On the Distributional Effects of Monetary Shocks

A model on the dynamics in the absence of helicopter drops

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Abstract

In the present work, I investigate whether monetary shocks that induce the re-shuffling of real balances across households can 1) produce real effects, 2) account for some degree of price stickiness, and 3) prevent the economy from immediately reaching the stationary equilibrium. I begin by developing a model with a cash-in-advance friction, monopolistic competition, and perfect foresight in which a market for bonds is absent. Next, I add bonds to the baseline model and study its implications both at the equilibrium interest rate and at the zero lower bound. The model confirms Friedman's argument that, in the absence of helicopter drops, the economy's return to a stationary, long-run, equilibrium should happen slowly (provided the shock is large enough). Besides, if more people have early access to the newly created money, the distortions generated by the shock are smaller. If bonds are present, the distributional effects may be permanent if the interest rate is allowed to reach its equilibrium value. Also, some degree of price stickiness can be produced by either 1) a gradual introduction of the new money in the bondless economy due to consumption smoothing, or 2) an urge by indebted agents to lower their prices relative to others in order to amortize their debt.

1 Introduction

Until very recently, the monetary literature has not paid much attention to the distributional effects of monetary policy. This is well-illustrated by the degree of success the helicopter drops assumption has had in the literature. By "helicopter drops", it is usually meant that money is introduced/removed from the economy through transfers/taxes so as to not distort the distribution of money holdings across the households. This assumption can be traced back to Friedman (1969). In that essay, he models monetary injections as if money were dropped by a helicopter, and then hastily collected by the people. Each person would then obtain an amount of money proportional to their current - equilibrium - monetary holdings. This way, prices could, at least at a conceptual level, grow by the same factor and keep the real allocations and relative prices identical to what they were before. Hence, the "helicopter drops" allegory ensures that distributional effects are abstracted away.

However, as Friedman points out, if agents obtain random amounts of the new money, then real balances would be reshuffled across them: the distribution of money holdings would become different from the one that would prevail in a stationary equilibrium. Then, it is no longer possible that the economy moves towards the new equilibrium immediately, since real monetary holdings would have to be rebalanced: some agents would choose to spend more than they earn, while others would do the opposite. Agents who have become richer through the money injections would likely want to smooth their consumption, which would make a prompt return to a stationary equilibrium even less likely. Relative prices would also be distorted to reflect the new distribution of real balances. In summary, Friedman argues that any deviation from the helicopter drops assumption should produce distributional effects in the short run and preclude prices, output, and individual consumption from reaching levels compatible with the long-run equilibrium of the economy in a fast manner.

However, a relatively new, but growing literature shows that there is a series of different channels through which monetary policy produces distributional effects. As will be explicitly argued in the literature review, although the joint effect of these channels may have an ambiguous sign - depending on the country and on the measure of inequality adopted, - the evidence on the individual channels is robust. This indicates that a reshuffling of monetary holdings is very likely to take place in the real world in the aftermath of a monetary shock, which, if Friedman is correct, should produce a sluggish path toward equilibrium.

In this work, I develop a fairly tractable model, in which I investigate the dynamics after a one-time monetary injection. In the baseline version of the model, agents are constrained to pay for their transactions only with cash, which generates a cash-in-advance friction. They buy and sell from a monopolistically competitive and centralized market. There is perfect foresight, which means that, once the monetary shock takes place, agents make decisions without any uncertainty with respect to how others will act and how aggregate quantities will evolve. Moreover, prices can be changed at the beginning of each period at no cost. Hence, I assume no pricing friction in the model. Then, the new money may be injected either through helicopter drops or as uneven transfers across individuals. I study each of the two cases in turn.

Although the model is potentially compatible with other channels through which monetary policy may have distributional effects, it is closer to the one proposed by Williamson (2008): the financial segmentation channel. In this paper, Williamson assumes that households are either *connected* or *unconnected* to financial markets, with no possibility of them moving between these groups over time. By *connected*, he means that these households operate frequently in financial markets and, hence, are the first to be affected by monetary shocks. These early effects of a monetary shock could take place, for instance, through their portfolio of assets. This heterogeneity can be well-illustrated by the fact that even in advanced economies such as the US, only 21.3% of families owned publicly traded stocks in 2001, and only 17.7% held mutual funds (Aizcorbe et al., 2003). Besides, distributional effects of monetary shocks that happen through re-valuations of portfolios have been found to have distributional effects in several empirical papers (Doepke and Schneider, 2006, Doepke et al., 2015, Montecino and Epstein, 2015, O'Farrell et al., 2016, Adam and Tzamourani, 2016, Casiraghi et al., 2018, Saiki and Frost, 2014, Ampudia et al., 2018, Auclert, 2019).

Williamson also assumes that goods markets are likewise segmented: connected agents trade *mostly* among themselves, and so do the unconnected, but eventual transactions do take place between individuals of the two groups. These goods markets are also assumed to be competitive. The economy in his model only returns to the stationary equilibrium asymptotically. Moreover, he identifies a particular pattern in individual prices: the price in the "connected goods market" overshoots, while the one in the "unconnected goods market" grows slowly. The reason for this is that demand in the former would then jump on impact in nominal terms, yielding the rise. Due to the imperfect segmentation in the goods market, the new money gradually flows from the connected households to the unconnected, which causes a gradual deflation in the market of the former and inflation in the market of the latter.

The same pattern in prices is observed in the model here presented, even though the setup is very different. This means that this heterogeneity in prices seems to be somewhat robust to changing the model specifications and to point to a very fundamental underlying phenomenon, namely, that the price of an agent will depend on the amount of money she will have access to. Still, the present work differs from Williamson (2008) by 1) assuming fewer market imperfections; 2) allowing for an endogenous amount of cash to be kept idle to smooth consumption of connected households - which ultimately makes the amount of money in circulation endogenous; - and 3) allowing the unconnected to borrow money from the connected, thereby easing the cash constraints of the latter. As a result, these models yield different results with respect to the path of output and aggregate prices and the mechanisms through which these paths are affected by the monetary shock.

Here, monetary shocks that are implemented through helicopter drops lead the economy to the new stationary equilibrium immediately. If the new money is injected unevenly and a bonds market is absent, I show that there is a relevant rise of inequality in consumption and income, which go in opposite directions: the consumption of connected households goes up relative to the unconnected, but the revenues of the latter are higher. This mechanism re-establishes the initial stationary equilibrium with homogeneity in monetary holdings eventually, since the new money - which initially remains partially idle - is gradually appropriated by unconnected households, as suggested by Friedman (1969) and Williamson (2008). Interestingly, output goes down with the shock, because the prices respond to the anticipated higher amount of money that will circulate in the next period. This happens only for a large enough shock: for smaller shocks, revenues, and prices go immediately to their long-run levels, although relative consumption is distorted in the first period.

I also study how financial development changes both the distributional effects of monetary policy as well as its consequences over aggregate variables. One of the ways in which I do so is by studying what happens when the fraction of connected households in the population changes. I show that distortions generated by a monetary shock are smaller when that fraction is bigger. Another way in which I address financial development here is by building a version of the model in which bonds can be bought and sold. In this version, if the equilibrium interest rate is attained, the model generates persistent - but small - differences in monetary holdings, consumption, and income. Unconnected agents incur some debt to smooth their consumption but roll this debt indefinitely. If the interest rate is set at a lower level, however, consumption inequality and price heterogeneity are mitigated, but revenue inequality is enhanced since the unconnected agents need to work harder in order to amortize their debt. Output grows in this case due to this increment in unconnected agents' production efforts. I also show that welfare is improved relative to the baseline specification by financial development, especially if the equilibrium interest rate is attained.

Finally, the model also produces some - small - endogenous price stickiness, which is yet another contribution to the literature. This bridges part of the gap between the New-Monetarist literature, in which money is necessary for transactions, and the New-Keynesian tradition, in which price rigidities are modeled in reduced form. This endogenous price stickiness takes place through two different mechanisms, which, to the best of my knowledge, are novel to the literature. The first takes place due to the fact that, in the absence of a market for credit, the new money is gradually put in circulation in the economy, and, in an effort to appropriate more of it, unconnected agents set a lower price to win over their connected competitors. Due to the presence of strategic complementarities in price-setting, this decreases the degree to which the price chosen by the connected agents overshoots. Secondly, unconnected agents may also be induced to set a lower price if they need to obtain even higher revenue in order to amortize their debt.

The findings here presented highlight the differences between developed and emerging economies. It ultimately means that the effects of a monetary shock can be very different in economies with well-developed markets for credit with respect to economies with strong financial frictions. In poorer economies, monetary policy can have much stronger distributional effects over consumption and produce a bigger welfare loss. Output would also be more volatile in these economies, at least as far as monetary policy goes. Hence, the present work stresses the importance of developing a well-functioning credit market for easing the possible distributional distortions caused by unforeseeable monetary shocks.

1.0.1 Literature review

According to the existing literature, monetary policy may affect differently agents who: 1) have different income compositions; 2) have different portfolios; 3) differ on whether they are net savers or net buyers; 4) frequently operate on financial markets relative from those who do not; and 5) differ with respect to preferences and skill levels (Hohberger et al.,2020; Coibion et al., 2017; Dolado et al., 2021). Distributional effects may also arise from the heterogeneity of price adjustment across goods (Cravino et al., 2020; Baqaee et al., 2022) and from potential risk sharing that monetary policy may facilitate/hinder (Berentsen et al., 2007; Rocheteau et al., 2018).

Several papers study the distributional effects of monetary policy through DSGEs. For instance, Hohberger et al. (2020) show that expansionist monetary policy structurally increases wealth inequality on impact, but reduces income and consumption inequality in the Euro Area. These effects are stronger and more persistent under QE. Another noteworthy example is Gornemann et al. (2016), who study the income composition and the risk-sharing channels very extensively through a HANK model featuring endogenous unemployment risk. They show that most households prefer a more accommodative monetary policy, which focuses on employment stability and reduce the risk they face. Richer people, on the other hand, would rather have the central bank focus on price stability instead.

These different channels may point in opposite directions: some may cause a monetary expansion to reduce inequality, others, to increase it. How they balance out in the real world is an empirical question. Using survey data from the U.S., Coibion et al. (2017) show that inequality in income and consumption historically followed after contractionary monetary shocks. Using a panel of 32 advanced and emerging countries, Furceri et al. (2018) back up this view for *unexpected* monetary shocks, but with the caveat that *expected* contractionary monetary policy (pushed by higher inflation or growth) *decreases* inequality. Conversely, Davtyan (2016) shows that contractionary monetary policy by the Fed decreases inequality, and Montecino and Epstein (2015), that QE induced inequality in the U.S.

However, although the evidence on how inequality is affected by monetary policy may be ambiguous in the empirical literature when one aggregates agents into income classes and percentiles, this does not rule out all the distributional effects that may take place at the individual level. For instance, Dolado et al. (2021) find contrary results to the one found by Coibion et al. (2017) despite using roughly the same data and method. The difference in the results is due to the measure of inequality: Dolado et al. (2021) use skill premium to gauge the disparity in earnings across high- and low-skilled workers, while the other authors use more general earnings, consumption, and income inequality measures. Also, Guerello (2018), using data for the Euro Area, find that results differ a lot for different individual countries, indicating that different channels might be stronger in some and weaker in others.

1.0.2 Structure of the present work

This work is organized as follows. In section 2, I develop the baseline version of the model, in which there is no market for bonds. I show some analytical results, but I also simulate the model to gain further insight into its implications. Next, in section 3, I add bonds to the model and begin by analyzing a version in which the interest rate is freely set. Then, I develop a version of the same model in which the interest rate is exogenously set to the zero lower bound and perform a short welfare analysis. Section 4 presents the concluding remarks. Most of the proofs are presented in Appendix A, while Appendix B contains further graphs and tables.

2 The Model

Consider a Lucas islands setup with a continuum of uniformly distributed islands with mass 1. I denote the population of islands with I. Every island has a seller and a buyer: sellers engage in production and selling activities within their own island and, thereby, earn money; buyers leave the island with the money accumulated until the last period in their pocket in order to buy from all the sellers. As in Lucas and Stokey (1985), this results in the presence of a cash-in-advance (CIA) constraint. Buyers visit all sellers, and sellers are visited by all buyers. These buyers see the goods as imperfect substitutes for one another.

I assume agents have identical preferences and start off with identical money holdings. This way, I have a clear benchmark for studying the distributional effects of an expansionist monetary shock. Moreover, as in Williamson (2008), I define *connected agents* as those who frequently trade in financial markets, being, thus, the first to to gain access to the new money. I assume they correspond to a fraction $\eta \in (0, 1)$ of the whole population. *Unconnected agents* are, thus, naturally, those who are affected indirectly by monetary policy, which corresponds to a fraction $1 - \eta$ of I.

The timing of the model goes as follows:

1. The island starts off with m_{it}^- units of money, and connected agents may receive an unanticipated (and unforeseeable) transfer τ_{it} from the government - an MIT shock, - financed through money creation. Thus, $m_{it} = m_{it}^- + \tau_{it}$;

2. The seller sets a price p_{it} for the good and chooses how much output to produce;

3. The buyer leaves the island and faces aggregate price $\hat{P}_t$. Given this price, she will choose how much of m_{it} to spend on consumption and how much to save. Also, she will decide how to allocate current consumption across the sellers by looking at their individual prices, $\hat{p}_{nt}$, with $n \in I$. The seller stays in the island to sell her output to incoming buyers.

4. Buyer and seller will meet again at the end of t. The money obtained through sales and the money unspent by the buyer will sum up to $m_{i,t+1}^-$.

Although monetary shocks are assumed to be *unanticipated*, they become immediately known by everyone whenever they take place, that is, before agents make any pricing and production decision for that period. Hence, there is no realm for uncertainty, meaning that no excess inventories are produced. As a result, whether pricing and production decisions are made before or concomitantly to purchases does not matter: the important element in events 2 and 3 above is the fact that *buyers cannot benefit* *from current sales*. Thus, there is a CIA friction, which ensures that money is relevant for agents since, as will be seen below, they start off cash-constrained in the initial stationary equilibrium. Moreover, as stated before, there is imperfect competition, which is necessary in order to allow for pricing decisions to be possible.

Here, a quick note is needed with respect to the proper interpretation of the role played by the sellers. They could also be seen as workers bargaining for a higher wage when demand is high. Naturally, it means that, if the prices chosen by other sellers are higher, this gives each individual seller more leeway to raise their price as well. The same should work for workers, since, whenever other workers are demanding higher wages, the outside option of their employer is worse, and hence their bargaining power is larger. In summary, sales revenues play the role of being the source of income for the inhabitants of the islands, but they can be interpreted more generally.

To begin, the seller's problem is:

$$V^S(s_{it}|\{\hat{C}_{nit}\}_{n\in I}, P_t) = max_{p_{it},h_{it},R_{it}} - \gamma h_{it} + \beta V^B(m_{i,t+1}, R_{i,t+1}|P_{t+1}) \tag{1}$$

$$\text{subject to} \quad R_{it} = p_{it}h_{it} \tag{2}$$

$$m_{i,t+1} = s_{it} + R_{it} \tag{3}$$

$$h_{it} \leq D(p_{it}) \tag{4}$$

where $V^B(m_{i,t+1}, R_{t+1}|P_{t+1})$ and $V^S(s_{it}|\{\hat{C}_{nit}\}_{n\in I}, P_t)$ are, respectively, the value function of the buyer and that of the seller; $\{\hat{C}_{nit}\}_{n\in I}$ corresponds to the demand of agents $n \in I$ for the good produced by island i; and R_{it}, to sales revenue. Besides, h_{it} corresponds to labor; γh_{it}, to labor disutility; s_{it}, to savings by the buyer; and $D(p_{it})$, to the demand faced given chosen price, p_{it}. Notice that, since there is perfect foresight, the buyer's savings are known beforehand and seen as a state variable by the seller. Also notice that I am assuming, for simplicity, linear technology, i.e. $y_t = h_{it}$, and labor disutility. Finally, notice that the future only matters to the seller to the extent that the money obtained today through sales affects the budget constraint of the buyer tomorrow. This specification generates a standard cash-in-advance solution (see Proposition 1).

