On the Distributional Effects of Monetary Shocks and Financial Development

Tulio Bouzas

Tilburg

Abstract

This paper aims to study the transmission of distortionary monetary policy shocks in an environment with incomplete markets by means of a heterogeneous agents general equilibrium model. It shows that the distributional effects of monetary shocks matter in the aggregate, but only in the absence of perfect risk sharing. I analytically show that there is a unique stationary equilibrium such that the distribution of monetary holdings fully reflects the productivity distribution. Then, I study the transition dynamics after an unforeseeable monetary shock that redistributes monetary holdings across agents. I begin with an economy without a credit market and, then, allow borrowing to take place. I show that only financially imperfect economies return to a stationary equilibrium that reflects differences in fundamentals between agents. These economies also display endogenous price stickiness. In financially developed economies, monetary shocks generate hysteresis in equilibrium by making the corresponding idiosyncratic shocks permanent. The development of well-functioning credit markets allows for nearly perfect risk sharing in consumption, but the resulting welfare improvement is limited by the irreversibility of the idiosyncratic shocks. All in all, this paper articulates a nuanced view on financial development, while still highlighting its positive net welfare effect.

1 Introduction

There is growing evidence that monetary policy shocks produce distributional effects along several channels. These shocks can affect relative income and benefit debtors to the detriment of creditors, but, more immediately, monetary shocks also affect asset prices, redistributing wealth between agents who own financial assets and those who do not. This is a particularly important concern given that, even in an economy as financially developed as the U.S., only 21% of households held publicly traded stocks directly, and only 1.1% held bonds in 2022 according to the Survey of Consumer Finances (SCF). Stock ownership has remained relatively stable over the years, but bond ownership has decreased since 1989. Moreover, excluded retirement funds, savings, and foreign bonds, households that own stocks and bonds, directly or indirectly, are nearly twice as rich as the average American household. Although well-functioning credit markets could protect households from these idiosyncratic wealth shocks by allowing for risk-sharing, access to credit markets is limited. Also according to the SCF, roughly 20% of U.S. families did not hold a credit card and 10% had their credit request turned down at least once in the twelve months before the 2022 survey.

Beyond concerns related to welfare, the presence of these distributional effects should also affect the very transmission of such shocks. As pointed out by Friedman (1969), in the classic essay where he develops the helicopter drops allegory, the random re-shuffling of monetary holdings across agents is incompatible with an immediate return of the economy to the initial, pre-shock equilibrium, even in a fairly frictionless economy. Agents made richer by the money injections would likely want to smooth their consumption, relative prices would be distorted to reflect the new distribution of real balances, and incomes would adjust to make the economy converge to the long-run equilibrium. As a result, the income and wealth redistribution led by monetary policy shocks should produce a sluggish path toward equilibrium and transient redistribution of consumption and income across agents especially in the absence of financial tools that allow for risk sharing. I investigate this claim and study how financial development - in the form of access to credit - affects post-shock dynamics.

I develop a fairly tractable general equilibrium model, which is still compatible with an endogenous demand for money and pricing decisions. It features a cash-in-advance friction and a monopolistically competitive goods market. There is perfect foresight, and prices can be changed at the beginning of each period at no cost. Furthermore, for a richer analysis of how the redistribution of monetary holdings affects inequality, I allow agents to differ with respect to productivity, which, in turn, generates heterogeneity in monetary holdings, consumption, output, and prices in the stationary equilibrium.

I begin by showing that there is a unique fundamental stationary equilibrium, that is, a stationary equilibrium in which, given the market structure and consumer preferences, differences in wealth fully reflect differences in productivity. However, for a given monetary base, any redistribution of monetary holdings across agents is compatible with a stationary equilibrium with borrowing from a frictionless bonds market. This is because agents whose assets are too low given their productivity become indebted and find it optimal to roll their debt indefinitely. On the other hand, entrepreneurs who own more assets than they can sustain through their income decide to permanently maintain savings and receive interest payments. This way, one-period bonds work as perpetuities.

Then, I investigate the dynamics after a one-time unforeseeable monetary shock that happens to an economy at the fundamental stationary equilibrium. I show that the distributional effects of monetary shocks indeed induce a more sluggish and distorted return to the fundamental stationary equilibrium in the presence of limited access to credit markets. On the other hand, and contrary to Friedman's hypothesis, if there is full enforcement of debt repayments, the economy does not return to the initial allocations, and the induced differences in monetary holdings become permanent. This implies that the effect of these shocks over wealth inequality can be persistent.

The model is closely related to the financial segmentation channel, proposed by Williamson (2008). In this paper, Williamson assumes that households are either *connected* or *unconnected* to financial markets, with no possibility of moving between groups. By *connected*, he means that these households operate frequently in financial markets and, hence, are the first to be affected by monetary shocks. This heterogeneity is well-illustrated by the aforementioned low levels of financial asset ownership observed in the U.S. Although any shock that affects the relative value of agents' liquid asset holdings unevenly could be modeled in a similar reduced form fashion, I will use the same classification of agents as in Williamson (2008) throughout the paper.

Moreover, I allow the whole productivity distribution between connected and unconnected agents to differ but assume, in line with the data, that connected agents are, on average, more productive. This allows me to capture a dimension of the disparity between both types of agent that was unexplored in Williamson (2006). For a big enough positive (negative) monetary shock, the consumption of connected households increases (decreases) relative to that of the unconnected, but the revenues of the latter are higher (lower). This is the mechanism that, in our model, allows for the endogenous convergence to the long-run equilibrium in economies with imperfect credit markets. I also show that a monetary shock can decrease inequality if it benefits agents that are, on average, poorer. However, the shock also creates a wedge between connected and unconnected agents *with the same productivity*. This source of inequality is diminished by the existence of a bonds market, which unambiguously increases welfare by allowing for risk sharing.

I show that consumption and wealth inequality move in the same direction, but income inequality moves in the opposite one in response to the shock. This means that the model links direct - portfolio-related - distributional effects of monetary policy shocks to the more indirect - general equilibrium- and income-related - ones in a process that re-establishes the initial equilibrium in the absence of borrowing. This happens since the new money - which initially remains partially idle - is gradually appropriated by unconnected households. Moreover, I show that allocative efficiency fluctuates if the connectedness status is correlated with productivity, as the agents that benefit from the shock cut their labor due to the higher liquid wealth.

The main contribution of the paper is to study how shocks to agents' relative monetary wealth that are not pushed by changes in fundamentals affect the transmission of monetary shocks in a context where agents can smooth consumption. Hence, it concentrates on the transition between stationary equilibria that results from the relaxation of the helicopter drops assumption. It also studies how the development of well-functioning credit markets reduces the distortions introduced by the monetary shock, for it not only allows for risk sharing but also solves the idle cash balances problem. Finally, the model is also able to generate real effects and endogenous price stickiness. This happens through two different mechanisms. Firstly, in the absence of borrowing, a positive monetary base shock leads unconnected agents to set a lower price to re-establish their real monetary holdings. Secondly, connected agents cut their production and, thereby, face lower marginal costs.

These findings highlight differences between developed and emerging economies. It ultimately means that the effects of a monetary shock can be different in economies with well- and poorly-developed credit markets. In poorer economies, monetary policy can have much stronger distributional effects over consumption and higher output volatility. Furthermore, I show that more widespread access to financial markets makes the economy less distorted. Hence, this paper stresses the importance of developing a well-functioning credit market to ease possible distortions caused by unforeseeable monetary shocks.

Lastly, I conduct a series of sensitivity exercises, where I study the effects of changing certain assumptions and parameters over the model's outcomes. First, I consider the cases of a negative monetary shock and CRRA utility. I also study how different

Frisch elasticities affect the transmission of the shock. Then, I show that distortions generated by a positive monetary shock are smaller when the fraction of connected agents in the economy is larger. Moreover, I show that the paths for most aggregate and individual-level variables are determined only by how individual-level monetary shocks compare to the aggregate shock. Finally, I show that the fraction of connected agents in the economy affects the speed of convergence of the bondless economy to the new stationary equilibrium.

The paper is organized as follows. In section 2, I develop the baseline version of the model and study its stationary equilibria analytically. In section 3, first, I study the dynamics after an MIT monetary policy shock that distorts the distribution of monetary holdings in the absence of a bonds market. Then, I introduce bonds to the model and begin by analyzing a version in which the market is in equilibrium. Then, I assume the interest rate is exogenously set at zero and perform a short welfare analysis. Section 4 presents the robustness/sensitivity analysis. Lastly, section 5 concludes. All proofs are presented in Appendix A, while Appendix B contains further graphs, and Appendix C, outstanding tables.

1.1 Related Literature

This paper is related to two strands of the literature. Firstly, it is related to the literature that studies the distributional effects of monetary policy shocks. According to it, monetary policy may affect differently agents who: 1) have different income compositions; 2) have different portfolios; 3) differ on whether they are net savers or net buyers; 4) frequently operate in financial markets or not; and 5) differ with respect to preferences and skill levels (Hohberger et al.,2020; Coibion et al., 2017; Dolado et al., 2021). Distributional effects may also arise from the heterogeneity of price adjustment across goods (Cravino et al., 2020; Baqaee et al., 2022) and from risk sharing that monetary policy may facilitate or hinder (Berentsen et al., 2007; Rocheteau et al., 2018).

These different channels may point in opposite directions. How they balance out in the real world is an empirical question. Using survey data from the U.S., Coibion et al. (2017) show that inequality in income and consumption historically followed after contractionary monetary shocks. Furceri et al. (2018) back up this view for *unexpected* monetary shocks, but with the caveat that *expected* contractionary monetary policy *decreases* inequality. Conversely, Davtyan (2016) shows that contractionary monetary policy by the Fed decreases inequality, and Montecino and Epstein (2015), that QE induced inequality in the U.S.

Ampudia et al. (2018) make, in a broad literature review, a distinction between direct and indirect, general-equilibrium, channels of monetary policy shocks. In my framework, the direct channels are modeled in reduced form through the financial segmentation channel proposed by Williamson (2008), but it can be interpreted more broadly as arising from increases in liquid wealth that arise from portfolio revaluations and variations in dividend income driven by monetary policy. These channels have been found to have distributional effects in several empirical papers (Doepke and Schneider, 2006, Doepke et al., 2015, Montecino and Epstein, 2015, O'Farrell et al., 2016, Adam and Tzamourani, 2016, Casiraghi et al., 2018, Saiki and Frost, 2014, Ampudia et al., 2018, Auclert, 2019).

Moreover, most papers have studied the income channel by looking at variations in wage, capital/dividends and transfers income (Hohberger et al., 2020; Gornemann et al., 2016;Areosa and Areosa,Areosa and Areosa; Coibion et al., 2017); skill premium (Dolado et al., 2021); and fluctuations in employment status across different income brackets (Hohberger et al., 2020; Casiraghi et al.,2018; Dolado et al., 2021) that are not necessarily linked to any tendency towards convergence.

Two papers in this literature are closely related to mine: Williamson and Grossman and Weiss. In the case of the former, my model differs in a couple of ways. He assumes goods markets segmentation[1], and he assumes away the possibility of consumption smoothing. Risk sharing is also assumed away, since, although there is a credit market, only connected agents have access to it. On the other hand, I assume no good market segmentation, competition is not assumed to be perfect, and I allow for consumption smoothing and borrowing between connected and unconnected.

Grossman and Weiss model open market operations by assuming that agents withdraw money from the bank in alternate periods. This means that those who are at the bank right after the monetary shock takes place benefit by making a large money withdrawal at a lower interest rate, financed with the interest-bearing savings of the agents who are not at the bank. They find that prices and the interest rate display an oscillatory and dampening path toward the long-run equilibrium, and money flows between the agents who benefit from the shock to those who do not. Since I model the distributional effects of monetary policy in a reduced form way, I can endogenize output - which is assumed exogenous in their paper; - model a role for a credit market in channeling idle cash to cash-constrained agents, and model an endogenous response in prices that accelerates the process of convergence.

In light of the discussion above, I contribute to this strand of the literature in several ways. First, I link the direct channels through which monetary shocks generate distri-

[1] In his paper, not only financial markets are segmented, but, moreover, connected agents trade *mostly* among themselves in a competitive goods market. The same goes for unconneccted, who also have a partially separate market of their own.

butional effects to the indirect, general equilibrium, ones, by modeling an endogenous response in revenues that re-establishes the initial allocations as in Friedman (1969). Second, I study the conditions under which this long-run money neutrality does not hold. Third, I provide an arguably more general and, still, tractable setup where the financial segmentation channel can be studied, which allows for borrowing between agents who benefit from the monetary shock and those who are harmed by it to play a relevant role.

Finally, I also contribute to the literature on incomplete markets, and, especially to the role of debt in such economies. In the New-Monetarist tradition, pioneered by Kiyotaki and Wright and Lagos and Wright, imperfect credit markets make money needed for transactions. When coupled with idiosyncratic shocks, this leads to idle balances (Rocheteau et al.,2018), which could be channeled from agents with low marginal utility of consumption to those with a higher one in the presence of a banking sector that allows for borrowing between agents (Berentsen et al., 2007). In my framework, as in Berentsen et al. (2007), the payment of interests to creditors improves allocations, but, here, the liquidity provided to cash-constrained households is also welfare-improving.

Access to liquidity through a frictionless credit market goes a long way in completing the market as in Telmer (1993), practically undoing the heterogeneity in consumption. I contribute to this literature by developing a tractable model that allows for the analytical characterization of the dynamics after a distortionary monetary shock both in an economy with and without a well-functioning credit market. Moreover, I show that the presence of such a market can lead to the rise of hysteresis in post-shock monetary holdings, which might significantly dampen the welfare improvement brought about by financial development.

2 The Model

Consider an economy with a continuum of entrepreneurs with unit mass, who differ in their time-invariant productivity, z. The productivity follows a cumulative distribution function $\mathbb{F}(\cdot)$. Every entrepreneur produces an intermediate good through her own work and derives utility from the consumption of the final good. There is also a final good firm, which operates in a competitive market and produces a composite final good out of the intermediate goods produced by the entrepreneurs. Moreover, I assume that there is a market for riskless one-period pure-discount bonds.

I assume that entrepreneurs have identical preferences. Moreover, as in Williamson (2008), I define *connected agents* as those who frequently trade in financial markets, being, therefore, directly affected by monetary shocks. They correspond to a fraction $\eta \in$

$(0, 1)$ of the whole population. The productivity of connected agents is distributed according to the c.d.f. $F_c(\cdot)$. *Unconnected agents* are, naturally, affected indirectly by monetary shocks and correspond to a fraction $1 - \eta$ of the population. Their productivity is distributed according to the c.d.f. $F_u(\cdot)$. Naturally, $\mathbb{F}(z) = \eta F_c(z) + (1 - \eta) F_u(z)$. Finally, I denote $\eta_i = \eta$ for $i = c$ and $\eta_i = 1 - \eta$ for $i = u$. I assume, for simplicity, a common support for these distributions Furthermore, I assume that connectedness status is fixed for each agent.

The timing of the model goes as follows:

1. All bonds, $b_{it}(z)$, purchased in the previous period reach maturity;

2. The entrepreneur with productivity z and connectedness status $i \in \{c, u\}$ starts with $m_{it}^{-}(z)$ units of money, and connected agents may receive an unanticipated and unforeseeable transfer (tax) $\tau m_{ct}^{-}(z)$ from the government, financed through money creation (destruction). Thus, $m_{it}(z) = (1 + \mathbb{1}_{i=c}\tau)(m_{it}^{-}(z) + b_{it}(z))$, where $\mathbb{1}_{i=c} = 1$ when the agent is connected, and $\mathbb{1}_{i=c} = 0$ otherwise;

3. The entrepreneur sets a price $p_{it}(z)$ for the good she produces. Given these prices, the final goods firm buys on credit the output of each entrepreneur, $y_{it}(z)$, and produces thereby a composite good Y_t;

4. Each entrepreneur then decides, given P_t, how much of $m_{it}(z)$ to spend on consumption, $C_{it}(z)$, how much to save as idle cash balances, $s_{it}(z) \geq 0$, and how much to save in terms of bonds. Agents have access to a centralized bonds market where they can buy/sell bonds at a price q_t.

5. The final goods firm pays the entrepreneurs for the purchases made in the same period with the money obtained through sales.

6. The money obtained through sales and the money unspent in the period will sum up to $m_{i,t+1}^{-}(z)$.

As usual, if the bond is bought, $b_{it}(z) > 0$. If it is sold, $b_{it}(z) < 0$. Moreover, although monetary shocks are assumed to be *unanticipated*, they become immediately known by everyone whenever they take place, that is, before agents make any pricing and production decision for that period. Importantly, this timing implies that *entrepreneurs cannot benefit from current sales* because purchases are only paid for at the end of the period. Thus, there is a cash-in-advance (CIA) friction, which ensures that money is relevant for agents and non-neutral. Moreover, there is imperfect competition in the intermediate goods market, which is needed to allow for pricing decisions.

