interest rate situation are shown in Table 7: (1) I impose the same fall in output as in the baseline economy while keeping the inequality level produced by the zero interest rate model; (2) I impose the inequality level in the baseline model, but retain the rise in output under the zero interest rate regime; and (3) I eliminate non-fundamental inequality. Compared to the baseline, the increase in welfare under the zero interest rate model is driven primarily by the reduction of the disequalizing effects of the monetary shock in the case of $M_{c0}/M_0 = 2.06$ - where it benefits relatively richer agents. Conversely for $M_{c0}/M_0 = 0.6$, welfare improves *despite* the resulting reduction in the redistribution promoted by the shock - that benefits poorer agents - because it reduces non-fundamental inequality.

4 Sensitivity Analysis

In this section, I conduct a sensitivity analysis. The primary goal is to check how robust or sensitive my findings are to changes in some of the assumptions or parameters. I begin by analyzing the case of a negative monetary policy shock. Secondly, I adopt a CRRA utility specification. Next, I let the Frisch elasticity of labor supply vary. Then, I change the size of the individual shocks, while keeping the aggregate one identical, and interpret the findings in the context of futures markets. Furthermore, I fix both individual and aggregate shock sizes, but change how spread across more (less) connected agents this shock is. Lastly, I study how the moment of convergence to the equilibrium varies with the fraction of connected agents. In all cases, I concentrate on the baseline economy, as it already gives a good idea of how the patterns change. Most graphs can be found in Appendix B.

4.1 A Negative Shock

In this section, I maintain the calibration in Table 1, except for the individual shock, τ, and the fraction of connected η. Instead, I give this economy an equivalent negative shock of $\tau = -0.1667$, which also makes the high-cash agents 20% richer than their low-cash counterparts. Figure 8 in Appendix B plots the graphs for the aggregate and individual variables. The patterns are very similar to the ones observed in the models above, except for the fact that the roles of connected and unconnected agents are now flipped and, as a result, the Gini coefficients and TFP go in the opposite direction as before. Interestingly, the aggregate price *undershoots* - as can be seen in Figure 8b. This is because money is kept idle.

4.2 CRRA Utility Specification

I now assume that the utility function is given by:

$$u(c) = \frac{C^{1-\alpha}}{1-\alpha},$$

and consider two possible values for α, namely, 2 and 0.5. These correspond, respectively, to an intertemporal elasticity of substitution (IES) of $IES_{\alpha=2} = 0.5$ and $IES_{\alpha=0.5} = 2$. I recalibrate the model and now assume, for simplicity, that there are no differences in average productivity between connected and unconnected. The basic patterns of the model are unchanged. Figure 9a shows that, under a lower IES, output goes down by more due to stronger consumption smoothing. Additionally, Figure 10 indicates that prices and revenues respond more sharply, as prices are more sensitive to changes in consumption for $IES_{\alpha=2} = 0.5$. Thus, despite connected agents' preference for extended consumption smoothing, higher prices accelerate convergence.

4.3 Varying Frisch Elasticity Of Labor Supply

I now let the inverse Frisch elasticity of labor supply vary. I consider four cases: $\zeta \in \{0, 0.5, 1, 2\}$, which correspond, respectively, to 1) an infinite Frisch elasticity, 2) a "macro" elasticity of 2, 3) the baseline value, and 4) a "micro" elasticity of 0.5. Figure 11 shows that there is close to no difference across cases in the speed of convergence. However, for large Frisch elasticities, there is a bigger initial response in prices, a larger fall in output, and more distortions in allocative efficiency. This is due to a higher responsiveness in connected agents' output and, thus, revenues.[14]

4.4 Varying Access To Financial Markets

I now let the fraction of connected households in the economy change, while fixing the aggregate. This can be interpreted as varying the access to financial markets. I consider $\eta \in \{0.1, 0.27, 0.5, 0.6\}$ To better control what parameters are varying, I assume no differences in average productivity between connected and unconnected, that is $M_{c0}/M_0 = 1$. As a result, for fractions of connected $\eta \in \{0.1, 0.27, 0.5, 0.6\}$, we have, respectively, individual proportional shocks of $\tau \in \{1.1124, 0.412, 0.2225, 0.1854\}$. As can be seen in Figure 12, the economy converges faster and prices react faster for higher values of η. Interestingly, output falls the least in the economy where η is the highest and the lowest. For $\eta = 0.1$, the combination of slow convergence and sluggish prices produces a lower fall in output than in the other cases.

Model	$\eta = 0.1$	$\eta = 0.27$	$\eta = 0.5$	$\eta = 0.6$
Baseline	-0.938%	-0.6945%	-0.456%	-0.359%
Full enforcement	-0.0234%	-0.007%	-0.0026%	-0.0017%
Zero interest rate	-0.3315%	-0.1737%	-0.1101%	-0.0907%

Table 3: Welfare analysis for different values of η

The heterogeneity in outcomes is highest when η is lower, due to a higher individual monetary shock. This is reflected in the Gini indexes, although the overall patterns are unchanged. Also, a tiny fall in TFP due to the diminishing returns introduced by the CES aggregator can be seen in Figure 12d. Finally, Table 3 shows, for $\eta \in \{0.1, 0.27, 0.5, 0.6\}$, that welfare falls the least in the economies with a higher η under all model specifications. Overall, the monetary shock is less distortionary when access to financial markets is more widespread, even though this means extend-

[14]Interestingly, Figure 11h shows that, under $\zeta = 0$, we observe the same pattern as in Williamson (2008): the price chosen by connected agents overshoots, while the price of unconnected agents' products grows slowly. In his model, demand in the goods market of connected agents jumps on impact in nominal terms, and the new money gradually flows into the unconnected goods market due to imperfect market segmentation. In my framework, under a perfectly elastic labor supply, marginal costs are fixed, meaning that individual prices become proportional to future consumption expenditure.

ing ownership of assets that are subject to monetary policy risk to more people, as it makes the shocks work more like helicopter drops.

4.5 Futures

Assume that a fraction θ_H of connected agents are hedgers, and $1-\theta_H$ are speculators. Moreover, assume for simplicity that there is a market-making company trading futures. At the end of a period, hedgers with productivity z can sign a contract whereby they accept to sell all their assets, at the beginning of the next period, to the market maker for a price $m_0(z)$. Speculators own a quantity of shares of this market-making company that is proportional to their monetary holdings. As a result, hedgers' monetary holdings are unchanged after the shock, making them formally identical to unconnected agents, while the speculators receive a proportional shock of $\tilde{\tau} = (1 + M^H_{c0}/M^S_{c0})\tau > \tau$, where M^H_{c0} and M^S_{c0} are, respectively, the average monetary holdings of hedgers and speculators.

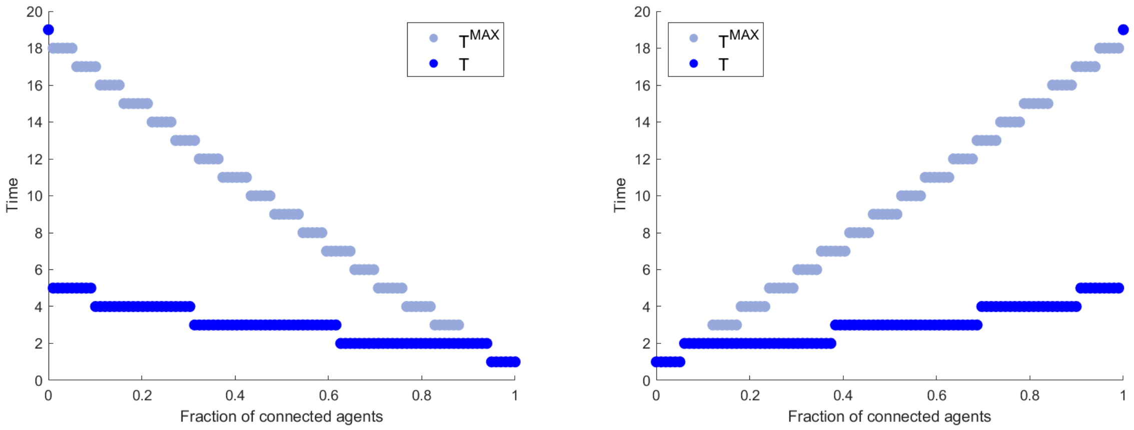
Since there is perfect foresight in this economy, this higher risk faced by speculators does not command a premium in the stationary equilibrium. Now, notice that the introduction of a futures market is isomorphic to lowering η, since only a fraction $\tilde{\eta}_c = \eta(1-\theta_H) < \eta$ receives monetary transfers from the government. As we have seen above, this implies that futures trading concentrates monetary policy risk on fewer agents, leading to slower convergence and more sluggish prices. The impact of the introduction of a futures market over output volatility seems inverse U-shaped, meaning that, while it can go up for high $\tilde{\eta}_c$, if the fraction of speculators is small enough, it can have a stabilizing effect. Lastly, the welfare implications are ambiguous, depending on the productivity distribution among hedgers and speculators.

4.6 Varying The Extensive And Intensive Margins Of Inequality

Now, I perform a similar exercise as the one above by varying η and maintaining the aggregate shock, $\tau^A = 0.135$. This time, I fix the individual monetary shock $\tau = 0.2$ and vary the relative average productivity of connected and unconnected.[15] I consider $M_{c0}/M_0 \in \{6.75, 2.5, 1.35, 1.125\}$ for, respectively, $\eta \in \{0.1, 0.27, 0.5, 0.6\}$. This exercise trades off two different margins of inequality: the extensive margin - namely, how many connected agents there are - and the intensive margin - how much richer than the unconnected they are on average. As can be seen in Figure 13, nothing changes,

[15] The reason why the aggregate shock is now higher than before is that I assume $M_{c0}/M_0 = 2.5$ in the economy with $\eta = 0.27$ to ensure that connected agents are on average more productive than the unconnected across all scenarios.

except for path of TFP and Gini coefficients. Naturally, for higher values of η, the productivity advantage of connected agents is lower by construction, reducing the fluctuation in allocative efficiency as well as inequality. Since aggregate variables and individual shocks are identical for all cases, so are individual choices.



(a) Positive shock, $\tau = 0.2$

(b) Negative shock, $\tau = -0.1667$

Figure 5: Number of periods until convergence

4.7 Financial Access And The Speed Of Convergence

Now, I let the fraction of connected vary more freely over the interval $\eta \in [0, 1]$.[16] I maintain the same individual shocks throughout - which can be positive or negative, $\tau \in \{0.2, -0.1667\}$ - and the relative initial average monetary holdings.[17] As a consequence, the value of η will govern the size of the aggregate shock. Figure 5 plots both T^{MAX}, computed as in (27), and the length of the transition path for both shock signs. The graphs indicate that general equilibrium effects explain most of the speed of convergence. In particular, when connected (unconnected) agents exist with mass zero, $T = T^{MAX}$ for a positive (negative) monetary shock. Moreover, the gap between T and T^{MAX} is smaller when there are a lot of high-cash agents. Intuitively, given a certain individual shock, a bigger aggregate shock increases the revenues that high-cash agents expect to obtain in the market, reducing their incentives to smooth consumption for very long.

[16] When $\eta = 0$ ($\eta = 1$), I consider the case of a discrete number of connected (unconnected) agents, with zero mass, meaning that we have no aggregate shock (nearly helicopter drops).

[17] I do not re-calibrate the model, as the normalization of the initial aggregate price is irrelevant here. Still, $M_{c0}/M_0 = 1$ for simplicity.