After observing P_t, the buyer faces a dynamic and a static problem. In the former, she chooses how much to consume now and how much to consume in the future. Hence, she solves:

$$V^B(m_{it}, R_{it}|P_t) = max_{C_{it},s_{it}} u(C_{it}) + \beta V^B(m_{i,t+1}, R_{it+1}|P_{t+1}) \tag{5}$$

$$\text{subject to} \quad P_t C_{it} + s_{it} \leq m_{it} \tag{6}$$

$$m_{i,t+1} = s_{it} + R_{it} \tag{7}$$

$$s_{it} \geq 0 \tag{8}$$

where (8) corresponds to the CIA constraint. Notice that the revenue obtained by the seller, which is known beforehand by the buyer, enters the problem of the latter as a state variable. Also notice that, if agent i received the transfers at the beginning of time t, then $m_{it} = m_{it}^{-} + \tau_{it}$, which means that transfers enter implicitly in the constraint. Moreover, I assume throughout, for simplicity, that $u(C_{it}) = \log(C_{it})$. Notice that, since monetary shocks are unforeseeable, agents assume that $m_{i,t+1}^{-} = m_{i,t+1}$. After deciding how much to consume in the current period, C_{it}, the buyer decides how much consumption to allocate to each of the sellers. I assume the consumer has preferences across these goods with the CES form, facing, thus, the static problem:

$$\max_{\{c_{int}\}_{n\in I}} C_{it} = \left(\int_0^1 c_{int}^{\frac{\epsilon-1}{\epsilon}} dn\right)^{\frac{\epsilon}{\epsilon-1}} \tag{9}$$

$$\text{subject to} \quad P_t C_{it} = \int_0^1 \hat{p}_{nt} c_{nt} dn$$

$$c_{int} \geq 0 \quad \forall n \in I$$

where $\hat{p}_{nt}$ and c_{int} are, respectively, the price and consumption of each individual good $n \in I$. Moreover, I assume that $\epsilon > 1$. Here, two notation notes are in order. First, I use upper case letters for aggregated consumption and prices (for each individual, that is, aggregated across goods). Second, I use $\hat{p}$, $\hat{c}$, $\hat{C}$ to denote the price chosen by and the consumption levels of other islands. Importantly, however, is that I use P_t to denote the aggregate price faced by all islands, which is common knowledge to all of them.

Finally, I define $\mathbb{T}$ as the set of periods at which the economy finds itself in a particular equilibrium. Then, the equilibrium of this economy is as follows:

Definition 1 (Equilibrium)**.** *An equilibrium in this economy is a series of individual and aggregate prices* $\{\{p_{it}\}_{i\in I}, P_t\}_{t\in\mathbb{T}}$*, individual consumption bundles and allocations across sellers* $\{\{C_{it}, \{c_{int}\}_{n\in I}\}\}_{i\in I}\}_{t\in\mathbb{T}}$*, individual and aggregate outputs* $\{\{y_{it}\}_{i\in I}, Y_t\}_{t\in\mathbb{T}}$*, and individual and aggregate monetary holdings* $\{\{m_{it}\}_{i\in I}, M_t\}_{t\in\mathbb{T}}$ *such that:*

1. *Given* $\{P_t\}_{t\in\mathbb{T}}$ *and an initial* m_{i,t_0} *(with* $t_0 := \min \mathbb{T}$*),* $\{C_{it}\}_{t\in\mathbb{T}}$ *solves the buyer i's intertemporal problem* (5)*;*

2. *Given* C_t *and* $\{\hat{p}_{nt}\}_{n\in I}$*,* $\{c_{int}\}_{n\in I}$ *solves the buyer i's static problem* (9) *for any given* $t \in \mathbb{T}$*;*

3. *Given* $\{m_{it}, C_{it}\}_{t\in\mathbb{T}}$*,* $\{p_{it}, y_{it}\}_{t\in\mathbb{T}}$ *solve the seller i's problem* (1)*;*

4. *The goods' market clears at an individual, i.e.* $\int_0^1 \hat{c}_{nit} dn = y_{it}$ *for* $i \in I$*, and aggregate level, i.e.* $\int_0^1 \hat{C}_{it} di = Y_t = \int_0^1 y_{it} di$ *for* $t \in \mathbb{T}$*;*

5. *Integrating individual monetary holdings equates the total monetary base, i.e.* $\int_0^1 m_{it} di = M_t$ *for* $t \in \mathbb{T}$*.*

2.1 Solution

To begin, consider the solution to the dynamic problem of the buyer. It yields the following results

$$P_t C_{it} = \begin{cases} \frac{1}{\beta} P_{t+1} C_{i,t+1} & \text{if } s_{it} > 0 \\ m_{it} & \text{otherwise} \end{cases}$$

where the latter case corresponds to full depletion, that is, the situation where the internal solution for current consumption is beyond the amount of money the buyer has available and, hence, the consumer spends all her money currently. Given C_{it}, then, the consumer solves her static problem, which yields:

$$c_{int} = C_{it} \left(\frac{\hat{p}_{nt}}{P_t} \right)^{-\epsilon} \qquad \forall n \in I \tag{10}$$

Moreover, as usual, the aggregate price is given by:

$$P_t = \left(\int_0^1 \hat{p}_{nt}^{1-\epsilon} dn \right)^{\frac{1}{1-\epsilon}}. \tag{11}$$

The seller's problem yields the solution:

$$\gamma D'(p_{it}) = \beta \frac{u'(C_{i,t+1})}{P_{t+1}} \left(h_{it} + p_{it} D'(p_{it}) \right) \tag{12}$$

This equation means that, as in the New-Keynesian models, agents are forward-looking when setting their prices. However, here, the rationale differs from the one in those models, since there are no pricing frictions in place here. For example, (and this point will become clearer later) when a positive monetary shock hits the economy, the islands will need to decide how much to adjust on prices and on quantities. When they adjust on quantities, they will have to face labor disutility in the present in order to produce more to accommodate the higher demand. When they adjust on prices, they will lose part of their customers' consumption expenditures to their competitors, which will mean that they will carry less money from sales to the next period. The value of this money is the value of relaxing the future budget constraint, which is directly related to the marginal utility and to the aggregate price.

Now, I impose market clearing in the market for seller i's good, that is $h_{it} = D(p_{it})$. Notice that $D(p_{it}) = \int_0^1 \hat{c}_{nit} dn = \int_0^1 \hat{C}_{nt} \left(\frac{p_{it}}{\hat{P}_t} \right)^{-\epsilon} dn$. This leads to the price-setting rule:

$$p_{it} = \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} \frac{P_{t+1}}{u'(C_{i,t+1})} = \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} P_{t+1} C_{i,t+1} \tag{13}$$

This equation means that there is a markup over marginal costs. However, there is also a forward-looking component to it, as alluded to before. It means that the higher the expenditure on consumption tomorrow, the higher the price the island will post today. The intuition is that, whenever consumption will be large in the future, the value of relaxing tomorrow's budget constraint by carrying more money to the future is lower. Hence, the island will adjust more on prices, due to the fact that she will care less about having a lot of money tomorrow than she does about the labor disutility she faces today. This value of relaxing the budget constraint in the future is also affected by future aggregate prices: when these prices are higher, the value is lower. Finally, as mentioned before, Proposition 1 shows that the solution produced by the model is compatible with a standard CIA model featuring monopolistic competition. The proof can be found in Appendix A.

Proposition 1. *The model here presented is isomorphic with a version of Lucas and Stokey (1985) with monopolistic competition in which there are no assets other than money.*

2.2 The stationary equilibrium

I assume the economy starts off, at $t = 0$, in a stationary equilibrium, since this is a natural departure point for understanding what is the impact of a one-time money injection. In this monetary equilibrium, $m_{it} = m_{i,t+1}$, $p_{it} = p_{i,t+1}$, $M_t = M_{t+1}$ and $P_t = P_{t+1}$, given that there is no entry. Let $\mathbb{T}^S$ be the set of periods at which the economy is in this kind of equilibrium. I define it below:

Definition 2 (Stationary equilibrium)**.** *A stationary equilibrium for this economy is a series of prices $\{\{p_{it}\}_{i\in I}, P_t\}_{t\in\mathbb{T}^S}$, consumption allocations $\{\{C_{it}, \{c_{int}\}_{n\in I}\}_{t\in\mathbb{T}^S}$ and output $\{\{y_{it}\}_{i\in I}, \}, Y_t\}_{t\in\mathbb{T}^S}$ which, given $\tau_{it} = 0$ for $i \in I$ and $t \in \mathbb{T}^S$, solve (5), (9) and (1) and make $m_{it} = m_{i,t+1}$ for $i \in I$ and, thus, $M_t = M_{t+1}$.*

Naturally, since the problem is symmetric, prices are the same for all islands, $p_{it} = \hat{p}_{nt}$ for all $n \in I$. Therefore, aggregate prices are simply:

$$P_t = 1^{\frac{1}{1-\epsilon}} p_{it} = p_{it}$$

Consumption is constant and given by:

$$C_0 = \left(\frac{\epsilon - 1}{\epsilon}\right) \frac{\beta}{\gamma}$$

Now, notice that, as in Lucas and Stokey (1985), the CIA constraint should bind in this economy, since, for inflation π, $1 = 1 + \pi > \beta$. This can be seen, here, by a simple proof by contradiction. Assume that this is not the case, and hence $s_{it} > 0$. Then, by the buyer's first order condition, $P_{t+1}C_{i,t+1} = \beta P_t C_{it}$. By the budget constraint, $P_t C_{it} < m_{it}$. Moreover, since the problem is symmetric, all islands post the same price, and, hence, each island gets exactly the average revenue in the economy, that is, $R_{it} = P_t C_{it}$. Therefore, $m_{i,t+1} = P_t C_{it} + s_{it} = m_{it}$. But $P_{t+1} C_{i,t+1} = \beta P_t C_{it} < P_t C_{it}$, which cannot hold, since the fact that the state, m, is the same at t and $t+1$ implies that choices should also be the same. Full depletion implies:

$$p_{it} = \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} m_0 = \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} M_0 \tag{14}$$

$$P_t = \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} M_0 \tag{15}$$

This means that prices rise one-to-one with the money supply, keeping, as seen above, output constant. Notice that these p_{it} and P_t at the stationary equilibrium are conditional on a certain money supply, which in this case equals M_0. I will, thus, whenever necessary, denote this dependence explicitly, by writing $p(M_t)$ and $P(M_t)$.

2.3 Helicopter drops of money

I assume, for now, that the central bank introduces $\eta\tau > 0$ units of money in the economy, where the initial money supply is M_0 at the stationary equilibrium. When helicopter drops take place, each agent gets $\tau_H = \eta\tau$, and this is known by all agents as soon as it happens, that is before they decide on their prices for the first period after the shock, $t = 1$. It is easy to see that the only possible equilibrium is one in which all agents fully deplete all of their money. Here, the argument is identical to the one made with respect to the stationary equilibrium. Essentially, the fact that all islands are identical means that agents start off with the same amount of money in every period. This, in turn, leads to the same pricing and consumption choices for every t, which is incompatible with an internal solution to the intertemporal consumption problem.

Since agents fully deplete their resources, prices are given by:

$$p_t^H = \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} (M_0 + \eta\tau) \tag{16}$$

$$P_t^H = \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} (M_0 + \eta\tau) \tag{17}$$

for $t = \{1, 2, ...\}$, where the H superscript refers to the "helicopter drops equilibrium", and consumption is given by:

$$C_t^H = \left(\frac{\epsilon - 1}{\epsilon}\right) \frac{\beta}{\gamma} = C_0$$

for $t = \{1, 2, ...\}$, which is identical to the consumption level in the initial, stationary, equilibrium, C_0. Thus, a monetary shock implemented through helicopter drops is neutral, since prices immediately rise enough to put the economy at the new stationary equilibrium already at $t = 1$.

2.4 Uneven access to the new money

For this part, I assume that the connected agents get access to the new money, whereas others do not. I will denote the former with subscript c, while the latter will be denoted with subscript u whenever necessary. Thus, $m_{c1} = m_0 + \tau$, with $\tau > 0$, whereas $m_{u1} = m_0$. Now, it may be the case that connected agents choose to fully deplete their money resources or not. I analyze each case in turn.

2.4.1 Full depletion by connected agents

Notice that, in this situation, since $s_{it} = 0$, connected agents will have at $t = 2$ only the amount of money that she manages to obtain through selling her product in the market at $t = 1$. This means that she will have the same amount of money holdings in the next period as the other agents and, hence, by symmetry, will make the same consumption decision in the future. By (13), all agents should, thus, post the same price and appropriate, each, $R_1 = m_0 + \eta\tau$. Therefore, individual and aggregate prices are identical to the ones in the helicopter drop case and, thus, given by (16) and (17) respectively.

Notice that, by the buyer's first order condition, full depletion happens when:

$$\frac{1}{\beta}(m_0 + \eta\tau) = \frac{1}{\beta}P_2 C_{c2} \geq P_1 C_{c1} = m_0 + \tau \tag{18}$$

$$\left(\frac{1-\beta}{\beta-\eta}\right) m_0 \geq \tau \tag{19}$$

This means that the connected agents will fully deplete their money holdings iff the monetary shock is "low enough". Notice that this expression is only defined for $\eta < \beta$. Thus, henceforth, I shall assume that $\eta \in (0, \beta)$, which is not very restrictive, since β is typically assumed to be fairly close to 1.

This means that, from period $t = 2$ onwards, the economy is at the new equilibrium. However, there are important distributional effects at the period $t = 1$. In fact, connected agents' consumption is given by:

$$C_{c1} = \frac{m_0 + \tau}{P_1} = \left(\frac{\epsilon - 1}{\epsilon}\right)\frac{\beta}{\gamma}\left(\frac{M_0 + \tau}{M_0 + \eta\tau}\right) = C_0\left(\frac{M_0 + \tau}{M_0 + \eta\tau}\right) > C_0$$

where I have used the fact that $m_0 = M_0$. For unconnected agents, however, we have:

$$C_{u1} = \frac{m_0}{P_1} = \left(\frac{\epsilon - 1}{\epsilon}\right)\frac{\beta}{\gamma}\left(\frac{M_0}{M_0 + \eta\tau}\right) = C_0\left(\frac{M_0}{M_0 + \eta\tau}\right) < C_0$$

Also, notice that aggregate production, in equilibrium, is given by:

$$Y_1 = \int_0^1 C_{i1} di = \left(\frac{\epsilon - 1}{\epsilon}\right)\frac{\beta}{\gamma}\left(\frac{M_0 + \eta\tau}{M_0 + \eta\tau}\right) = C_0 = Y_0 \tag{20}$$

This means that, in the situation where connected agents fully deplete their monetary holdings at $t = 1$, money is neutral in the aggregate. However, connected agents do benefit by consuming more than they did at the stationary equilibrium in the first period after the shock, while others consume less.

2.4.2 Partial depletion by connected agents

When the shock is large enough - (19) not satisfied, - connected agents smooth their consumption. Then, the adjustment towards equilibrium is a bit more sluggish. More explicitly, prices and output do not go to their equilibrium values immediately anymore. To begin, I define agent i's revenue relative to the average revenue as:

$$\theta_{it} := \frac{p_{it} D(p_{it})}{P_t Y_t} = \left(\frac{p_{it}}{P_t}\right)^{1-\epsilon} \tag{21}$$

Naturally, this means that connected agents' joint market share is given by $\eta\theta_{ct}$, while that of the unconnected is given by $(1-\eta)\theta_{ut}$. I also define M_t^C as the money in circulation at time t, that is, the amount of money that is demanded in the economy for transaction motive. The "C" superscript is used to differentiate it from the monetary base, $M_t = M_0 + \tau$, and stands for " *in circulation*". Notice that, for as long as connected agents do not fully deplete their resources, we must have $M_0 < M_t^C < M_0 + \eta\tau$. In Proposition 2, I characterize the dynamics after the shock. The proof for the proposition can be found in the appendix, in section A.1.2.

Proposition 2. *There is a certain time $T < \infty$ satisfying:*

$$\beta^{T-1} > \frac{M_0 + \eta\tau}{M_0 + \tau} \tag{22}$$

at which connected agents choose $P_T C_{cT} = m_{cT}$, that is, they fully deplete their money holdings. All other agents choose $P_T C_{uT} = m_{uT}$ as well. Moreover, $m_{cT} > m_{uT}$, and, thus, $C_{cT} > C_{uT}$. For $t = \{T, T+1, ...\}$, we have $p_{it} = p^H(M_0 + \eta\tau)$ for all agents and, thus, $P_t = P^H(M_0 + \eta\tau)$. In particular, for $t = \{T+1, ...\}$, $C_{it} = C_0$ and $m_{it} = m_0 + \eta\tau$ for all agents and $Y_t = Y_0$.

For $t = \{1, ..., T-1\}$, unconnected agents fully deplete their money holdings: $P_t C_{ut} = m_{ut}$, but connected agents do not, and choose $P_t C_{ct} = \beta P_{t+1} C_{c,t+1}$. We have prices:

$$p_{ut} = \left[\left(\frac{\epsilon}{\epsilon - 1}\right)\frac{\gamma Y_t}{\beta}\right]^{\frac{1}{\epsilon}} P_t > p^H(M_t^C) \tag{23}$$

$$p_{ct} = \left(\frac{\epsilon}{\epsilon - 1}\right)\gamma P_t C_{ct} > p^H(M_t^C) \tag{24}$$

and $p_{ct} > p_{ut}$. *Therefore,* $P(M_t^C) > P^H(M_t^C)$. *Moreover,* $p_{ct} = \frac{1}{\beta} p_{c,t+1} > p_{c,t+1}$ *and* $p_{ut} < p_{u,t+1}$. *With respect to how the revenue of each type of agent compares to the average, we have:*

$$\theta_{ut} = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{\beta}{\gamma} \frac{1}{Y_t} \right]^{\frac{\epsilon-1}{\epsilon}} > 1 > \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{1}{\gamma C_{ct}} \right]^{\epsilon - 1} = \theta_{ct}$$

and we have $\theta_{c,t+1} > \theta_{ct}$ *and* $\theta_{u,t+1} < \theta_{ut}$. *This means that:*

$$C_0 < C_{c,t+1} < C_{ct} \tag{25}$$

and

$$Y_0 \geq Y_{t+1} > Y_t \tag{26}$$

where the weak inequality is satisfied with equality only for $t + 1 = T$. *In particular, for every* $t = \{1, 2, ..., T - 1\}$, $C_{ct} > C_0 > C_{ut}$ *and* $y_{ut} > y_{ct}$. *Money is slowly injected into the economy, that is,* $M_0 + \eta\tau \geq M_{t+1}^C > M_t^C > M_0$, *where the weak inequality is satisfied with equality only if* $t + 1 = T$. *In particular, we have:*

$$\frac{1}{\beta} (m_0 + \eta\tau) > m_{u,t+1} = R_{ut} > R_{u,t-1} = m_{ut} \geq m_0 \tag{27}$$

where the weak inequality is satisfied with equality only if $t = 1$. *And, with respect to connected agents:*

$$m_0 + \eta\tau < m_{c,t+1} < m_{ct} \leq m_0 + \tau \tag{28}$$

where the weak inequality is, again, satisfied with equality only if $t = 1$. *For all* $t = \{1, 2, ..., T - 1\}$, $m_{ct} > m_{ut}$.