Furthermore, I assume that the monetary shock that each entrepreneur receives is proportional to their current monetary holdings. I make this assumption for two reasons. First, it is fairly tractable given the pre-existing heterogeneity in cash holdings. Second, it seems more plausible to assume that monetary shock affects agents proportionally. For example, if a fall in the interest rate increases the price of a connected agent's assets, this valuation shock should be proportional to their asset holdings. Importantly, for as long as $\eta < 1$, we do not have helicopter drops of money.

Since $q_t > 1$ is not possible, $s_{i,t}(z) > 0$ can only happen if: 1) $q_t = 1$ or 2) the bonds market is shut down. I will also assume, henceforth, that, if the nominal interest rate is equal to zero, that is, $q_t = 1$, all the savings will still take place through the bonds market. Hence, savings will never take the form of idle money if agents can buy and sell bonds. This can be rationalized as being the only choice that is robust to small upward trembles in the real interest rate, which would always produce $s_{it}(z) = 0$ for any arbitrary $z \in [\underline{z}, \overline{z}]$.

The problem of an entrepreneur with productivity z and connectedness status $i \in \{c, u\}$ is:

$$\max_{\{C_{it}(z), m^-_{i,t+1}(z), h_{it}(z), p_{it}(z), b_{i,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(C_{it}(z)) - \gamma \frac{h_{it}(z)^{1+\zeta}}{1+\zeta} \right] \tag{1}$$

$$\begin{aligned}
\text{subject to} \quad & P_t C_{it}(z) + q_t b_{i,t+1} \leq m_{it}(z) && (2)\\
& m^-_{i,t+1}(z) + P_t C_{it}(z) + q_t b_{i,t+1}(z) \leq \\
& \qquad m_{it}(z) + p_{it}(z) y_{it}(z) && (3)\\
& m_{i,t+1}(z) = m^-_{i,t+1}(z) + b_{i,t+1}(z) && (4)\\
& y_{it}(z) = z h_{it}(z) \leq D(p_{it}(z), P_t, Y_t) && (5)\\
& b_{i,t+1}(z) \geq -l_t(z, m_{it}(z)) && (6)\\
& C_{it}(z) \geq 0 && (7)
\end{aligned}$$

where $h_{it}(z)$ is labor; $D(p_{it}(z), P_t, Y_t)$ is the demand faced given the chosen price, $p_{it}(z)$; and $l_t(z, m_{it}(z)) \geq 0$ is the borrowing limit. Moreover, let $R_{it}(z) := p_{it}(z) y_{it}(z)$ be the entrepreneur's revenue. As usual, I assume that the utility function satisfies $u \in \mathcal{C}^2$, $u'(\cdot) > 0$, and $u''(\cdot) < 0$. I assume isoelastic labor disutility for the sake of tractability and that $\zeta \geq 0$. Also for simplicity, I assume linear technology, i.e. $y_{it}(z) = z h_{it}(z)$. Moreover, notice that transfers enter implicitly in the CIA constraint (2), since, if an agent receives the transfers at the beginning of time t, $m_{ct}(z) = (1+\tau) m^-_{ct}(z)$. Lastly, given that monetary shocks are unforeseeable, agents assume that $m^{-}_{i,t+1}(z) = m_{i,t+1}(z)$. The final good's firm faces the static problem:

$$\max_{\{y^D_{it}(z)\}_{z\in[\underline{z},\overline{z}],i\in\{c,u\}}} \quad Y_t = \left(\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} y^D_{it}(z)^{\frac{\epsilon-1}{\epsilon}} dF_i(z) \right)^{\frac{\epsilon}{\epsilon-1}} \tag{8}$$

$$\text{subject to} \quad P_t Y_t = \sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} p_{it}(z) y^D_{it}(z) dF_i(z) \tag{9}$$

$$y_t(z) \geq 0 \quad \forall z \in [\underline{z}, \overline{z}] \tag{10}$$

where $y^D_{it}(z)$ is the demand for the intermediate good produced by the entrepreneur with productivity z and connectedness status $i \in \{c, u\}$. Moreover, I assume that $\epsilon > 1$. Finally, I define $\mathbb{T}$ as the set of periods at which the economy finds itself in a particular equilibrium path, and $t_0 := \min \mathbb{T}$. Then, the equilibrium path of this economy is defined as follows:

Definition 1 (Equilibrium path)**.** *An equilibrium path is a series of intermediate good, final good and bond prices* $\{\{p_{it}(z)\}_{z\in[\underline{z},\overline{z}]}, P_t, q_t\}_{i\in\{c,u\},t\in\mathbb{T}}$*, individual consumption bundles* $\{C_{it}(z)\}_{z\in[\underline{z},\overline{z}],i\in\{c,u\},t\in\mathbb{T}}$*, intermediate and final good outputs* $\{\{y_{it}(z)\}_{z\in[\underline{z},\overline{z}]}, Y_t\}_{i\in\{c,u\},t\in\mathbb{T}}$*, individual labor* $\{h_{it}(z)\}_{z\in[\underline{z},\overline{z}],i\in\{c,u\},t\in\mathbb{T}}$*, individual monetary holdings and monetary base* $\{\{m_{it}(z)\}_{z\in[\underline{z},\overline{z}]}, M_t\}_{i\in\{c,u\},t\in\mathbb{T}}$*, and net bonds holdings* $\{b_{i,t+1}(z)\}_{z\in[\underline{z},\overline{z}],i\in\{c,u\},t\in\mathbb{T}}$ *such that, for every* $t \in \mathbb{T}$*:*

1. *Given* $\{q_t, P_t\}_{t\in\mathbb{T}}$ *and the initial* $m_{t_0}(z)$*,* $\{C_{it}(z), p_{it}(z), y_{it}(z), h_{it}(z), b_{i,t+1}(z)\}_{t\in\mathbb{T}}$ *solve the problem* (1) *of the entrepreneur with* $z \in [\underline{z}, \overline{z}]$ *and* $i \in \{c, u\}$*;*

2. *Given prices for the intermediate and final goods,* $\{\{p_{it}(z)\}_{z\in[\underline{z},\overline{z}]}, P_t\}_{i\in\{c,u\},t\in\mathbb{T}}$*, intermediate goods demand and final good output* $\{\{y^D_{it}(z)\}_{z\in[\underline{z},\overline{z}]}, Y_t\}_{i\in\{c,u\},t\in\mathbb{T}}$ *solve the problem* (8) *of the final good firm;*

3. *The intermediate goods markets clear, i.e.* $y^D_{it}(z) = y_{it}(z)$ *for* $z \in [\underline{z}, \overline{z}]$*,* $i \in \{c, u\}$*;*

4. *The final good's market clears, i.e.* $\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} C_{it}(z) dF_i(z) = Y_t$*;*

5. *The bonds market clears, i.e.* $\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} b_{i,t+1}(z) dF_i(z) = 0$*;*

6. *The monetary base is owned by entrepreneurs, i.e.* $\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} m_{it}(z) dF_i(z) = M_t$*.*

2.1 Solution

The solution to the problem of the final good firm, (8), takes the standard form:

$$D(p_{it}(z), P_t, Y_t) = \left(\frac{p_{it}(z)}{P_t} \right)^{-\epsilon} Y_t \qquad \forall z \in [\underline{z}, \overline{z}] \tag{11}$$

and the final good price is given by:

$$P_t = \left(\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} p_{it}(z)^{1-\epsilon} dF_i(z) \right)^{\frac{1}{1-\epsilon}}, \tag{12}$$

which I will, henceforth, refer to as aggregate price. The problem of the entrepreneur with productivity $z \in [\underline{z}, \overline{z}]$ and connectedness status $i \in \{c, u\}$, (1), yields the consumption schedule:

$$C_{it}(z) \begin{cases} = (u')^{-1} \left(\beta \frac{P_t}{P_{t+1}} u'(C_{i,t+1}(z)) \right) & \text{if } s_{i,t}(z) > 0 \\ = \frac{m_{it}(z)}{P_t} & \text{if } s_{i,t} = 0 \text{ and } b_{i,t+1}(z) = 0 \\ \leq (u')^{-1} \left(\frac{\beta}{q_t} \frac{P_t}{P_{t+1}} u'(C_{i,t+1}(z)) \right) & \text{otherwise} \end{cases}$$

where $(u')^{-1}(\cdot)$ is the inverse of the marginal utility function. This function is well defined because $u'(\cdot)$ is injective and continuous. The cases above correspond, respectively, to 1) partial depletion (that is, the monetary holdings are not fully spent) with idle cash, 2) full depletion (the consumer spends all her money currently), and 3) partial depletion with non-zero bond holdings. Notice that the first case can only happen if the bonds market is shut down, which corresponds to a fully imperfect financial system. Moreover, the strict inequality in the third case is satisfied with equality if, and only if, (6) binds. Furthermore, the price chosen by the entrepreneur is given by:

$$p_{it}(z) = \underbrace{\left(\frac{\epsilon}{\epsilon - 1} \right)}_{\text{(i)}} \underbrace{\frac{\gamma h_{it}(z)^{\zeta}}{z}}_{\text{(ii)}} \underbrace{\frac{P_{t+1}}{\beta u'(C_{i,t+1}(z))}}_{\text{(iii)}} \tag{13}$$

This equation implies that there is a markup, (i), over marginal costs, (ii), and a forward-looking component, (iii), to pricing decisions. To understand the intuition behind this equation, notice that revenues affect how much money the entrepreneur carries to the next period. The value of this money is the value of relaxing the future budget constraint, which is directly related to the marginal utility and the aggregate price in the next period. Whenever consumption will be large in the future, the value of relaxing the next period's budget constraint by carrying more money to the future is lower. Hence, the agent will choose a relatively high price to balance the intertemporal trade-off towards lower current labor disutility. Furthermore, the value of relaxing the budget constraint in the future is decreasing on future aggregate prices. Now, I define the revenue of an arbitrary entrepreneur relative to the average revenue as:

$$\theta_{it(z)} := \frac{p_{it}(z) D(p_{it}(z), P_t, Y_t)}{P_t Y_t} = \left(\frac{p_{it}(z)}{P_t} \right)^{1-\epsilon}, \tag{14}$$

which can be interpreted as the equivalent of a market share in the continuous case, since $R_{it}(z) = \theta_{it}(z) M_t$. It takes values $\theta_{it}(z) \in (0, 1)$ if the revenue of the entrepreneur with productivity $z \in [\underline{z}, \overline{z}]$ and connectedness status $i \in \{c, u\}$ is below the average revenue, $\theta_{it}(z) = 1$ if it is equal to average, and $\theta_{it}(z) > 1$ if it is larger. I study now the stationary equilibria of this economy.

2.2 The Stationary Equilibrium

I now define a stable price stationary equilibrium. The term "stable price" aims to restrict attention to stationary equilibria where the monetary base is constant. In these monetary equilibria, $m_{it}(z) = m_{i,t+1}(z)$ and $p_{it}(z) = p_{i,t+1}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$, $M_t = M_{t+1}$ and $P_t = P_{t+1}$. Let $\mathbb{T}^S$ be the set of periods in which the economy is at this kind of equilibrium. I define it as follows:

Definition 2 (Stable price stationary equilibrium)**.** *A stable price stationary equilibrium for this economy is a series of prices* $\{\{p_{it}(z)\}_{z \in [\underline{z}, \overline{z}]}, P_t, q_t\}_{i \in \{c,u\}, t \in \mathbb{T}^S}$*, consumption, labor and output allocations* $\{\{C_{it}(z), h_{it}(z), y_{it}(z)\}_{z \in [\underline{z}, \overline{z}]}\}, Y_t\}_{i \in \{c,u\}, t \in \mathbb{T}^S}$*, and bond holdings* $\{b_{i,t+1}(z)\}_{z \in [\underline{z}, \overline{z}], i \in \{c,u\}, t \in \mathbb{T}^S}$ *which, given* $\tau = 0$ *for and* $t \in \mathbb{T}^S$*, solve* (1) *and* (8) *and make* $m_{it}(z) = m_{i,t+1}(z)$ *for every* $z \in [\underline{z}, \overline{z}]$ *and* $i \in \{c, u\}$ *and, thus,* $M_t = M_{t+1}$*.*

Notice that the above definition implies that real allocations and relative prices are constant in all periods in stable price stationary equilibria. Moreover, since all the stationary equilibria studied throughout the paper are stable price, I will, henceforth, call them simply "stationary equilibria". As will be shown in Proposition 1, there are infinite such equilibria that are compatible with a given monetary base, M_t. I will, therefore, refine this concept further by defining *fundamental stationary equilibria* as:

Definition 3 (Fundamental stationary equilibrium)**.** *A fundamental stationary equilibrium for this economy is a stable price stationary equilibrium where* $m_{it}(z) = R_{it}(z)$ *for every* $z \in [\underline{z}, \overline{z}]$*,* $i \in \{c, u\}$ *and* $t \in \mathbb{T}^S$*.*

In other words, a fundamental stationary equilibrium is a stationary equilibrium where differences in monetary holdings across agents reflect differences in fundamentals between them. Given our assumptions, differences in fundamentals boil down to differences in productivity. Intuitively, agents are just as rich as their capacity to make money allows them to be. As a result, for every $z \in [\underline{z}, \overline{z}]$ and $t \in \mathbb{T}^S$, $m_{ct}(z) = m_{ut}(z)$, that is, connected and unconnected agents with the same productivity have the same monetary holdings. The proposition below shows that such a fundamental stationary equilibrium exists and is unique.

Proposition 1. *There is a unique fundamental stationary equilibrium, which requires that* $b_{i,t+1}(z) = 0$ *for all* $z \in [\underline{z}, \bar{z}]$, $i \in \{c, u\}$ *and* $t \in \mathbb{T}^S$. *Moreover, given the fundamental stationary equilibrium distribution of monetary holdings, it is the only possible equilibrium. Lastly, for any function* $m_{ct}(\cdot) > 0$ *and* $m_{lt}(\cdot) > 0$ *defined over the domain* $[\underline{z}, \bar{z}]$ *and satisfying* $\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\bar{z}} m_{it}(z) dF_i(z) = M_t$, *if* $l_t(z, m_{it}(z)) = R_{it}(z)$ *for all* $z \in [\underline{z}, \bar{z}]$ *and* $i \in \{c, u\}$, *there is a unique (non-fundamental) stationary equilibrium with borrowing compatible with it. This equilibrium requires* $q_t = \beta$ *for every* $t \in \mathbb{T}^S$.

Apart from the uniqueness of the fundamental stationary equilibrium, Proposition 1 implies that there are infinite non-fundamental stationary equilibria compatible with any given monetary base for as long as the borrowing limit requires only that the agent can repay her debt at the beginning of the next period. Any distribution of money is made permanent in such a stationary equilibrium through borrowing. Conditional on a given distribution, however, the equilibrium is unique. This means that, in the absence of any financial frictions, no mechanism ensures that the economy converges to a stationary equilibrium where differences in monetary holdings, allocations, and prices fully reflect differences in fundamentals.

The intuition is that indebted agents roll their debt and pay the interest with their revenue from sales indefinitely. Hence, these one-time bonds end up working just like perpetuities. To put it simply, under the natural borrowing limit $l_t(z, m_{it}(z)) = R_{it}(z)$, which ensures the capacity debt repayment, consumption should either decrease, increase or stay constant for *all* agents according to their Euler equation. Since the monetary base after the shock is constant, this can only be satisfied for constant consumption. Moreover, the proposition implies that the amount of cash borrowed is always strictly below the amount of cash that the indebted entrepreneur will earn in sales in the current period. In the next section, I investigate the case of an MIT monetary shock to an economy that starts at the fundamental stationary equilibrium.

3 Transition Dynamics

In what follows, I consider an economy that starts at the fundamental stationary equilibrium at $t = 0$ and receives an MIT monetary shock at $t = 1$ in the fashion described at the beginning of section 2. Whenever I refer to the stationary equilibrium, I use the notation X_0 for $X \in \{Y, P, M\}$ and $x_0(z)$ for $x \in \{m, b, C, p, y, h, R, \theta\}$, which does not depend on connectedness status by the definition of fundamental stationary equilibrium. I start by studying a baseline economy with perfect financial frictions and, only after, allow the bonds market to become operational to study how the transmission of the shock is affected by financial development.

3.1 Baseline Economy

In the baseline economy, I assume that no borrowing can take place, that is, $l_t(z, m_{it}(z)) = 0$ for $z \in [\underline{z}, \bar{z}]$ and $i \in \{c, u\}$. This amounts to shutting off the bond market completely. I assume that the central bank introduces (withdraws) $\eta\tau M_{c0} > 0\,(< 0)$ units of money in the economy at $t = 1$, where $M_{c0} := \int_{\underline{z}}^{\bar{z}} m_0(z) dF_c(z)$ is the average monetary holdings of connected agents in the stationary equilibrium. Moreover, let $\tau^A := \eta\tau M_{co}/M_0$ be the proportional aggregate shock. I also define M_t^C as the money in circulation at time t, that is, the amount of money that is demanded in the economy for transaction motive, and $M_t = (1 + \tau^A)M_0$ is the monetary base. I now consider an economy that receives a monetary shock operated through helicopter drops.