5 Concluding remarks

I have shown that the distributional effects of monetary shocks can prevent the economy's immediate return to the long-run equilibrium. These effects might be significant if a well-functioning bonds market is not in place to channel savings to the agents who got the shortest straw of the shock. Prices adjust sluggishly, but output is depressed due to the existence of idle cash balances. Trade in a bonds market is enough to make inequality in consumption nearly unresponsive to monetary shocks, but these shocks produce hysteresis in monetary holdings, and the economy does not return to a fundamental equilibrium unless the interest is set below its equilibrium value.

My contribution to the literature on the distributional effects of monetary shocks is a tractable model that links portfolio-related channels to general equilibrium ones in a process that can endogenously restore the original allocations. I also show how distributional effects matter for the transmission of such shocks in the absence of a well-developed credit market. I also contribute to the literature on market incompleteness by showing that one-period bonds can go a long way in allowing risk sharing. Lastly, my framework offers new insights into price stickiness, which can be driven by (i) beneficiaries of monetary shocks lowering marginal costs and (ii) others setting lower prices to reestablish their wealth.

Four key policy implications arise. First, reducing financial frictions that hinder access to credit goes a long way in completing the market, as in Telmer (1993). Second, expanding the access to financial markets can reduce post-shock distortions by diluting the effects of such shocks across more people. More research is required on why this access is relatively restricted even in some advanced economies. Third, although credit markets reduce distributional distortions created by monetary shocks, they can lead to persistent, non-fundamental, wealth disparities, suggesting that financial development should be coupled with measures to address long-term inequality. Fourth, given that the welfare losses due to distortions are small in the long run, distributional effects are not relevant for monetary policy in developed economies, with well-functioning credit markets.

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A Proofs of Propositions

A.1 Proposition 1

Existence and uniqueness of the fundamental stationary equilibrium:

To begin, notice that, as in Lucas and Stokey (1985), the CIA constraint should bind in this economy. To prove this explicitly, notice that, for any arbitrary $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$, by imposing the fundamental stationary equilibrium conditions onto (3), we obtain:

$$P_t C_{it}(z) = m_{it}(z) + (1 - q_t) b_{i,t+1}(z). \tag{36}$$

Imposing (2) gives:

$$m_{it}(z) - q_t b_{i,t+1}(z) = m_{it}(z) + (1 - q_t) b_{i,t+1}(z), \tag{37}$$

which can only be satisfied for $b_{i,t+1}(z) = 0$. Now, by aggregating (13), and imposing $P_t = P_{t+1}$ and $C_{i,t+1}(z) = C_{it}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$, we find that:

$$\left[\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\epsilon-1} \frac{u'\left(C_{it}(z)\right)^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}} dF_i(z) \right]^{\frac{1}{\epsilon-1}} = \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} \tag{38}$$

By exploiting the continuity of the $u'(\cdot)$ function, I define z^* as:

$$(z^*)^{\epsilon-1} \frac{u'\left(C_{it}(z^*)\right)^{\epsilon-1}}{h_{it}(z^*)^{\zeta(\epsilon-1)}} = \sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\epsilon-1} \frac{u'\left(C_{it}(z)\right)^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}} dF_i(z)$$

Now, notice that:

$$\theta_{it}(z) = \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} \right]^{1-\epsilon} \frac{z^{\epsilon-1} u'\left(C_{i,t+1}(z)\right)^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}} = \left[\frac{z u'\left(C_{i,t+1}(z)/h_{it}(z)^{\zeta}\right)}{z^* u'\left(C_{i,t+1}(z^*)/h_{it}(z^*)^{\zeta}\right)} \right]^{\epsilon-1},$$

where the last equality follows from (38) and (A.1). This means that $\theta_{it}(z^*) = 1$. Moreover, notice that, with a bit of algebra, and using the fact that $Y_t = M_t/P_t$, we have:

$$h_{it}(z) = \left(\frac{p_{it}(z)}{P_t} \right)^{-\epsilon} Y_t = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{z}{\gamma} \beta u'(C_{i,t+1}(z)) \right]^{\frac{\epsilon}{1+\zeta\epsilon}} \left(\frac{M_t}{P_t} \right)^{\frac{1}{1+\zeta\epsilon}} \tag{39}$$

Since, in the stationary equilibrium there is no borrowing and $m_{it}(z) = R_{it}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$, we must, therefore have that $C_{i,t+1}(z^*) = C_{it}(z^*) = \theta_{it}(z^*) M_t/P_t = M_t/P_t$. This means that:

$$\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} = z^* \frac{u'\left(M_t/P_t\right)}{h_{it}(z^*)^{\zeta}} \tag{40}$$

By using (39), one can show that:

$$u'\left(\frac{M_t}{P_t}\right)\left[\frac{M_t}{P_t}\right]^{-\zeta} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta z^*}, \tag{41}$$

which means that M_t/P_t is defined uniquely, as the left-hand side is strictly decreasing on it. Therefore, P_t is a linear function of M_t, and aggregate output $Y_t = M_t/P_t$ is defined by parameters and z^* alone, being, therefore, independent of P_t and M_t. Finally, $\theta_{it}(z)$ is also well and uniquely defined for every $z \in [\underline{z}, \overline{z}]$ and is independent of connectedness status. This means that $m_{it}(z)$, $C_{it}(z)$, $p_{it}(z)$, $y_{it}(z)$ and $R_{it}(z)$ are also well- and uniquely-defined. Now, I look into non-fundamental stationary equilibria.

Finding a stationary equilibrium with $q_t = \beta$:

To begin, let us define "high-cash" agents as being agents for whom $b_{h,t+1}(z) > 0$ and "low-cash" as having $b_{l,t+1}(z) \leq 0$. We can then build $F_h(\cdot)$ and $F_l(\cdot)$ to be the corresponding cumulative distribution functions. Finally, let η_h and $\eta_l = 1 - \eta_h$ be the corresponding fractions of the population that falls into either category.

The structure of the proof is as follows: I will begin by assuming that $l_t(z, m_{lt}(z)) = \infty$. This allows for the possibility of some low-cash agents ending up with an outstanding debt after the bonds' market closes at the beginning of the period. Then, I will show that the equilibrium implemented by the unconstrained economy is feasible in the constrained economy, meaning that the constraint does not bind. So, to begin, notice that one can re-write the Euler equation in the unconstrained case as:

$$\frac{u'(C_{it}(z))}{u'(C_{i,t+1}(z))} = \frac{\beta}{q_t}\frac{P_t}{P_{t+1}} \tag{42}$$

Since, by assumption, $u(\cdot)$ is increasing and strictly concave, the marginal utility is such that $u'(\cdot) \geq 0$ and is strictly decreasing. This means that, given certain values of the aggregate variables at the right-hand side, this means that either the ratio $C_{it}(z)/C_{i,t+1}(z)$ is decreasing, constant, or increasing over time for all agents, regardless of productivity and high-or-low-cash status. Besides, notice that all the money not used by high-cash agents to buy bonds in any period $t = 0, 1, 2, ...$ must be used for consumption, that is $P_t C_{ht}(z) = m_{ht}(z) - q_t b_{h,t+1}(z)$. Moreover, notice that, if $q_t = 1$, $s_{it}(z) = 0$ for all agents by assumption, and, if $q_t < 1$, it cannot be optimal for low-cash agents to sell bonds at a price q_t and save in cash (*i.e.*, $s_{lt}(z) > 0$), then we must have $M_t^C = M_t$ for any $t \in \{1, 2, ...\}$.

Now, I will show that there is a stationary equilibrium in which $q_t = \beta$. To begin, notice that, if we set $q_t = \beta$, (42) implies that $\frac{u'(C_{i1}(z))}{u'(C_{i2}(z))} = \frac{P_1}{P_2}$. Moreover, notice that:

$$P_t \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} C_{it}(z) dF_i(z) = M_t = P_{t+1} \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} C_{i,t+1}(z) dF_i(z), \tag{43}$$

Naturally, if $C_{it}(z) = C_{i,t+1}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$, the equation above implies that $P_t = P_{t+1}$, meaning that the first-order condition (42) is satisfied for every agent. I will prove below that this requires $m_{it}(z) = m_{i,t+1}(z)$ for all agents and for any arbitrary $t = 1, 2, ...$, but, for now, I will take this result as a given for simplicity. In this case, it must be the case that this equilibrium lasts forever, proving the existence of this stationary equilibrium with borrowing. Let us denote the demand/supply of bonds by each agent under this equilibrium with a star, that is, $b^*_{it}(z)$

Proving that $q_t = \beta$ for every period t:

Now assume, by contradiction, that $1 \geq q_t > \beta$. The argument above proves that an equilibrium with $q_t = \beta$ always exists, meaning that $\sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b^*_{h,t+1}(z)dF_h(z) = -\frac{(1-\eta_h)}{\eta_h} \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b^*_{l,t+1}(z)dF_l(z)$. Now, notice that (42) means that buying/selling bonds trades off current and future consumption. I will consider two cases now. To begin, I consider the case where $q_t/P_t > \beta/P^*_t$, meaning that the price of bonds relative to current consumption rises when $q_t > \beta$. As a result, bond demand by the high-cash agents must decrease, $b_{h,t+1}(z) < b^*_{h,t+1}(z)$, and low-cash agents' supply of bonds must increase, $-b_{l,t+1}(z) > -b^*_{l,t+1}(z)$, to make the marginal value of holding money in the future, $\beta^{t+1}u'(C_{i,t+1}(z))/P_{t+1}$, higher for both types of agent. Now, notice that the former condition implies:

$$\begin{aligned}
-\frac{(1-\eta_h)}{\eta_h} \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b_{l,t+1}(z)dF_l(z) &= \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b_{h,t+1}(z)dF_h(z) \\
&< \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b^*_{h,t+1}(z)dF_h(z) \\
&= -\frac{(1-\eta_h)}{\eta_h} \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b^*_{l,t+1}(z)dF_l(z),
\end{aligned}$$

which contradicts the latter condition. This rules out any equilibrium with $1 \geq q_t > \beta$ under $q_t/P_t > \beta/P^*_t$. Now, let us consider the case where $q_t/P_t \leq \beta/P^*_t$. By a similar argument to the one made above, we must have $b_{i,t+1}(z) \geq b^*_{i,t+1}(z)$ and, therefore, $q_t b_{i,t+1} > \beta b^*_{i,t+1}(z)$ for $i \in \{h, l\}$. Integrating both sides and imposing the market clearing condition for the bonds market, gives us:

$$0 = q_t \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b_{i,t+1}(z)dF_i(z) > \beta \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b^*_{i,t+1}(z)dF_i(z) = 0, \tag{44}$$

a contradiction. This means that no equilibrium with $1 \geq q_t > \beta$ can exist. A similar argument can be made to rule out equilibria with $q_t < \beta$.

Equilibrium uniqueness:

Now, I will show that, under $q_t = \beta$, there can be no equilibrium that is not stationary.