Proposition 2 implies that the economy eventually goes back to a new stationary equilibrium, where the allocation is identical to the initial one, but prices are higher. The number of periods that are needed for the economy to reach the new equilibrium must satisfy condition (22). To get some intuition on what this condition means, notice that $m_{c1} = M_0 + \tau > M_0 + \eta\tau = m_{c,T+1}$. So, essentially, by fully depleting her money at T, a connected agent is willing to accept having lower monetary holdings in the next period. This willingness comes from their impatience: as in the full depletion case, if the resulting decrement in the next period's monetary holdings (and on consumption) is small enough, the agents are willing to accept full depletion in order to anticipate consumption. So, if a connected agent chooses, at $t = 1$, a consumption path that leads her to fully deplete her resources $T - 1$ periods into the future, it must be the case that the strength of her time discount over the whole interval of T periods must be higher than the factor by which monetary holdings fall.

Importantly, the proposition also implies that, as connected agents gradually inject new money into the economy, the other agents acquire an increasing fraction of the monetary base. This ultimately reduces inequality in money holdings over time, until all agents become equally wealthy at time $T+1$. However, it is important to note that, as in the case where connected agents fully deplete their post-shock monetary holdings, here it is also the case that the uneven injection of new money produces transient distributional effects on consumption. More explicitly, connected agents consume more than in the stationary equilibrium at all periods $t = \{1, 2, ..., T-1\}$, whereas other agents consume less.

The fundamental difference between both of the cases analyzed is that, when connected agents smooth consumption, *money is not neutral in the aggregate*, for it causes GDP to *fall*. The cause for this fall is the fact that nominal GDP must equate the amount of money in circulation, M_t^C, since the velocity of money is constant. Given that aggregate prices rise beyond the level compatible with a stationary equilibrium (at which output is equal to Y_0 and $P_t = P^H(M_t^C)$), output must fall.

To understand why prices rise excessively when connected agents smooth consumption, notice that the price chosen by them actually overshoots on impact and falls gradually towards $p^H(M_0 + \eta\tau)$, reaching this level only at time T. The reason for this excessive response in p_{c1} is precisely the fact that connected agents can afford to consume more, both currently as well as in the future, which ultimately reduces the marginal utility of future consumption.

This, in turn, reduces the value of holding cash (relaxing, thereby, the budget constraint) in the future. This makes connected agents more prone to adjust to the higher demand on prices than other agents, incurring, thus, lower labor disutility currently. This is the reason why $y_{ut} > y_{ct}$ for $t = \{1, 2, ..., T-1\}$, meaning that other agents make more of an effort in production than the connected ones. As will be seen below, y_{ut} does not respond to the shock in this setup: unconnected sellers respond fully in prices. This means that the whole fall in output is attributable to connected agents.

The prices chosen by unconnected agents follow very different dynamics: they actually grow over time. The reason why they adjust sluggishly to the monetary shock is directly related to the CIA friction, to the extent that these individuals are constrained by their money holdings. This active constraint implies that their future expenditures with consumption (and, hence, their current prices) are determined by the fraction of the circulating money, M_t^C that they manage to appropriate through sales in the present.

To understand this point better, notice that, since $C_{ut} < C_{ct}$ for $t = \{1, 2, ..., T\}$, unconnected agents are individually responsible for *less than* the amount of money circulating on average, that is, $P_t C_{ut} = m_{ut} < M_t^C$. However, they appropriate *more than* the average, since $R_{ut} = \theta_{ut} M_t^C > M_t^C$. This difference is made possible by the fact that connected agents put more money in circulation than they get back from sales. Therefore, the stickiness in p_{ut} is caused by the gradual introduction of new money into the economy through connected agents' expenditures with consumption. In summary, unconnected agents have a high marginal utility for holding money in the future, which encourages them to set lower prices relative to connected agents in order to appropriate more of the total amount of money in circulation.

Importantly, as already mentioned before, this same heterogeneity in prices is present in Williamson (2008). This is remarkable since his setup is very different from the one presented here: in his paper, goods markets are segmented and competitive. Also, all the money in his model is immediately put in circulation. However, there appears to be a more fundamental reason why this pattern is observed in both setups, namely, the fact that the new money goes slowly from connected agents to the unconnected. More specifically, in his model, connected agents charge higher prices, because, due to market segmentation, they face higher demand than the unconnected. The resulting sustained higher demand (in nominal terms) makes connected agents *wealthier* because they also consume cheaper goods produced by the unconnected. In my framework, this effect is produced by money hoarding.

Finally, notice that even the prices chosen by unconnected agents are such that $p_{ut} > p^H(M_t^C)$, meaning that their prices are also *too high* to allow for output to be as large as Y_0. The reason for this is that connected agents' price overshoots on impact, which pushes the aggregate price upwards. This gives some lee-way for other agents to also raise their prices while still remaining highly competitive. Hence, this complementarity in prices[1] makes inflation *contagious*, partially counteracting the sluggishness in the adjustment of p_{ut}. Nothing can be said analytically, however, about how P_t compares to the final price level $P^H(M_0 + \eta\tau)$. In other words, it is not clear whether the *aggregate* price grows gradually or overshoots on impact and then falls. To gain some insight into it, I perform a simple simulation.

I set $\epsilon = 11$ in order to get a 10% markup, as usual, and $\beta = 0.95$. I normalize $m_0 = 1$ and choose γ in order to ensure that the initial aggregate price is normalized to 1 as well, i.e. $P_0 = 1$. As a result of this normalization, $C_0 = 1$. I give to the connected agents a $\tau = 0.5m_0$ shock. Besides, I set $\eta = 0.25$. A sensitivity analysis on this parameter will be conducted at the end of this section. I select the T - among

[1]The complementarity in prices arises here because, in spite of each agent's weight in the aggregate being zero, choices are identical among the connected/unconnected. As a result, when the connected agents choose a higher price, this affects the aggregate price since these agents have mass $\eta > 0$.

Parameter	Value
ϵ	11
β	0.95
γ	0.8636
η	0.25
m_0	1
τ	0.5

Table 1: Parameter values for the simulation

the ones that satisfy (22) - that is compatible with the analytical dynamics derived. Higher values eventually generate negative money holdings for connected agents, a possibility that is assumed away. The values used are summarized in Table 1.

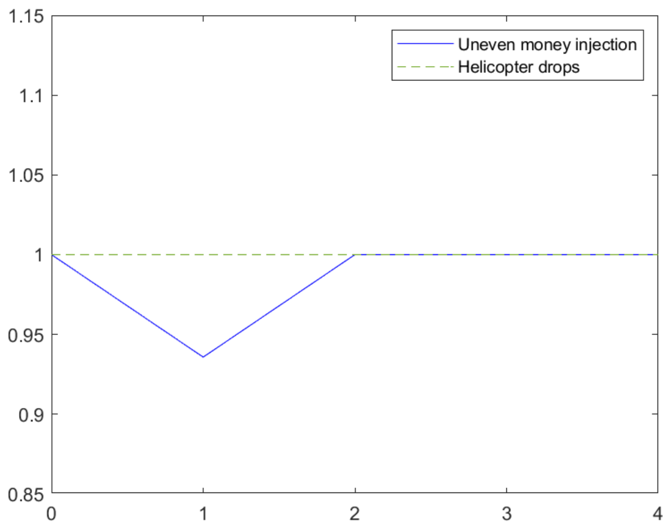
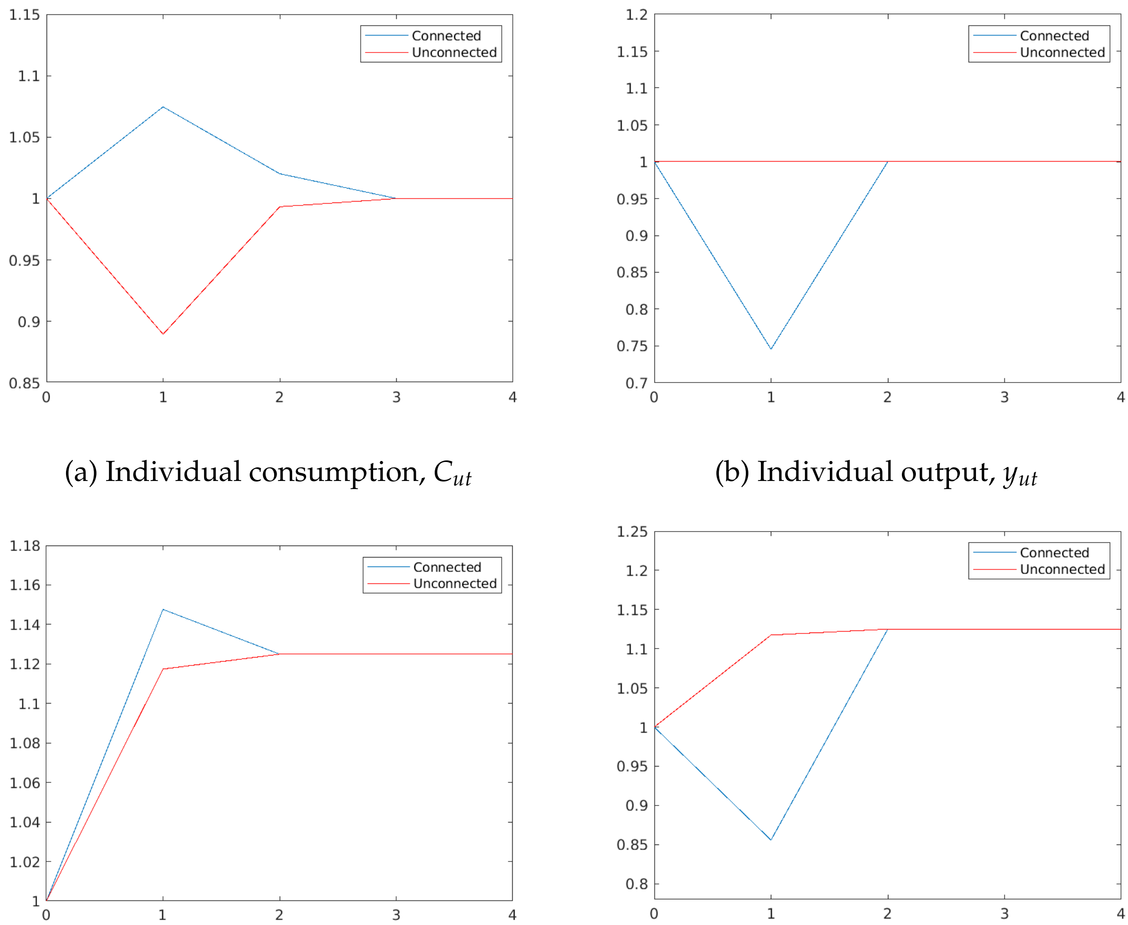


Figure 1: Path of output, Y_t

Given the parameter values chosen here, $T = 2$, which indicates that connected agents do not fully deplete their money holdings only at $t = 1$, in spite of the fairly large 50% shock to m_0. Figure 1 shows the path of GDP; Figure 2 displays how individual consumption, output, prices, and revenues vary between connected and unconnected agents; and Figure 3 compares the money in circulation and the price level under helicopter drops and under an uneven distribution of money. To begin, notice that Figure 1 indicates that output falls approximately 6.42% on impact, and recovers at $t = 2$. This fall is concentrated on the production by connected agents, as can be seen in Figure 2a. To understand why, notice that, since unconnected agents fully deplete their resources, (13) implies that:

$$p_{u1} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}P_2 C_{u2} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}p_{u1}D(p_{u1})$$
$$y_{ut} = D(p_{u1}) = \left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma} \tag{29}$$

that is, unconnected agents set the price so as to keep the demand they face constant. However, if bonds are present, as will be seen in section 3, this will not be the case.



(a) Individual consumption, C_{ut}

(b) Individual output, y_{ut}

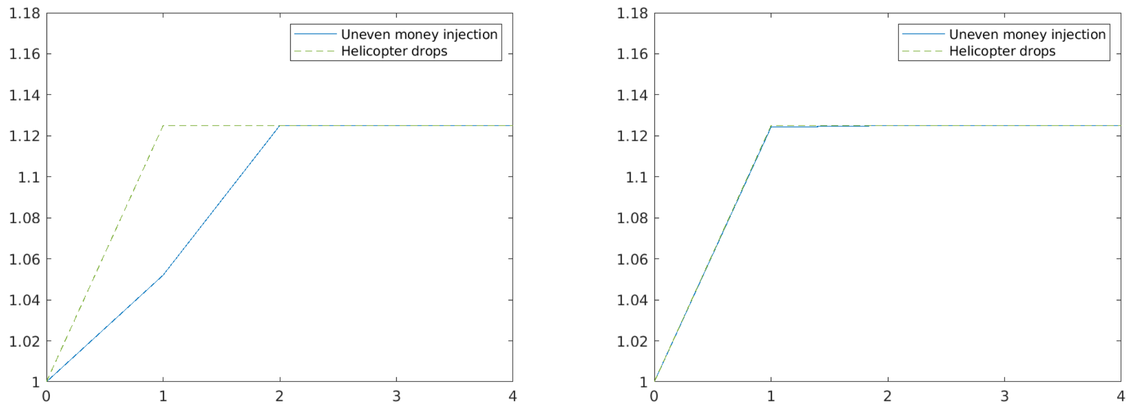
(c) Individual prices, p_{ut}

(d) Individual revenues, R_{ut}

Figure 2: Comparison across agents

Figure 2a shows exactly the dynamics described above for individual consumption. connected agents' consumption rises by approximately 7.46% on impact: a quite significant variation. The consumption of the other agents falls by around 11.05%. With respect to individual prices, the ones chosen by unconnected agents display very little sluggishness. In fact, approximately 94% of the total rise in p_{ut} takes place already at $t = 1$. Furthermore, Figure 3b indicates that aggregate prices display close to no sluggishness. In fact, the path for aggregate prices is almost identical to the one that takes place under helicopter drops. Around 99.4% of the whole rise in aggregate prices takes place at $t = 1$.

Interestingly, Figure 3a shows that indeed, money is put in circulation in the economy more slowly than the response in prices would suggest. In fact, 41.61% of the injected money is put in circulation at $t = 1$, and, thus, the majority is introduced later. The reason behind this is the fact that, as can be seen in Figure 2d, connected agents' revenues fall significantly at $t = 1$. It goes from $R_{10} = 1$ to $R_{c1} \approx 0.8134$. This fall takes place due to the fact that the overshooting price chosen by her at $t = 1$ leads to a drastic reduction in her output. So, even though she fully depletes τ in two periods, the fact that she anticipates a fall in her income at $t = 1$ makes her save around 0.584τ to afford to consume $P_2 C_{c2} = \beta P_1 C_{c1}$.



(a) Money in circulation, M_t^C

(b) Aggregate price, P_t

Figure 3: Comparison between helicopter drops and uneven injection of money

Proposition 3. *For any period t, the Laspeyres price index with $t = 0$ as the base period is given by:*

$$\mathbb{P}_t^L = \frac{M_{t+1}^C}{M_0} = \frac{P^H(M_{t+1}^C)}{P_0} \tag{30}$$

Moreover, if we normalize $P_0 = 1$ and the aggregate price, P_t, is not too far from the helicopter drops level, $P^H(M_t^C)$, then:

$$P_t \approx \mathbb{P}_t^L$$

Proposition 3 implies that, if the aggregate price is close enough to the helicopter drops level, it is well-approximated by the Laspeyres price index. Besides, the Laspeyres index is proportional to the amount of money in circulation in the next period, M_{t+1}^C. The reason for this is the forward-looking behavior of prices, which anticipates the equilibrium in the consecutive period. As a result, the reason why the aggregate price gets so close to its final level already at $t = 1$ is the fact that the amount of money in circulation at $t = 2$ is $M_2^C = M_0 + \eta\tau$. To make this point clearer, I simulate the evolution of aggregate prices under a bigger shock ($\tau = 1.5$), to induce connected agents to fully deplete their resources only at $t = 3$. Figure 4 compares aggregate prices under helicopter drops and an uneven injection of money for this bigger shock. As can be seen, this induces slightly more sluggishness to P_t.