3.1.1 Helicopter Drops Of Money

When helicopter drops take place, each agent gets a proportional $\tau_H = \tau^A$ over their monetary holdings. It is easy to see that the only possible equilibrium is one in which all agents fully deplete their money. Essentially, relative monetary holdings are not distorted by the shock since every agent gets the same proportional shock. Thus, the economy goes immediately to the new fundamental stationary equilibrium, in which the monetary base is $M_t = (1 + \tau^A)M_0$ for $t \in \{1, 2, ...\}$. The corollary below formalizes that. The proof can be found in Appendix A.

Corollary 1.1. *After helicopter drops of money, the economy goes immediately to the new fundamental stationary equilibrium.*

Since all agents fully deplete their resources, prices are given by:

$$P_t^H = (1 + \tau^A)P_0 \qquad \text{and} \qquad p_t^H(z) = (1 + \tau^A)p_0(z) \tag{15}$$

for $z \in [\underline{z}, \bar{z}]$, $i \in \{c, u\}$ and $t = \{1, 2, ...\}$, where the H superscript refers to the "helicopter drops equilibrium", and consumption is identical to the consumption level in the initial stationary equilibrium level, that is, $C_{it}^H(z) = C_0(z)$ for $t = \{1, 2, ...\}$. Thus, a monetary shock implemented through helicopter drops is neutral, since prices immediately rise uniformly and enough to put the economy at the new fundamental stationary equilibrium already at $t = 1$.

3.1.2 Uneven Access To The New Money

Now, I assume that the connected agents are the first to have their monetary holdings affected by the monetary shock. With some abuse of notation, I denote the agents who are made richer, in relative terms, by the monetary shock as *high-cash* and the ones that are made relatively poorer as *low-cash*. I denote the former with subscript h and the latter with subscript l. Naturally, if $\tau > 0$, then $h = c$, $m_{h1}(z) = m_{c1}(z) = (1+\tau)m_0(z)$ and $m_{l1}(z) = m_{u1}(z) = m_0(z)$ for any arbitrary $z \in [\underline{z}, \overline{z}]$; whereas $h = u$, $m_{l1}(z) = m_{c1}(z) = (1+\tau)m_0(z)$ and $m_{h1}(z) = m_{u1}(z) = m_0(z)$ when $\tau < 0$. Moreover, I define:

$$\overline{U}_t^{GAP} = \left(\frac{\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\epsilon-1} \frac{u'(C_{it}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}} dF_i(z))}{\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\epsilon-1} \frac{u'(C_{i0}(z))^{\epsilon-1}}{h_{i0}(z)^{\zeta(\epsilon-1)}} dF_i(z)} \right)^{\frac{1}{\epsilon-1}}, \tag{16}$$

which captures the deviation, at a given moment, of a kind of weighted mean of the ratio of marginal utility of consumption over marginal labor disutility relative to the stationary equilibrium value, where the weights are a function of productivity. This expression is well-defined since the utility function is assumed to be equal across agents, allowing for comparison across them. It will be useful to also define an analogous individual-level measure as:

$$U_{it}^{GAP}(z) = \left(\frac{z^{\epsilon-1} \frac{u'(C_{it}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}}}{z^{\epsilon-1} \frac{u'(C_{i0}(z))^{\epsilon-1}}{h_{i0}(z)^{\zeta(\epsilon-1)}}} \right)^{\frac{1}{\epsilon-1}}. \tag{17}$$

In the following proposition, I characterize the dynamics after the shock.

Proposition 2. *After a monetary shock. there is a certain time $T < \infty$ in which the economy converges to the new fundamental stationary equilibrium. For $t = \{T, T+1, ...\}$, $p_{it}(z) = p^H(z, (1+\tau^A)M_0)$ for $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$, $P_t = P^H((1+\tau^A)M_0)$, and $Y_t = Y_0$. Moreover, for $t = \{T+1, T+2, ...\}$, $C_{it}(z) = C_{i0}(z)$ and $m_{it}(z) = (1+\tau^A)m_0$ for all $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$. Along the transition, that is, for $t = \{1, ..., T-1\}$, we have $C_{lt}(z) \leq C_{ht}(z)$, $\theta_{lt}(z) \geq \theta_{ht}(z)$, $p_{lt}(z) \leq p_{ht}(z)$, $y_{lt}(z) \geq y_{ht}(z)$, $R_{lt}(z) \geq R_{ht}(z)$, $m_{l,t+1}(z) \leq m_{h,t+1}(z)$, and $m_{h,t+1}(z) - m_{ht}(z) \leq m_{l,t+1}(z) - m_{lt}(z)$ for all $z \in [\underline{z}, \overline{z}]$, with strict inequality whenever the high-cash agent does not fully deplete. Moreover, low-cash agents are always more likely to fully deplete than their high-cash counterparts. Finally:*

$$1 + \pi_{t+1} = \overline{U}_{t+1}^{GAP}, \tag{18}$$

and

$$\frac{\theta_{it}(z)}{\theta_{i0}(z)} = \left(\frac{1}{1+\pi_{t+1}}\right)^{\epsilon-1} \frac{\frac{u'(C_{i,t+1}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}}}{\frac{u'(C_{i0}(z))^{\epsilon-1}}{h_{i0}(z)^{\zeta(\epsilon-1)}}} = \left(\frac{U_{i,t+1}^{GAP}(z)}{\overline{U}_{t+1}^{GAP}}\right)^{\epsilon-1}. \tag{19}$$

Proposition 2 implies that the economy eventually goes to the new fundamental stationary equilibrium, where the allocation is identical to the initial one, but prices are different. To understand this, notice that (19) means that the "market share" of a given entrepreneur, $\theta_{it}(z)$, will be higher (lower) whenever her ratio of marginal utility of consumption over labor disutility in the next period will be higher (lower) than the weighted mean, $\overline{U}_{t+1}^{GAP}$. This means that agents who are *artificially* poorer - meaning that their real balances are below its fundamental level - will *tend* to have a higher market share than their fundamentals would suggest[2].

This result follows, because lower future consumption, *ceteris paribus*, increases the value of having money, encouraging poorer agents to choose lower prices and work relatively harder than their high-cash counterparts. This latter effect can increase the marginal disutility of labor, which partially counteracts the effect of lower consumption. As a result, price and output differences between agents with the same productivity arise due to a wealth effect, which gradually reduces the differences in monetary holdings across high- and low-cash agents along the transition path. This is, in essence, the mechanism described in Friedman (1969), and it gradually reduces inequality. This is captured by the fact that the monetary holdings of low-cash agents grow faster than that of their high-cash counterparts, *i.e.* $m_{h,t+1}(z) - m_{ht}(z) \leq m_{l,t+1}(z) - m_{lt}(z)$.

Finally, (18) suggests that inflation (or deflation) still occurs for as long as the mean weighted marginal utility of consumption is distorted away from its fundamental stationary equilibrium level, and it is stronger whenever the distortion is large. To understand this expression, we need to take the forward-looking behavior of prices into account. Consider, for instance, the case where $\overline{U}_{t+1}^{GAP} > 1$. In this case, the marginal future value of money is relatively high on average, meaning that prices would tend to be *artificially* low - that is, lower than their final stationary equilibrium level, - since many agents need to accumulate cash to replenish their real balances. As the economy becomes less distorted over time, future prices will tend to be closer to their final level than current prices, which, in the case of our example, means inflation.

For the sake of tractability[3], I assume, henceforth, a logarithmic utility function, *i.e.* $u(\cdot) = \log(\cdot)$. Before I characterize the fundamental stationary equilibrium under the new specification, I define, respectively, the following aggregate and connectedness

[2]What is meant with the word "tend" here is that some artificially poorer agents can still have a lower market share than in the stationary equilibrium if the rise in her future marginal utility is still lower than the rise in the weighted mean, $\overline{U}_{t+1}^{GAP}$, especially if marginal labor goes up enough. As will be shown later, log-utility (and homothetic utility functions in general) rules that possibility out.

[3]The tractability arises mainly due to the elimination of difficulties related to potential non-homothety. However, the property that income and substitution effects cancel out also helps in the analytical proofs but does not seem essential for obtaining the results, as is shown in section 4.

status-specific measures of productivity:

$$\mathcal{Z} := \left(\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\frac{\epsilon-1}{\epsilon}} dF_i(z) \right)^{\frac{\epsilon}{\epsilon-1}} \qquad \text{and} \qquad \mathcal{Z}_i := \left(\int_{\underline{z}_i}^{\overline{z}_i} z^{\frac{\epsilon-1}{\epsilon}} dF_i(z) \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{20}$$

where $i \in \{c, u\}$. The characterization of the log-specific fundamental stationary equilibrium follows as a corollary to Proposition 1.

Corollary 1.2. *In the case of log-utility, for every $t \in \mathbb{T}^S$, aggregate output and final good prices are given by:*

$$Y_t = \mathcal{Z} \left(\frac{\epsilon-1}{\epsilon} \right) \frac{\beta}{\gamma} \qquad\qquad P_t = \left(\frac{\epsilon}{\epsilon-1} \right) \frac{\gamma}{\beta} \frac{1}{\mathcal{Z}} M_t \tag{21}$$

and individual revenue relative to the average, monetary holdings, consumption, and prices are given by:

$$\theta_{it}(z) = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}^{\frac{\epsilon-1}{\epsilon}}} \qquad\qquad m_{it}(z) = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}^{\frac{\epsilon-1}{\epsilon}}} M_t \tag{22}$$

$$C_{it}(z) = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}^{\frac{\epsilon-1}{\epsilon}}} Y_t \qquad\qquad p_{it}(z) = \left(\frac{z}{\mathcal{Z}} \right)^{-\frac{1}{\epsilon}} P_t \tag{23}$$

for an arbitrary $z \in [\underline{z}, \overline{z}]$

Corollary 1.2 shows that changes in the money supply are fully absorbed into prices and, hence, aggregate output is constant across these equilibria[4]. Moreover, notice that these $p_{it}(z)$ and P_t at the stationary equilibrium are conditional on a certain money supply, which in this case equals M_0. I will, thus, whenever convenient, denote this dependence of prices on the money supply explicitly, by writing $p_i(z, M_t)$ and $P(M_t)$. Furthermore, notice that, for an arbitrary $z \in [\underline{z}, \overline{z}]$, $\theta_t(z), m_{it}(z), C_{it}(z)$ and $p_{it}(z)$ are rescaled versions of their average counterparts, where the rescaling factor depends only on z. This illustrates how these stationary equilibria reflect fundamentals. More explicitly, more productive entrepreneurs tend to have higher revenues, monetary holdings and consumption, and lower prices than the average.

3.1.3 Immediate convergence to the stationary equilibrium

In this subsection, I study under what circumstances high-cash agents fully deplete their money already at $t = 1$. Notice that, in this situation, since $s_{it}(z) = 0$ for all

[4]This feature is common to all fundamental stationary equilibria, being, thereby, not specific to the logarithmic functional specification of $u(\cdot)$.

$z \in [\underline{z}, \bar{z}]$ and $i \in \{c, u\}$, high-cash agents will have at $t = 2$ only the amount of money that they manage to obtain through selling their products in the market at $t = 1$, as is the case for their low-cash counterparts. By (13), the price choice does not differ between equally productive high- and low-cash agents, whereby they appropriate, each, $R_1(z) = (1 + \tau^A)m_0(z)$. Therefore, individual and aggregate prices are identical to the ones in the helicopter drop case. Output also satisfies $Y_t = Y_0$.

Notice that, by the buyer's first order condition, full depletion happens when:

$$\frac{1}{\beta}\left(1 + \tau^A\right) m_0(z) = \frac{1}{\beta} P_2 C_{h2} \geq P_1 C_{h1} = m_{h1} \tag{24}$$

The condition (24) implies that high-cash agents fully deplete their money if:

$$|\tau| = \tau \leq \frac{1-\beta}{\beta - \eta \frac{M_{c0}}{M_0}} \qquad \text{if } \tau > 0 \tag{25}$$

$$|\tau| = -\tau \leq \frac{1-\beta}{\eta \frac{M_{c0}}{M_0}} \qquad \text{if } \tau > 0 \tag{26}$$

This means that the high-cash agents will fully deplete their money holdings if, and only if, the monetary shock is "low enough". Notice that (25) is only defined for $\eta < \beta \frac{M_0}{M_{co}}$. To understand why, notice that, if $\eta \in \left[\beta \frac{M_0}{M_{c0}}, 1\right]$, then:

$$\frac{1}{\beta}\left(1 + \tau^A\right) m_0(z) > (1 + \tau) m_0$$

which means that the full depletion condition is satisfied for any value of τ and for all $z \in [\underline{z}, \bar{z}]$. Intuitively, this means that the fall in monetary holdings from one period to the next when the connected agent fully depletes her cash holdings is too small to encourage her to save a positive amount. Thus, henceforth, I shall assume that $\eta \in \left(0, \beta \frac{M_0}{M_{co}}\right)$, which is the most interesting case.

When (24) is satisfied, the economy is at the new equilibrium from period $t = 2$ onwards. However, there are important distributional effects at the period $t = 1$. High-cash and low-cash agents' consumption is given, respectively, by:

$$C_{i1}(z) = C_0(z) \left(\frac{m_{i1}(z)}{(1 + \tau^A) m_0(z)}\right) \quad for i \in \{c, u\}.$$

with $C_{h1}(z) > C_0(z)$ and $C_{l1}(z) < C_0(z)$. I now analyze the situation in which (24) is not satisfied.

3.1.4 Gradual convergence to the stationary equilibrium

When the shock is large enough - that is, (24) is not satisfied, - high-cash agents smooth their consumption. Then, prices and output do not go to their equilibrium values immediately anymore. Notice that, for as long as high-cash agents do not fully deplete their resources, we must have $M_t^C < (1+\tau^A)M_0$. Evidently, for any $t \in \{1, 2, ...\}$, $M_t^C > M_0$ for $\tau > 0$ and $M_t^C < M_0$ for $\tau < 0$. In the following proposition, I fully characterize the transition dynamics after the shock. Most of these findings will be shown to generalize to a setup with CRRA utility function in section 4.

Proposition 3. *Let $T^{MAX} < \infty$ be defined as:*

$$T^{MAX} := \arg\max_{T \in \mathbb{N}} \left[\beta^{T-1} > \frac{1+\tau^A}{1+\mathbb{1}_{\tau>0}\tau} \right] \tag{27}$$

Under log-utility, the moment where prices achieve their new stationary equilibrium level satisfies $T \leq T^{MAX}$. Moreover, if we define:

$$\overline{X_{i1}} := X_{i1}(\mathcal{Z}_i) \qquad \text{for } X \in \{C, \theta, m\} \text{ and } i \in \{h, l\} \tag{28}$$

as the average level of consumption, relative revenues, and monetary holdings among either high- or low-cash agents, we have:

$$X_{i1}(z) = \overline{X_{i1}} \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_i^{\frac{\epsilon-1}{\epsilon}}} \qquad \text{for } X \in \{C, \theta, m\} \text{ and } i \in \{h, l\} \tag{29}$$

for all $t = \{0, 1, ...\}$ due to the homothety of log-utility. Moreover, the following results hold for $t = \{1, ..., T-1\}$ and for any arbitrary $z \in [\underline{z}, \overline{z}]$:

$$1) \quad \frac{m_{ht}(z)}{P_t} > C_{ht}(z) > C_0(z) > C_{lt}(z) = \frac{m_{lt}(z)}{P_t} \tag{30}$$

$$2) \quad C_{h,t+1}(z) < C_{ht}(z) \qquad \text{and} \qquad m_{h,t+1}(z) < m_{ht}(z) \tag{31}$$

$$3) \quad C_{l,t+1}(z) > C_{lt}(z) \qquad \text{and} \qquad m_{l,t+1}(z) > m_{lt}(z) \tag{32}$$

$$4) \quad p_{ht}(z) > p^H(z, M_t) > p_{lt}(z) > p^H(z, M_t^C) \tag{33}$$

$$5) \quad P_t > P^H(M_t^C) \tag{34}$$

$$6) \quad \theta_0(z) > \theta_{h,t+1}(z) > \theta_{ht}(z) \tag{35}$$

$$7) \quad \theta_0(z) < \theta_{l,t+1}(z) < \theta_{lt}(z) \qquad \text{and} \qquad p_{l,t+1}(z) > p_{lt}(z) \tag{36}$$

$$8) \quad h_{l,t+1}(z) = h_{lt}(z) = h_{lt}(z') = h_{l,t+1}(z') \qquad \text{foreveryz}, z' \in [\underline{z}, \overline{z}] \tag{37}$$

$$9) \quad Y_0 \geq Y_{t+1} > Y_t \qquad \text{with strict inequality for } t = \{1, ..., T-2\}. \tag{38}$$

Proposition 3 shows that the number of periods that are needed for the economy to reach the new equilibrium must satisfy condition (27). To build intuition on this result, notice that high-cash agents' monetary holdings fall between $t = 1$, when any arbitrary high-cash agent owns $m_{h1}(z) = \theta_0(z)(1 + \mathbb{1}_{\tau>0}\tau)M_0$, and $t = T + 1$, when she owns $m_{h,T+1}(z) = \theta_0(z)(1+\tau^A)M_0$. For a fixed individual level proportional shock τ, this fall depends crucially on how big the aggregate proportional shock, τ^A, is, as it affects the demand that high-cash agents expect to encounter in the market. If the aggregate shock is relatively small, high-cash agents' revenues are lower, which means that they need to keep savings for longer to smooth consumption.