This will prove the uniqueness of the equilibrium described above under full enforcement. By integrating the individual price choices and imposing the result that the full depletion first-order condition holds for a non-negligible mass of agents, we obtain

$$P_t = P_{t+1}\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\left[\sum_{i\in\{h,l\}}\eta_i\int_{\underline{z}}^{\overline{z}}\frac{z^{\epsilon-1}u'(C_{i,t+1}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}}dF_i(z)\right]^{\frac{1}{1-\epsilon}} \tag{45}$$

Now, notice that labor effort in production by the agent with $z\in[\underline{z},\overline{z}]$ and $i\in\{h,l\}$ is, out of a stationary equilibrium, given by:

$$h_{it}(z) = \left(\frac{p_{it}(z)}{P_t}\right)^{-\epsilon}Y_t = \left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{z}{\gamma}\beta\frac{P_t}{P_{t+1}}u'(C_{i,t+1}(z))\right]^{\frac{\epsilon}{1+\zeta\epsilon}}Y_t^{\frac{1}{1+\zeta\epsilon}}, \tag{46}$$

which can be plugged into (45) to give:

$$P_t = P_{t+1}\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}Y_t^{\zeta}\left[\sum_{i\in\{h,l\}}\eta_i\int_{\underline{z}}^{\overline{z}}z^{\frac{\epsilon-1}{1+\zeta\epsilon}}u'(C_{i,t+1}(z))^{\frac{\epsilon-1}{1+\zeta\epsilon}}dF_i(z)\right]^{\frac{1+\zeta\epsilon}{1-\epsilon}} \tag{47}$$

Now, notice that, at any period, and since $q_t=\beta$, (42) can be written with a weak inequality, that is $u'(C_{i,t+1}(z))\le\frac{P_{t+1}}{P_t}u'(C_{it}(z))$, which should also be valid even if the economy ends up in a fundamental stationary equilibrium in the next period. By plugging this version of the first-order condition into the equation above, we obtain:

$$\left[\sum_{i\in\{h,l\}}\eta_i\int_{\underline{z}}^{\overline{z}}z^{\frac{\epsilon-1}{1+\zeta\epsilon}}u'(C_{it}(z))^{\frac{\epsilon-1}{1+\zeta\epsilon}}dF_i(z)\right]^{\frac{1+\zeta\epsilon}{\epsilon-1}}\ge\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{q_t}Y_t^{\zeta}=\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}Y_t^{\zeta}$$

which is valid even if the economy converges to the stationary equilibrium (and, hence, no borrowing occurs) between t and $t+1$. This allows us to re-write (47) as:

$$P_t\le P_{t+1}\left(\frac{Y_t}{Y_{t+1}}\right)^{\zeta}, \tag{48}$$

Since, in the presence of borrowing, $P_t=M_t/Y_t$, we can re-write the condition above as:

$$\left(\frac{Y_t}{Y_{t+1}}\right)^{1+\zeta}\ge 1 \qquad \therefore \qquad Y_t\ge Y_{t+1} \tag{49}$$

Notice that this immediately rules out the possibility of deflation. Thus, we must either have inflation or constant prices. Assume, by contradiction, that $P_{t+1}>P_t$ for a given period $t\in\{1,2,...\}$. Then, by using the definition (16) and (45), we can see that:

$$\frac{P_{t+1}}{P_t}=U_{t+1}^{GAP}, \tag{50}$$

meaning that $\overline{U}^{GAP}_{t+1} > 1$. I am going to show that this implies that $P_{t+2} > P_{t+1}$. To see this, notice that, since $P_{t+2} \geq P_{t+1}$ by the condition above, and given the first-order condition (42) and $q_t = \beta$, then $C_{i,t+2}(z) \leq C_{i,t+1}(z)$ for any $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$. Now, notice that, by imposing (46), we can obtain:

$$\frac{z^{\epsilon-1}u'(C_{i,t+2}(z))^{\epsilon-1}}{h_{i,t+1}(z)^{\zeta(\epsilon-1)}} = \left[\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\frac{P_{t+2}}{P_{t+1}}\right]^{\frac{\zeta\epsilon(\epsilon-1)}{1+\zeta\epsilon}} Y_{t+1}^{\frac{\zeta(1-\epsilon)}{1+\zeta\epsilon}} z^{\frac{\epsilon-1}{1+\zeta\epsilon}} u'(C_{i,t+2}(z))^{\frac{\epsilon-1}{1+\zeta\epsilon}} \tag{51}$$

$$> \left[\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\frac{P_{t+2}}{P_{t+1}}\right]^{\frac{\zeta\epsilon(\epsilon-1)}{1+\zeta\epsilon}} Y_{t}^{\frac{\zeta(1-\epsilon)}{1+\zeta\epsilon}} z^{\frac{\epsilon-1}{1+\zeta\epsilon}} u'(C_{i,t+1}(z))^{\frac{\epsilon-1}{1+\zeta\epsilon}} \tag{52}$$

$$= \frac{z^{\epsilon-1}u'(C_{i,t+1}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}}\left(\frac{P_{t+2}}{P_{t+1}}\frac{P_t}{P_{t+1}}\right)^{\frac{\zeta\epsilon(\epsilon-1)}{1+\zeta\epsilon}} \tag{53}$$

since $Y_{t+1} < Y_t$ due to our contradiction assumption. Aggregating it, by (50), we obtain:

$$\frac{P_{t+2}}{P_{t+1}} > \frac{P_{t+1}}{P_t},$$

which shows that prices cannot be constant, given (18). By a simple induction argument, we can see that prices must grow forever and that the inflation rate is bounded below by a positive constant, since $1 + \pi_{t+s} = \overline{U}^{GAP}_{t+s} \geq \overline{U}^{GAP}_{t+1} > 1$ for $s \in \{2, 3, ...\}$, meaning that the price level diverges. This, implies, however, by equation (43), that:

$$\lim_{t\to\infty} \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} C_{it}(z) dF_i(z) = 0$$

as the price level diverges. However, the fact that there is borrowing in any equilibrium along this path, we know that there is a non-negligible mass of high-cash agents that are strictly richer than their low-cash counterparts, that is, $C_{ht}(z) > C_{lt}(z) \geq 0$. This means that aggregate consumption must be bounded away from zero. As a result, the only possible equilibrium is the one in which $P_{t+1} = P_t$ and $C_{i,t+1}(z) = C_{it}(z)$ for every $z \in [\underline{z}, \overline{z}]$, $i \in \{h, l\}$ and $t \in \mathbb{T}^S$. Evidently, we must have $P_t C_{ht}(z) > P_t C_{lt}(z)$, $p_{ht}(z) > p_{lt}(z)$ and, thus, $R_{lt}(z) > R_{ht(z)}$ for all $t \in \mathbb{T}^S$.

Constancy of borrowing/lending decisions:

Now, notice that, by imposing (2) into the budget constraint, (3), of any arbitrary entrepreneur in period $t \in \mathbb{T}^S$, we see that $m^-_{i,t+1}(z) = R_{it}(z)$. Now, notice that, since $u'(C_{i,t+1})/P_{t+1} = u'(C_{i,t+2})/P_{t+2}$, by (13), $p_{it}(z) = p_{i,t+1}(z)$ and, thus, $R_{it}(z) = R_{i,t+1}(z)$. This means that $m^-_{it}(z) = m^-_{i,t+1}(z)$ for $t = 2, 3, ...$ Therefore, for $t = 2, 3, ...,$ by the budget constraint, (3), we have:

$$P_t C_{it}(z) = R_{it}(z) + b_{it}(z) - \beta b_{i,t+1}(z) \tag{54}$$

Together with (54) and the fact that $P_t C_{it}(z) = P_{t+1} C_{i,t+1}(z)$, this implies that:

$$b_{it}(z) - \beta b_{i,t+1}(z) = b_{i,t+1}(z) - \beta b_{i,t+2}(z) \tag{55}$$

for every $t \in \mathbb{T}^S$. Now, let us define α_s such that $b_{i,t+s}(z) = \alpha_s b_{i,t+s-1}(z)$, for $s = 1, 2, ...$. Thus, for an arbitrary $r = 1, 2, ...$, we can apply this definition iteratively to obtain:

$$b_{i,t+r}(z) = \left(\prod_{s=1}^{r} \alpha_s \right) b_{it}(z) \tag{56}$$

I will now prove that $\alpha_s = 1$ for $s = 1, 2, ...$. I will concentrate on α_1 without any loss of generality. Assume, by contradiction, that $\alpha_1 < 1$. Then, by (55), we have:

$$\begin{aligned} b_{it}(z) - \beta b_{i,t+1}(z) &= b_{i,t+1}(z) - \beta b_{i,t+2}(z) \\ (1 - \alpha_1 \beta) b_{it}(z) &= (1 - \alpha_2 \beta) \alpha_1 b_{it}(z) \\ (1 - \alpha_1 \beta) &= (1 - \alpha_2 \beta) \alpha_1 < (1 - \alpha_2 \beta) \\ \alpha_2 &< \alpha_1 \end{aligned}$$

Obviously, the same reasoning applies to show that $\alpha_s < \alpha_1$. Therefore, $\alpha_s < 1$ for all $s \geq 1$. However, this means that:

$$\lim_{r \to \infty} b_{i,t+r}(z) = 0$$

and, thus:

$$\lim_{r \to \infty} P_{t+r} C_{i,t+r}(z) = R_{i,t+r}(z)$$

However, since $R_{l,t+r}(z) > R_{h,t+r}(z)$, this means that $P_{t+r} C_{l,t+r}(z) > P_t C_{h,t+r}(z)$ at the limit: an absurd, since $P_{t+r} C_{h,t+r}(z)$ is constant for all r and larger than $P_{t+r} C_{l,t+r}(z)$.

Now, I proceed to the second case: assume, by contradiction, that $\alpha_1 > 1$. Similarly to before, this implies that $\alpha_s > \alpha_1 > 1$ for any $s > 1$. Now, this implies that:

$$\lim_{r \to \infty} b_{c,t+r} = \lim_{r \to \infty} \left(\prod_{s=1}^{r} \alpha_s \right) b_{ct} > \lim_{r \to \infty} \alpha_1^r b_{ct} = \infty$$

which is not possible, since $M_t < \infty$. Therefore, I conclude that $\alpha_s = 1$ for $s = 1, 2, ...$ and, thus, $b_{it}(z) = b_{i,t+1}(z)$ for $z \in [\underline{z}, \overline{z}]$, $i \in \{h, l\}$ and $t \in \mathbb{T}^S$.

Proving that the unconstrained optimum is feasible:

Using agents' cash-in-advance constraint, (2), at t, this result implies that:

$$b_{it}(z) = \frac{m_{it}(z) - P_t C_{it}(z)}{\beta}$$

for $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$. From this expression, one can easily see that $m_{it}(z) = m_{i,t+1}(z)$ for $t \in \mathbb{T}^S$. However, since $m_{i1}(z) > 0$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$, this implies that no agent enters the bonds market with outstanding debt, meaning that $b_{l,t+1}(z) < m^{-}_{l,t+1}(z) = R_{lt}(z)$ for every $z \in [\underline{z}, \overline{z}]$, which proves that the borrowing constraint does not bind.