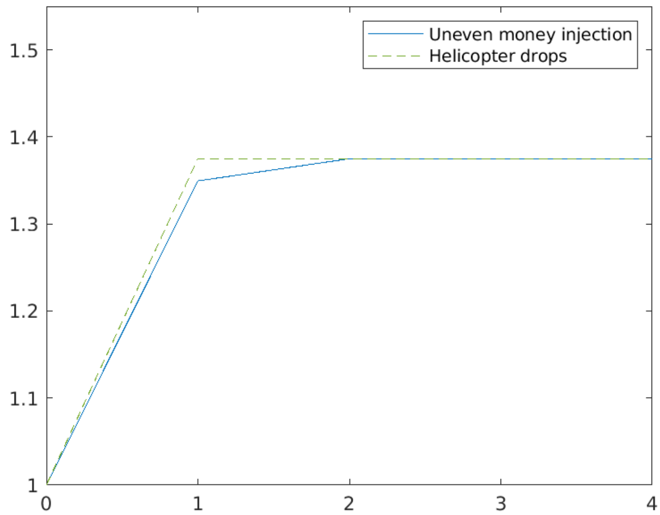


Figure 4: Comparison of P_t between helicopter drops and uneven injection of money under a bigger shock

Furthermore, since the Laspeyres index does not take into account the fact that consumption will move towards the goods with the lowest price, it will overstate the price level. Therefore, the Laspeyres index can be seen as an upper bound for any price index. And, thus, for $t = \{1, ..., T-1\}$:

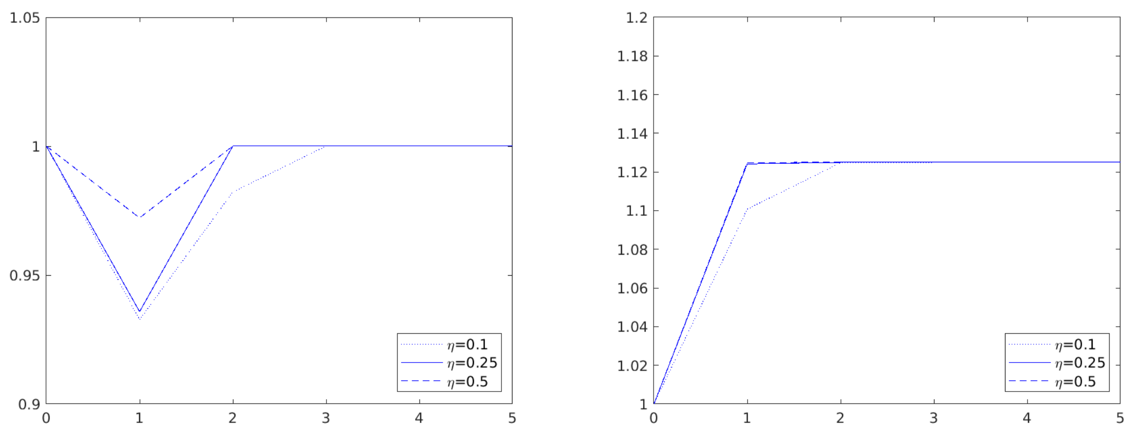
$$\frac{P_t}{P_0} < \mathbb{P}_t^L = \frac{P^H(M_{t+1}^C)}{P_0}$$
$$P_t < P^H(M_{t+1}^C)$$

This means the model produces endogenous aggregate price stickiness, although the results from the simulation seem to indicate that the degree of this stickiness is small.

It is important to make a clarifying note here. In another point of the exposition, it was shown that the aggregate price grows *more* than what should be the case, given the amount of money put in circulation, that is, $P_t > P^H(M_t^C)$ for the periods $t = 1, ..., T$. This is what causes the aggregate output to fall. Nevertheless, the stickiness comes from the fact that $P_t < P^H(M_0 + \eta\tau)$ for $t = 1, ..., T$, that is, the price is *lower* than the final level it attains when the whole money supply is in circulation. This stickiness, therefore, arises from the fact that money is gradually put into circulation due to the consumption-smoothing behavior of connected agents. Thus, the most natural next step is to allow for borrowing to take place. In an economy where connected agents can lend money to unconnected agents, all money can be put in circulation immediately. In the next section, I present a version of the model with credit. However, before that, I need to analyze how the results change for different values of η.

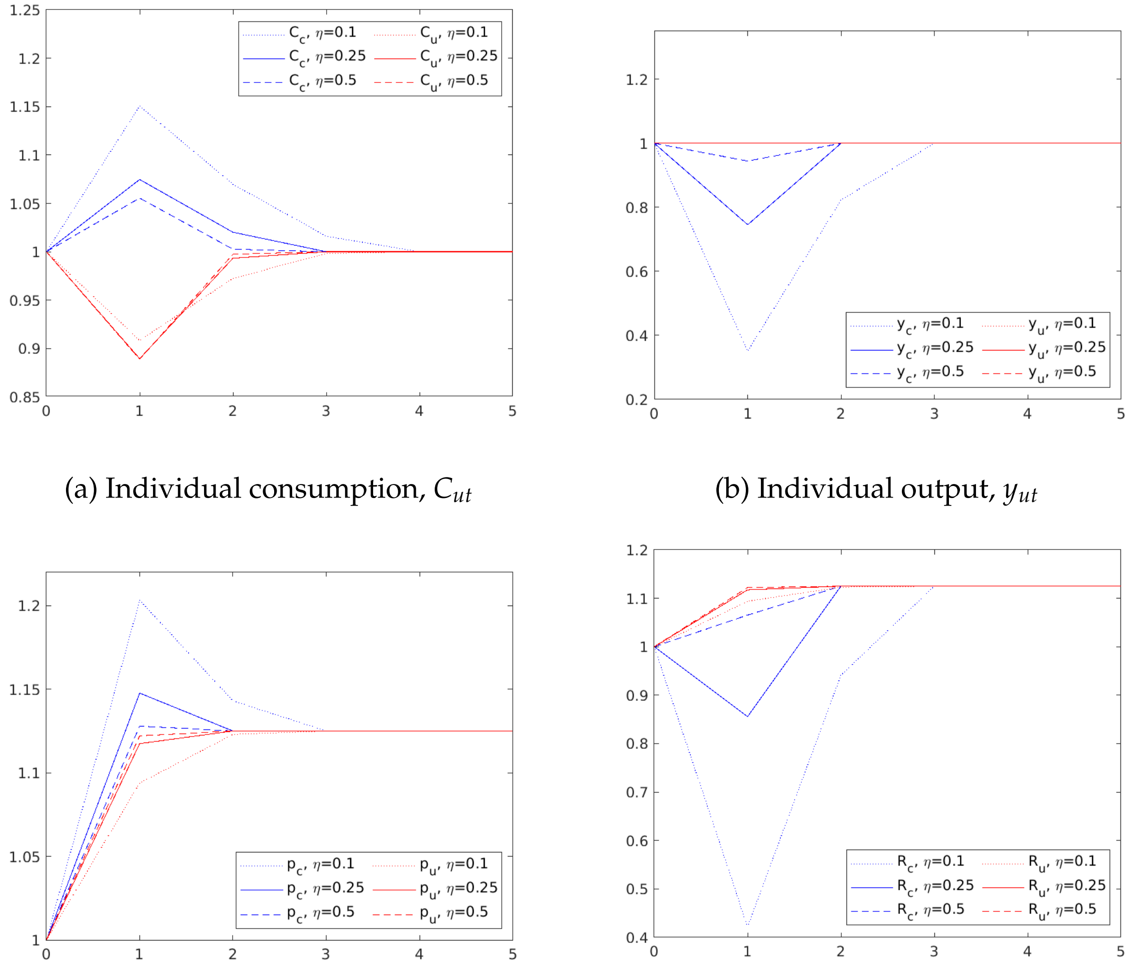
2.5 Sensitivity analysis

Here, I analyze how the results presented above are affected by changes in the fraction of connected households in the economy, η. This exercise has the natural interpretation of exploring the consequences of changes in the degree of access to financial markets in the population. In addition to $\eta = 0.25$, I also present in the figures below the paths for the variables of interest for $\eta = 0.1$ and $\eta = 0.5$. Throughout this analysis, I keep the aggregate monetary shock, $\eta\tau = 0.125$ constant. Therefore, if the fraction of connected households in the population is $\eta = 0.1$, the individual shock that those connected households receive is higher than in the cases where $\eta = 0.25$ and $\eta = 0.5$. As can be seen in Figure 5, this means that the economy takes one period longer to reach equilibrium for the lowest value of η than for the other two. As a result, the aggregate price displays more sluggishness. The fall of output at $t = 1$ is very close for $\eta = 0.1$ and $\eta = 0.25$. Output falls the least for the economy where η is the highest.



(a) Output, Y_t (b) Aggregate price, P_t

Figure 5: Paths of output and the aggregate price for different values of η



(a) Individual consumption, C_{ut}

(b) Individual output, y_{ut}

(c) Individual prices, p_{ut}

(d) Individual revenues, R_{ut}

Figure 6: Comparison across agents for different values of η

As expected, Figure 6 shows that the heterogeneity in outcomes is highest when η is lower. The reason for this is that, in that case, the individual shock to the connected households is much bigger ($\tau = 1.25$ in this case, which is in contrast to $\tau = 0.5$ and $\tau = 0.25$ for, respectively, $\eta = 0.25$ and $\eta = 0.5$). Interestingly, since the aggregate price grows to approximately the same extent for $\eta = 0.25$ and $\eta = 0.5$, the consumption of unconnected households (which, given m_0, depends only on the aggregate price) is not much different across both situations.

The stronger sluggishness in the aggregate price for $\eta = 0.1$ is what leads to a smaller fall in the consumption of the unconnected at $t = 1$ for this parameter value. Also in this case, the response in consumption, prices, output, and revenues of the connected households is very large. The absence of any friction in the adjustment of these quantities is what allows for this result under a - quite substantial - 125% one-time increase in their monetary holdings. The rise in the consumption of the connected households is a bit lower for $\eta = 0.5$. However, aggregate consumption is still higher for this value of η, since a bigger fraction of the population increases their consumption than for other values. Overall, this analysis shows that, when more agents have access to financial markets, the monetary shock has less deleterious effects.

3 Financial Development

In this section, I introduce riskless (due to perfect foresight) one-period pure-discount bonds. In Williamson (2008), although there is a market for credit, it exists only among connected agents. Since the solution is symmetric, there is no actual borrowing between them. The role of the credit market in his model is to allow for the study of how the interest rate responds to monetary policy. Here, however, I allow for borrowing to take place *between* connected and unconnected agents. Hence, it will be possible for the former to receive interest payments on their previously idle cash balances. To begin, I assume that the timing goes as follows:

1. First, all bonds purchased in the previous period reach maturity;
2. Then, as before, transfers from the government financed through the creation of new money may take place. Thus, $m_{it} = m_t^- + b_t + \tau_{it}$;
3. The rest happens exactly in the same order as before, except for the fact that the bonds market reopens for consumers as they start trading in the goods market.

where b_t corresponds to the amount of bonds purchased/sold in the previous period. As usual, if the bond is bought, $b_t > 0$; if it is sold, however, $b_t < 0$.

The buyer's intertemporal problem becomes:

$$
\begin{aligned}
V^B(m_{it}|P_t) =& max_{C_{it},s_{it},b_{i,t+1}} u(C_{it}) + \beta V^B(m_{i,t+1}|P_{t+1}) && (31)\\
\text{subject to} \quad & P_t C_{it} + s_{i,t} + q_t b_{i,t+1} \leq m_{it} \\
& m_{i,t+1} = s_{i,t} + R_{i,t} + b_{i,t+1} \\
& s_{i,t} \geq 0 && (32)\\
& b_{i,t+1} > -l_t && (33)
\end{aligned}
$$

where q_t is the price of the bond and $l_t > 0$. Naturally, (33) is the borrowing constraint. Again, I assume that the buyer supposes that $\tau_{t+1} = 0$ and, thus, $m^-_{i,t+1} = m_{i,t+1}$. The buyer's intratemporal problem, the price-setting problem and the seller's problem remain more or less unchanged.

Now, the first order conditions of the buyer's intertemporal problem yield:

$$P_tC_{it}\begin{cases} = \frac{1}{\beta}P_{t+1}C_{i,t+1} & \text{if } s_{i,t}>0 \\ = m_{it} & \text{if } s_{i,t}=0 \text{ and } b_{i,t+1}=0 \\ \geq \frac{q_t}{\beta}P_{t+1}C_{i,t+1} & \text{otherwise} \end{cases}$$

where the weak inequality is satisfied with equality if, and only if, (33) does not bind. I will begin by assuming that there is full enforcement of debt contracts, and, hence, $l_t = \infty$ for all periods, meaning that:

$$q_t = \beta\frac{P_tC_{it}}{P_{t+1}C_{i,t+1}} \tag{34}$$

The bonds market clears, meaning that:

$$\int_0^1 b_{i,t+1}di = 0 \tag{35}$$

for every t. Naturally, the nominal interest rate is defined as:

$$i_t := \frac{1-q_t}{q_t}$$

Since $q_t > 1$ is not possible, $s_{i,t} > 0$ can only happen if $q_t = 1$. I will also assume, henceforth, that, if the nominal interest rate is equal to zero, that is, $q_t = 1$, $s_{i,t} = 0$, and all the savings will still take place through the bonds' market. Hence, savings will never take the form of idle money here. Now, notice that, given (34), in the stationary equilibrium, we must have $q_0 = \beta$, since $P_tC_{it} = P_{t+1}C_{i,t+1}$ for all individuals i. However, $b_{i,0} = 0$ for all islands, since they have the same preferences, technology, and monetary holdings in such an equilibrium.

3.1 Helicopter drops of money

Since the helicopter drops case does not distort relative monetary balances across islands, and since it brings the economy immediately to the new stationary equilibrium, the dynamics would be identical here as in the case where the bonds were absent. In fact, in the case of helicopter drops, all agents remain identical after the monetary shock. Hence, in this case, there would be no role for borrowing. Besides, $q^H = \beta = q_0$. Next, I analyze the situation where there is an uneven monetary injection.

3.2 Uneven access to the new money and full enforcement of bond contracts

Again, as before, I assume that only connected agents get access to the newly created money, $\eta\tau > 0$. Proposition 4 summarizes the dynamics after the monetary shock.

Proposition 4. *For any $\tau > 0$, if there is full enforcement of bond contracts, the economy goes immediately to the new stationary equilibrium at $t = 1$. The equilibrium bond price and interest rate are, respectively, $q_t = \beta$ and $i_t = (1-\beta)/\beta$ for $t = 1, 2, ...$ Moreover, for all periods $t = 1, 2, ...,$ we have:*

- $C_{it} = C_{i,t+1}$ *and* $m_{it} = m_{i,t+1}$ *for* $i \in I$;
- $C_{ct} > C_{ut}$, $p_{ct} > p^H(M_0 + \eta\tau) > p_{ut}$ *and* $R_{ut} > R_{ct}$;

Moreover, for $t = 2, 3, ...,$ we have $b_{it} = b_{i,t+1}$ for $i \in I$ with:

$$b_{ct} = \frac{m_0 + \tau - P_1 C_{ct}}{\beta} \tag{36}$$

$$b_{ut} = \frac{m_0 - P_1 C_{ut}}{\beta} \tag{37}$$

for unconnected agents. Expenditures with consumption are a convex combination between m_{it} and R_{it}, that is:

$$P_t C_{ct} = (1-\beta)(m_0 + \tau) + \beta R_{ct} \tag{38}$$

$$P_t C_{ut} = (1-\beta) m_0 + \beta R_{ut} \tag{39}$$

There are an upper bound and lower bound to the difference in revenues between connected and unconnected agents:

$$\left(\frac{1-\beta}{\beta}\right)\tau > R_{ut} - R_{ct} > 0 \tag{40}$$

and for the difference in consumption expenditures between them:

$$(1-\beta)\tau > P_t C_{ct} - P_t C_{ut} = (1-\beta)\tau - \beta(R_{ut} - R_{ct}) > 0 \tag{41}$$

According to the proposition above, the economy immediately goes to the new stationary equilibrium, which is characterized by persistent consumption differences between connected and unconnected agents. Essentially, the latter keeps on rolling their debt and paying the interests with their revenue from sales indefinitely. Hence, these one-time bonds end up working in a way identical to perpetuities. One way to understand this is that, if there is full enforcement of bond contracts, consumption should either decrease, increase or stay constant for *all* agents (connected and unconnected alike) according to their Euler equation. Since the monetary base after the shock is constant, this can only be satisfied if their consumption stays the same over time.

Another way to look at this result is that, at the equilibrium interest rate, all the consumers must be indifferent between consuming more today and tomorrow. Since all agents have identical preferences, this means that the compensation, in terms of future consumption, that unconnected agents are willing to pay for the benefit of consuming more today is identical to the one that connected agents are willing to receive for giving up on current consumption. The equilibrium interest rate is fully determined by the parameter β. Hence, this result should still hold whenever $q_t = \beta$, regardless of the particular value it takes.