Unlike the case of immediate convergence to the stationary equilibrium, here, when high-cash agents smooth consumption, *money is not neutral in the aggregate*. The proposition above shows that GDP *falls* under log-utility after the shock, for contractionary or expansionist shocks alike. The cause for this fall is the fact that the price level remains above the helicopter drops price level compatible with the amount of money in circulation, that is, $P_t > P^H(M_t^C)$ throughout the transition period. This means that the aggregate price is higher than the one that ensures money neutrality.

For an expansionist shock, that is, for $\tau > 0$, the price chosen by unconnected agents grows along the transition path because the value of holding money in the next period is given by $\beta/m_{l,t+1}(z)$ for them, that is, it is proportional to the inverse of their future monetary holdings. As they get richer, the value of money decreases, and they allow their prices to get closer to their final, higher, level. As their "market share" falls over time, the rise in their monetary wealth is due to a higher amount of money in circulation, M_t^C. Finally, notice that even the prices chosen by unconnected agents are such that $p_{ut}(z) > p^H(z, M_t^C)$, meaning that their prices are also *too high* to allow for output to be as large as Y_0.

On the other hand, for a contractionary shock, that is, for $\tau < 0$, the price of the goods chosen by connected agents *undershoots*, since connected agents are made relatively poorer, and they need to decrease their price on impact more than the unconnected to remain competitive. They gradually increase their prices as their monetary holdings grow. They also make a higher production effort than the unconnected, that is $y_{ct}(z) > y_{ut}(z)$ for $t \in \{1, ..., T-1\}$. To study the quantitative implications of the model, I now conduct a simulation of the model.

3.1.5 Simulation

For the simulation, I assume a uniform productivity distribution for simplicity. I also do away with the common support assumption to facilitate calibration[5]. In particular, I fix the lower bound for the productivity of both types of agent at $\underline{z}_c = \underline{z}_c = 0.2$ and

[5]This assumption is useful - though not essential - for the proofs. However, due to the homotheticity of the utility function assumed here, the relaxation of this assumption is inconsequential. Alternatively, we can assume that $F_u(z) = 1$ for $z \in [\overline{z}_u, \overline{z}_c]$ without having to relax it.

calibrate the upper bounds as will be described in more detail below. I normalize the initial monetary base and aggregate prices to $M_0 = 1$ and $P_0 = 1$, which means that the aggregate output is also normalized to $Y_0 = 1$. The parameter values used are summarized in Table 1.

Parameter	Definition	Value
ϵ	Elasticity of substitution	11
β	Rate of time discount	0.99
γ	Labor disutility	8.1
η	Fraction of connected agents	0.27
M_0	Initial money supply	1
τ	Individual monetary shock	0.2
$\underline{z}_c$	Minimum productivity among connected	0.2
$\underline{z}_u$	Minimum productivity among unconnected	0.2
$\overline{z}_c$	Maximum productivity among connected	12.4467
$\overline{z}_u$	Maximum productivity among unconnected	3.6023

Table 1: Parameter values for the simulation

As usual, I set the elasticity of substitution to $\epsilon = 11$ to get a 10% markup. Moreover, each period is assumed to be a quarter, and, thus, I set the rate of time discount to $\beta = 0.99$ to get a 1% quarterly real interest rate in equilibrium. I also set the shock size to $\tau = 0.2$, which amounts to connected agents becoming 20% richer than the unconnected with the same productivity. In our calibration, this leads to, roughly, a $10,5\%$ aggregate shock. Although this is fairly large, 1) the shock is not persistent, and 2) as can be seen in Table 4 (see Appendix C), since 1999, shocks to the monetary base of at least 10% occurred in a total of nine quarters, eight of which were positive shocks.

I use data from the Survey of Consumer Finances (SCF) to retrieve the fraction of connected agents in the economy. The SCF contains information on the fraction of the U.S. population that owns each type of financial asset (*e.g.* bonds, stocks, investment funds, etc.). As usual in limited participation literature[6], I consider both stock and bond ownership, direct or through mutual funds. I disregard, whenever possible, foreign bonds, since, by our definition, asset owners should get richer with interest

[6]In fact, Mankiw and Zeldes (1991) and Vissing-Jørgensen (2002) show that the consumption of stock owners is significantly more reactive to fluctuations in the excess return on the stock market than the general population, which is in line with our definition of the connected agents. In particular, in the case of bondholders, according to the latter author, the intertemporal elasticity of substitution (IES) is even larger than for stockholders.

rate cuts conducted by the Fed. I also disregard retirement funds and US savings bonds, which are highly illiquid.

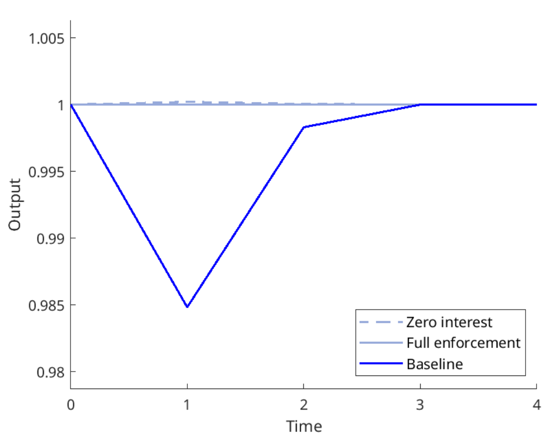
In 2022, the fraction of American households that fit our definitions, according to the SCF, was 27%, and their income was, on average, 1, 9439 higher than that of the average American household. I therefore, set $\eta = 0.27$ and calibrate $\bar{z}_c$, $\bar{z}_u$ and γ to obtain 1) the normalizations described above, 2) a stationary equilibrium labor supply of $1/3$[7], corresponding to an 8-hour workday, and 3) a ratio of the monetary wealth of connected agents and the whole population of $M_{c0}/M_0 = 1.94$. These targets are achieved exactly by construction. By setting $\underline{z}_c = \underline{z}_u = 0.2$, we also obtain a ratio of the income of the 90^{th} to the 10^{th} of approximately 10.0899, which is close to the actual ratio, according to the SCF, of 10, 4762.

I compute Gini indexes for consumption, revenues, and monetary holdings. The stationary equilibrium income Gini obtained is $Gini_0 \approx 0.4127$. This is quite close to the actual one, which has fluctuated around 0.4 since the 1990s. I also compute the total factor productivity (TFP) in the simulations. The idea behind the TFP measure is to see how productive a representative household would need to be in order to produce the aggregate output given the average amount of labor in the economy $\bar{h}_t = \sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\bar{z}} l_{it}(z) dF_i(z)$. Since the technology is linear, this means that $TFP_t := Y_t/\bar{h}_t$.

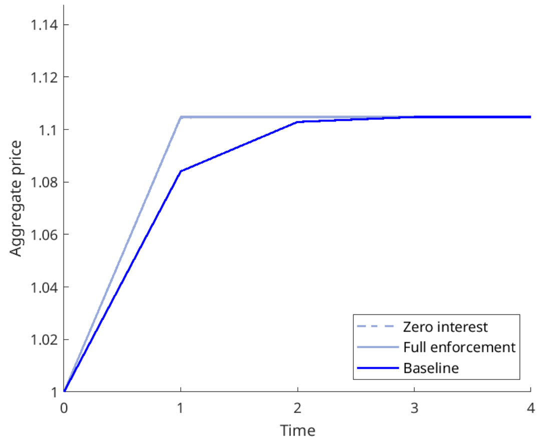
Figure 1 and Figure 2 show, respectively, the paths for aggregate and individual level variables under the baseline economy, the economy with full enforcement of bond contracts - discussed in subsection 3.2, - and the zero interest rate economy - discussed in subsection 3.3. Figure 2 considers connected and unconnected agents with productivity $z_c = z_u = \mathcal{Z}$. Since the logarithmic utility is homothetic, these graphs are identical for other productivity levels up to a re-scaling.

In our calibration, connected agents do not fully deplete their cash only for the first two quarters. Figure 1a indicates that output falls approximately 1.5% on impact, and nearly recovers at $t = 2$. This fall is concentrated on the production of connected agents, as can be seen in Figure 2a. This is because, as shown in Proposition 3, labor - and, hence, output - of unconnected agents is constant, since they set the price to keep the demand they face unchanged. Notice that, since connected agents are more efficient on average, allocative efficiency goes down with the shock - as can be seen in Figure 1d. This means that monetary shocks can generate fluctuations in TFP if they redistribute resources towards more/less productive agents.

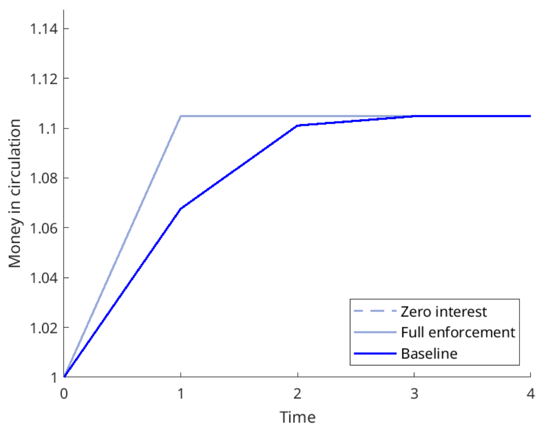
[7]In the model, for simplicity, I do not assume any upper bound to the labor of the entrepreneur. Nevertheless, for the simulation, I set the labor endowment to 1. This does not affect the solutions for as long as the labor constraint does not bind.



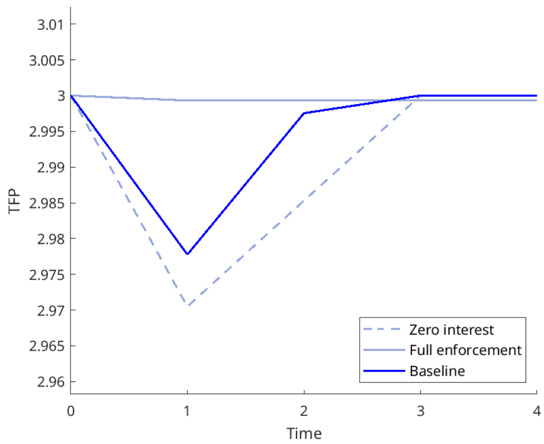
(a) Path of output, Y_t



(b) Aggregate price, P_t



(c) Money in circulation, M_t^C

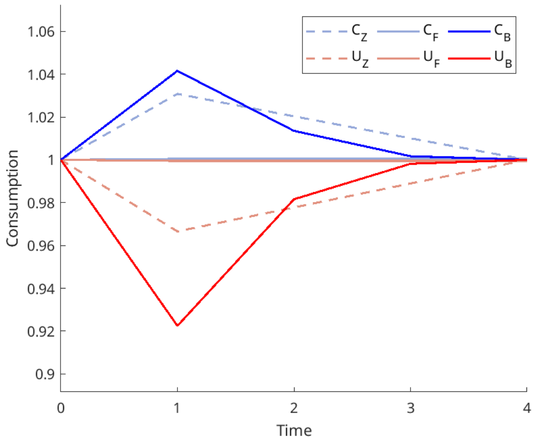


(d) Total Factor Productivity

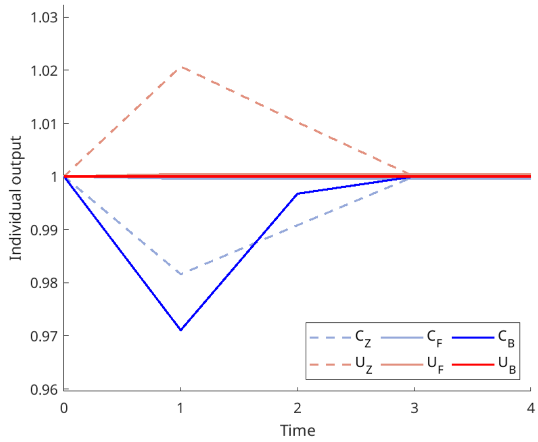
Figure 1: Paths for aggregate variables in the three economies

Figure 2a shows exactly the dynamics described above for individual consumption. Connected agents' consumption rises by approximately 4.2% on impact. The consumption of the other agents falls by around 7.8%, due to the rise in the price of the final good. Moreover, Figure 1c shows that 64.54% of the injected money is put in circulation at $t = 1$. The reason behind this is the fact that, as can be seen in Figure 2d, connected agents' revenues fall by 5.4%, due to the reduction in their output. So, even though they fully deplete the extra money in three periods, the fact that they anticipate a fall in their income at $t = 1$ makes them save around $0.2955\tau m_0(\mathcal{Z})$ to afford to consume $P_2 C_{c2}(\mathcal{Z}) = \beta P_1 C_{c1}(\mathcal{Z})$.

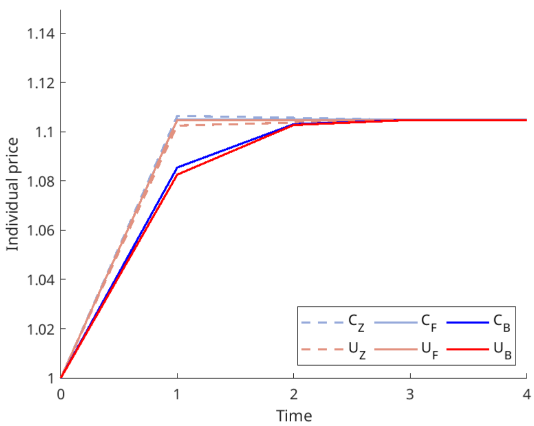
All these patterns are reflected in the Gini indexes, as can be seen in Figure 3. In this graph, I use the following notation: I denote each Gini index as G_k, where $G \in \{C, R, M\}$ corresponds, respectively, to the consumption, revenue, and monetary holdings Gini; and $k \in \{B, F, Z\}$ correspond, respectively to the baseline, full enforce-



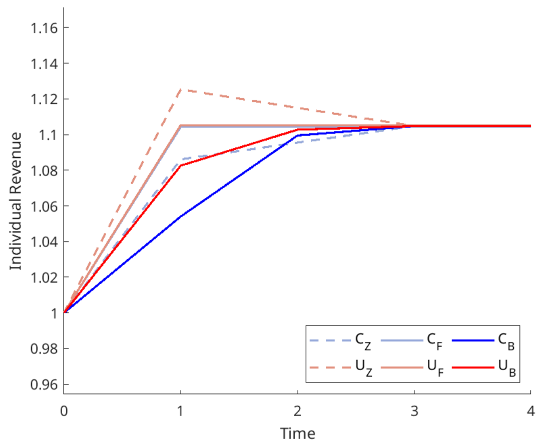
(a) Individual consumption, $C_{it}(\mathcal{Z})$



(b) Individual output, $y_{it}(\mathcal{Z})$



(c) Individual prices, $p_{it}(\mathcal{Z})$



(d) Individual revenues, $R_{it}(\mathcal{Z})$

Figure 2: Paths for individual variables in the three economies

ment and zero interest rate economies. Notice that the Gini for monetary wealth is the one that goes up the most, indicating a big increment in wealth inequality, as - the relatively rich - connected agents become richer with the shock.

The Gini for income goes down, which reflects the mechanism that re-establishes the equilibrium, since - the relatively poor - unconnected agents choose low prices to raise their revenues and recover their real monetary balances. As a result, consumption inequality grows, but less than wealth inequality. Concerning the aggregate price, Figure 1b indicates that roughly 80% of the increment in the aggregate prices happens already at the first period after the shock. This means the model produces endogenous aggregate price stickiness since money is gradually put into circulation due to the consumption-smoothing behavior of connected agents[8].