Equilibrium Uniqueness:

Finally, I can now prove that, if money is distributed such as in the fundamental stationary equilibrium, this is not only the unique stationary equilibrium but also the unique equilibrium in this economy. I denote the fundamental stationary equilibrium with the subscript 0 and, moreover, drop the i subscript since agents with the same productivity do not differ in monetary holdings and, as a result, in their choices. Notice that the only circumstance in which the economy would not be in a fundamental stationary equilibrium is if $C_t(z) < m_t(z) = R_0(z)$ for a non-negligible mass of agents. Assume this is the case by means of contradiction. This signifies that some agents save acquire bonds and, by the market clearing condition for the bonds market, some agents must sell bonds. Thus, $b_{t+1}(z) \neq 0$ for a positive mass of agents.
As proven above, under $l_t(z, m_t(z) = m_{t+1}(z)$, $q_t = \beta$ and $P_{t+1} \geq P_t$ for every t. Now, although consumption cannot differ across two agents with the same productivity, we must still have $C_t(z) > C_t(z') \geq 0$ for $z > z'$, since $m_t(z) > m_t(z')$ for all t. As a result, we can still rule out diverging prices and must, thus, conclude that $P_{t+1} = P_t$ for every period t. By the first-order condition, therefore, $C_{t+1}(z) = C_t(z)$ for all agents. For constant prices, this can only happen if $m_{t+1}(z) = m_t(z) = R_0(z)$ for all agents and all periods, since consumption is strictly increasing on monetary holdings. As show in the beginning of the proof to this proposition, this requires $b_{t+1}(z) = 0$ for all $z \in [\underline{z}, \overline{z}]$, which violates our contradiction assumption. I conclude, therefore, that $m_t(z) = R_0(z)$ implies that the economy is in the fundamental stationary equilibrium. $\square$

A.1.1 Corollary 1.1

To begin, notice that, by assumption, the monetary holdings function, $m_0(\cdot)$, is such that the economy starts off at the fundamental stationary equilibrium. Moreover, the monetary base grows by a factor τ^A, that is, $M_1 = (1 + \tau^A)M_0$. If revenues are undistorted relative to the fundamental equilibrium, *i.e.* $m_1(z) = (1 + \tau_H)m_0(z) = \theta_0(z)(1 + \tau^A)M_0 = R_1(z)$, since $\tau_H = \tau^A$. However, this implies that the new monetary holdigns are compatible with a new fundamental stationary equilibrium. Given Proposition 1, this is the unique equilibrium that can arise in this economy. Now, notice that, in this case, if $l_t(z, m_t(z)) = m_{t+1}(z)$, by Proposition 1, we must still be in a fundamental equilibrium. Moreover, notice that, even if borrowing is possible and the interest rate is positive, in the fundamental equilibrium, $b_{t+1}(z) = 0$ for all $z \in [\underline{z}, \overline{z}]$. Since buying bonds dominates saving in idle cash for $q_t = \beta$, the fact that $b_{t+1}(z) = 0$ in the economy with full enforcement of bond contracts implies that $s_t(z) = 0$ for all $z \in [\underline{z}, \overline{z}]$ in the economy where $l_t(z, m_t(z)) = 0$. This proves that the economy must be in the fundamental stationary equilibrium after the helicopter drops shock. □

A.1.2 Corollary 1.2

To begin, by definition, $m_{it}(z) = m_{i,t+1}(z)$ for all $z \in [\underline{z}, \overline{z}]$, $i \in \{c, u\}$, and $t \in \mathbb{T}^S$ at the stationary equilibrium. By (13), we must, thus, have:

$$\begin{aligned} p_{it}(z) &= \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} \frac{[D(p_{it}(z), P_t, Y_t)/z]^{\zeta}}{z} p_{it}(z) D(p_{it}(z), P_t, Y_t) \\ &= \left[\left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} \frac{Y_t^{1+\zeta}}{z^{1+\zeta}}\right]^{\frac{1}{\epsilon(1+\zeta)}} P_t \end{aligned} \tag{57}$$

for $t \in \mathbb{T}^S$. Aggregating prices according to (12) gives:

$$Y_t = \left[\left(\frac{\epsilon - 1}{\epsilon}\right) \frac{\beta}{\gamma}\right]^{\frac{1}{1+\zeta}} \left(\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\frac{\epsilon-1}{\epsilon}} dF_i(z)\right)^{\frac{\epsilon}{\epsilon-1}} \tag{58}$$

for $t \in \mathbb{T}^S$. This way, we can retrieve the aggregate prices as:

$$P_t = \frac{M_t}{Y_t} = \left[\left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta}\right]^{\frac{1}{1+\zeta}} \frac{1}{\left(\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\frac{\epsilon-1}{\epsilon}} dF_i(z)\right)^{\frac{\epsilon}{\epsilon-1}}} M_t \tag{59}$$

for $t \in \mathbb{T}^S$. Now, using (57), and plugging (58) into it gives:

$$\theta_{it}(z) = \left(\frac{p_{it}(z)}{P_t}\right)^{1-\epsilon} = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} \hat{z}^{\frac{\epsilon-1}{\epsilon}} dF_i(\hat{z})} \tag{60}$$

for $t \in \mathbb{T}^S$. By plugging (59) and (58) in, this means that:

$$m_{it}(z) = \theta_t(z) M_t = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} \hat{z}^{\frac{\epsilon-1}{\epsilon}} dF_i(\hat{z})} M_t \tag{61}$$

$$C_{it}(z) = \frac{m_{it}(z)}{P_t} = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} \hat{z}^{\frac{\epsilon-1}{\epsilon}} dF_i(\hat{z})} Y_t \tag{62}$$

$$p_{it}(z) = \left[\frac{z}{\left(\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} \hat{z}^{\frac{\epsilon-1}{\epsilon}} dF_i(\hat{z})\right)^{\frac{\epsilon}{\epsilon-1}}}\right]^{-\frac{1}{\epsilon}} P_t \tag{63}$$

for $t \in \mathbb{T}^S$. □

A.1.3 Corollary 1.3

Notice that, at $t = 1$, the cash-in-advance and the budget constraints, (2) and (3), can be written, respectively, as:

$$P_1 C_{i1}(z) = m_{it}(z) - \beta b_{i2}(z)$$
$$P_2 C_{i2}(z) = R_{i1}(z) + (1-\beta) b_{i2}(z)$$

Isolating $b_{i2}(z)$, using the fact that $P_t C_{it}(z) = P_s C_{is}(z)$ for $i \in \{h, l\}$ and $t, s = 1, 2, ...$ and re-arranging the terms gives us:

$$P_t C_{ct}(z) = (1-\beta)(1+\tau) m_0(z) + \beta R_{ct}(z)$$
$$P_t C_{ut}(z) = (1-\beta) m_0(z) + \beta R_{ut}(z)$$

Subtracting the latter expression from the former yields:

$$P_t C_{ct}(z) - P_t C_{ut}(z) = (1-\beta)\tau m_0(z) - \beta(R_{ut}(z) - R_{ct}(z))$$

Since we know that $C_{ct}(z) > (<) C_{ut}(z)$, then $P_t C_{ct}(z) - P_t C_{ut}(z) > (<) 0$ for $\tau > (<) 0$. As a result, we obtain:

$$|R_{ut} - R_{ct}| < \left| \left(\frac{1-\beta}{\beta} \right) \tau m_0(z) \right|$$

Equations (34) and (35) follow immediately from the facts that $R_{ut}(z) - R_{ct}(z) > (<) 0$ and $C_{ct}(z) > (<) C_{ut}(z)$ for $\tau > (<) 0$. □

A.2 Proposition 2

To begin, assume that, if no high-cash agent chooses to partially deplete, then $T = 1$ and, hence, the result follows trivially. I will, then, concentrate on the case where a positive mass of high-cash agents partially depletes. The case where a zero mass of high-cash agents partially depletes also follows as a combination of both cases.

Low-cash agents are more likely to fully deplete their cash at $t = 1$:

First, I need to prove that, for an arbitrary $z \in [\underline{z}, \overline{z}]$, the low-cash agent is more likely to fully deplete at $t = 1$. This requires proving that she will fully deplete if the high-cash agent chooses full depletion at $t = 1$. To see this, assume that the high-cash agent saves nothing at $t = 1$. Now, notice that $m_{h1}(z) > m_{l1}(z)$. Then:

$$C_{h1}(z) = \frac{m_{h1}(z)}{P_1} > \frac{m_{l1}(z)}{P_1} \geq C_{l1}(z)$$

By the first order condition of the, fully-depleting, high-cash agent:

$$\frac{u'(C_{h1}(z))}{P_1} \geq \beta \frac{u'(R_{h1}(z)/P_2)}{P_2},$$

but then:

$$\frac{u'(C_{l1}(z))}{P_1} > \beta \frac{u'(R_{h1}(z)/P_2)}{P_2}$$

Since $R_{l1}(z) = R_{h1}(z)$ in the case where both decide to fully deplete at $t = 1$, the full depletion condition is satisfied for the low-cash agent as well. As a result, for an arbitrary $z \in [\underline{z}, \overline{z}]$, there are three possibilities: 1) both kinds of agent fully deplete; 2) both partially deplete; or 3) only the high-cash type partially depletes.

Revenues are not lower for low-cash agents than for high-cash ones:

I will show here that $\theta_{l1}(z) \geq \theta_{h1}(z)$. We analyze each of the possible cases in turn in turn:

1) Everyone fully depletes for $z \in [\underline{z}, \overline{z}]$: Naturally, in this case, $\theta_{l1}(z) = \theta_{h1}(z)$.

2) Everyone partially depletes for $z \in [\underline{z}, \overline{z}]$: Now, we have, for $i \in \{h, l\}$:

$$\theta_{i1}(z) = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{z^{1+\zeta} u'(C_{i1}(z))}{\gamma Y_t^{\zeta}} \right]^{\frac{\epsilon-1}{1+\zeta\epsilon}}$$

Since, naturally, $C_{i1}(z)$ strictly increases on $m_{i1}(z)$ given P_1, $\theta_{l1}(z) > \theta_{h1}(z)$.

3) Only the high-cash agent partially depletes for $z \in [\underline{z}, \overline{z}]$: Notice that, in this case, we have:

$$\theta_{i1}(z) = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{z}{\gamma} \beta \frac{P_1}{P_2} \frac{u'(C_{i2}(z))}{h_{i1}^{\zeta}}) \right]^{\epsilon-1}, \tag{64}$$

Finally, assume by contradiction that $\theta_{l1}(z) \leq \theta_{h1}(z)$. This requires $p_{l1}(z) \geq p_{h1}(z)$ and, therefore, by (13):

$$\frac{u'(C_{h2}(z))}{h_{h1}^{\zeta}} \geq \frac{u'(C_{l2}(z))}{h_{l1}^{\zeta}},$$

As shown before, for $i \in \{h, l\}$:

$$\frac{u'(C_{i2}(z))}{h_{i1}(z)^{\zeta}} = \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta z} \frac{P_2}{P_1} \right]^{\frac{\zeta\epsilon}{1+\zeta\epsilon}} Y_1^{\frac{-\zeta}{1+\zeta\epsilon}} u'(C_{i2}(z))^{\frac{1}{1+\zeta\epsilon}},$$

which, in turn, implies that $C_{l2}(z) \geq C_{h2}(z)$. Since the low-cash agent fully depletes, we must have $R_{l1}(Z) \geq P_2 C_{l2}(z)$. Moreover, $\theta_{l1}(z) \leq \theta_{h1}(z)$ implies that $R_{h1}(z) \geq$

$R_{l1}(z)$. However, since $s_{h1}(z) > 0$ and $s_{l1}(z) = 0$, $C_{l2}(z) \geq C_{h2}(z)$ cannot happen, since consumption at $t = 2$ is strictly increasing on $m_{i2}(z)$, given the price P_2. As a result, $\theta_{l1}(z) > \theta_{h1}(z)$.

I have proved above that $\theta_{l1}(z) \geq \theta_{h1}(z)$. This implies that $p_{l1}(z) \leq p_{h1}(z)$, $y_{l1}(z) \geq y_{h1}(z)$ and $R_{l1}(z) \geq R_{h1}(z)$ for all $z \in [\underline{z}, \overline{z}]$, where the inequalities hold strictly whenever the high-cash agent partially depletes. Now, I will prove, by an induction argument, that these results hold for an arbitrary $t = \{1, 2, ...\}$ in which the economy is not in a stationary equilibrium. Therefore, it suffices to show that no reversion occurs, that is, if the inequalities above hold for an arbitrary t, then $m_{h,t+1}(z) \geq m_{l,t+1}(z)$, which will, in turn, imply that $\theta_{l,t+1}(z) \geq \theta_{h,t+1}(z)$.

Again, I will show it by cases. To begin, the case where both high- and low-cash agents fully deplete their cash at t for some $z \in [\underline{z}, \overline{z}]$ is trivial, as it directly implies that $m_{l,t+1}(z) = m_{h,t+1}(z)$. Now, consider the other two cases - that is, either both types partially deplete at t or only the high-cash agent does. Assume, by contradiction, that $t + 1$ is the moment of reversion in roles, meaning that $m_{lt}(z) < m_{ht}(z)$ and $m_{l,t+1}(z) > m_{h,t+1}(z)$. This means that $\theta_{lt}(z) > \theta_{ht}(z)$. This requires $C_{l,t+1}(z) < C_{h,t+1}(z)$, which contradicts our assumption of reversion. I, therefore, conclude that $\theta_{lt}(z) \geq \theta_{ht}(z)$, $p_{lt}(z) \leq p_{ht}(z)$, $y_{lt}(z) \geq y_{ht}(z)$ and $R_{lt}(z) \geq R_{ht}(z)$ for all $z \in [\underline{z}, \overline{z}]$ and for all t such that the economy is not in a stationary equilibrium.