This way, in every period, the initial monetary holdings are forever identical to those at the beginning of time $t = 1$. The fact that connected agents will receive interest payments in the next period is precisely what allows her to set a higher price, work less and receive a lower revenue today, while still maintaining a higher consumption standard indefinitely: she essentially has a future real claim on part of other agents' current revenues. The facts that 1) connected agents' desire to buy bonds and other agents' desire to supply them are exactly off-set in equilibrium due to the identical preferences and 2) the absence of any borrowing constraint allows the bonds' market to reach its equilibrium is what does away with the liquidity effect, keeping the real interest rate unchanged. Besides, the nominal interest rate does not change either, since all inflation takes place at $t = 1$ already, meaning that the Fisher effect also does not play a role here.

Furthermore, $p_{ct} > p^H(M_0 + \tau) > p_{ut}$ and $R_{ut} > R_{ct}$. By Proposition 3, one should expect the helicopter drops aggregate price, $P^H(M_0 + \tau)$, to be a good approximation to P_t, which should lead output to be nearly unchanged, that is $Y_t = \frac{M_0+\eta\tau}{P_t} \approx Y_0$. Finally, notice that, with the parameter values employed in the simulations throughout the present work and for $\tau = 0.5$, Proposition 4 also implies that $R_{ut} - R_{ct} < 0.026316$ and $P_t C_{ct} - P_t C_{ut} < 0.025$. For reference, in the bondless version of the model, at $t = 1$, these gaps were $R_{ut} - R_{ct} = 0.2617$ and $P_t C_{ct} - P_t C_{ut} = 0.2081$ at $t = 1$. Thus, as expected, the presence of bonds reduces the heterogeneity between agents dramatically, but at the expense of making these smaller differences permanent. Finally, Corollary 4.1 ensures that these results hold for any of the usual utility representations.

Corollary 4.1. *The same results in Proposition 3 hold for any utility specification such that $u(\cdot)$ is non-decreasing and strictly concave.*

To get more insights on the results produced by the model, I perform a simulation in exactly the same conditions as before, that is, using the same parameter values as is Table 1. I also keep the scale in the graphs identical for a clean comparison across both scenarios. As expected, Figure 7 shows that aggregate output is unaffected by the monetary shock, meaning that, if bonds are present and there is perfect enforcement of bond contracts, monetary policy is neutral in the aggregate. Figure 8 shows that the same holds for the aggregate price, which is identical to its helicopter drops level.

Therefore, in this setup, a representative agent model would not be a bad approximation of the aggregate effects of the shock.

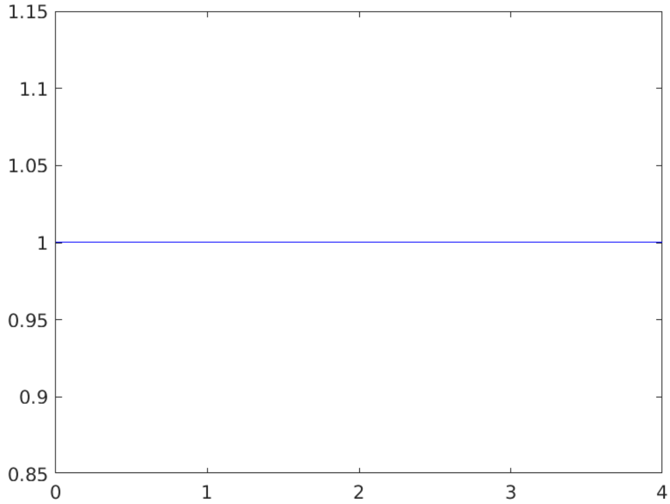


Figure 7: Path of output, Y_t

Figure 9 shows the evolution of individual consumption, output, prices, and revenues. As can be seen in Figure 9a, the gap in consumption is almost unnoticeable. As a result, individual prices are also not very different, as can be seen in Figure 9c. The differences in revenues and output are, however, more perceptible. In particular, unlike before, the output produced by unconnected agents is now *larger* than in the initial stationary equilibrium, at $t = 0$, which is in contrast to the flat line obtained in Figure 2b.

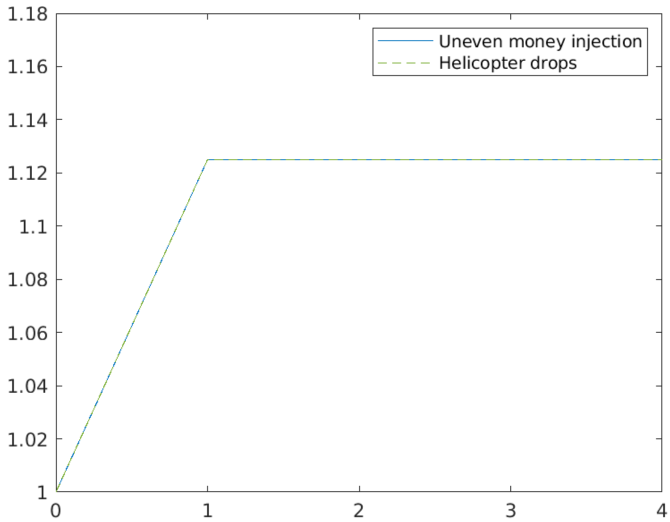
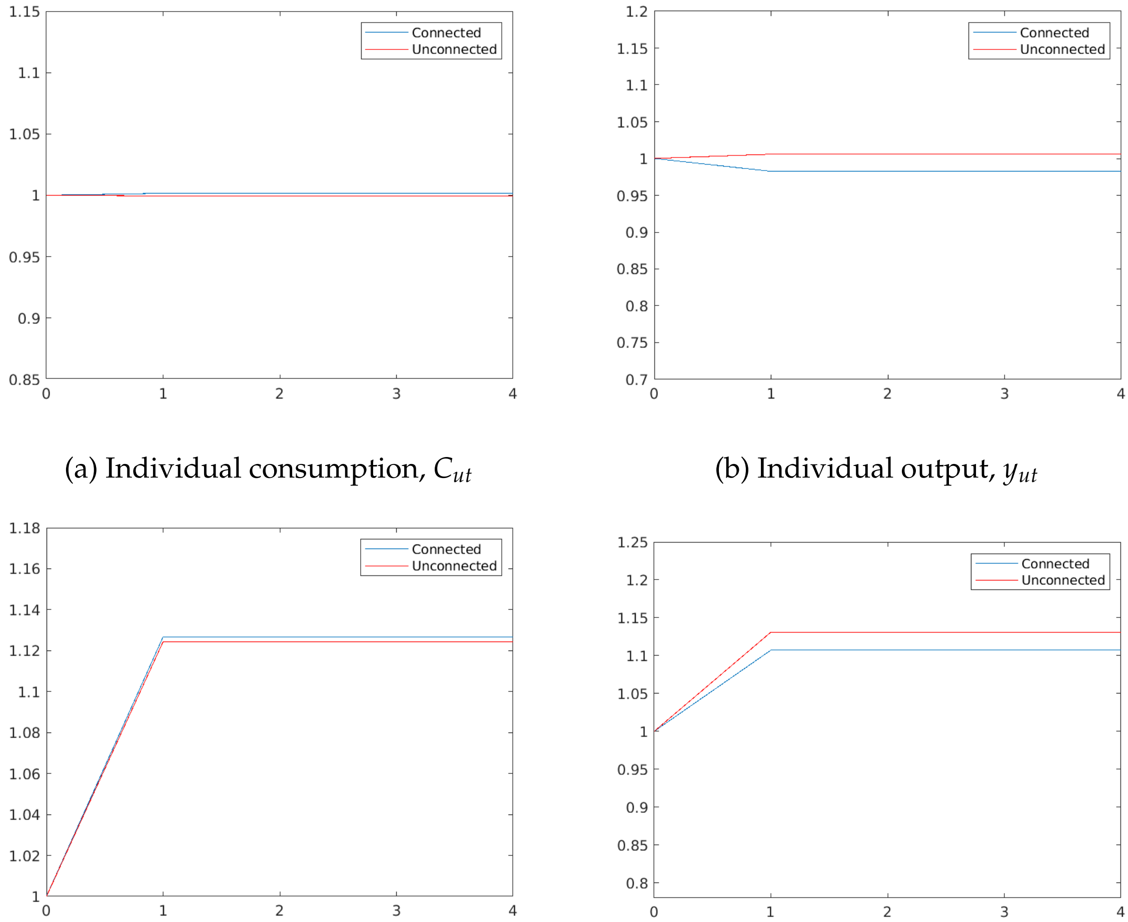


Figure 8: Comparison of P_t between helicopter drops and uneven injection of money



(a) Individual consumption, C_{ut}

(b) Individual output, y_{ut}

(c) Individual prices, p_{ut}

(d) Individual revenues, R_{ut}

Figure 9: Comparison across agents

To understand better why y_{ut} responds to the monetary shock here, notice that the results in Proposition 4 imply that agent i's budget constraint for $t = 2, 3, ...$ can be written as:

$$P_t C_{it} = R_{it} + (1 - \beta) b_{it}$$

meaning that, for unconnected agents, since $b_{ut} < 0$ for $t \geq 2$, $P_t C_{it} < R_{it}$, and the difference between both is precisely given by interest payments. Thus, although $P_1 C_{u1} > R_{u0} = m_0$ precisely due to the bonds sold in period $t = 1$, from that moment on, unconnected agents will need to sell more than the amount they will consume to pay interests. Now notice that:

$$\begin{aligned} p_{ut} &= \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} P_{t+1} C_{u,t+1} \\ &= \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} (p_{ut} y_{ut} + (1 - \beta) b_{it}) \end{aligned}$$

which can be re-written as:

$$y_{ut} = \left(\frac{\epsilon - 1}{\epsilon}\right)\frac{\beta}{\gamma} \underbrace{-(1-\beta)\frac{b_{it}}{p_{ut}}}_{>0}$$

This means that output, here, must be higher than the one obtained in the bondless version of the model, seen in equation (29). The first term in the expression is the amount of output necessary to balance the trade-off between working less today and consuming more tomorrow in the absence of interest payments. The increment is precisely the extra output that needs to be produced and sold in order for the agent to pay these interests on the bond sold. Hence, the reason for the monetary policy to be neutral in the aggregate in the presence of a market for bonds results from 1) the fact that connected agents' prices do not overshoot - which means that they will produce more in equilibrium than in the first period after the shock in the bondless model, - and 2) the increment in production by other agents to pay interests.

3.3 Exogenously set interest rate

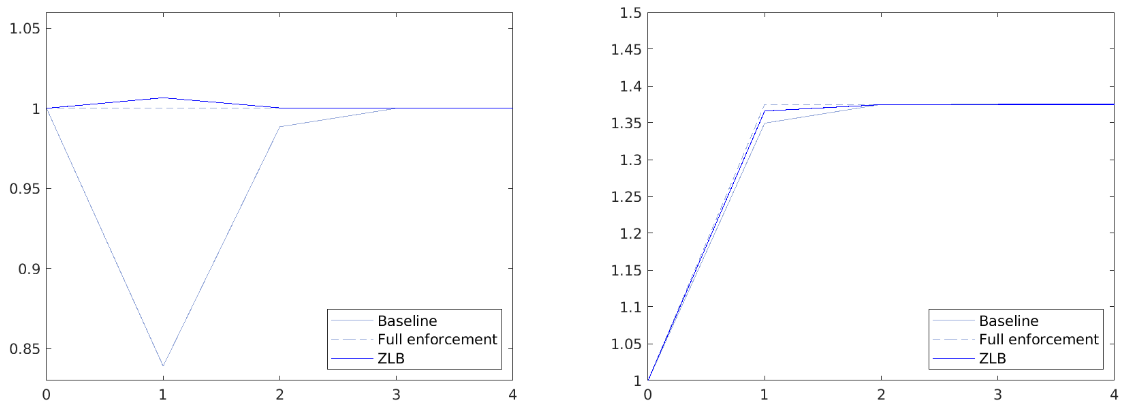
Finally, a question, however, is in order here: what would happen in the situation in which the nominal interest rate is exogenously set at a $\beta < q_t \leq 1$? This is a relevant question since the concept that underlies it is that, in the real world, monetary policy directly affects the interest rate. From the results for the full enforcement equilibrium, we know that, for $q_t > \beta$, the demand for bonds would be lower than the supply. Thus, bond sales would be restricted by the demand for an exogenously higher price. By assuming a symmetric solution for unconnected agents, naturally, we have $l_t = \int_{i \in I^C} b_{i,t+1} di/(1-\eta)$ where $I^C \subset I$ is the set of agents who are connected to financial markets.

Since the nominal interest rate is exogenous, it does not return to its equilibrium value endogenously. Hence, I will, for simplicity, assume that q_t and, thus, i_t will remain constant, at the exogenously set level, for as long as connected agents are still smoothing their consumption. Proposition 5 ensures that, in this setup, the economy should eventually return to a stationary equilibrium with homogeneous agents. The intuition for this result is the following: since the interest rate here is lower than its equilibrium value, the optimal consumption path for the connected agents features decreasing consumption while the economy is out of the stationary equilibrium.

As a result, their optimal monetary holdings also decrease over time. Eventually, these holdings will be so close to their stationary equilibrium level, $m + \eta\tau$, that these agents are better off by spending it all at once, just like in the bondless economy. This is possible, because, for an interest rate below $(1-\beta)/\beta$, connected agents are not compensated enough for giving up on current consumption. For the remainder of this subsection, I will, thus, concentrate on $q_t = 1$, which implies that the economy is at the zero lower bound, that is, $i_t = 0$. In this situation, cash and bonds are perfect substitutes as saving devices, but, by assumption, connected agents should save in bonds only.

Proposition 5. *If there is a constant $\beta < q_t \leq 1$, then there must be a period $T < \infty$ at which connected agents decide to fully deplete their extra money.*

If this economy is hit by a monetary shock of the size $\tau = 0.5$ as before, full depletion by connected agents will take place already at $t = 2$. Thus, the dynamics do not differ too much from those in the baseline economy. In order to illustrate how this new situation differs from the economy in the baseline - bondless - situation, I will hit this economy with a higher shock, so as to ensure that connected agents take one period longer to fully deplete the entirety of τ. I will, thus, use $\tau = 1.5$.



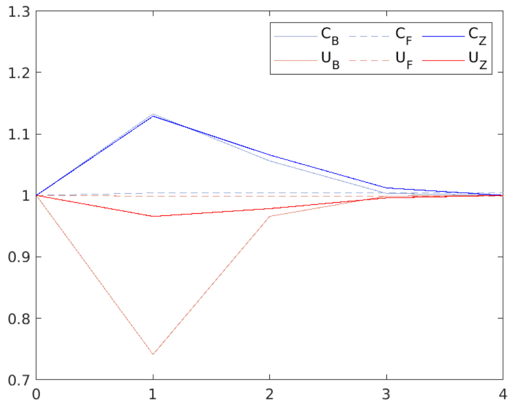
(a) Output, Y_t

(b) Aggregate price, P_t

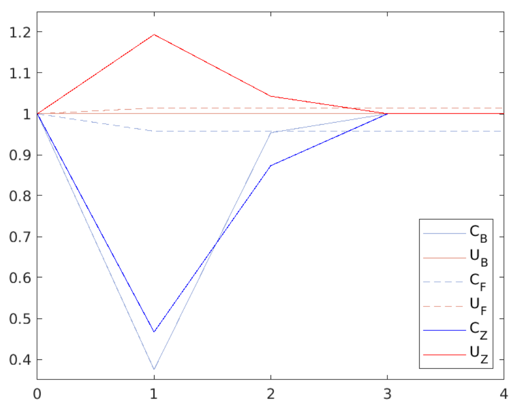
Figure 10: Paths of output and aggregate price

Figure 10 shows the paths of the aggregate output and price under the baseline economy, the economy with full enforcement of bond contracts, and the economy at the zero lower bound (ZLB). As expected, the aggregate price displays less stickiness than the one in the baseline economy. However, even though the whole monetary base is put in circulation already at $t = 1$, P_t still displays some small degree of stickiness in the ZLB economy, unlike the economy without a borrowing constraint. Since the aggregate output is given by $Y_t = M_t^C / P_t$, it will inevitably increase a little bit due to this mild aggregate price stickiness.

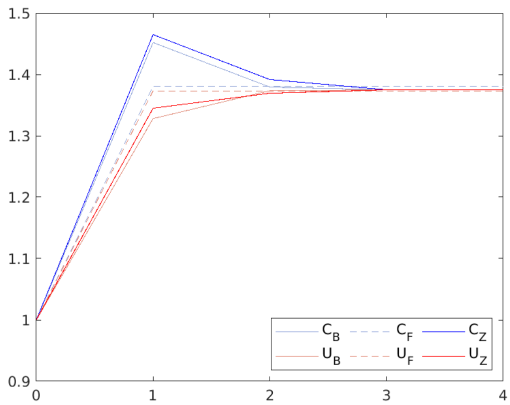
In Figure 11, I plot the paths of individual consumption, output, prices, and revenues across the three versions of the model. I use the following notation in the legends: i_k stands for individual $i \in \{C, U\}$ (that is, respectively, a "representative" connected or unconnected agent) in the economy k, where $k = B$ stands for the baseline economy, $k = FE$ stands for the unconstrained (by an exogenously low interest rate) economy with full enforcement of bond contracts, and $k = Z$ stands for the economy at the zero lower bound. Figure 11a shows that, as expected, the path of C_{ct} is very similar to the one in the baseline economy since saving in terms of bonds is equivalent to saving in terms of cash. It was also to be expected that the consumption of unconnected agents would be much closer to connected agents' consumption at the ZLB than in the baseline economy and farther from it than in the full enforcement economy. In fact, C_{ut} falls by much less than in the economy in which a market for bonds is totally absent.