[8]Elsewhere, it was shown that the aggregate price grows *more* than what should be the case, given the amount of money put in circulation, that is, $P_t > P^H(M_t^C)$ for the periods $t = 1, ..., T$. Nevertheless,

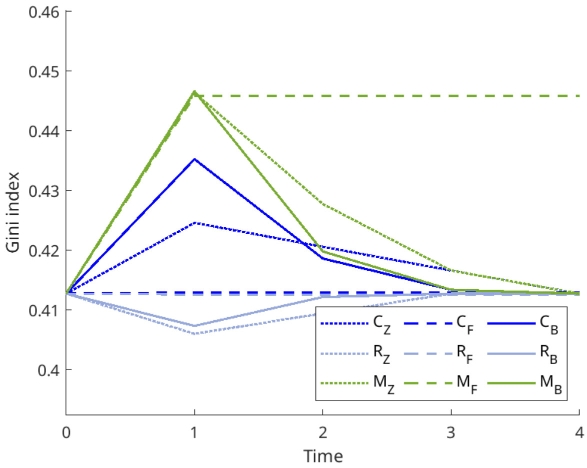
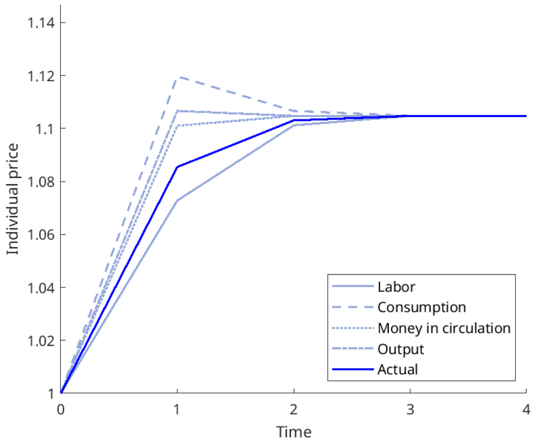
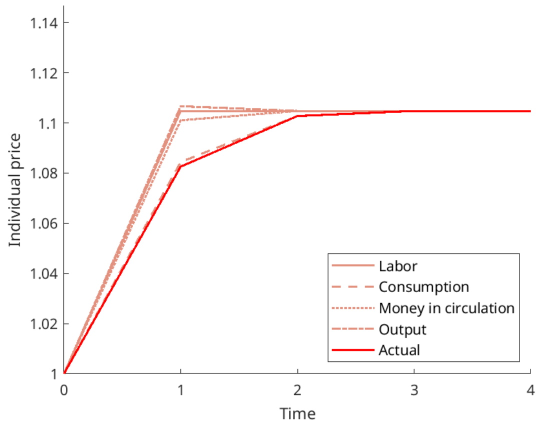


Figure 3: Gini indexes in all economies



(a) Connected agents' prices, $p_{ct}(\mathcal{Z})$



(b) Unconnected agents' prices, $p_{ut}(\mathcal{Z})$

Figure 4: Decomposition of individual prices in the baseline economy

As for individual prices, as can be seen in Figure 2c, the differences between connected and unconnected agents are small, and, hence, the bulk of the disparities in revenue is due to the adjustment on the labor margin. To better understand this, I decompose the price series into four components: labor, future consumption, future money in circulation, and future output. The idea of this decomposition is that, I plug into the price-setting equation, (13), the actual paths for, respectively, $h_{it}(z)$, $C_{i,t+1}(z)$, M^C_{t+1} and Y_{t+1} - since $P_{t+1} = M^C_{t+1}/Y_{t+1}$, - while keeping the other variables constant at, respectively, $h_0(z)$, $C_0(z)$, M_T and Y_0. I plot the results on Figure 4.

As can be seen in the graph, for connected agents, the fall in labor is the main source of price sluggishness since it implies lower marginal disutility of labor and,

the stickiness comes from the fact that $P_t < P^H((1+\tau^A)M_0)$ for $t = 1, ..., T$, that is, the price is *lower* than the final level it attains when the whole money supply is in circulation.

hence, lower marginal costs. The gradual introduction of money also contributes to the sluggishness to a lesser extent, since it increases the future value of money relative to the case where the money is injected all at once. Consumption smoothing by connected agents, however, produces an overshooting pattern since a higher future consumption lowers the value of having money in the next period for them.

For the unconnected agents, on the other hand, beyond the money in circulation, the fall in consumption is the main factor pushing their prices downwards, since they need to remain competitive by lowering their prices to replenish their purchasing power through higher revenues. Now, given that part of the dynamics described in this section are, directly or indirectly, due to the gradual introduction of money, the most natural next step is to allow for borrowing to take place, which means that no amount of cash is kept idle. In the next subsection, I present a version of the model with credit.

3.2 Full Enforcement

Now, I consider the case full enforcement of debt contracts. As a result, bond sales are subject to a natural borrowing limit, namely, $l_t(z, m_{it}(z)) = m^-_{i,t+1}(z) = R_{it}(z)$ for all periods, meaning that, since the repayment of one's debt is always enforced, the entrepreneur must only have enough cash at the beginning of $t+1$ to repay her debt. This means that, for $i \in \{h, l\}$:

$$q_t = \beta \frac{P_t C_{it}(z)}{P_{t+1} C_{i,t+1}(z)} \tag{39}$$

In the stationary equilibrium, we must have $q_0 = \beta$, since $P_t C_{it}(z) = P_{t+1} C_{i,t+1}(z)$ for all individuals with $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$. However, $b_0(z) = 0$ for all entrepreneurs, since the economy starts at a fundamental stationary equilibrium.

3.2.1 Helicopter drops of money

Since the helicopter drops case does not distort relative monetary balances, and since it brings the economy immediately to the new stationary equilibrium, the dynamics would be identical here as in the case where the bonds were absent. In the case of helicopter drops, equally productive agents remain identical after the shock. Hence, in this case, there would be no role for borrowing. Besides, $q^H = \beta = q_0$. Next, I analyze the situation where there is an uneven monetary injection.

3.2.2 Uneven access to the new money and full enforcement of bond contracts

As before, I assume that connected agents are the first to see their monetary holdings change. Then, the following corollary to Proposition 1 holds.

Corollary 1.3. *For any $\tau \neq 0$, if there is full enforcement of bond contracts, the economy goes immediately to the new stationary equilibrium at $t = 1$. The equilibrium bond price and interest rate are, respectively, $q_t = \beta$ and $i_t = (1-\beta)/\beta$ for $t = 1, 2, ...$ Moreover, for all periods $t = 1, 2, ...$, we have:*

- $C_{it}(z) = C_{i,t+1}(z)$ *and* $m_{it}(z) = m_{i,t+1}(z)$ *for* $i \in \{c, u\}$ *and* $z \in [\underline{z}, \overline{z}]$;
- $C_{ht}(z) > C_{lt}(z)$, $p_{ht}(z) > p^H(z, (1+\tau^A)M_0) > p_{lt}(z)$ *and* $R_{lt}(z) > \theta_0(z)M_t > R_{ht}(z)$;

Moreover, for $t = 2, 3, ...$, we have $b_{it}(z) = b_{i,t+1}(z)$ for $i \in \{c, u\}$ and $z \in [\underline{z}, \overline{z}]$ with:

$$b_{ct}(z) = \frac{(1+\tau)m_0(z) - P_1 C_{ct}(z)}{\beta} \tag{40}$$

$$b_{ut}(z) = \frac{m_0(z) - P_1 C_{ut}(z)}{\beta} \tag{41}$$

Expenditures with consumption are a convex combination between $m_{it}(z)$ and $R_{it}(z)$, that is:

$$P_t C_{ct}(z) = (1-\beta)(1+\tau)m_0(z) + \beta R_{ct}(z) \tag{42}$$

$$P_t C_{ut}(z) = (1-\beta)m_0(z) + \beta R_{ut}(z) \tag{43}$$

There is an upper bound and lower bound to the difference in revenues between connected and unconnected agents:

$$\left(\frac{1-\beta}{\beta}\right)\tau m_0(z) > R_{ut}(z) - R_{ct}(z) > 0 \qquad \text{if } \tau > 0 \tag{44}$$

$$\left(\frac{1-\beta}{\beta}\right)\tau m_0(z) < R_{ut}(z) - R_{ct}(z) < 0 \qquad \text{if } \tau < 0 \tag{45}$$

and for the difference in consumption expenditures between them, if $\tau > 0$:

$$(1-\beta)\tau m_0(z) > P_t C_{ct}(z) - P_t C_{ut}(z) = (1-\beta)\tau m_0(z) - \beta(R_{ut}(z) - R_{ct}(z)) > 0, \tag{46}$$

and, if $\tau < 0$:

$$(1-\beta)\tau m_0(z) < P_t C_{ct}(z) - P_t C_{ut}(z) = (1-\beta)\tau m_0(z) - \beta(R_{ut(z)} - R_{ct}(z)) < 0. \tag{47}$$

According to the corollary above, the economy immediately goes to the new stationary equilibrium, which is characterized by persistent consumption differences between connected and unconnected agents. Thus, monetary holdings remain forever identical to those at the beginning of time $t = 1$. The fact that high-cash agents will receive interest payments in the next period is precisely what allows them to set a higher price, work less, and receive a lower revenue today, while still maintaining a higher consumption standard indefinitely: they essentially have a future real claim on part of other agents' current revenues. Notice that, Corollary 1.2 also implies that $|R_{ut}(\mathcal{Z}) - R_{ct}(\mathcal{Z})| < 0.002$ and $|P_t C_{ct}(\mathcal{Z}) - P_t C_{ut}(\mathcal{Z})| < 0.002$. For reference, in the bondless version of the model, at $t = 1$, these gaps were $|R_{ut}(\mathcal{Z}) - R_{ct}(\mathcal{Z})| = 0.0285$ and $|P_t C_{ct}(\mathcal{Z}) - P_t C_{ut}(\mathcal{Z})| = 0.1291$ at $t = 1$. Thus, as expected, the presence of bonds reduces the heterogeneity between agents dramatically, but at the expense of making these smaller differences permanent.

As expected, Figure 1a shows that aggregate output is unaffected by the monetary shock, meaning that, if bonds are present and there is perfect enforcement of bond contracts, monetary policy is neutral in the aggregate. Figure 1b shows that the same holds for the aggregate price. Therefore, in this setup, a representative agent model would not be a bad approximation of the aggregate effects of the shock. As can be seen in Figure 2a, the gap in consumption is almost unnoticeable. As a result, individual prices are also not very different, as can be seen in Figure 2c. The differences in revenues and output are, however, more perceptible. In particular, unlike before, the output produced by unconnected agents is now slightly *larger* than in the initial stationary equilibrium, at $t = 0$, in contrast to the flat line obtained in Figure 2b, although this increment is quantitatively insignificant. It is straightforward to show that the output produced by the unconnected agents is:

$$y_{ut}(z) = \underbrace{z\left(\frac{\epsilon - 1}{\epsilon}\right)\frac{\beta}{\gamma}}_{y_0(z)} \underbrace{-(1-\beta)\frac{b_{it}(z)}{p_{ut}(z)}}_{>0},$$

The first term in the expression is the output needed to balance the trade-off between current labor and future consumption in the absence of interest payments. The increment is the extra output that needs to be produced and sold for the agent to pay these interests on the bonds sold. In fact, Corollary 1.2 implies that $P_t C_{ut}(z) < R_{ut}(z)$, and the difference between both is precisely given by coupon payments. Hence, the reason for the monetary policy to be neutral in the aggregate in the presence of a market for bonds results from 1) a smaller increment in connected agents' consumption offsetting part of the fall in output by increasing the value of future consumption, and 2) the increment in production by unconnected agents to pay interests.

3.3 Exogenously Set Interest Rate

In this section, we study the situation in which the nominal interest rate is exogenously set at a $\beta < q_t \leq 1$. This is relevant, not only because interest rates fall with an expansionist shock, but also because it gives us a middle ground between the two previous cases. In fact, I will concentrate on the case where $q_t = 1$, meaning that there are no interest payments, but, still, the idle cash balances of the connected agents are channeled to the unconnected. For $q_t > \beta$, there would be an excess supply of bonds. Thus, bond sales would be restricted by the demand. I assume:

$$l_t(z, m_{ut}(z)) = \frac{\theta_{ut}(z)}{\theta_{ut}(\mathcal{Z}_u)} \left(\frac{\eta}{1-\eta} \int_{\underline{z}}^{\bar{z}} b_{c,t+1}(z) dF_c(z) \right), \tag{48}$$

meaning that each unconnected agent can obtain a fraction of total bonds that is proportional to their "market share" relative to that of the average unconnected agent. An analogous assumption can be made for the connected agents in the case of $\tau < 0$. Since the nominal interest rate is exogenous, it does not return to its equilibrium value endogenously. Hence, I will, for simplicity, assume that q_t remains constant at the exogenously set level for as long as the economy is in the transition path to the new stationary equilibrium. Proposition 4 ensures that, in this setup, the economy should, indeed, eventually return to the fundamental stationary equilibrium.

Proposition 4. *If there is a constant $\beta < q_t \leq 1$, then there must be a period $T < \infty$ at which high-cash agents decide to fully deplete their extra money.*

The intuition for this result is the following: since the interest rate is lower than its equilibrium value, connected agents are not compensated enough for giving up on current consumption. Thus, their optimal consumption path features decreasing consumption during the transition and, thus, decreasing monetary holdings. Eventually, these holdings will be so close to their stationary equilibrium level that these agents are better off spending it all at once, as in the bondless economy.

Figure 1b shows that the aggregate price displays nearly zero stickiness. As can be seen in Figure 6 in Appendix B, the labor and consumption components of prices cancel out for both connected and unconnected agents. This is made possible by the fact that unconnected agents need to work harder to pay interests, meaning that they face higher labor disutility and, hence, marginal costs, offsetting the incentives to set lower prices due to lower consumption. As a result, $Y_t = M_t^C / P_t$ is also nearly unaffected and faces, if anything, a negligible increase. Figure 2a shows that the consumption of connected agents grows by less than in the baseline economy due to the higher prices. Also, the consumption of unconnected agents is much closer to connected agents' consumption than in the baseline economy and farther from it than in the full enforcement economy.

Figure 2b shows that the output and revenues of all agents are larger than in the baseline economy. Also, notice that, unlike the other situations, the revenue of unconnected agents overshoots. In Appendix B, I show in Figure 7 the path of monetary holdings, m_{it}. It shows that the overshooting behavior of unconnected agents' revenue is not sufficient to make the monetary holdings of connected and unconnected converge as fast as in the baseline economy, due to the need for debt amortization. Overall, the aggregate variables, except for TFP, are compatible with a representative agent model, but the heterogeneities between connected and unconnected agents remain significant, which is reflected in the Gini indexes. Still, Figure 3 shows that, although the Gini for monetary wealth and income do not change by very much relative to the baseline, the consumption Gini increases by much less now, which illustrates the role played by bonds in allowing for risk sharing.

3.4 Welfare

I now move on to analyze the welfare consequences of the models here presented. To begin, I adopt a utilitarian specification of the welfare function and give all individuals equal weight. The function is, then, given by:

$$W_t = \sum_{s=t}^{\infty} \beta^{s-1} \sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} \omega_i(z) \left(log(C_{is}(z)) - \gamma \frac{h_{is}(z)^{1+\zeta}}{1+\zeta} \right) dF_i(z) \tag{49}$$

where the individual weight $\omega_i(z) = 1$ for all $z \in [\underline{z}, \overline{z}]$, $i \in \{c, u\}$ and $t = 1, 2, ...$. Now I define the short-run consumption equivalent as follows:

$$W_t = log(\Phi C_0(z)) - \gamma \frac{h_0(z)^{1+\zeta}}{1+\zeta} + \frac{\beta}{1-\beta} W_0, \tag{50}$$

where $\Phi - 1$ is the uniform increment/decrement to the fundamental stationary equilibrium consumption of all agents *in the first period* that would yield the same welfare level as the actual allocation. This measure is used for the sake of readability, since, due to the transitory nature of the shock, it does not matter in the long run. As a result, the usual consumption equivalent measure in terms of lifetime consumption is two orders of magnitude smaller.

Table 2 shows the results. Appendix C contains tables for some counterfactual exercises aimed at better understanding what drives the welfare differences. I have included the cases for $M_{c0}/M_0 \in \{1.94, 1, 0.6\}$ for ease of comparison. The individual level shocks are kept at $\tau = 0.2$ throughout, and the aggregate shock is $\tau^A =$

0.10476. Hence, I let the value of the fraction of connected agents be, respectively, $\eta \in \{0.27, 0.5238, 0.873\}$ to ensure that the aggregate shock is constant[9].