Characterization of inflation and relative revenues:

Integrating the individual price choices and imposing the result that the full depletion first-order condition holds for a non-negligible mass of agents implies that:

$$P_t = P_{t+1} \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} \left[\sum_{i \in \{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} \frac{z^{\epsilon-1} u'(C_{i,t+1}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}} dF_i(z) \right]^{\frac{1}{1-\epsilon}} \tag{65}$$

Now, notice that we can re-write:

$$\theta_{it}(z) = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{\beta}{\gamma} \frac{P_t}{P_{t+1}} \frac{z u'(C_{i,t+1}(z))}{h_{it}(z)^{\zeta}} \right]^{\epsilon - 1}$$

Therefore:

$$\frac{\theta_{it}(z)}{\theta_{i0}(z)} = \left(\frac{1}{1 + \pi_{t+1}} \right)^{\epsilon-1} \frac{z^{\epsilon-1} u'(C_{i,t+1}(z))^{\epsilon-1} / h_{it}(z)^{\zeta(\epsilon-1)}}{z^{\epsilon-1} u'(C_{io}(z))^{\epsilon-1} / h_{i0}(z)^{\zeta(\epsilon-1)}} = \frac{U^{GAP}_{i,t+1}(z)}{U^{GAP}_{t+1}} \tag{66}$$

Characterization of relative monetary holdings per income bracket:

Notice that, for any $z \in [\underline{z}, \overline{z}]$, the result above implies that $P_t C_{ht}(z) > P_t C_{lt}(z)$ and

$R_{ht}(z) < R_{lt}(z)$ for as long as the high-cash agent does not fully deplete, which means, by (3), that:

$$\Delta m_{h,t+1}(z) = R_{ht}(z) - P_t C_{ht}(z) < R_{lt}(z) - P_t C_{lt}(z) = \Delta m_{l,t+1}(z), \qquad (67)$$

where $\Delta m_{i,t+1}(z) = m_{i,t+1}(z) - m_{it}(z)$ for $i \in \{h, l\}$.

The economy converges to the new stationary equilibrium in finite time:
Assume, by contradiction, that the mass of high-cash partially depleting agents remains forever bounded away from 0. Then, for these agents:

$$\lim_{t\to\infty} \frac{u'(C_{ht}(z))}{P_t} = \lim_{t\to\infty} \frac{u'(C_{h1}(z))}{\beta^{t-1} P_1} = \infty,$$

which implies that either $P_t \to 0$ or $C_{ht}(z) \to 0$. However, notice that, for partially depleting high-cash agents (who exist in positive mass by the contradiction assumption), $p_{ht}(z) > p_{lt}(z) \geq 0$, which implies that $P_t > 0$ for every $t = \{1, 2, ...\}$ by our contradiction assumption. This implies that $C_{ht} \to 0$, which cannot be the case either, since $m_{ht}(z) > m_{lt}(z) \geq 0$, which means that positive consumption is always available for these high-cash agents.

This proves that the mass of partially depleting agents goes to 0 in finite time, meaning that, at some time $T < \infty$, the economy reaches the new fundamental stationary equilibrium, in which the aggregate price is equal to $P_T = P^H(M_T) > 0$. This ensures that no agent, even with negligible mass, can partially deplete forever. Naturally, for any high-cash agent that partially depletes until $T-1$, it is the case that $m_{hT}(z) > m_{lT}(z)$, meaning that $C_{hT}(z) = m_{hT}(z)/P^H((1+\tau^A)M_0) > C_{h0}(z)$. As a result, consumption only returns to the stationary equilibrium level for all agents at $T+1$. Finally, $Y_T = M_T / P^H(M_T) = Y_0$. □

A.3 Proposition 3

There is a non-negligible mass of fully depleting agents:
Now, I go on to prove that there is a non-negligible mass of fully depleting agents. Assume otherwise by means of contradiction. I begin by showing that, if there is no positive mass of fully depleting agents at t, then this must also be the case for $t+1$. Again, by contradiction, assume that there is some low-cash agent that fully depletes at $t+1$. This suffices as we have already shown that low-cash agents are more likely to fully deplete. As will be shown in more details below, due to the homothety of preferences, $R_{lt}(z) = \overline{R_{lt}} z^{\frac{\epsilon-1}{\epsilon}} / \mathcal{Z}_l^{\frac{\epsilon-1}{\epsilon}}$ and $C_{lt}(z) = \overline{C_{lt}} z^{\frac{\epsilon-1}{\epsilon}} / \mathcal{Z}_l^{\frac{\epsilon-1}{\epsilon}}$, where $\overline{R_{lt}}$ and $\overline{C_{lt}}$ are

respectively the average revenue and consumption among low-cash agents[18]. As a result:

$$(\overline{R_{lt}} - P_t\overline{C_{lt}})\frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_l^{\frac{\epsilon-1}{\epsilon}}} = R_{lt}(z) - P_tC_{lt}(z) = \Delta m_{l,t+1}(z)$$

Now, notice that, since $\Delta m_{l,t+1}(z)$ must aggregate to 0 and that $\Delta m_{l,t+1}(z) > \Delta m_{h,t+1}(z)$ for every $z \in [\underline{z}, \overline{z}]$ according to Proposition 2, then it must be the case that $\Delta m_{l,t+1}(z) > 0$ for all $z \in [\underline{z}, \overline{z}]$. Now, notice that, for the low-cash agent that partially depletes her money at t, but fully depletes at $t+1$, it must be the case, by the first-order condition, that:

$$P_{t+1}C_{l,t+1}(z) = \beta P_tC_{lt}(z) < m_{lt}(z) < m_{l,t+1}(z),$$

a contradiction. Therefore, all low-cash agents that partially deplete at t must also partially deplete at $t+1$. As a consequence, if there is no positive mass of partially depleting agents at t, then there must be no positive mass of partially depleting agents at $t+1$ as well. This, however, means that the economy will never converge to the fundamental stationary equilibrium, which contradicts Proposition 2. Therefore, there must be, at any given t, a positive mass of fully depleting agents.

Relative revenues obtained by low- and high-cash agents:

To begin, notice that, since 1) some agents need to fully deplete in equilibrium, and 2) low-cash agents are more likely to do so, then a non-negligible mass of low-cash agents must choose full depletion. However, due to the homothety of preferences, this means that all of the low cash agents choose to do so too. I will prove so, by showing that all low-cash agents must choose to fully deplete whenever any of them finds it optimal to do so. I start by showing that, for any fully depleting low-cash agent, $p_{lt}(z)$ must satisfy (57). Therefore, using (11), (14) and the fact that $C_{l,t+1}(z) = \theta_{lt}\frac{M_t^C}{P_{t+1}}$, we obtain, for the low-cash agents:

$$\begin{aligned}\theta_{lt}(z) &= \left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}\frac{z}{\left(\theta_{lt}(z)^{\frac{\epsilon}{\epsilon-1}}Y_t/z\right)^{\zeta}}\frac{P_t}{P_t+1}\frac{P_{t+1}}{\theta_{lt}(z)M_t^C}\right]^{\epsilon-1} \\ &= \left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}\frac{1}{Y_t^{1+\zeta}}\right]^{\frac{\epsilon-1}{\epsilon(1+\zeta)}} z^{\frac{\epsilon-1}{\epsilon}} \qquad (68)\end{aligned}$$

[18] I will show later in this proof that this occurs both for fully and partially depleting agents, and it means that all agents of the same type, that is, either low- or high-cash, must make the same choice between full and partial depletion.

Notice that it is decreasing on aggregate output. Now, notice that, at $t = 1$, full depletion requires:

$$\frac{1}{\beta}\theta_{l1}(z)M_1^C = \frac{1}{\beta}R_{l1}(z) \geq m_{l1}(z) = (1 + \mathbb{1}_{\tau<0}\tau)\theta_{l0}(z)M_0$$
$$\frac{1}{\beta}Y_t^{\frac{1-\epsilon}{\epsilon}}M_t^C \geq \frac{1}{\beta}Y_0^{\frac{1-\epsilon}{\epsilon}}M_0,$$

where $\mathbb{1}_{\tau<0}$ takes the value of 1 in the case of a contractionary shock, and 0 otherwise. Notice that this condition does not depend on z, meaning that it holds for all low-cash agents. A simple proof by induction generalizes this result for an arbitrary t along the transition toward the new stationary equilibrium. Now, I look into high-cash agents. Notice that, by imposing the partial depletion first-order condition to (13) and computing the resulting relative revenue, we obtain:

$$\theta_{ht}(z) = \left[\left(\frac{\epsilon - 1}{\epsilon}\right)\frac{z^{1+\zeta}}{\gamma C_{ht}(z)}\frac{1}{Y_t^{\zeta}}\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}} \tag{69}$$

Thus, it decreases their current consumption. As before, homothety implies that all high-cash agents choose to save a positive amount whenever one of them does so. A similar argument to the one made above suffices to prove it, as the condition for partial depletion by high cash agents does not depend on z. I will now arrive at an expression for the consumption, at $t = 1$, of any high-cash agent as a function of the contemporaneous average high-cash consumption.

Average individual variables

Notice that the intertemporal budget constraint of any high-cash agent along the transition path can be written as:

$$P_1 C_{h1}(z)\sum_{t=1}^{T}\beta^{t-1} = m_{h1}(z) + \sum_{t=1}^{T-1}\theta_{ht}(z)P_t Y_t,$$

where I have imposed the partial depletion first-order condition. Moreover, we can also impose it onto (69), which yields:

$$C_{h1}(z) = \left(\frac{1-\beta}{1-\beta^T}\right)\frac{1}{P_1}$$
$$\left\{(1 + \mathbb{1}_{\tau>0}\tau)m_0(z) + \left(\frac{z^{1+\zeta}}{C_{h1}(z)}\right)^{\frac{\epsilon-1}{1+\zeta\epsilon}}\left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{1}{\gamma P_1}\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}}\sum_{t=1}^{T-1}\beta^{\frac{(t-1)(1-\epsilon)}{1+\zeta\epsilon}}P_t^{\epsilon}Y_t^{\frac{1+\zeta}{1+\zeta\epsilon}}\right\},$$

which, using (22), can be simplified to:

$$C_{h1}(z) = Az^{\frac{\epsilon-1}{\epsilon}} + Bz^{\frac{(1+\zeta)(\epsilon-1)}{1+\zeta\epsilon}}C_{h1}(z)^{\frac{1-\epsilon}{1+\zeta\epsilon}}, \tag{70}$$

where A and B correspond to expressions that include only parameters and aggregate variables, which are common across all high-cash agents. Now, one can assume that $C_{h1}(z) = \overline{C_{h1}} z^D$, where $\overline{C_{h1}}$ is the average consumption by high-cash agents, and D is a constant. By plugging this in the equation above, we obtain:

$$C_{h1}(z) = \overline{C_{h1}} \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_h^{\frac{\epsilon-1}{\epsilon}}},$$

where $\overline{C_{h1}} = C_{h1}(\mathcal{Z}_h)$. By the first-order condition of partially depleting agents, this means that the whole consumption path is the same across high-cash agents as well as the moment where they decide to fully deplete, T. This, in turn, implies that $\theta_{ht}(z) = \overline{\theta_{ht}} \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_h^{\frac{\epsilon-1}{\epsilon}}}$, $m_{ht}(z) = \overline{m_{ht}} \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_h^{\frac{\epsilon-1}{\epsilon}}}$, $\overline{\theta_{ht}} = \theta_{ht}(\mathcal{Z}_h)$, and $\overline{m_{ht}} = m_{ht}(\mathcal{Z}_h)$. Moreover, (68) implies that $X_{l1}(z) = \overline{X_{l1}} \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_i^{\frac{\epsilon-1}{\epsilon}}}$ and $\overline{X_{l1}} = X_{l1}(\mathcal{Z}_h)$ for $X \in \{C, \theta, m\}$ as well.