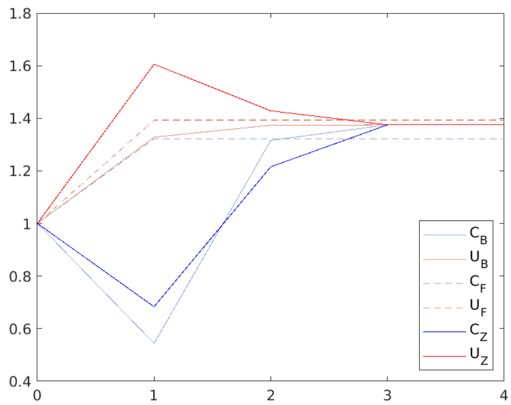
(a) Individual consumption, C_{ut}



(b) Individual output, y_{ut}



(c) Individual prices, p_{ut}



(d) Individual revenues, R_{ut}

Figure 11: Comparison across agents

Figure 11c shows that p_{ut} grows more on impact than in the baseline economy. This reflects the fact that unconnected agents get to spend more with consumption than in the situation where bonds are lacking. However, notice that p_{u1} reflects the expenditure with consumption at $t = 2$. Hence, the reason why p_{u1} is not as different in these economies as the gap in C_{u1} would suggest is the fact that the gap in C_{u2} is much smaller. Interestingly, the price chosen by connected agents overshoots by *more* than in the baseline economy. This illustrates the *contagion* of inflation that is present when there is strategic complementarity in price setting: it is due to the lower degree of sluggishness in the aggregate price under a ZLB that p_{ct} may grow by more than in the economy without bonds.

Figure 11b shows that the output of both types of agent is larger than in the baseline economy. This is especially true for unconnected agents since their output grows by much more than in the economy with full enforcement of bond contracts. The same patterns are also observable for revenues. Importantly, notice that, unlike in the other situations, the revenue of unconnected agents actually overshoots. In Appendix B, I show in Figure 13 the path of monetary holdings, m_{it}. It shows that the evolution of m_{it} is not very different in the baseline and in the ZLB economies. Therefore, this overshooting behavior of revenues - which is pushed mostly by the behavior of output than by prices - is necessary for unconnected agents to amortize their debt, since here they no longer roll it indefinitely. The reason why this happens is that connected agents are actually less willing to buy bonds as time passes and their monetary holdings fall.

This clarifies why the aggregate price does not reach its final level immediately at $t = 1$: unconnected agents need to post a low enough price to ensure they will be able to get a revenue high enough to amortize their debt while still maintaining a high enough consumption standard. Moreover, output responds *positively* to the monetary shock in this economy - unlike in the baseline economy, where it decreases, - precisely because indebted agents work harder in the model. Hence, indebtedness works here in a way that compels agents to produce more and post lower prices - generating a source of price stickiness that is absent in the baseline version of the model.

3.4 Sensitivity analysis

Now, I perform again a sensitivity analysis with respect to the parameter η, but this time for the financially developed economy. Again, I study the cases where the fraction in the population of connected agents is $\eta \in \{0.1, 0.25, 0.5\}$. Besides, throughout this exercise, I use a shock of $\tau = 0.5$. The figures can be found in Appendix B. Figure 14 shows that, for the standard financially developed economy, as before, we observe less heterogeneity across connected and unconnected agents for higher values of η. The reasoning is the same as before: for the same aggregate shock, if more people have early access to the injected money, the amount each individual can get will be smaller.

Two things are noteworthy here. Firstly, the graph for the paths of individual output is a mirror image of the one on the individual consumption paths. The amount that unconnected agents consume and produce is very similar across different values of η - like in the baseline economy. Also, the cases where connected agents receive higher interest payments (*i.e.*, for a lower η) are also the ones in which these agents need to work the least to afford their high consumption standards. Secondly, there is very little heterogeneity in prices. Thus, the bulk of the differences in revenue come from differences in output.

Figure 15 shows the paths of output and aggregate price for the different values of η at the zero lower bound. The response in aggregate output and the sluggishness in the aggregate price is nearly unnoticeable. For the former, the rise in output is below 0.5% in all of the analyzed cases. Besides, the strongest response in output is observed for $\eta = 0.1$. Figure 16 shows the paths for the individual variables. For all of them, we observe more heterogeneity, as before, for lower values of η.

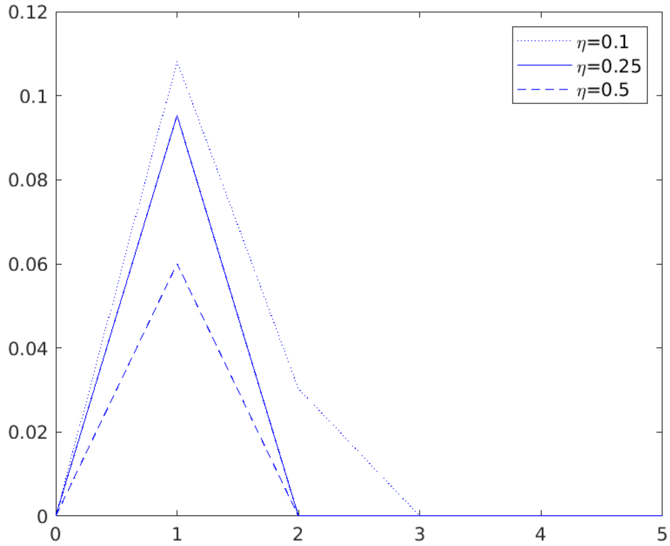


Figure 12: Borrowing, b_{ut}, for different values of η

Interestingly, the fall in consumption by the unconnected households is strongest for $\eta = 0.5$. Since prices grow by nearly the same extent in all cases, this means that the unconnected households manage to borrow *less* when there are more connected people. This can be seen in Figure 12, which plots the absolute amount of bonds that the unconnected sell for different values of η. The intuition for this result is straightforward: when there are more connected agents, the individual amount of money each of them gets access to is smaller for the same aggregate shock. Thus, they save a smaller fraction of the new money than in the economies where η is lower.

3.5 Welfare

I now move on to analyze the welfare consequences of the models here presented for $\eta = 0.25$. Since these models yield several qualitative implications analytically, but little can be said quantitatively without simulations, this welfare analysis will be conducted in the context of the simulations performed here. The results related to welfare should, then, be interpreted as possible - though not necessary - implications of these models. To begin, I adopt a utilitarian specification of the welfare function and give all individuals equal weight. The function is, then, given by:

$$W(\{\{C_{it}, h_{it}\}_{i\in I}\}_{t=1}^{\infty}) = \sum_{t=1}^{\infty} \beta^{t-1} \int_0^1 \omega_i (log(C_{it}) - \gamma h_{it}) di \tag{42}$$

where the individual weight $\omega_i = 1$ for all $i \in I$ and $t = 1, 2, ...$ Notice that I focus on post-shock paths of individual consumption and labor, and ignore the initial equilibrium, since it is identical across all specifications. Table 2 shows the results for each of the possible scenarios under both at $\tau = 0.5$ and a $\tau = 1.5$ shock. Appendix B contains tables for some counterfactual exercises, which were performed to better understand what drives the welfare differences.

Shock size	No shock	Baseline	Full enforcement	ZLB
$\tau = 0.5$	-17.2719	-17.2869	-17.2720	-17.2737
$\tau = 1.5$	-17.2719	-17.3329	-17.2723	-17.2788

Table 2: Welfare analysis

Notice that welfare unequivocally falls with the monetary shock across all scenarios relative to the situation in which the shock does not occur. This pattern is more pronounced for the bigger shock. The baseline economy is the one at which welfare falls the most. The full enforcement economy stays quite close to the "no shock" scenario - even for the big monetary shock. The zero lower bound model generates a middle ground between both limiting cases, although it is much closer to the full enforcement scenario. In fact, welfare falls by less in the ZLB model under the big monetary injection than a baseline economy that receives the small shock. This indicates that: 1) financial frictions that impede borrowing make the distributional effects of monetary policy very consequential for its welfare effects, and 2) an exogenously low interest rate reduces welfare relative to the equilibrium - *natural* - rate.

Table 4, in Appendix B, shows that most of the fall in welfare in the baseline economy is owed to the fall in output. However, to the extent that poorer agents have a higher marginal utility of consumption, the fall in welfare caused by the monetary shocks' disequalizing effects is also very large. Moreover, if I perform the exercise of imposing that, after T periods, the full enforcement economy goes to the same equilibrium as the other ones, the counterfactual scenario gets very close to the situation in which the shock does not take place. This means that the most important element behind the fall in welfare under full enforcement of bond contracts is the fact that it induces permanent inequality.

Next, I perform three exercises regarding the ZLB situation (see Table 6): (1) I impose the same fall in output that takes place in the baseline economy while keeping the degree of inequality produced by the ZLB model; (2) I impose the inequality level in the baseline model, but keep the rise in output under the ZLB regime; and (3) I eliminate inequality altogether. Case (1) produces the biggest fall in welfare, reinforcing the finding that most of the welfare loss is due to the falling output. Case (2) produces the second-largest fall among the three. Naturally, both cases increase the welfare loss relative to the benchmark ZLB economy. Lastly, case (3) implies that welfare falls with the rise in output. This result means that, under these parameter values, labor disutility matters more than the increase in consumption produced by the rise in production.

	Shock size	$\eta = 0.1$	$\eta = 0.25$	$\eta = 0.5$
Baseline	$\tau = 0.5$	-17.29120	-17.28685	-17.27979
	$\tau = 1.5$	-17.39342	-17.33284	-17.31036
Full Enforcement	$\tau = 0.5$	-17.27211	-17.27196	-17.27191
	$\tau = 1.5$	-17.27328	-17.27234	-17.27204
ZLB	$\tau = 0.5$	-17.27487	-17.27372	-17.27366
	$\tau = 1.5$	-17.28300	-17.27881	-17.27598

Table 3: Welfare analysis under the zero lower bound for different values of η

Finally, Table 3 shows, for both shock sizes and for $\eta \in \{0.1, 0.25, 0.5\}$, the welfare under the baseline, full enforcement, and ZLB economies. Overall, welfare is highest in the economies with a higher value of η. This means that financial development in the form of amplified access to financial markets makes the monetary shock less deleterious. Furthermore, economies with fewer financially connected people are the ones that benefit the most from financial development in the form of a well-functioning credit market. This can be seen through the fact that welfare improves the most between the baseline and the full enforcement/ZLB scenarios in the economy where $\eta = 0.1$. As seen before, the new money is put in circulation more slowly and borrowing is more widespread if the benefits of the monetary shock are highly concentrated. This partially offsets the large distortions introduced by the shock for a low η.

4 Concluding remarks

Only recently, the literature on the distributional effects of monetary policy has started to pick up steam. Still, few of the papers in this literature argue that these disequalizing effects may generate out-of-equilibrium dynamics that preclude the economy's prompt return to the long-term equilibrium - as pointed out by Friedman in 1969. Still, the literature points to the fact that there are several different channels through which monetary policy has distributional effects. These distributional effects may promote a re-shuffling of monetary holdings across agents, which is not linked to fundamentals, and may produce a sluggish process of re-balancing of monetary holdings.

This process is especially important if the agents who get access to the new money earlier decide to smooth consumption by spending it slowly. I have shown in the present work that, if a well-functioning bonds market is not in place to channel these excess reserves to the agents who got the shortest straw of the monetary shock (a fall in real money balances), the disequalizing effects in the short-run may be significant. Also, if agents smooth consumption out of their newly created (by the monetary shock) purchasing power, the new money is slowly injected into the economy in the absence of a bonds market. If agents are forward-looking, prices may rise excessively in response, bringing about a fall in output. Moreover, the model here presented reinforces the price heterogeneity found in Williamson (2008) under a very different specification: the price of connected agents overshoots, while other prices rise slowly.

If credit markets are well-developed enough, and connected agents can lend their excessive reserves to unconnected ones, inequality grows by much less. However, if the interest rate is at its equilibrium value and unconnected agents are as impatient as connected ones, the former will roll over their debt indefinitely, meaning that the differences in monetary holdings become persistent. Moreover, unconnected agents work indefinitely more than the connected ones in order to afford the interest payments and maintain the optimal level of consumption. Hence, connected agents gain a permanent claim over part of the revenues of the unconnected even if only one-period bonds are traded. If the economy is at the zero lower bound, it will eventually reach the new equilibrium, where the inequality induced by monetary policy is undone. Importantly, output goes up in the model due to unconnected agents' effort to get enough money to amortize their debt while still keeping a high enough consumption standard.

There is yet another dimension of financial development that is relevant in the present work, namely, how widespread participation in financial markets is. For a given aggregate monetary shock, the distributional effects and distortions generated are smaller in economies where more agents are connected to financial markets. Importantly, if these agents are too few in a given economy, welfare can be even more dramatically improved through the development of a well-functioning market for credit. The analysis conducted here leads to the recommendation of two sets of policies if one is interested in mitigating potentially deleterious effects of monetary shocks. First, reducing financial frictions that hinder access to credit seems to be very relevant, because it allows unconnected agents to smooth their consumption. Second, policies targeted at amplifying access to financial markets are also welcome. This requires more research to be done on the causes why this access is relatively restricted even in some advanced economies.

Finally, the models here presented bring new insight into the phenomenon of price stickiness. Here, a mild degree of endogenous price stickiness is induced by two different mechanisms. If part of the new money remains idle and is put in circulation slowly, unconnected agents will raise their prices more slowly. The reason for this is the fact that their relatively low monetary holdings make the marginal utility of holding money in the future higher, which strengthens their motivation to remain competitive and get high revenues today. In the presence of strategic complementarity in price setting, this will make the price of connected households overshoot less. The second mechanism operates through debt: indebted agents will work harder and set lower prices to get high enough revenues to amortize their debt.

In summary, the present work confirms Friedman's argument that, in the absence of helicopter drops, the economy's return to a stationary, long-run, equilibrium should happen slowly (provided that the shock is large enough). The distortions are, however, eased by a bonds market, which channels excess reserves toward impoverished households. However, some of these results are reliant on the assumption that all households are initially identical. Also, if households were *unidentical* concerning their preferences, relative prices could be distorted even for a small uneven monetary shock, since it would benefit the producers of goods that are preferred by the agents made richer. Moreover, if connected and unconnected agents differ in terms of initial income and time preferences, the effects could potentially be different in the version of the model which features a bonds market. These questions require future research to be elucidated. Finally, it may also be worthwhile to study how the effects of a monetary injection would change if the distributional effects were generated by channels other than the financial segmentation one.