Model	$M_{c0}/M_0 = 1.94$	$M_{c0}/M_0 = 1$	$M_{c0}/M_0 = 0.6$
Baseline	-4.9684%	-0.3965%	6.2557%
Full enforcement	-4.4928%	-0.0021%	6.5237%
Zero interest rate	-4.5545%	-0.1%	6.3706%

Table 2: Welfare analysis

The table implies that the consumers in this economy would be just as well off by accepting a one-period fall of 4.97% in their consumption as in the baseline economy under the calibration adopted above. Notice that welfare falls with the monetary shock for $M_{c0}/M_0 = 1.94$ and $M_{c0}/M_0 = 1$, but it goes up for $M_{c0}/M_0 = 0.6$ relative to the situation where the shock does not occur. The reason is that, in the latter scenario, connected agents are, on average, poorer. As a result, a positive monetary shock reduces inequality and increases the consumption of agents that, on average, derive a higher marginal utility from consumption. The baseline economy is the one with the lowest welfare among the shocked economies.

The full enforcement economy is where welfare is the highest among the shocked economies. The improvement in welfare is not substantial, being below 0.5% for the baseline calibration. However, it nearly undoes the fall in welfare caused by the shock in the case where $M_{c0}/M_0 = 1$. The zero interest rate model generates a middle ground between both limiting cases, although it is closer to the full enforcement scenario. This indicates that: 1) financial frictions drive the welfare after a monetary shock down, and 2) an exogenously low-interest rate reduces welfare relative to the equilibrium - *natural* - rate.

Moreover, the table also shows that monetary policy has two kinds of distributional effects: on the one hand, it unequivocally *decreases* welfare by increasing inequality between connected and unconnected agents with the same productivity; and, on the other hand, it may benefit mostly agents that have on average higher/lower productivity, yielding thereby an ambiguous effect over inequality, depending on how well-off connected agents are relative to the unconnected.

Table 5, in Appendix C, shows that the main factor driving welfare down in the baseline economy is the increment in inequality for $M_{c0}/M_0 \in \{1.94, 1\}$, although the fall in output marginally lowers welfare as well. For $M_{c0}/M_{u0} = 0.6$, the reduction

[9] As will be shown in section 4, changing η and M_{c0}/M_0 leads to the same aggregate and individual level paths.

in inequality between the average connected and the average unconnected produced by the shock improves welfare. Table 6 shows the counterfactual exercises conducted on the full enforcement economy. If I perform the exercise of imposing that, after T periods, the full enforcement economy goes to the same equilibrium as the other ones, the welfare loss becomes very small. This means that the most important element behind the fall in welfare under full enforcement is the fact that it induces permanent non-fundamental inequality.

Finally, I perform three exercises regarding the zero interest rate situation (see Table 7): (1) I impose the same fall in output that takes place in the baseline economy while keeping the degree of inequality produced by the zero interest rate model; (2) I impose the inequality level in the baseline model, but keep the rise in output under the zero interest rate regime; and (3) I eliminate non-fundamental inequality between connected and unconnected. When compared to the baseline economy, the increase in welfare under the zero interest rate model is driven mostly by the reduction of the disequalizing effects of the monetary shock in the case of $M_{c0}/M_0 = 2$ (where it benefits relatively richer agents), whereas, for $M_{c0}/M_0 = 0.6$, welfare improves *in spite* of the fact that it reduces the inequality between the average connected and the average unconnected agent (that benefits relatively poorer agents), since it reduces non-fundamental inequality.

4 Sensitivity and Robustness Analysis

In this section, I conduct a sensitivity analysis. The primary goal is to check 1) how robust our findings are to changes in some of the assumptions, and 2) how the results change if we change certain parameters. I begin by analyzing the case of a negative monetary policy shock. Secondly, I study the case of a CRRA utility specification. Next, I let the Frisch elasticity of labor supply vary. Then, I change the fraction of connected agents while maintaining the aggregate shock identical - which amounts to also changing the individual shock. Furthermore, I study how results change if we maintain the same aggregate and individual shock, but change how spread across more (less) connected agents this shock is. Lastly, I study how the moment of convergence to the equilibrium varies with the fraction of connected agents in the economy. In all cases, I concentrate on the baseline economy, as it already gives a good idea of how the patterns change. Most graphs can be found in Appendix B.

4.1 A Negative Shock

In this section, I maintain the calibration in Table 1, except for the individual shock, τ, and the fraction of connected η. Instead, I give this economy an equivalent negative shock of $\tau = -0.1667$, which also makes the high-cash agents 20% richer than their low-cash counterparts. Figure 8 in Appendix B plots the graphs for the aggregate and individual variables. The patterns are very similar to the ones observed in the models above, except for the fact that the roles of connected and unconnected agents are now flipped and, as a result, the Gini coefficients and TFP go in the opposite direction as before. Interestingly, the aggregate price *undershoots* - as can be seen in Figure 8b. This is because money is, also here, kept idle.

4.2 CRRA Utility Specification

I now assume that the utility function is given by:

$$u(c) = \frac{C^{1-\alpha}}{1-\alpha}, \tag{51}$$

and consider two possible values for α, namely, 2 and 0.5. These correspond, respectively, to an intertemporal elasticity of substitution (IES) of $IES_{\alpha=2} = 0.5$ and $IES_{\alpha=0.5} = 2$. I then recalibrate the model under the same targets as before, except for the fact that I now assume, for simplicity, that there are no differences in average productivity between connected and unconnected, that is, $M_{c0}/M_0 = 1$.

The basic patterns of the model are robust to changing the utility specification. Figure 9a shows that, under a lower IES, output goes down by more, since the consumption of connected agents increases by less, as can be seen in Figure 10, due to a heightened desire for consumption smoothing. Given to a similar response in aggregate prices, the consumption of unconnected is similar across both cases. Moreover, prices and revenues respond more strongly under a lower IES. This is because prices are more responsive to variations in consumption under a lower IES. Thus, although connected agents with a lower IES *want* to smooth consumption for longer, the higher prices ultimately curb that intention, leading to faster convergence for $IES = 0.5$.

4.3 Varying The Frisch Elasticity Of Labor Supply

I now let the inverse Frisch elasticity of labor supply vary. I consider four cases: $\zeta \in \{0, 0.5, 1, 2\}$, which correspond, respectively, to 1) an infinite Frisch elasticity, 2) a "macro" elasticity of 2, 3) the baseline value, and 4) a "micro" elasticity of 0.5. Figure 11 shows that there is close to no difference across cases in the speed of consumption smoothing. However, for large Frisch elasticities, the aggregate price rises much faster, while output receives a much bigger hit. This also means higher distortions in allocative efficiency for a more elastic labor supply. This naturally happens because the output of connected agents is more responsive under a high elasticity. As a result, revenues are much lower for connected agents in that case. Together with the higher response in the aggregate price, this makes the consumption of connected and unconnected agents alike much lower than for lower elasticities.

Importantly, Figure 11h shows that, under $\zeta = 0$, we observe the same pattern in the price of connected and unconnected agents as in Williamson (2008): the price chosen by the former *overshoots*, while the price of the later grows slowly. This happens because demand in the goods market of connected agents jumps on impact in nominal terms. Due to the imperfect segmentation in the goods market, the new money gradually flows from connected to unconnected agents, which causes a gradual deflation in the former market and inflation in the latter.

4.4 Varying The Fraction Of Connected Agents And The Individual Shocks

Here, I analyze how the results presented above are affected by changes in the fraction of connected households in the economy, η. This exercise has the natural interpretation of exploring the consequences of changes in the degree of access to financial markets in the population. In addition to $\eta = 0.27$, I also present in the figures below the paths for the variables of interest for $\eta = 0.1$, $\eta = 0.5$ and $\eta = 0.6$. Throughout this analysis, I keep the same aggregate shock as before. Moreover, to better control what parameters are varying, I assume no differences in average productivity between connected and unconnected, that is $M_{c0}/M_0 = 1$. As a result, for fractions of connected $\eta \in \{0.1, 0.27, 0.5, 0.6\}$, we have, respectively, individual proportional shocks of $\tau \in \{1.0476, 0.388, 0.20952, 0.1746\}$.

As can be seen in Figure 12, this means that the economy converges to the equilibrium prices and output at $t = 2$ for $\eta = 0.6$, at $t = 3$ for $\eta = 0.5$, at $t = 4$ for $\eta = 0.27$, and at $t = 7$ for $\eta = 0.1$. As a result, the aggregate price displays more sluggishness in the latter case. Moreover, output falls the least for the economy where η is the highest and the lowest. In fact, the slower convergence in the case of $\eta = 0.1$ and the lower response in prices, surprisingly, produces a lower fall in output than in the other cases. As expected, the heterogeneity in outcomes is highest when η is lower, since, in that case, the individual shock to the connected households is much bigger.

Moreover, the patterns in the Gini index are kept unchanged here, and we observe higher variations in inequality for lower values of η, since this implies a bigger

Model	$\eta = 0.1$	$\eta = 0.27$	$\eta = 0.5$	$\eta = 0.6$
Baseline	-0.3681%	-0.2591%	-0.1574%	-0.1185%
Full enforcement	-0.0061%	-0.0018%	-0.0007%	-0.0005%
Zero interest rate	-0.1145%	-0.065%	-0.0411%	-0.0355%

Table 3: Welfare analysis for different values of η

shock that is concentrated on fewer people. Lastly, there is a tiny fall in TFP due to the diminishing returns introduced by the CES aggregator, which can be seen in Figure 12d. Overall, this analysis shows that, when more agents have access to financial markets, the monetary shock is less distortionary. Finally, Table 3 shows, for $\eta \in \{0.1, 0.27, 0.5, 0.6\}$, the welfare under the baseline, full enforcement, and zero interest rate economies. Overall, welfare falls the least in the economies with a higher value of η. This means that financial development in the form of amplified access to financial markets makes the distortions caused by the shock less severe.

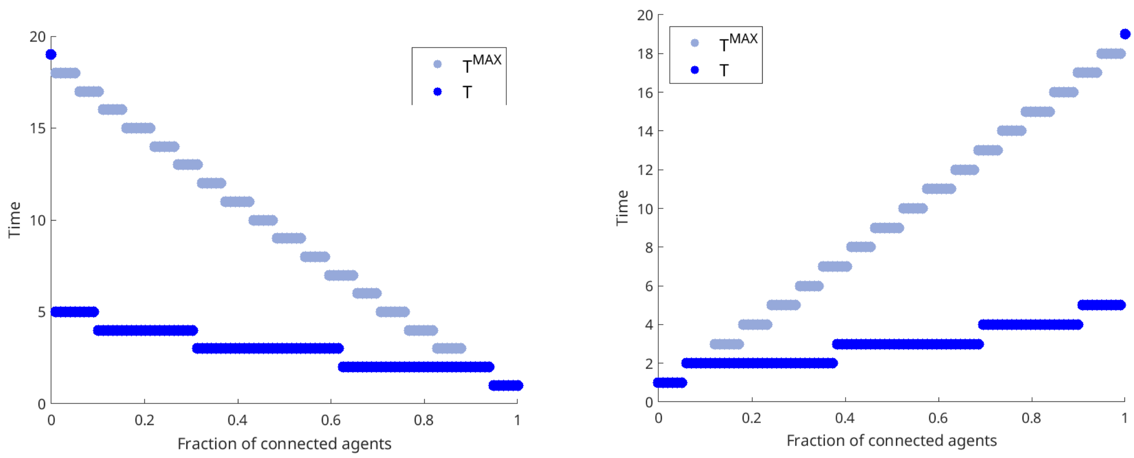
4.5 Varying The Fraction Of Connected Agents And Average Relative Productivity

Now, I perform a similar exercise as the one above, whereby I vary η and maintain the aggregate shock, $\tau^A = 0.135$. However, this time, I keep the individual monetary shock $\tau = 0.2$ and vary the relative average productivity of connected and unconnected. The reason why the aggregate shock is now higher than before is that I assume $M_{c0}/M_0 = 2.5$ in the baseline economy, with $\eta = 0.27$, to ensure that connected agents are on average more productive than the unconnected across all scenarios. As a result, we have $M_{c0}/M_0 \in \{6.75, 2.5, 1.35, 1.125\}$ for, respectively, $\eta \in \{0.1, 0.27, 0.5, 0.6\}$. This exercise trades off two different margins of inequality: the extensive margin - namely, how many connected agents there are - and the intensive margin - how much richer than the unconnected they are on average.

As can be seen in Figure 13, there is no change in the path of aggregate variables, except for TFP. Naturally, for higher values of η, the productivity advantage of connected agents is lower by construction and, as a result, the fluctuation in allocative efficiency is much lower. As a result of this equivalence in the relevant aggregate variables, and the same individual shock size, the choices made by all agents are identical across the different values of η. The Gini coefficients, however, are different in level and percentage variations, as can be seen in Figure 13e, with higher inequality in the economy with fewer and richer connected agents.

4.6 The Impact Of The Fraction Of Connected Agents Over The Speed Of Convergence

Now, I let the fraction of connected vary more freely over the interval $\eta \in [0, 1]$. I maintain the same individual shocks throughout, which can be positive or negative, $\tau \in \{0.2, -0.1667\}$, and keep the relative initial average monetary holdings $M_{c0}/M_0 = 1$ for simplicity. Moreover, I do not re-calibrate the model, as the normalization of the initial aggregate price is not important here. As a consequence, the value of η will govern the size of the aggregate shock, while keeping the individual shocks constant. Importantly, when $\eta = 0$ ($\eta = 1$), I consider the case of a discrete number of connected (unconnected) agents, with zero mass, meaning that we have no aggregate shock (nearly helicopter drops).



(a) Positive shock, $\tau = 0.2$ (b) Negative shock, $\tau = -0.1667$

Figure 5: Number of periods until convergence

Figure 5 plots both T^{MAX}, which is computed as in Equation 27, and the actual number of periods it takes for the economy to converge both for a positive and a negative shocks. The graphs indicate that general equilibrium effects play an important role in explaining the relatively fast convergence. In particular, this can be seen more clearly by the fact that, when connected agents exist with mass zero, $T = T^{MAX}$ for a positive monetary shock, meaning that the few connected agents that exist smooth their consumption as much as is indicated by Equation 27. The same happens to the few unconnected that exist under $\eta = 1$ and $\tau < 0$. Moreover, the gap between T and T^{MAX} is smaller when there are a lot of high-cash agents. The intuition is straightforward: given a certain individual shock, the bigger the aggregate shock, the bigger the revenues that the high-cash agents expect to obtain in the market - which ultimately reduces their incentives to smooth consumption for very long.

5 Concluding remarks

I have shown that disequalizing effects induced by monetary shocks may generate out-of-equilibrium dynamics that preclude the economy's immediate return to the long-run equilibrium. This process is especially important if the agents who get access to the new money earlier decide to smooth consumption out of their wealth. These disequalizing effects might be significant if a well-functioning bonds market is not in place to channel this idle cash to the agents who got the shortest straw of the monetary shock. Prices may rise excessively in response, bringing about a fall in output.

If credit markets are well-developed, and connected agents can lend their excessive reserves to unconnected ones, inequality in consumption and income change by much less. However, if the interest rate is at its equilibrium value, unconnected agents will roll over their debt indefinitely, meaning that the differences in monetary holdings become persistent, and the economy does not return to the same stationary equilibrium. If the interest rate is lower than the equilibrium rate, the economy will eventually reach the new fundamental stationary equilibrium. Moreover, higher participation in financial markets entails lower distortions from monetary shocks and smaller variations in welfare.

As far as the distributional effects of monetary shocks are concerned, two main policy recommendations seem to follow. First, reducing financial frictions that hinder access to credit seems to be very relevant, because it allows for risk sharing. Second, policies targeted at enhancing access to financial markets are also welcome. More research is required on the causes of why this access is relatively restricted even in some advanced economies. Finally, the model here presented brings new insight into the phenomenon of price stickiness. Here, a mild degree of endogenous price stickiness is induced by two different mechanisms. If part of the new money remains idle and is put in circulation slowly, unconnected agents will raise their prices more slowly to remain competitive and replenish their real monetary balances faster. The second mechanism operates through lower marginal costs in the production of the connected agents after they cut their production.

References

Adam, K. and Tzamourani, P. (2016). Distributional consequences of asset price inflation in the euro area. *European Economic Review*, 89:172–192.

Aizcorbe, A. M., Kennickell, A. B., and Moore, K. B. (2003). Recent changes in us family finances: Evidence from the 1998 and 2001 survey of consumer finances. *Fed. Res. Bull.*, 89:1.

Ampudia, M., Georgarakos, D., Slacalek, J., Tristani, O., Vermeulen, P., and Violante, G. (2018). Monetary policy and household inequality.

Areosa, W. D. and Areosa, M. B. (2016). The inequality channel of monetary transmission. *Journal of Macroeconomics*, 48:214–230.

Auclert, A. (2019). Monetary policy and the redistribution channel. *American Economic Review*, 109(6):2333–2367.

Baqaee, D., Farhi, E., and Sangani, K. (2022). The Supply-Side Effects of Monetary Policy. NBER Working Paper No. 28345.