Output and consumption paths:

The results above imply that $\theta_{lt}(z) > \theta_0(z) > \theta_{ht}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $t = \{1, ..., T-1\}$. In turn, equations (68) and (69) imply, respectively, that $Y_t < Y_0$ and $C_{ht}(z) > C_0(z) > C_{lt}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and t along the transition to the new stationary equilibrium, where the latter inequality relies on the former. Moreover, notice that, by (13):

$$p_{it}(z) = \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{z^{1+\zeta}\beta} P_t^{\zeta\epsilon} Y_t^{\zeta} P_{t+1} C_{i,t+1}(z) \right]^{\frac{1}{1+\zeta\epsilon}}$$

for $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$. Aggregating it, and computing the corresponding $\theta_{it}(z)$ gives:

$$\theta_{it}(z) = \frac{\left(\frac{z^{1+\zeta}}{P_{t+1}C_{i,t+1}(z)} \right)^{\frac{\epsilon-1}{1+\zeta\epsilon}}}{\sum_{j\in\{h,l\}} \eta_j \int_{\underline{z}}^{\overline{z}} \left(\frac{\hat{z}^{1+\zeta}}{P_{t+1}C_{j,t+1}(\hat{z})} \right)^{\frac{\epsilon-1}{1+\zeta\epsilon}} dF_j(\hat{z})}, \tag{71}$$

which means that, for any $z \in [\underline{z}, \overline{z}]$:

$$\frac{\theta_{lt}(z)}{\theta_{ht}(z)} = \left(\frac{P_{t+1}C_{h,t+1}(z)}{P_{t+1}C_{l,t+1}(z)} \right)^{\frac{\epsilon-1}{1+\zeta\epsilon}}$$

Since $R_{lt}(z) > R_{ht}(z)$ and $P_tC_{lt}(z) < P_tC_{ht}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $t = \{1, ..., T-1\}$, then we must have $R_{lt}(z) - P_tC_{lt}(z) > 0 > R_{ht}(z) - P_tC_{ht}(z)$ given the homothety of the utility function. As a result, $m_{l,t+1}(z) > m_{lt}(z)$ and $m_{h,t+1}(z) < m_{ht}(z)$ for every $z \in [\underline{z}, \overline{z}]$. This means that, for low-cash agents, $P_tC_{lt}(z) = m_{lt}(z)$ grows over time. For high-cash agents, on the other hand, $P_tC_{lt}(z) = \beta P_{t-1}C_{h,t_1}(z)$ for $t \in \{2, 3, ..., T-1\}$, meaning that it decreases over time. As a result, $\theta_{l0}(z)/\theta_{h0}(z) \leq \theta_{l,t+1}(z)/\theta_{h,t+1}(z) < \theta_{lt}(z)/\theta_{ht}(z)$ for $t \in \{1, 2, ..., T-1\}$.

Due to the homothety of preferences and the fact that $\theta_{it}(z)$ must integrate to 1, this means that $\theta_0(z) \leq \theta_{l,t+1}(z) < \theta_{lt}(z)$ and $\theta_0(z) \geq \theta_{h,t+1}(z) > \theta_{ht}(z)$. By (68), this implies that $Y_0 \geq Y_{t+1} > Y_t$ with strict inequality for $t = \{1, ..., T-2\}$ and equality for $t = T-1$. Finally, with this in mind, (69) implies that $C_0(z) \leq C_{h,t+1}(z) < C_{ht}(z)$.

Characterization of T:

Notice that $P_T C_{hT}(z) < \beta^{T-1} m_{h1}(z)$ for $z \in [\underline{z}, \overline{z}]$ by the first-order condition and budget constraint of high-cash agents. Given that, as proven in Proposition 2, $C_{hT}(z) > \theta_0(z)(M_0 + \eta\tau M_{c0})/P_T$, we obtain the condition:

$$\theta_0(z)(1+\tau^A)M_0 < \beta^{T-1} P_1 C_{h1}(z) < \beta^{T-1}\theta_0(z)(1+\mathbb{1}_{\tau>0}\tau)M_0$$
$$\beta^{T-1} > \frac{1+\tau^A}{1+\mathbb{1}_{\tau>0}\tau} \tag{72}$$

Let the maximum value of T that satisfies this condition be denoted by T^{MAX}. Then, $T \leq T^{MAX}$.

Characterization of prices:

Since low-cash agents always fully deplete their resources, we must have that, for $t = \{1, 2, ...T-1\}$:

$$P_{t+1} C_{l,t+1}(z) = R_{lt}(z) = \theta_{lt}(z) M_t^C > \theta_0(z) M_t^C$$

which means that:

$$p_{ht}(z) > p_{lt}(z) > p^H(z, M_t^C). \tag{73}$$

This means that $P(M_t^C) > P^H(M_t^C)$. Moreover, notice that (13) implies that:

$$p_{lt}(z) = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\frac{\left(\frac{y_{lt}(z)}{z}\right)^{\zeta}}{z} p_{lt}(z) y_{lt}(z) \iff y_{lt}(z) = z\left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}\right]^{\frac{1}{1+\zeta}}, \tag{74}$$

meaning that $h_{l,t+1}(z) = h_{lt}(z) = h_{lt}(z')$ for all periods and any $z, z' \in [\underline{z}, \overline{z}]$. However, since low-cash agents fully deplete, this means that their prices are proportional to $p_{lt}(z) y_{lt}(z) = m_{l,t+1}(z)$. As a result, $p_{lt}(z) \leq p_{l,t+1}(z)$, with strict inequality for $t \in \{1, 2, ..., T-1\}$.

Lastly, by an argument already made in the proof of Proposition 1, one can show that:

$$\left[\sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} \frac{z^{\frac{\epsilon-1}{1+\zeta\epsilon}}}{C_{it}(z)^{\frac{\epsilon-1}{1+\zeta\epsilon}}} dF_i(z)\right]^{\frac{1+\zeta\epsilon}{\epsilon-1}} > \left(\frac{\epsilon}{\epsilon-1}\right)\gamma Y_t^{\zeta}$$

for every $t \in \{1, 2, ...\}$, where the strong inequality follows from the fact that, by the first-order condition of low-cash agents, $C_{l,t+1}(z) > \beta \frac{P_t}{P_{t+1}} C_{lt}(z)$. We can plug this into the aggregate price expression (47), which gives us:

$$\frac{P_{t+1}}{P_t} > \beta \left(\frac{Y_{t+1}}{Y_t} \right)^{\zeta} \geq \beta, \tag{75}$$

where the last inequality holds strictly for $t \in \{1, 2, ..., T-1\}$, since $Y_{t+1} > Y_t$ then. $\square$

A.4 Proposition 4

I want to show that $T = \infty$ is not possible. I will do so through a simple proof by contradiction. Assume that $T = \infty$. As the borrowing constraint only binds for low-cash agents, we have:

$$\frac{u'(C_{ht}(z))}{P_t} = \frac{\beta}{q_t} \frac{u'(C_{h,t+1}(z))}{P_{t+1}} \quad \text{and} \quad \frac{u'(C_{lt}(z))}{P_t} > \frac{\beta}{q_t} \frac{u'(C_{l,t+1}(z))}{P_{t+1}} \tag{76}$$

which is valid for any $t = 1, 2, ...$ if $q_t \neq \beta$. This means that, (18) can be re-written as:

$$\frac{P_{t+1}}{P_t} > \frac{\beta}{q_{t+1}} \tag{77}$$

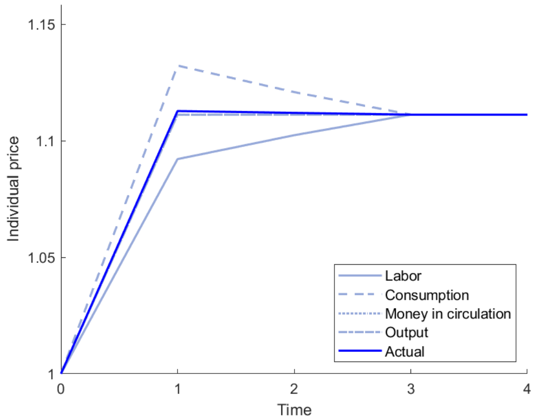
which follows from the fact that $\overline{U}^{GAP}_{t+2} < \overline{U}^{GAP}_{t+1} \frac{\beta}{q_{t+1}} \frac{P_{t+1}}{P_{t+2}}$ by the first-order conditions of above. Now, notice that that, by our simplifying assumption that $q_t = q_{t+1}$, which means that $P_{t+1} q_t / \beta > P_t$. This can be plugged into the first-order condition for the high-cash agent, as in (76), to obtain $u'(C_{ht}(z)) < u'(C_{h,t+1}(z))$, which means that $C_{h,t+1}(z) < C_{ht}(z)$ for all $t \in \{1, 2, ...\}$. I will now show that $\lim_{t\to\infty} C_{ht}(z) = 0$ for all high-cash agents. Assume that there is some high-cash agent with productivity $z \in [\underline{z}, \overline{z}]$ whose consumption does not converge to 0. Then, let $\lim_{t\to\infty} C_{ht}(z) =: C^{MIN}_{hT}(z) > 0$, meaning that $C^{MIN}_{hT}(z) = C^{MIN}_{h,T+1}(z)$[19] This means that, by (76), at the limit, $1 + \pi_{T+1} = \beta / q_T < 1$, meaning that the deflation rate remains bounded away from zero even at the limit. This, in turn, means that $P_t \to 0$ and, thus, by (43), we must have:

$$\lim_{t\to\infty} \sum_{i \in \{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} C_{it}(z) dF_i(z) = \infty,$$

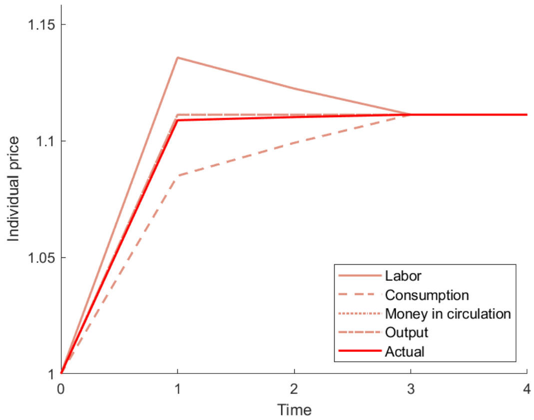
which cannot happen, as the consumption of all high cash agents is finite and below $C_{h1}(z)$. Hence (77) implies that $\lim_{t\to\infty} C_{ht}(z) = 0$ for every $z \in [\underline{z}, \overline{z}]$. This is a contradiction, however, since $C_{ht}(z) > C_{lt}(z) \geq 0$ for as long as high-cash agents do not fully deplete, which leads to the conclusion that $T < \infty$.

[19] I look at the limit for simplicity, but this intuitively means that, after some $t < \infty$, the differences between $C_{ht}(z)$ and $C_{h,t+1}(z)$ are nearly zero.