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A Appendix

A.1 Proofs of Propositions

A.1.1 Proposition 1

Let us redefine the problem faced by the islands as in Lucas and Stokey (1985). Here, I do away with the separation between buyer and seller and assume that each island corresponds to an entrepreneur, who solves the problem:

$$\begin{aligned}
\max_{\{C_{it}, m_{i,t+1}, h_{it}, p_{it}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - \gamma h_t\right] \\
\text{subject to} \quad & P_t C_{it} \leq m_{it} && (\lambda_{it}) \\
& m_{i,t+1} + P_t C_{it} \leq m_{it} + p_{it} h_{it} && (\mu_{it}) \\
& h_{it} \leq D(p_{it}) && (\zeta_{it}) \\
& C_{it} \geq 0
\end{aligned}$$

The resulting first-order conditions for, respectively, C_{it}, h_{it}, p_{it} and $m_{i,t+1}$ are:

$$\lambda_{it} + \mu_{it} = \beta^t \frac{u'(C_{it})}{P_t} \tag{43}$$

$$p_{it}\mu_{it} - \gamma\beta^t = \zeta_{it} \tag{44}$$

$$h_{it}\mu_{it} = -D'(p_{it})\zeta_{it} \tag{45}$$

$$\mu_{it} = \lambda_{i,t+1} + \mu_{i,t+1} \tag{46}$$

First, notice that (43) for $t+1$, together with (46) produces:

$$\frac{u'(C_{it})}{P_{it}} = \beta \frac{u'(C_{i,t+1})}{P_{t+1}} + \frac{\lambda_{it}}{\beta^t} \tag{47}$$

Now, notice that (47), (44) and (45) produce (12). Moreover, for $u(\cdot) = \log(\cdot)$, (47) yields:

$$P_t C_{it} = \begin{cases} \frac{1}{\beta} P_{t+1} C_{i,t+1} & \text{if } \lambda_{it} = 0 \\ m_{it} & \text{if } \lambda_{it} > 0 \end{cases}$$

since $\lambda_{it} = 0$ implies that $s_{it} = m_{it} - P_t C_{it} > 0$, this is the same result as the one obtained before. □

A.1.2 Proposition 2

Prices and allocation under consumption smoothing by connected agents:

To begin, notice that, since unconnected agents own m_0 units of money each at $t = 1$, but expect to earn more than that amount in sales in the same period, they choose to fully deplete their resources. In fact, the same holds for any t, as will be seen below. For now, I will assume this is the case, and verify it later. By (13), we must, thus, have:

$$p_{it} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}R_{it} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}p_{it}D(p_{it})$$
$$p_{it}^{\epsilon} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma P_t^{\epsilon}}{\beta}\int_0^1 \hat{C}_{nt}dn = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma P_t^{\epsilon}}{\beta}Y_t$$

Therefore, using (21), we obtain, for the unconnected agents:

$$\theta_{ut} = \left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}\frac{1}{Y_t}\right]^{\frac{\epsilon-1}{\epsilon}} \tag{48}$$

which means that when real output is higher, unconnected agents' revenues are smaller relative to the average revenue. Now, using connected agents' first order condition and (13):

$$p_{ct} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\beta P_t C_{ct} = \left(\frac{\epsilon}{\epsilon-1}\right)\gamma P_t C_{ct}$$

Therefore, the fraction of connected agents' revenue in nominal GDP is given by:

$$\theta_{ct} = \left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{1}{\gamma C_{ct}}\right]^{\epsilon-1} \tag{49}$$

Thus, it decreases connected agents' current consumption. Now, assume by means of contradiction that, at $t=1$, $\theta_{c1} \geq 1 = \theta_{c0}$. Since θ_{it} is decreasing on own price, p_{it}, and $\int_0^1 \theta_{it}di = 1$, then $\theta_{u1} \leq \theta_{c1}$ and, thus, $p_{u1} \geq p_{c1}$ for the unconnected. By (13), this can only happen if $P_2C_{u2} \geq P_2C_{c2}$. However, since unconnected agents fully deplete m_{u1}, their budget constraint at $t=2$ requires $R_{u1} \geq P_2C_{u2}$. Furthermore, given that $\theta_{c1} \geq \theta_{u1}$, we have:

$$R_{c1} = \theta_{c1}P_1Y_1 \geq \theta_{u1}P_1Y_1 = R_{u1}$$

Thus, $R_{c1} \geq R_{u1}$ and $s_{c1} > 0 = s_{u1}$, which means that $m_{c2} > m_{u2}$ and, therefore, it cannot be optimal for connected agents to choose $C_{c2} \leq C_{u2}$. It must be the case, then, that $\theta_{c1} < 1 = \theta_{c0}$. By (49), this means that:

$$C_{c1} > \left(\frac{\epsilon-1}{\epsilon}\right)\frac{1}{\gamma}$$
$$C_{c1} > \beta C_{c1} > \left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma} = C_{c0}$$

By the same token, (48) implies that:

$$Y_1 < \left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}$$
$$Y_1 < Y_0$$

And, therefore, $C_{u1} < C_0$ for unconnected agents. Moreover, notice that, since $C_{c1} > C_{u1} = m_{u1}/P_1$ and $\theta_{u1} > 1$:

$$R_{u1} = \theta_{u1} P_1 \left((1-\eta)\frac{m_{u1}}{P_1} + \eta C_{c1} \right) > P_1 \left(\frac{m_{u1}}{P_1} \right) = m_{u1}$$

for unconnected agents, which confirms the hypothesis previously made that these agents must fully deplete their money resources. Finally, notice that, since $p_{u1} < p_{c1}$:

$$y_{u1} = Y_1 \left(\frac{p_{u1}}{P_1} \right)^{-\epsilon} > Y_1 \left(\frac{p_{c1}}{P_1} \right)^{-\epsilon} = y_{c1} \tag{50}$$

in equilibrium. The same argument presented so far for $t = 1$ is also valid for every t at which connected agents fully deplete their resources.

Also, notice that, since $P_{t+1}C_{c,t+1} = \beta P_t C_{ct}$, by (13), $p_{c,t+1} = \beta p_{ct}$, hence the price of connected agents falls over time, at least for as long as they do not fully deplete their resources. Also, notice that, for unconnected agents, $P_t C_{ut} = m_{ut}$ grows over time, since $\theta_{ut} > 1$ and $P_t Y_t = (1-\eta)m_{ut} + \eta P_t C_{ct} > m_{ut}$. This implies that $m_{u,t+1} = R_{ut} = \theta_{ut} P_t Y_t > m_{ut}$. As a result, their prices also grow over time, i.e. $p_{u,t+1} > p_{ut}$, for as long as connected agents fully deplete their resources.

Connected agents must optimally choose to fully deplete their money at finite time:

Let T be the date when connected agents decide to fully deplete their resources. Given connected agents' first-order conditions, this must take place when:

$$\frac{1}{\beta}(m_0 + \eta\tau) \geq \frac{1}{\beta} P_{c,t+1} C_{c,t+1} \geq P_{ct} C_{ct} = m_{ct}$$
$$m_{ct} \leq \frac{1}{\beta}(m_0 + \eta\tau)$$

I will show that $T < \infty$ through a simple argument by contradiction. Assume that such a $T < \infty$ does not exist. Thus, by connected agents' first-order condition, we must have:

$$\beta^{t-1} P_1 C_{c1} = P_t C_{ct}$$

Naturally, as $t \to \infty$, $\beta^{t-1} \to 0$ and, thus, $P_t C_{ct} \to 0$. However, since p_{ut} grows over time and $p_{ct} > p_{ut}$, $P_t > 0$ always. Thus, $P_t C_{ct} \to 0$ can only happen if $C_{ct} \to 0$. This should lead $\theta_{ct} \to \infty$, and, thus, connected agents' market share should be $\eta\theta_{ct} \to \infty$: a contradiction, since $\eta\theta_{ct} \leq 1$.

Characterization of T:

I will now prove that other agents still fully deplete their resources at time T. I will begin by showing that $P_T C_{cT} > m_0 + \eta\tau$. Assume that it is not the case, that is, $P_T C_{cT} \leq m_0 + \eta\tau$. Thus, we must have:

$$\begin{aligned} m_0 + \eta\tau \geq P_T C_{cT} = \beta P_{T-1} C_{c,T-1} \\ \frac{1}{\beta}(m_0 + \eta\tau) \geq P_{T-1} C_{c,T-1} \end{aligned}$$

Since, at $T-1$, unconnected agents fully deplete their resources, this condition means that connected agents should have depleted their resources as well at time $T-1$, a contradiction. This proves the statement. But notice that this implies that $m_{cT} = P_T C_{cT} > m_0 + \eta\tau$ and, therefore:

$$\begin{aligned} m_{uT} &= \frac{1}{1-\eta}(M_0 + \eta\tau - \eta m_{cT}) < \frac{1}{1-\eta}\left(m_0 + \eta\tau - \eta m_0 - \eta^2\tau\right) \\ &< m_0 + \eta\tau < \frac{1}{\beta}(m_0 + \eta\tau) \end{aligned}$$

which means, by the first order conditions of unconnected agents, that they must also fully deplete their resources at time T.

Also, notice that $P_T C_{cT} = \beta^{T-1} P_1 C_{c1} < \beta^{T-1}(m_0 + \tau)$ by the budget constraint of connected agents at $t = 1$. Given that $P_T C_{cT} > m_0 + \eta\tau$, we obtain the condition:

$$\begin{aligned} m_0 + \eta\tau < \beta^{T-1} P_1 C_{c1} < \beta^{T-1}(m_0 + \tau) \\ \beta^{T-1} > \frac{M_0 + \eta\tau}{M_0 + \tau} \end{aligned} \tag{51}$$

Let the maximum value of T that satisfies this condition be denoted by T^{MAX}. Then, $T \leq T^{MAX}$. Also, notice that evidently the equilibrium becomes identical to the helicopter drops one after from time $T+1$ onwards, and the price level is already identical to that under helicopter drops already at T.

Further characterization of prices:

Since unconnected agents fully deplete their resources for every $t = 1, 2, ...$, we must have that, for $t = \{1, 2, ...T-1\}$:

$$P_{t+1} C_{u,t+1} = R_{ut} = \theta_{ut} P_t Y_t > P_t Y_t = M_t^C$$

which means that:

$$p_{u,t} > \left(\frac{\epsilon}{\epsilon - 1}\right)\frac{\gamma}{\beta} M_t^C = p^H(M_t^C) > p_0 \tag{52}$$

that is, p_{ut}, for unconnected agents is larger than the price level that would take place under helicopter drops (or under full depletion by connected agents) if the money supply were M_t^C. Therefore, we must also have $p_{ct} > p_{ut} > p_t^H(M_t^C) > p_0$, which, in turn, means, that the price level is larger than under helicopter drops of $\tau' = M_t^C - M_0$, i.e. $P_t(M_t^C) > P_t^H(M_t^C) > P_0$. Also, since prices from time T onwards are identical to $p^H(M_0 + \eta\tau)$ and $P^H(M_0 + \eta\tau)$ and $p_{ut} < p_{u,t+1}$ for $t = \{1, 2, ..., T-1\}$, then we must have:

$$p_{ut} < p_{uT} = p^H(M_0 + \eta\tau) \tag{53}$$

for every $t = \{1, 2, ..., T-1\}$. For connected agents, the opposite happens:

$$p_{ct} > p_{cT} = p^H(M_0 + \eta\tau) \tag{54}$$

again, for every $t = \{1, 2, ..., T-1\}$. □

A.1.3 Proposition 3

Taking $t = 0$ as the base period, the Laspeyres price index for period t can be computed as:

$$\begin{aligned}
\mathbb{P}_t^L &= \frac{\eta p_{it} y_0 + (1-\eta) p_{ut} y_0}{p_0 y_0} \\
&= \frac{\eta p_{it} + (1-\eta) p_{ut}}{p_0} \\
&= \frac{p_t^H}{p_0} + \frac{1}{p_0}\left[\eta(p_{it} - p_t^H) + (1-\eta)(p_{ut} - p_t^H)\right] \\
&= P^H(M_t^C) + \eta(p_{it} - p_t^H) + (1-\eta)(p_{ut} - p_t^H)
\end{aligned}$$

where the fourth equality comes from the normalization made that $P_0 = 1$ and from (11). Also, I denoted the helicopter drops price for an amount of money in circulation M_t^C as $p_t^H := p^H(M_t^C)$ for short. Now, notice that a first-order Taylor approximation around the helicopter drops price level is given by:

$$\begin{aligned}
P_t &= \int_0^1 p_{it} di = \left[\eta p_{ct}^{1-\epsilon} + (1-\eta) p_{ut}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \\
&\approx P^H(M_t^C) + \eta\left(\frac{p_t^H}{P_t^H}\right)^{-\epsilon}(p_{ct} - p_t^H) + (1-\eta)\left(\frac{p_t^H}{P_t^H}\right)^{-\epsilon}(p_{ct} - p_t^H) \\
&= P^H(M_t^C) + \eta(p_{it} - p_t^H) + (1-\eta)(p_{ut} - p_t^H) \\
&= \mathbb{P}_t^L
\end{aligned}$$

where the equality in the third line follows from (11). Notice that I have also simplified the notation for aggregate prices under helicopter drops as $P_t^H := P^H(M_t^C)$. Moreover, notice that the Laspeyres index can be re-written as:

$$\begin{aligned}\mathbb{P}_t^L &= \frac{\eta p_{it} y_0 + (1-\eta) p_{ut} y_0}{P_0 y_0} \\ &= \frac{\eta p_{it} y_0 + (1-\eta) p_{ut} y_0}{P_0 Y_0} \\ &= \frac{(\eta p_{it} + (1-\eta) p_{ut})\, y_0}{M_0}\end{aligned}$$

where the first equality, again, follows from (11); the second follows from the fact that $y_0 = Y_0$; and the third, from the fact that nominal output is equal to the total monetary base in the stationary equilibrium. Also, notice that, by (13):

$$\eta p_{it} + (1-\eta) p_{ut} = \eta \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} P_{t+1} C_{c,t+1} + (1-\eta) \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} P_{t+1} C_{u,t+1}$$

Thus:

$$\begin{aligned}\mathbb{P}_t^L &= \left(\frac{\epsilon}{\epsilon-1}\right) \frac{\gamma}{\beta} \left(\frac{\eta P_{t+1} C_{c,t+1} + (1-\eta) P_{t+1} C_{u,t+1}}{M_0}\right) y_0 \\ &= \left(\frac{\epsilon}{\epsilon-1}\right) \frac{\gamma}{\beta} \left(\frac{P_{t+1} Y_{t+1}}{M_0}\right) y_0 \\ &= \left(\frac{\epsilon}{\epsilon-1}\right) \frac{\gamma}{\beta} \left(\frac{M_{t+1}^C}{M_0}\right) y_0 \\ &= \frac{M_{t+1}^C}{M_0}\end{aligned}$$

where the second equality follows from the market clearing condition, and the fourth comes from (2.2) and the fact that $y_0 = C_0$ by the market clearing condition of the stationary equilibrium. Finally, given (17), we have:

$$\mathbb{P}_t^L = \frac{M_{t+1}^C}{M_0} = \frac{P^H(M_{t+1}^C)}{P_0} \tag{55}$$

□

A.1.4 Proposition 4

To begin, notice that one can re-write the Euler equation (34) as:

$$\frac{C_{it}}{C_{i,t+1}} = \frac{q_t P_{t+1}}{\beta P_t}$$

Notice that the right-hand side contains only aggregate variables. This implies that the ratio $C_{it}/C_{i,t+1}$ must be identical across all buyers. Besides, notice that, since $s_{ct} = 0$ for all t and $b_{c1} = 0$, all the money not used by connected agents to buy bonds in period $t = 1$ must be used for consumption, that is $P_1 C_{c1} = m_0 + \tau - q_1 b_{c2}$. Moreover, notice that, if $q_1 = 1$, $s_{i1} = 0$ for all agents by assumption, and, if $q_1 < 1$, it cannot be optimal for unconnected agents to sell bonds at a price q_1 and save in cash (*i.e.*, $s_{u1} > 0$), then we must have $M_1^C = M_0 + \eta\tau$. Since the same line of reasoning applies for $t = 2$, we must also have $M_2^C = M_0 + \eta\tau$. This implies that:

$$M_0 + \eta\tau = P_1 \int_0^1 C_{i1} di = \frac{q_t}{\beta} P_2 \int_0^1 C_{i2} di = \frac{q_1}{\beta}(M_0 + \eta\tau)$$

where the third equality naturally follows from (34). This implies that $q_1 = \beta$. The same argument can be applied for any $t = 1, 2, ...$, meaning that we must have $q_t = \beta$. It follows immediately that $C_{it} = C_{i,t+1}$ for $i \in I$. Evidently, we must have $P_t C_{ct} > P_t C_{ut}$ for unconnected agents and $t = 1, 2, ...$, which implies that $p_{ct} > p_{ut}$ and, thus, $R_{ut} > R_{ct}$ for all $t \geq 1$. Since $P_t C_{ct} > P_t C_{ut}$ for all periods t and $P_t \int_0^1 C_{it} di = M_0 + \eta\tau$, then we must have $P_t C_{ct} > M_0 + \eta\tau > P_t C_{ut}$. Given (13) and (16), it immediately follows that $p_{ct} > p^H(M_0 + \eta\tau) > p_{ut}$.

Now, notice that, by agent i's budget constraint in period $t = 2, 3, ...$, we have:

$$P_t C_{it} = R_{i,t-1} + b_{it} - \beta b_{i,t+1} \tag{56}$$

Notice that, since $P_{t+1} C_{i,t+1} = P_{t+2} C_{i,t+2}$ for any $t = 1, 2, ...$ By (13), this implies that $p_{i1} = p_{i2}$ for $i \in I$. Since all prices remain identical over time, so must the revenues each agent obtains, that is, $R_{it} = R_{i,t+1}$. Together with (56) and with the fact that $P_t C_{it} = P_{t+1} C_{i,t+1}$, this implies that:

$$b_{it} - \beta b_{i,t+1} = b_{i,t+1} - \beta b_{i,t+2} \tag{57}$$

for every $t = 2, ...$ Now, let us define α_s such that $b_{i,s} = \alpha_s b_{i,s-1}$, for $s = 3, 4, ...$ Thus, for an arbitrary $t \geq 3$, we can apply this definition iteratively to obtain:

$$b_{it} = \left(\prod_{s=3}^{t} \alpha_s \right) b_{i2} \tag{58}$$

I will now prove that $\alpha_t = 1$ for $t = 3, 4, ...$ I will concentrate on α_3 without any loss of generality. Assume, by contradiction, that $\alpha_3 < 1$. Then, by (57), we have:

$$\begin{aligned}
b_{i2} - \beta b_{i3} &= b_{i3} - \beta b_{i4} \\
(1 - \alpha_3\beta) b_{i2} &= (1 - \alpha_4\beta)\alpha_3 b_{i2} \\
(1 - \alpha_3\beta) &= (1 - \alpha_4\beta)\alpha_3 < (1 - \alpha_4\beta) \\
\alpha_4 &< \alpha_3
\end{aligned}$$

Obviously, the same reasoning applies to show that $\alpha_t < \alpha_3$. Therefore, $\alpha_t < 1$ for all $t \geq 3$. However, this means that:

$$\lim_{t \to \infty} b_{it} = 0$$

and, thus:

$$\lim_{t\to\infty} P_t C_{it} = R_{it}$$

However, since $R_{ut} > R_{ct}$ for unconnected agents, this means that $P_t C_{ut} > P_t C_{ct}$ at the limit: an absurd, since $P_t C_{it}$ is constant for all t after the shock and larger than $P_t C_{ut}$.