Berentsen, A., Camera, G., and Waller, C. (2007). Money, credit and banking. *Journal of Economic theory*, 135(1):171–195.

Casiraghi, M., Gaiotti, E., Rodano, L., and Secchi, A. (2018). A "reverse robin hood"? the distributional implications of non-standard monetary policy for italian households. *Journal of International Money and Finance*, 85:215–235.

Coibion, O., Gorodnichenko, Y., Kueng, L., and Silvia, J. (2017). Innocent bystanders? monetary policy and inequality. *Journal of Monetary Economics*, 88:70–89.

Cravino, J., Lan, T., and Levchenko, A. A. (2020). Price stickiness along the income distribution and the effects of monetary policy. *Journal of Monetary Economics*, 110:19–32.

Davtyan, K. (2016). Income inequality and monetary policy: An analysis on the long run relation. *AQR–Working Papers, 2016, AQR16/04*.

Doepke, M. and Schneider, M. (2006). Inflation and the redistribution of nominal wealth. *Journal of Political Economy*, 114(6):1069–1097.

Doepke, M., Schneider, M., and Selezneva, V. (2015). Distributional effects of monetary policy. *Unpublished manuscript*.

Dolado, J. J., Motyovszki, G., and Pappa, E. (2021). Monetary policy and inequality under labor market frictions and capital-skill complementarity. *American economic journal: macroeconomics*, 13(2):292–332.

Friedman, M. (1969). The optimum quantity of money, and other essays. Technical report.

Furceri, D., Loungani, P., and Zdzienicka, A. (2018). The effects of monetary policy shocks on inequality. *Journal of International Money and Finance*, 85:168–186.

Gornemann, N., Kuester, K., and Nakajima, M. (2016). Doves for the rich, hawks for the poor? distributional consequences of monetary policy. *Distributional Consequences of Monetary Policy (April 2016)*.

Grossman, S. and Weiss, L. (1983). A transactions-based model of the monetary transmission mechanism. *The American Economic Review*, 73(5):871–880.

Guerello, C. (2018). Conventional and unconventional monetary policy vs. households income distribution: An empirical analysis for the euro area. *Journal of International Money and Finance*, 85:187–214.

Havranek, T. and Rusnak, M. (2012). Transmission lags of monetary policy: A meta-analysis.

Herrenbrueck, L. (2019). Frictional asset markets and the liquidity channel of monetary policy. *Journal of Economic Theory*, 181:82–120.

Hohberger, S., Priftis, R., and Vogel, L. (2020). The distributional effects of conventional monetary policy and quantitative easing: Evidence from an estimated dsge model. *Journal of Banking & Finance*, 113:105483.

Kiyotaki, N. and Wright, R. (1993). A search-theoretic approach to monetary economics. *The American Economic Review*, pages 63–77.

Lagos, R. and Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of political Economy*, 113(3):463–484.

Lucas, R. E. and Stokey, N. L. (1985). Money and interest in a cash-in-advance economy. Technical report, National Bureau of Economic Research.

Mankiw, N. G. and Zeldes, S. P. (1991). The consumption of stockholders and non-stockholders. *Journal of financial Economics*, 29(1):97–112.

Montecino, J. and Epstein, G. (2015). Did quantitative easing increase income inequality? *Institute for New Economic Thinking working paper series*, (28).

O'Farrell, R., Rawdanowicz, Ł., and Inaba, K.-I. (2016). Monetary policy and inequality.

Rocheteau, G., Weill, P.-O., and Wong, T.-N. (2018). A tractable model of monetary exchange with ex post heterogeneity. *Theoretical Economics*, 13(3):1369–1423.

Romer, C. D. and Romer, D. H. (2004). A new measure of monetary shocks: Derivation and implications. *American economic review*, 94(4):1055–1084.

Saiki, A. and Frost, J. (2014). Does unconventional monetary policy affect inequality? evidence from japan. *Applied Economics*, 46(36):4445–4454.

Telmer, C. I. (1993). Asset-pricing puzzles and incomplete markets. *The Journal of Finance*, 48(5):1803–1832.

Vissing-Jørgensen, A. (2002). Limited asset market participation and the elasticity of intertemporal substitution. *Journal of political Economy*, 110(4):825–853.

Williamson, S. and Wright, R. (2010). New monetarist economics: Models. In *Handbook of monetary economics*, volume 3, pages 25–96. Elsevier.

Williamson, S. D. (2006). Search, limited participation, and monetary policy. *International Economic Review*, 47(1):107–128.

Williamson, S. D. (2008). Monetary policy and distribution. *Journal of monetary economics*, 55(6):1038–1053.

A Proofs of Propositions

A.1 Proposition 1

Existence and uniqueness of the fundamental stationary equilibrium:

To begin, notice that, as in Lucas and Stokey (1985), the CIA constraint should bind in this economy. To prove this explicitly, notice that, for any arbitrary $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$, by imposing the fundamental stationary equilibrium conditions onto (3), we obtain:

$$P_t C_{it}(z) = m_{it}(z) + (1 - q_t) b_{i,t+1}(z). \tag{52}$$

Imposing (2) gives:

$$m_{it}(z) - q_t b_{i,t+1}(z) = m_{it}(z) + (1 - q_t) b_{i,t+1}(z), \tag{53}$$

which can only be satisfied for $b_{i,t+1}(z) = 0$. Now, by aggregating (13), and imposing $P_t = P_{t+1}$ and $C_{i,t+1}(z) = C_{it}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$, we find that:

$$\left[\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\epsilon - 1} \frac{u'\left(C_{it}(z)\right)^{\epsilon - 1}}{h_{it}(z)^{\zeta(\epsilon - 1)}} dF_i(z) \right]^{\frac{1}{\epsilon - 1}} = \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} \tag{54}$$

By exploiting the continuity of the $u'(\cdot)$ function, I define z^* as:

$$(z^*)^{\epsilon - 1} \frac{u'\left(C_{it}(z^*)\right)^{\epsilon - 1}}{h_{it}(z^*)^{\zeta(\epsilon - 1)}} = \sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\epsilon - 1} \frac{u'\left(C_{it}(z)\right)^{\epsilon - 1}}{h_{it}(z)^{\zeta(\epsilon - 1)}} dF_i(z)$$

Now, notice that:

$$\theta_{it}(z) = \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} \right]^{1 - \epsilon} \frac{z^{\epsilon - 1} u'\left(C_{i,t+1}(z)\right)^{\epsilon - 1}}{h_{it}(z)^{\zeta(\epsilon - 1)}} = \left[\frac{z u'\left(C_{i,t+1}(z) / h_{it}(z)^{\zeta}\right)}{z^* u'\left(C_{i,t+1}(z^*) / h_{it}(z^*)^{\zeta}\right)} \right]^{\epsilon - 1},$$

where the last equality follows from (54) and (A.1). This means that $\theta_{it}(z^*) = 1$. Moreover, notice that, with a bit of algebra, and using the fact that $Y_t = M_t / P_t$, we have:

$$h_{it}(z) = \left(\frac{p_{it}(z)}{P_t} \right)^{-\epsilon} Y_t = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{z}{\gamma} \beta u'(C_{i,t+1}(z)) \right]^{\frac{\epsilon}{1 + \zeta\epsilon}} \left(\frac{M_t}{P_t} \right)^{\frac{1}{1 + \zeta\epsilon}} \tag{55}$$

Since, in the stationary equilibrium there is no borrowing and $m_{it}(z) = R_{it}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$, we must, therefore have that $C_{i,t+1}(z^*) = C_{it}(z^*) = \theta_{it}(z^*) M_t / P_t = M_t / P_t$. This means that:

$$\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} = z^* \frac{u'\left(M_t / P_t\right)}{h_{it}(z^*)^{\zeta}} \tag{56}$$

By using (55), one can show that:

$$u'\left(\frac{M_t}{P_t}\right)\left[\frac{M_t}{P_t}\right]^{-\zeta} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta z^*}, \tag{57}$$

which means that M_t/P_t is defined uniquely, as the left-hand side is strictly decreasing on it. Therefore, P_t is a linear function of M_t, and aggregate output $Y_t = M_t/P_t$ is defined by parameters and z^* alone, being, therefore, independent of P_t and M_t. Finally, $\theta_{it}(z)$ is also well and uniquely defined for every $z \in [\underline{z}, \overline{z}]$ and is independent of connectedness status. This means that $m_{it}(z)$, $C_{it}(z)$, $p_{it}(z)$, $y_{it}(z)$ and $R_{it}(z)$ are also well- and uniquely-defined. Now, I look into non-fundamental stationary equilibria.

Finding a stationary equilibrium with $q_t = \beta$:

To begin, let us define "high-cash" agents as being agents for whom $b_{h,t+1}(z) > 0$ and "low-cash" as having $b_{l,t+1}(z) \leq 0$. We can then build $F_h(\cdot)$ and $F_l(\cdot)$ to be the corresponding cumulative distribution functions. Finally, let η_h and $\eta_l = 1 - \eta_h$ be the corresponding fractions of the population that falls into either category.

The structure of the proof is as follows: I will begin by assuming that $l_t(z, m_{lt}(z)) = \infty$. This allows for the possibility of some low-cash agents ending up with an outstanding debt after the bonds' market closes at the beginning of the period. Then, I will show that the equilibrium implemented by the unconstrained economy is feasible in the constrained economy, meaning that the constraint does not bind. So, to begin, notice that one can re-write the Euler equation in the unconstrained case as:

$$\frac{u'(C_{it}(z))}{u'(C_{i,t+1}(z))} = \frac{\beta}{q_t}\frac{P_t}{P_{t+1}} \tag{58}$$

Since, by assumption, $u(\cdot)$ is increasing and strictly concave, the marginal utility is such that $u'(\cdot) \geq 0$ and is strictly decreasing. This means that, given certain values of the aggregate variables at the right-hand side, this means that either the ratio $C_{it}(z)/C_{i,t+1}(z)$ is decreasing, constant, or increasing over time for all agents, regardless of productivity and high-or-low-cash status. Besides, notice that all the money not used by high-cash agents to buy bonds in any period $t = 0, 1, 2, ...$ must be used for consumption, that is $P_t C_{ht}(z) = m_{ht}(z) - q_t b_{h,t+1}(z)$. Moreover, notice that, if $q_t = 1$, $s_{it}(z) = 0$ for all agents by assumption, and, if $q_t < 1$, it cannot be optimal for low-cash agents to sell bonds at a price q_t and save in cash (*i.e.*, $s_{lt}(z) > 0$), then we must have $M_t^C = M_t$ for any $t \in \{1, 2, ...\}$.

Now, I will show that there is a stationary equilibrium in which $q_t = \beta$. To begin, notice that, if we set $q_t = \beta$, (58) implies that $\frac{u'(C_{i1}(z))}{u'(C_{i2}(z))} = \frac{P_1}{P_2}$. Moreover, notice that:

$$P_t \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} C_{it}(z) dF_i(z) = M_t = P_{t+1} \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} C_{i,t+1}(z) dF_i(z), \tag{59}$$

Naturally, if $C_{it}(z) = C_{i,t+1}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$, the equation above implies that $P_t = P_{t+1}$, meaning that the first-order condition (58) is satisfied for every agent. I will prove below that this requires $m_{it}(z) = m_{i,t+1}(z)$ for all agents and for any arbitrary $t = 1, 2, ...$, but, for now, I will take this result as a given for simplicity. In this case, it must be the case that this equilibrium lasts forever, proving the existence of this stationary equilibrium with borrowing. Let us denote the demand/supply of bonds by each agent under this equilibrium with a star, that is, $b^*_{it}(z)$

Proving that $q_t = \beta$ for every period t:

Now assume, by contradiction, that $1 \geq q_t > \beta$. The argument above proves that an equilibrium with $q_t = \beta$ always exists, meaning that $\sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b^*_{h,t+1}(z) dF_h(z) = -\frac{(1-\eta_h)}{\eta_h} \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b^*_{l,t+1}(z) dF_l(z)$. Now, notice that (58) means that buying/selling bonds trades off current and future consumption. I will consider two cases now. To begin, I consider the case where $q_t/P_t > \beta/P^*_t$, meaning that the price of bonds relative to current consumption rises when $q_t > \beta$. As a result, bond demand by the high-cash agents must decrease, $b_{h,t+1}(z) < b^*_{h,t+1}(z)$, and low-cash agents' supply of bonds must increase, $-b_{l,t+1}(z) > -b^*_{l,t+1}(z)$, to make the marginal value of holding money in the future, $\beta^{t+1} u'(C_{i,t+1}(z))/P_{t+1}$, higher for both types of agent. Now, notice that the former condition implies:

$$\begin{aligned}
-\frac{(1-\eta_h)}{\eta_h} \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b_{l,t+1}(z) dF_l(z) &= \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b_{h,t+1}(z) dF_h(z) \\
&< \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b^*_{h,t+1}(z) dF_h(z) \\
&= -\frac{(1-\eta_h)}{\eta_h} \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b^*_{l,t+1}(z) dF_l(z),
\end{aligned}$$

which contradicts the latter condition. This rules out any equilibrium with $1 \geq q_t > \beta$ under $q_t/P_t > \beta/P^*_t$. Now, let us consider the case where $q_t/P_t \leq \beta/P^*_t$. By a similar argument to the one made above, we must have $b_{i,t+1}(z) \geq b^*_{i,t+1}(z)$ and, therefore, $q_t b_{i,t+1} > \beta b^*_{i,t+1}(z)$ for $i \in \{h, l\}$. Integrating both sides and imposing the market clearing condition for the bonds market, gives us:

$$0 = q_t \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b_{i,t+1}(z) dF_i(z) > \beta \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b^*_{i,t+1}(z) dF_i(z) = 0, \tag{60}$$

a contradiction. This means that no equilibrium with $1 \geq q_t > \beta$ can exist. A similar argument can be made to rule out equilibria with $q_t < \beta$.

Equilibrium uniqueness:

Now, I will show that, under $q_t = \beta$, there can be no equilibrium that is not stationary.

This will prove the uniqueness of the equilibrium described above under full enforcement. By integrating the individual price choices and imposing the result that the full depletion first-order condition holds for a non-negligible mass of agents, we obtain

$$P_t = P_{t+1}\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\left[\sum_{i\in\{h,l\}}\eta_i\int_{\underline{z}}^{\overline{z}}\frac{z^{\epsilon-1}u'(C_{i,t+1}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}}dF_i(z)\right]^{\frac{1}{1-\epsilon}} \tag{61}$$

Now, notice that labor effort in production by the agent with $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$ is, out of a stationary equilibrium, given by:

$$h_{it}(z) = \left(\frac{p_{it}(z)}{P_t}\right)^{-\epsilon}Y_t = \left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{z}{\gamma}\beta\frac{P_t}{P_{t+1}}u'(C_{i,t+1}(z))\right]^{\frac{\epsilon}{1+\zeta\epsilon}}Y_t^{\frac{1}{1+\zeta\epsilon}}, \tag{62}$$

which can be plugged into (61) to give:

$$P_t = P_{t+1}\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}Y_t^{\zeta}\left[\sum_{i\in\{h,l\}}\eta_i\int_{\underline{z}}^{\overline{z}}z^{\frac{\epsilon-1}{1+\zeta\epsilon}}u'(C_{i,t+1}(z))^{\frac{\epsilon-1}{1+\zeta\epsilon}}dF_i(z)\right]^{\frac{1+\zeta\epsilon}{1-\epsilon}} \tag{63}$$

Now, notice that, at any period, and since $q_t = \beta$, (58) can be written with a weak inequality, that is $u'(C_{i,t+1}(z)) \leq \frac{P_{t+1}}{P_t}u'(C_{it}(z))$, which should also be valid even if the economy ends up in a fundamental stationary equilibrium in the next period. By plugging this version of the first-order condition into the equation above, we obtain:

$$\left[\sum_{i\in\{h,l\}}\eta_i\int_{\underline{z}}^{\overline{z}}z^{\frac{\epsilon-1}{1+\zeta\epsilon}}u'(C_{it}(z))^{\frac{\epsilon-1}{1+\zeta\epsilon}}dF_i(z)\right]^{\frac{1+\zeta\epsilon}{\epsilon-1}} \geq \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{q_t}Y_t^{\zeta} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}Y_t^{\zeta}$$

which is valid even if the economy converges to the stationary equilibrium (and, hence, no borrowing occurs) between t and $t+1$. This allows us to re-write (63) as:

$$P_t \leq P_{t+1}\left(\frac{Y_t}{Y_{t+1}}\right)^{\zeta}, \tag{64}$$

Since, in the presence of borrowing, $P_t = M_t/Y_t$, we can re-write the condition above as:

$$\left(\frac{Y_t}{Y_{t+1}}\right)^{1+\zeta} \geq 1 \qquad \therefore \qquad Y_t \geq Y_{t+1} \tag{65}$$