B Outstanding Graphs



(a) Connected agents' prices, $p_{ct}(\mathcal{Z})$



(b) Unconnected agents' prices, $p_{ut}(\mathcal{Z})$

Figure 6: Decomposition of individual prices in the baseline economy

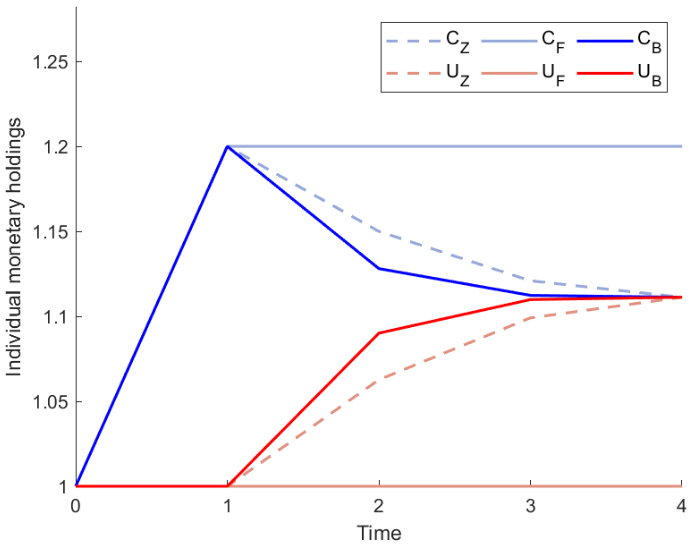
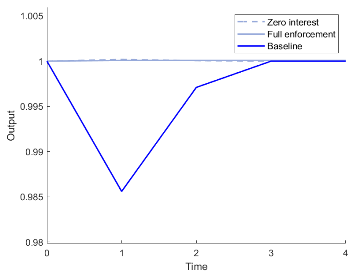
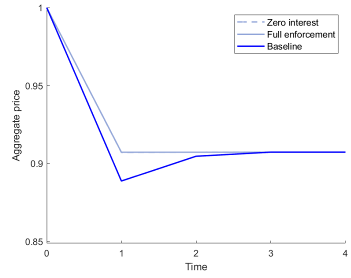


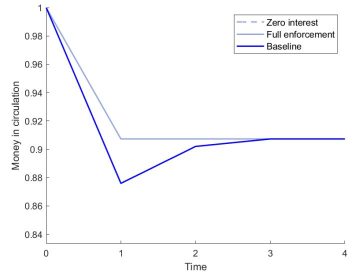
Figure 7: Comparison of monetary holdings, m_t, across agents



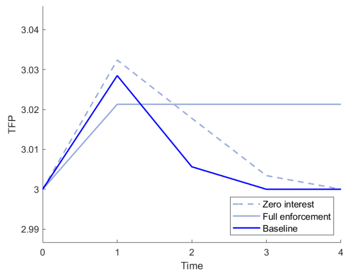
(a) Path of output, Y_t



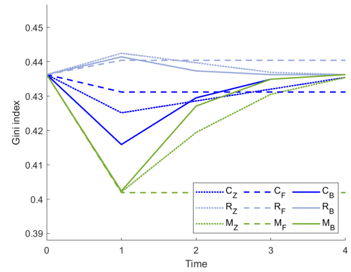
(b) Path of output, P_t



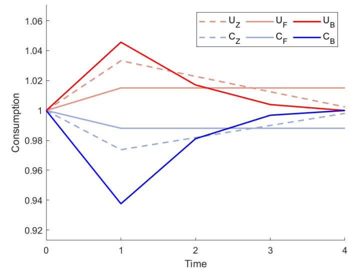
(c) Money in circul., M_t^C



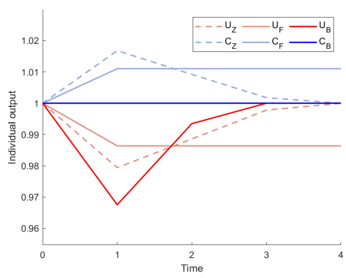
(d) Total Factor Productivity



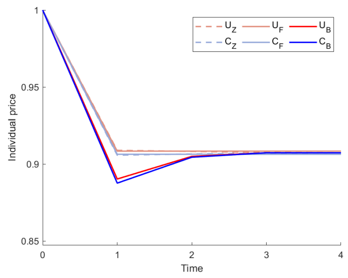
(e) Gini indexes



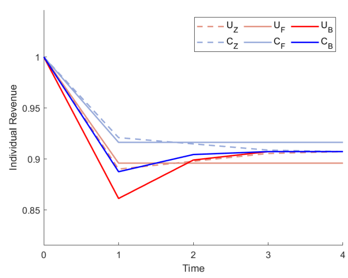
(f) Indiv. consumption, $C_{it}(z)$



(g) Individual output, $y_{it}(z)$

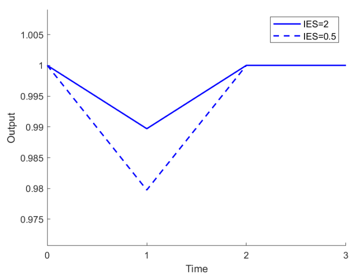


(h) Individual prices, $p_{it}(z)$

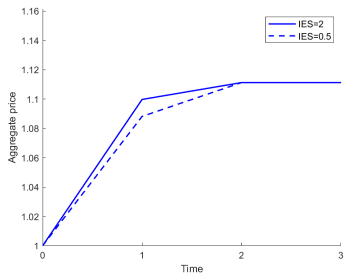


(i) Indiv. revenues, $R_{it}(z)$

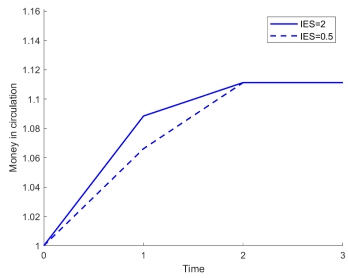
Figure 8: Paths for aggregate and individual variables under a negative shock



(a) Path of output, Y_t

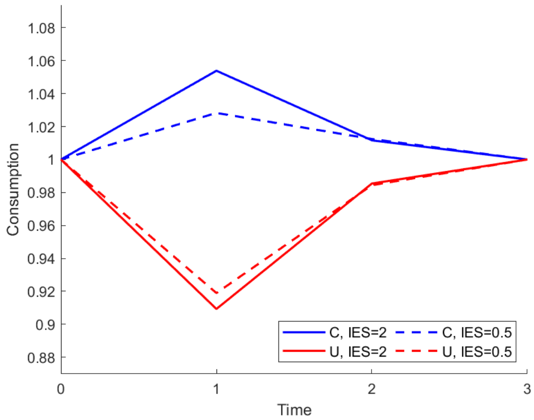


(b) Aggregate price, P_t

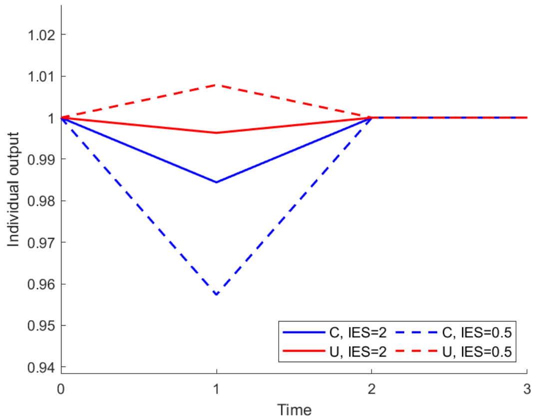


(c) Money in circul., M_t^C

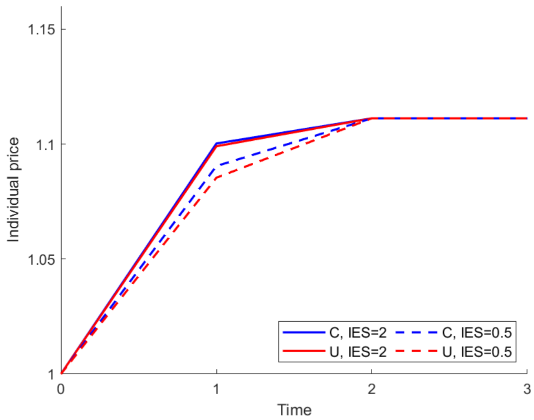
Figure 9: Paths for aggregate variables under CRRA utility



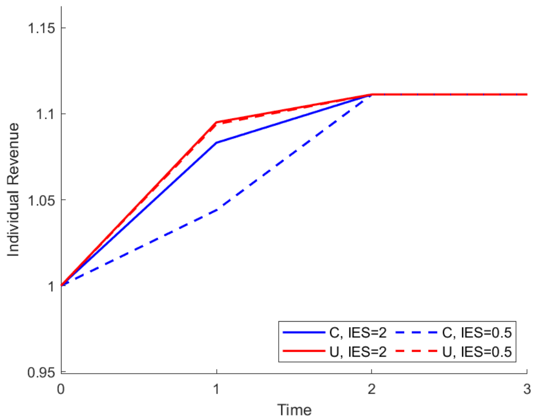
(a) Individual consumption, $C_{it}(\mathcal{Z})$



(b) Individual output, $y_{it}(\mathcal{Z})$

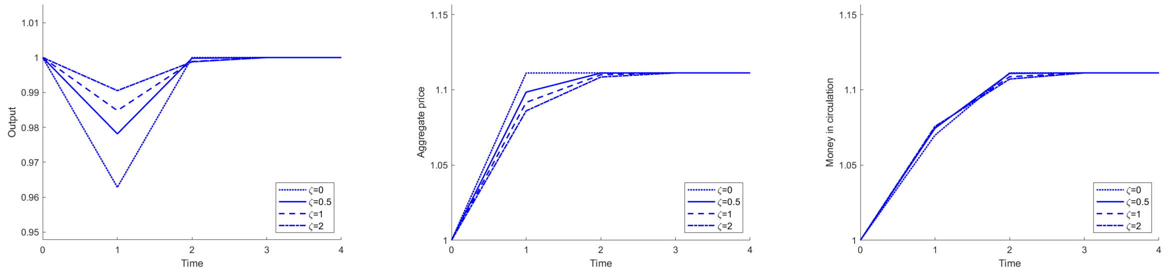


(c) Individual prices, $p_{it}(\mathcal{Z})$

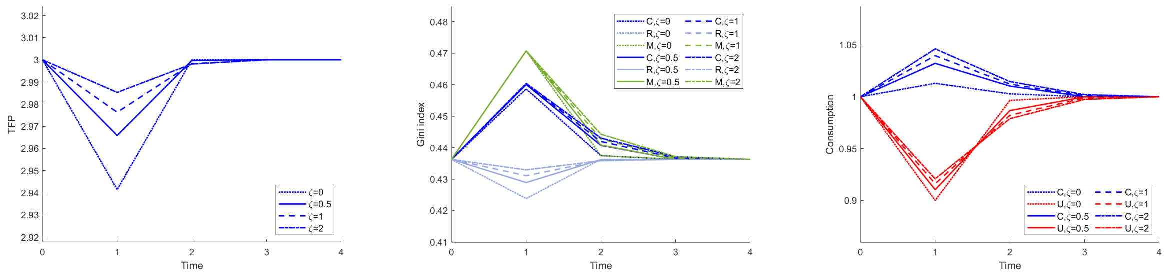


(d) Indiv. revenues, $R_{it}(\mathcal{Z})$

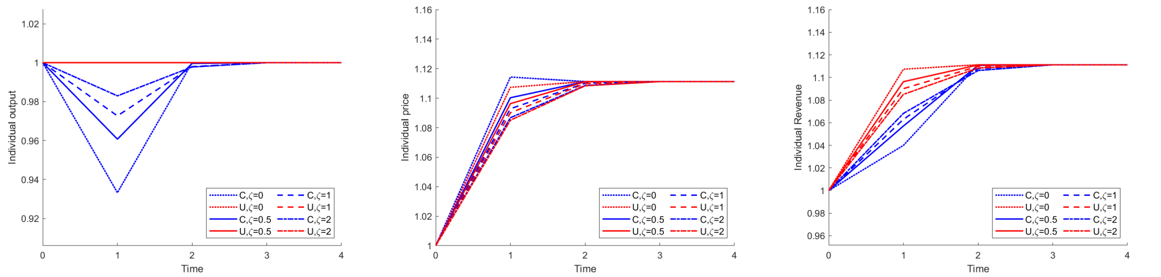
Figure 10: Paths for individual variables under CRRA utility