Now, I proceed to the second case: assume, by contradiction, that $\alpha_3 > 1$. Similarly to before, this implies that $\alpha_t > \alpha_3 > 1$ for any $t > 3$. Now, this implies that:

$$\lim_{t\to\infty} b_{ct} = \lim_{t\to\infty} \left(\prod_{s=3}^{t} \alpha_s \right) b_{c2} > \lim_{t\to\infty} \alpha_3^t b_{c2} = \infty$$

which is not possible, since $M_0 + \eta\tau < \infty$. Therefore, I conclude that $\alpha_t = 1$ for $t = 3, 4, ...$ and, thus, $b_{it} = b_{i,t+1}$ for $i \in I$ and $t \geq 3$. Using agents' budget constraint at $t = 1$, this result implies that:

$$\begin{aligned} b_{ct} &= \frac{m_0 + \tau - P_1 C_{ct}}{\beta} \\ b_{ut} &= \frac{m_0 - P_1 C_{ut}}{\beta} \end{aligned}$$

for unconnected agents. From this expression, one can easily see that $m_{it} = m_{i,t+1}$ for $t = 1, 2, ...$

Finally, notice that, at $t = \{1, 2\}$, the budget constraint can be written as:

$$\begin{aligned} P_1 C_{i1} &= m_{it} - \beta b_{i2} \\ P_2 C_{i2} &= R_{i1} + (1-\beta) b_{i2} \end{aligned}$$

Isolating b_{i2}, using the fact that $P_t C_{it} = P_s C_{is}$ for $i \in I$ and $t, s = 1, 2, ...$ and re-arranging the terms gives us:

$$\begin{aligned} P_t C_{ct} &= (1-\beta)(m_0 + \tau) + \beta R_{ct} \\ P_t C_{ut} &= (1-\beta) m_0 + \beta R_{ut} \end{aligned}$$

Subtracting the latter expression from the former yields:

$$P_t C_{ct} - P_t C_{ut} = (1-\beta)\tau - \beta(R_{ut} - R_{ct})$$

Since we know that $C_{ct} > C_{ut}$, then $P_t C_{ct} - P_t C_{ut} > 0$. As a result, we obtain:

$$R_{ut} - R_{ct} < \left(\frac{1-\beta}{\beta} \right) \tau$$

Equation 41 follows immediately from the facts that $R_{ut} - R_{ct} > 0$ and $C_{ct} > C_{ut}$. $\square$

A.1.4.1 Corollary 4.1

With the new utility specification, (34) can be re-written as:

$$\frac{u'(C_{it})}{u'(C_{i,t+1})} = \frac{\beta}{q_t}\frac{P_t}{P_{t+1}}$$

Since, by assumption, $u(\cdot)$ is increasing and strictly concave, the marginal utility is such that $u'(\cdot) \geq 0$ and is strictly decreasing. This means that, given certain values of the aggregate variables at the right-hand side, there is a unique ratio $C_{it}/C_{i,t+1}$ that satisfies buyer i's first order condition above. The rest of the argument goes exactly as in the proof of Proposition 3, which relies only on the buyers' budget constraints and market clearing conditions. $\square$

A.1.5 Proposition 5

To begin, notice that, if $T = 2$, then evidently $T < \infty$, which proves the result immediately. I will hence, consider only $T > 2$. I want to show that $T = \infty$ is not possible. Now, let us pick $t \geq 2$. Notice that, from (34), if $\beta < q_t \leq 1$, we will have:

$$P_{t+1}C_{c,t+1} < P_t C_{ct} \leq \frac{P_{t+1}C_{c,t+1}}{\beta} \tag{59}$$

since $b_{ct} \geq 0$ for a monetary expansion and, thus, connected agents are never constrained by (33) in this case, meaning that their expenditure with consumption will actually decrease over time. Since $P_t C_{ut} = (M_0 + \eta\tau - \eta P_t C_{ct})/(1-\eta)$, then $P_t C_{ut}$ must grow over time. Naturally, from (13), it follows that $p_{ct} < p_{c,t-1}$ and $p_{ut} > p_{u,t-1}$, which means that $R_{ct} > R_{c,t-1}$ and $R_{ut} < R_{u,t-1}$. By (56) and (59), we must have:

$$\begin{aligned} R_{c,t-1} + b_{ct} - \beta b_{c,t+1} &> R_{ct} + b_{c,t+1} - \beta b_{c,t+2} \\ b_{ct} - \beta b_{c,t+1} &> b_{c,t+1} - \beta b_{c,t+2} \end{aligned}$$

Then, let α_t be as defined in the Proof A.1.4. Assume, by contradiction, that $\alpha_{t+1} \geq 1$. The above equation must, then, imply that, since $b_{c2} > 0$:

$$\begin{aligned} 1 - \alpha_{t+1}\beta > \alpha_{t+1}(1 - \alpha_{t+2}\beta) &\geq 1 - \alpha_{t+2}\beta \\ \alpha_{t+2} &> \alpha_{t+1} \end{aligned}$$

Then, as before, by (58), we must have $\lim_{t\to\infty} b_{ct} = \infty$: an absurd, since the monetary base is finite. I conclude that $\alpha_{t+1} < \alpha_t$, meaning that $b_{c,t+1} < b_{ct}$. Therefore,

$$\lim_{t\to\infty} b_{it} = 0$$

Now, notice that, at the limit, we must have:

$$\lim_{t\to\infty} P_t C_{it} = \lim_{t\to\infty} R_{i,t-1} + b_{it} - \beta b_{i,t+1} = \lim_{t\to\infty} R_{i,t-1}$$

Hence, we must have full depletion at the limit, meaning that:

$$\lim_{t\to\infty} m_{it} = \lim_{t\to\infty} R_{it} = m_0 + \eta\tau$$

Now, notice that connected agents will choose to fully deplete if, and only if:

$$m_0 + \eta\tau = P_{t+1} C_{c,t+1} \geq \frac{\beta}{q_t} P_t C_{ct} \geq \frac{\beta}{q_t} m_{ct} \tag{60}$$

However, since $q_t > \beta$ is constant, then $\beta / q_t < 1$. Therefore:

$$\lim_{t\to\infty} \frac{\beta}{q_t} m_{ct} = \frac{\beta}{q_t} (m_0 + \eta\tau) < m_0 + \eta\tau$$

Therefore, (60) is satisfied with strict inequality. Since $m_{c1} > m_0 + \eta\tau$, then this means that m_{ct} is decreasing over time, and, therefore, there must be $T < \infty$ such that $\frac{\beta}{q_T} m_{cT} \leq m_0 + \eta\tau$, inducing connected agents to fully deplete their money.

B Outstanding Graphs and Tables

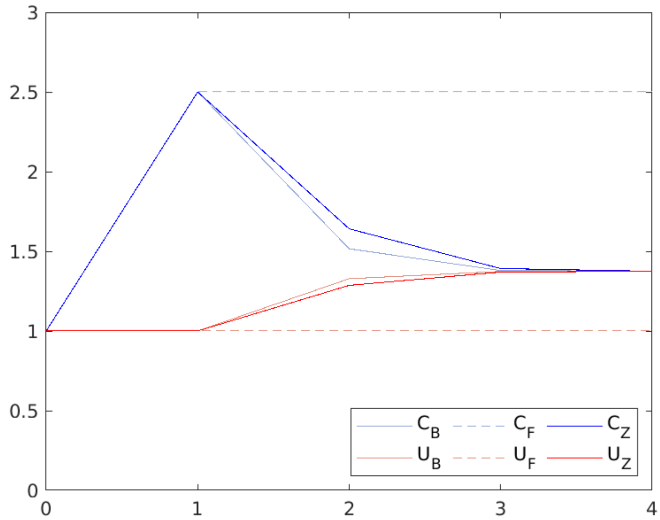
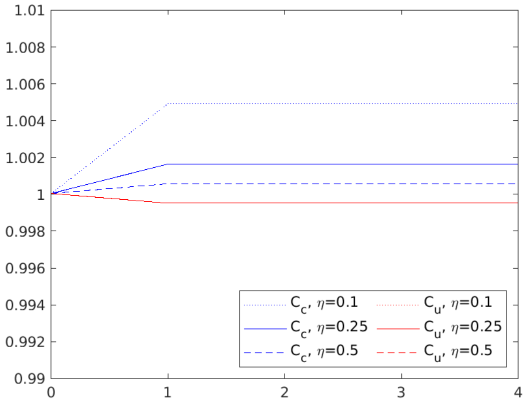


Figure 13: Comparison of monetary holdings, m_t, across agents

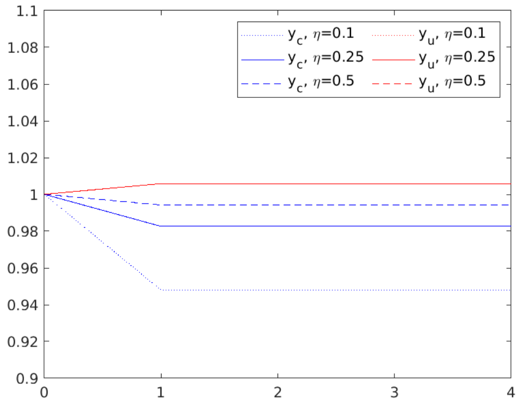
Shock size	Model	Constant output	No inequality
$\tau = 0.5$	-17.2869	-17.2760	-17.2833
$\tau = 1.5$	-17.3328	-17.2955	-17.3141

The second column contains the values for the welfare function under the benchmark specification of the baseline economy. The third column presents the counterfactual exercise of assuming that output is constant at the initial level, but keeping the degree of inequality across the agents. The last column stands for the opposite exercise: it removes inequality but maintains the fall in output.

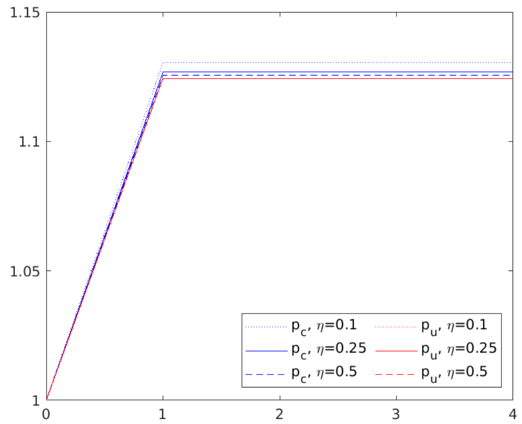
Table 4: Counterfactual welfare analysis of the baseline economy



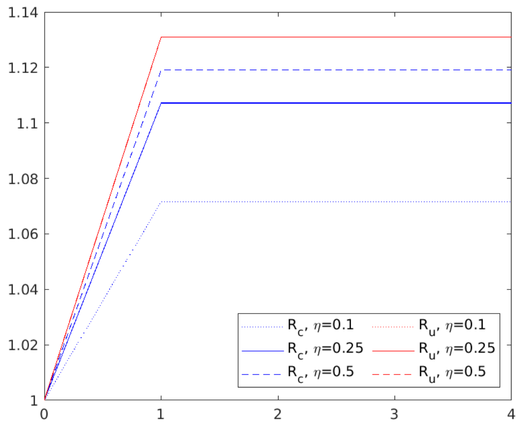
(a) Individual consumption, C_{ut}



(b) Individual output, y_{ut}



(c) Individual prices, p_{ut}



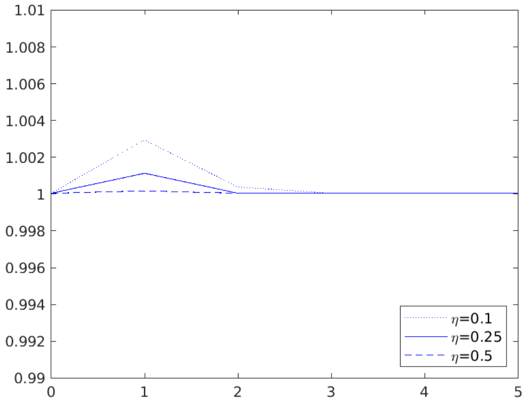
(d) Individual revenues, R_{ut}

Figure 14: Comparison across agents for the full enforcement economy for different values of η

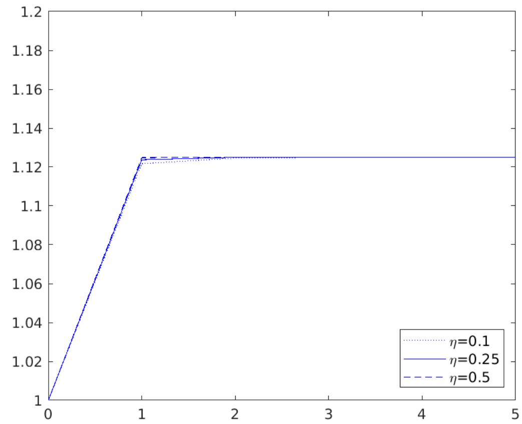
Shock size	Model	Counterfactual
$\tau = 0.5$	-17.2720	-17.2719
$\tau = 1.5$	-17.2723	-17.2720

The second column contains the values for the welfare function under the benchmark specification of the full enforcement economy. The counterfactual corresponds to the exercise of assuming that, after period T, as the baseline economy returns to equilibrium, the full enforcement economy returns as well.

Table 5: Counterfactual welfare analysis of the full enforcement economy

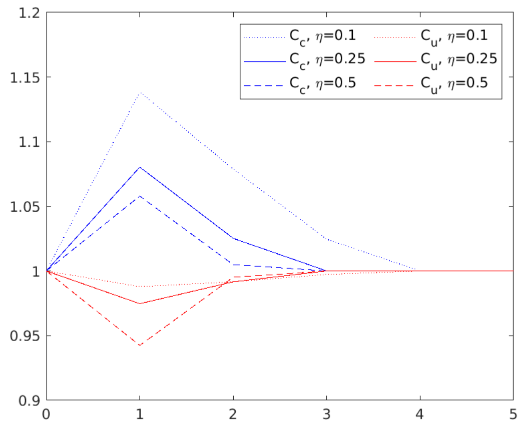


(a) Output, Y_t

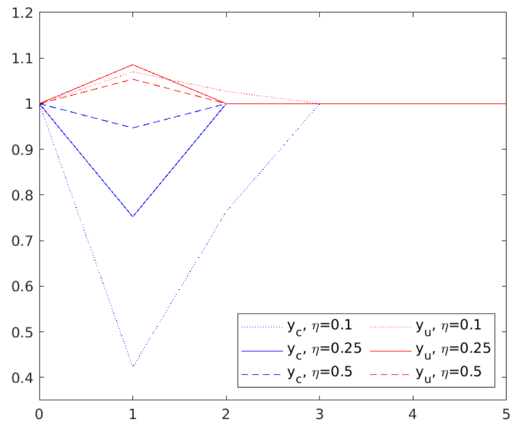


(b) Aggregate price, P_t

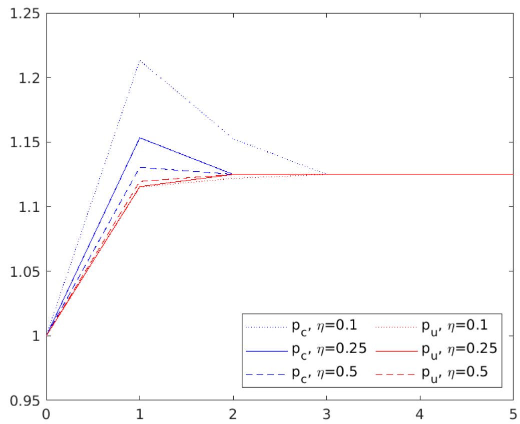
Figure 15: Paths of output and aggregate price at the zero lower bound for different values of η



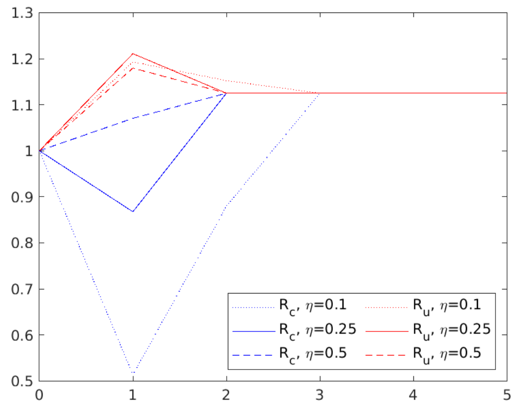
(a) Individual consumption, C_{ut}



(b) Individual output, y_{ut}



(c) Individual prices, p_{ut}



(d) Individual revenues, R_{ut}

Figure 16: Comparison across agents at the zero lower bound for different values of η

Shock size	Model	Counterfactual output	Counterfactual inequality	No inequality
$\tau = 0.5$	-17.2737	-17.2847	-17.2761	-17.2726
$\tau = 1.5$	-17.2788	-17.3170	-17.2945	-17.2758

The second column contains the values for the welfare function under the benchmark specification of the ZLB economy. The third column stands for the exercise of keeping the degree of consumption inequality in the ZLB, but imposing that the aggregate output be equal to the one in the baseline economy. The penultimate column stands for the opposite exercise: keeping the output level, but modifying consumption inequality. The last column stands for the counterfactual removal of inequality across agents.

Table 6: Counterfactual welfare analysis of the ZLB economy