(a) Path of output, Y_t (b) Aggregate price, P_t (c) Money in circulation, M_t^C

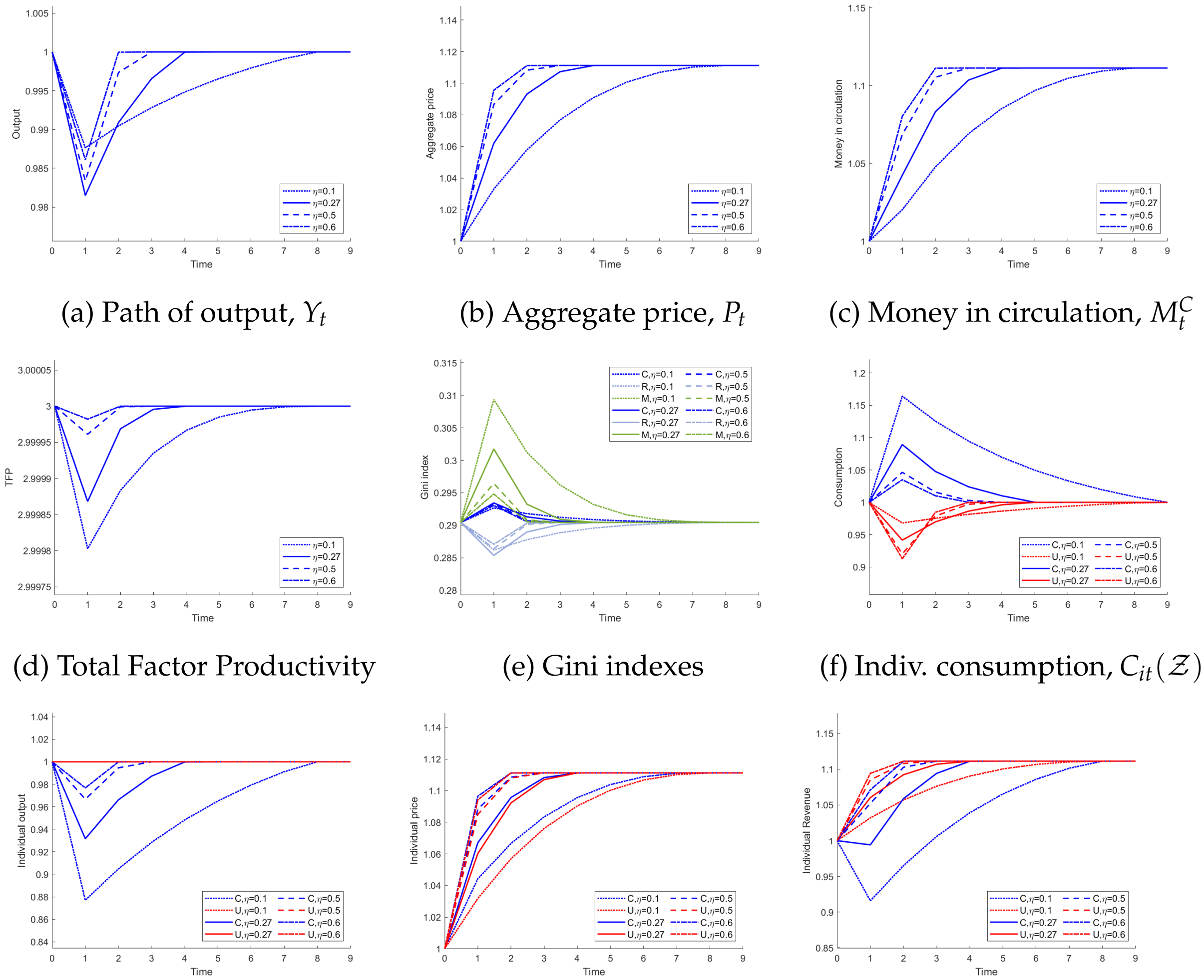


(d) Total Factor Productivity (e) Gini indexes (f) Indiv. consumption, $C_{it}(\mathcal{Z})$



(g) Individual output, $y_{it}(\mathcal{Z})$ (h) Individual prices, $p_{it}(\mathcal{Z})$ (i) Indiv. revenues, $R_{it}(\mathcal{Z})$

Figure 11: Paths for aggregate and individual variables under different Frisch elasticities

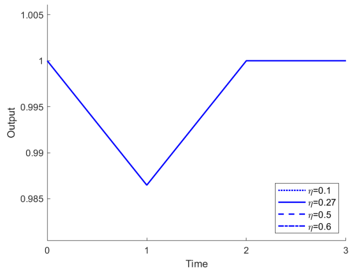


(a) Path of output, Y_t (b) Aggregate price, P_t (c) Money in circulation, M_t^C

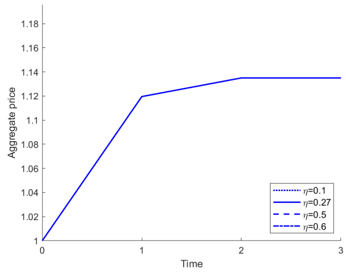
(d) Total Factor Productivity (e) Gini indexes (f) Indiv. consumption, $C_{it}(\mathcal{Z})$

(g) Individual output, $y_{it}(\mathcal{Z})$ (h) Individual prices, $p_{it}(\mathcal{Z})$ (i) Indiv. revenues, $R_{it}(\mathcal{Z})$

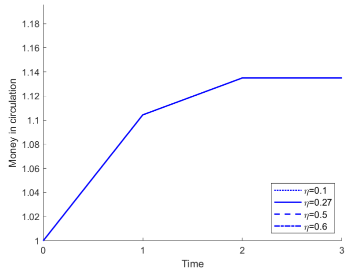
Figure 12: Paths for aggregate and individual variables for different values of η and τ



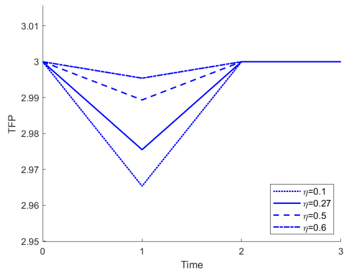
(a) Path of output, Y_t



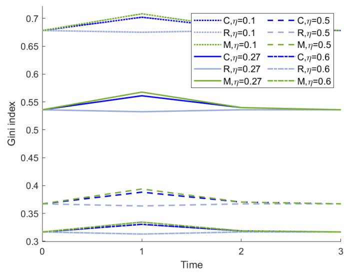
(b) Aggregate price, P_t



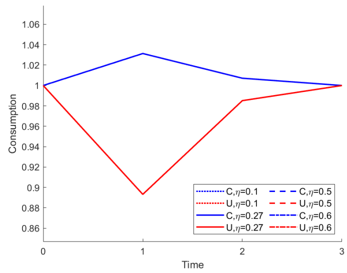
(c) Money in circulation, M_t^C



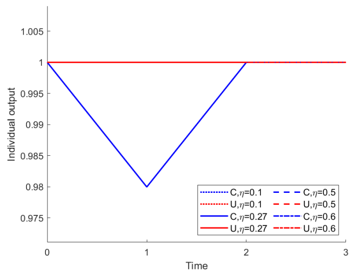
(d) Total Factor Productivity



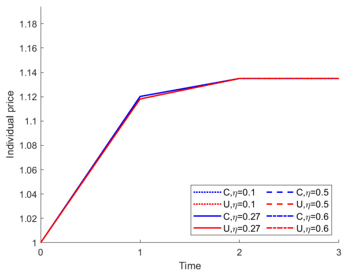
(e) Gini indexes



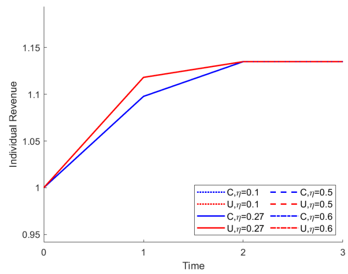
(f) Indiv. consumption, $C_{it}(\mathcal{Z})$



(g) Individual output, $y_{it}(\mathcal{Z})$



(h) Individual prices, $p_{it}(\mathcal{Z})$



(i) Indiv. revenues, $R_{it}(\mathcal{Z})$

Figure 13: Paths for aggregate and individual variables for different values of η and $\frac{M_{c0}}{M_0}$

C Outstanding Tables

Quarter	ΔM_t
1999-Q4	10.0036%
2008-Q4	83.1813%
2009-Q4	12.4917%
2011-Q1	18.7556%
2011-Q2	10.5707%
2020-Q1	13.3256%
2020-Q2	28.8095%
2021-Q1	12.1483%
2022-Q2	-10.2375%

Table 4: Shocks to the U.S. monetary base of more than 10%

Source: Board of Governors of the Federal Reserve System (US), retrieved from FRED, Federal Reserve Bank of St. Louis

Fraction of Connected	Model	Constant output	No inequality
$M_{c0}/M_0 = 2.06$	-5.5304%	-5.3758%	-0.1853%
$M_{c0}/M_0 = 1$	-0.4016%	-0.217%	-0.1853%
$M_{c0}/M_0 = 0.6$	6.6595%	6.8873%	-0.1853%

The second column shows the welfare in the economy that does not receive a monetary shock. The third column contains the values for the welfare function under the benchmark specification of the baseline economy. The fourth column presents the counterfactual exercise of assuming that output is constant at the initial level, but keeping the degree of inequality across the connected and unconnected agents. The last column stands for the opposite exercise: it removes inequality between connected and unconnected agents with the same productivity but maintains the fall in output.

Table 5: Counterfactual welfare analysis of the baseline economy

Fraction of Connected	Model	Counterfactual
$M_{c0}/M_0 = 2.06$	-5.0228%	-0.1025%
$M_{c0}/M_0 = 1$	-0.0020807%	-4.14e-05%
$M_{c0}/M_0 = 0.6$	6.8995%	0.13286%

The second column shows the welfare in the economy that does not receive a monetary shock. The third column contains the values for the welfare function under the benchmark specification of the full enforcement economy. The counterfactual corresponds to the exercise of assuming that, after period T, as the baseline economy returns to equilibrium, the full enforcement economy returns as well.

Table 6: Counterfactual welfare analysis of the full enforcement economy

Fraction of Connected	Model	Baseline output	Baseline inequality	No inequality
$M_{c0}/M_0 = 2.06$	-5.0894%	-5.2367%	-6.0506%	0.0025%
$M_{c0}/M_0 = 1$	-0.1006%	-0.2876%	-0.2276%	0.0025%
$M_{c0}/M_0 = 0.6$	6.755%	6.5109%	7.8565%	0.0025%

The second column shows the welfare in the economy that does not receive a monetary shock. The third column contains the values for the welfare function under the benchmark specification of the zero interest rate economy. The fourth column stands for the exercise of keeping the degree of consumption and labor inequality in the zero interest rate, but imposing that the aggregate output be equal to the one in the baseline economy. The penultimate column stands for the opposite exercise: keeping the output level, but modifying consumption inequality between connected and unconnected. The last column stands for the counterfactual removal of inequality across connected and unconnected agents with the same productivity.

Table 7: Counterfactual welfare analysis of the zero interest rate economy