Notice that this immediately rules out the possibility of deflation. Thus, we must either have inflation or constant prices. Assume, by contradiction, that $P_{t+1} > P_t$ for a given period $t \in \{1, 2, ...\}$. Then, by using the definition (16) and (61), we can see that:

$$\frac{P_{t+1}}{P_t} = U_{t+1}^{GAP}, \tag{66}$$

meaning that $\overline{U}_{t+1}^{GAP} > 1$. I am going to show that this implies that $P_{t+2} > P_{t+1}$. To see this, notice that, since $P_{t+2} \geq P_{t+1}$ by the condition above, and given the first-order condition (58) and $q_t = \beta$, then $C_{i,t+2}(z) \leq C_{i,t+1}(z)$ for any $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$. Now, notice that, by imposing (62), we can obtain:

$$\frac{z^{\epsilon-1} u'(C_{i,t+2}(z))^{\epsilon-1}}{h_{i,t+1}(z)^{\zeta(\epsilon-1)}} = \left[\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\frac{P_{t+2}}{P_{t+1}}\right]^{\frac{\zeta\epsilon(\epsilon-1)}{1+\zeta\epsilon}} Y_{t+1}^{\frac{\zeta(1-\epsilon)}{1+\zeta\epsilon}} z^{\frac{\epsilon-1}{1+\zeta\epsilon}} u'(C_{i,t+2}(z))^{\frac{\epsilon-1}{1+\zeta\epsilon}} \tag{67}$$

$$> \left[\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\frac{P_{t+2}}{P_{t+1}}\right]^{\frac{\zeta\epsilon(\epsilon-1)}{1+\zeta\epsilon}} Y_{t}^{\frac{\zeta(1-\epsilon)}{1+\zeta\epsilon}} z^{\frac{\epsilon-1}{1+\zeta\epsilon}} u'(C_{i,t+1}(z))^{\frac{\epsilon-1}{1+\zeta\epsilon}} \tag{68}$$

$$= \frac{z^{\epsilon-1} u'(C_{i,t+1}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}} \left(\frac{P_{t+2}}{P_{t+1}}\frac{P_t}{P_{t+1}}\right)^{\frac{\zeta\epsilon(\epsilon-1)}{1+\zeta\epsilon}} \tag{69}$$

since $Y_{t+1} < Y_t$ due to our contradiction assumption. Aggregating it, by (66), we obtain:

$$\frac{P_{t+2}}{P_{t+1}} > \frac{P_{t+1}}{P_t},$$

which shows that prices cannot be constant, given (18). By a simple induction argument, we can see that prices must grow forever and that the inflation rate is bounded below by a positive constant, since $1 + \pi_{t+s} = \overline{U}_{t+s}^{GAP} \geq \overline{U}_{t+1}^{GAP} > 1$ for $s \in \{2, 3, ...\}$, meaning that the price level diverges. This, implies, however, by equation (59), that:

$$\lim_{t\to\infty} \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} C_{it}(z) dF_i(z) = 0$$

as the price level diverges. However, the fact that there is borrowing in any equilibrium along this path, we know that there is a non-negligible mass of high-cash agents that are strictly richer than their low-cash counterparts, that is, $C_{ht}(z) > C_{lt}(z) \geq 0$. This means that aggregate consumption must be bounded away from zero. As a result, the only possible equilibrium is the one in which $P_{t+1} = P_t$ and $C_{i,t+1}(z) = C_{it}(z)$ for every $z \in [\underline{z}, \overline{z}]$, $i \in \{h, l\}$ and $t \in \mathbb{T}^S$. Evidently, we must have $P_t C_{ht}(z) > P_t C_{lt}(z)$, $p_{ht}(z) > p_{lt}(z)$ and, thus, $R_{lt}(z) > R_{ht(z)}$ for all $t \in \mathbb{T}^S$.

Constancy of borrowing/lending decisions:

Now, notice that, by imposing (2) into the budget constraint, (3), of any arbitrary entrepreneur in period $t \in \mathbb{T}^S$, we see that $m_{i,t+1}^-(z) = R_{it}(z)$. Now, notice that, since $u'(C_{i,t+1})/P_{t+1} = u'(C_{i,t+2})/P_{t+2}$, by (13), $p_{it}(z) = p_{i,t+1}(z)$ and, thus, $R_{it}(z) = R_{i,t+1}(z)$. This means that $m_{it}^-(z) = m_{i,t+1}^-(z)$ for $t = 2, 3, ...$ Therefore, for $t = 2, 3, ...,$ by the budget constraint, (3), we have:

$$P_t C_{it}(z) = R_{it}(z) + b_{it}(z) - \beta b_{i,t+1}(z) \tag{70}$$

Together with (70) and the fact that $P_tC_{it}(z) = P_{t+1}C_{i,t+1}(z)$, this implies that:

$$b_{it}(z) - \beta b_{i,t+1}(z) = b_{i,t+1}(z) - \beta b_{i,t+2}(z) \tag{71}$$

for every $t \in \mathbb{T}^S$. Now, let us define α_s such that $b_{i,t+s}(z) = \alpha_s b_{i,t+s-1}(z)$, for $s = 1, 2, \ldots$. Thus, for an arbitrary $r = 1, 2, \ldots$, we can apply this definition iteratively to obtain:

$$b_{i,t+r}(z) = \left(\prod_{s=1}^{r} \alpha_s\right) b_{it}(z) \tag{72}$$

I will now prove that $\alpha_s = 1$ for $s = 1, 2, \ldots$. I will concentrate on α_1 without any loss of generality. Assume, by contradiction, that $\alpha_1 < 1$. Then, by (71), we have:

$$\begin{aligned}
b_{it}(z) - \beta b_{i,t+1}(z) &= b_{i,t+1}(z) - \beta b_{i,t+2}(z) \\
(1 - \alpha_1\beta) b_{it}(z) &= (1 - \alpha_2\beta)\alpha_1 b_{it}(z) \\
(1 - \alpha_1\beta) &= (1 - \alpha_2\beta)\alpha_1 < (1 - \alpha_2\beta) \\
\alpha_2 &< \alpha_1
\end{aligned}$$

Obviously, the same reasoning applies to show that $\alpha_s < \alpha_1$. Therefore, $\alpha_s < 1$ for all $s \geq 1$. However, this means that:

$$\lim_{r\to\infty} b_{i,t+r}(z) = 0$$

and, thus:

$$\lim_{r\to\infty} P_{t+r}C_{i,t+r}(z) = R_{i,t+r}(z)$$

However, since $R_{l,t+r}(z) > R_{h,t+r}(z)$, this means that $P_{t+r}C_{l,t+r}(z) > P_tC_{h,t+r}(z)$ at the limit: an absurd, since $P_{t+r}C_{h,t+r}(z)$ is constant for all r and larger than $P_{t+r}C_{l,t+r}(z)$.

Now, I proceed to the second case: assume, by contradiction, that $\alpha_1 > 1$. Similarly to before, this implies that $\alpha_s > \alpha_1 > 1$ for any $s > 1$. Now, this implies that:

$$\lim_{r\to\infty} b_{c,t+r} = \lim_{r\to\infty} \left(\prod_{s=1}^{r} \alpha_s\right) b_{ct} > \lim_{r\to\infty} \alpha_1^r b_{ct} = \infty$$

which is not possible, since $M_t < \infty$. Therefore, I conclude that $\alpha_s = 1$ for $s = 1, 2, \ldots$ and, thus, $b_{it}(z) = b_{i,t+1}(z)$ for $z \in [\underline{z}, \overline{z}]$, $i \in \{h, l\}$ and $t \in \mathbb{T}^S$.

Proving that the unconstrained optimum is feasible:

Using agents' cash-in-advance constraint, (2), at t, this result implies that:

$$b_{it}(z) = \frac{m_{it}(z) - P_tC_{it}(z)}{\beta}$$

for $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$. From this expression, one can easily see that $m_{it}(z) = m_{i,t+1}(z)$ for $t \in \mathbb{T}^S$. However, since $m_{i1}(z) > 0$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$, this implies that no agent enters the bonds market with outstanding debt, meaning that $b_{l,t+1}(z) < m^{-}_{l,t+1}(z) = R_{lt}(z)$ for every $z \in [\underline{z}, \overline{z}]$, which proves that the borrowing constraint does not bind.

Equilibrium Uniqueness:

Finally, I can now prove that, if money is distributed such as in the fundamental stationary equilibrium, this is not only the unique stationary equilibrium but also the unique equilibrium in this economy. I denote the fundamental stationary equilibrium with the subscript 0 and, moreover, drop the i subscript since agents with the same productivity do not differ in monetary holdings and, as a result, in their choices. Notice that the only circumstance in which the economy would not be in a fundamental stationary equilibrium is if $C_t(z) < m_t(z) = R_0(z)$ for a non-negligible mass of agents. Assume this is the case by means of contradiction. This signifies that some agents save acquire bonds and, by the market clearing condition for the bonds market, some agents must sell bonds. Thus, $b_{t+1}(z) \neq 0$ for a positive mass of agents.

As proven above, under $l_t(z, m_t(z) = m_{t+1}(z)$, $q_t = \beta$ and $P_{t+1} \geq P_t$ for every t. Now, although consumption cannot differ across two agents with the same productivity, we must still have $C_t(z) > C_t(z') \geq 0$ for $z > z'$, since $m_t(z) > m_t(z')$ for all t. As a result, we can still rule out diverging prices and must, thus, conclude that $P_{t+1} = P_t$ for every period t. By the first-order condition, therefore, $C_{t+1}(z) = C_t(z)$ for all agents. For constant prices, this can only happen if $m_{t+1}(z) = m_t(z) = R_0(z)$ for all agents and all periods, since consumption is strictly increasing on monetary holdings. As show in the beginning of the proof to this proposition, this requires $b_{t+1}(z) = 0$ for all $z \in [\underline{z}, \overline{z}]$, which violates our contradiction assumption. I conclude, therefore, that $m_t(z) = R_0(z)$ implies that the economy is in the fundamental stationary equilibrium. $\square$

A.1.1 Corollary 1.1

To begin, notice that, by assumption, the monetary holdings function, $m_0(\cdot)$, is such that the economy starts off at the fundamental stationary equilibrium. Moreover, the monetary base grows by a factor τ^A, that is, $M_1 = (1 + \tau^A)M_0$. If revenues are undistorted relative to the fundamental equilibrium, *i.e.* $m_1(z) = (1 + \tau_H)m_0(z) = \theta_0(z)(1 + \tau^A)M_0 = R_1(z)$, since $\tau_H = \tau^A$. However, this implies that the new monetary holdigns are compatible with a new fundamental stationary equilibrium. Given Proposition 1, this is the unique equilibrium that can arise in this economy. Now, notice that, in this case, if $l_t(z, m_t(z)) = m_{t+1}(z)$, by Proposition 1, we must still be in a fundamental equilibrium. Moreover, notice that, even if borrowing is possible and the interest rate is positive, in the fundamental equilibrium, $b_{t+1}(z) = 0$ for all $z \in [\underline{z}, \overline{z}]$. Since buying bonds dominates saving in idle cash for $q_t = \beta$, the fact that $b_{t+1}(z) = 0$ in the economy with full enforcement of bond contracts implies that $s_t(z) = 0$ for all $z \in [\underline{z}, \overline{z}]$ in the economy where $l_t(z, m_t(z)) = 0$. This proves that the economy must be in the fundamental stationary equilibrium after the helicopter drops shock. $\square$

A.1.2 Corollary 1.2

To begin, by definition, $m_{it}(z) = m_{i,t+1}(z)$ for all $z \in [\underline{z}, \overline{z}]$, $i \in \{c, u\}$, and $t \in \mathbb{T}^S$ at the stationary equilibrium. By (13), we must, thus, have:

$$\begin{aligned} p_{it}(z) &= \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} \frac{[D(p_{it}(z), P_t, Y_t)/z]^{\zeta}}{z} p_{it}(z) D(p_{it}(z), P_t, Y_t) \\ &= \left[\left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} \frac{Y_t^{1+\zeta}}{z^{1+\zeta}}\right]^{\frac{1}{\epsilon(1+\zeta)}} P_t \end{aligned} \tag{73}$$

for $t \in \mathbb{T}^S$. Aggregating prices according to (12) gives:

$$Y_t = \left[\left(\frac{\epsilon - 1}{\epsilon}\right) \frac{\beta}{\gamma}\right]^{\frac{1}{1+\zeta}} \left(\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\frac{\epsilon-1}{\epsilon}} dF_i(z)\right)^{\frac{\epsilon}{\epsilon-1}} \tag{74}$$

for $t \in \mathbb{T}^S$. This way, we can retrieve the aggregate prices as:

$$P_t = \frac{M_t}{Y_t} = \left[\left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta}\right]^{\frac{1}{1+\zeta}} \frac{1}{\left(\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\frac{\epsilon-1}{\epsilon}} dF_i(z)\right)^{\frac{\epsilon}{\epsilon-1}}} M_t \tag{75}$$

for $t \in \mathbb{T}^S$. Now, using (73), and plugging (74) into it gives:

$$\theta_{it}(z) = \left(\frac{p_{it}(z)}{P_t}\right)^{1-\epsilon} = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} \hat{z}^{\frac{\epsilon-1}{\epsilon}} dF_i(\hat{z})} \tag{76}$$

for $t \in \mathbb{T}^S$. By plugging (75) and (74) in, this means that:

$$m_{it}(z) = \theta_t(z) M_t = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} \hat{z}^{\frac{\epsilon-1}{\epsilon}} dF_i(\hat{z})} M_t \tag{77}$$

$$C_{it}(z) = \frac{m_{it}(z)}{P_t} = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} \hat{z}^{\frac{\epsilon-1}{\epsilon}} dF_i(\hat{z})} Y_t \tag{78}$$

$$p_{it}(z) = \left[\frac{z}{\left(\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} \hat{z}^{\frac{\epsilon-1}{\epsilon}} dF_i(\hat{z})\right)^{\frac{\epsilon}{\epsilon-1}}}\right]^{-\frac{1}{\epsilon}} P_t \tag{79}$$

for $t \in \mathbb{T}^S$. $\square$

A.1.3 Corollary 1.3

Notice that, at $t = 1$, the cash-in-advance and the budget constraints, (2) and (3), can be written, respectively, as:

$$P_1 C_{i1}(z) = m_{it}(z) - \beta b_{i2}(z)$$
$$P_2 C_{i2}(z) = R_{i1}(z) + (1-\beta) b_{i2}(z)$$

Isolating $b_{i2}(z)$, using the fact that $P_t C_{it}(z) = P_s C_{is}(z)$ for $i \in \{h, l\}$ and $t, s = 1, 2, ...$ and re-arranging the terms gives us:

$$P_t C_{ct}(z) = (1-\beta)(1+\tau) m_0(z) + \beta R_{ct}(z)$$
$$P_t C_{ut}(z) = (1-\beta) m_0(z) + \beta R_{ut}(z)$$

Subtracting the latter expression from the former yields:

$$P_t C_{ct}(z) - P_t C_{ut}(z) = (1-\beta)\tau m_0(z) - \beta(R_{ut}(z) - R_{ct}(z))$$

Since we know that $C_{ct}(z) > (<) C_{ut}(z)$, then $P_t C_{ct}(z) - P_t C_{ut}(z) > (<) 0$ for $\tau > (<) 0$. As a result, we obtain:

$$|R_{ut} - R_{ct}| < \left| \left(\frac{1-\beta}{\beta}\right) \tau m_0(z) \right|$$

Equations (46) and (47) follows immediately from the facts that $R_{ut}(z) - R_{ct}(z) > (<) 0$ and $C_{ct}(z) > (<) C_{ut}(z)$ for $\tau > (<) 0$. □

A.2 Proposition 2

To begin, assume that, if no high-cash agent chooses to partially deplete, then $T = 1$ and, hence, the result follows trivially. I will, then, concentrate on the case where a positive mass of high-cash agents partially depletes. The case where a zero mass of high-cash agents partially depletes also follows as a combination of both cases.

Low-cash agents are more likely to fully deplete their cash at $t = 1$:

First, I need to prove that, for an arbitrary $z \in [\underline{z}, \overline{z}]$, the low-cash agent is more likely to fully deplete at $t = 1$. This requires proving that she will fully deplete if the high-cash agent chooses full depletion at $t = 1$. To see this, assume that the high-cash agent saves nothing at $t = 1$. Now, notice that $m_{h1}(z) > m_{l1}(z)$. Then:

$$C_{h1}(z) = \frac{m_{h1}(z)}{P_1} > \frac{m_{l1}(z)}{P_1} \geq C_{l1}(z)$$

By the first order condition of the, fully-depleting, high-cash agent:

$$\frac{u'(C_{h1}(z))}{P_1} \geq \beta \frac{u'(R_{h1}(z)/P_2)}{P_2},$$

but then:

$$\frac{u'(C_{l1}(z))}{P_1} > \beta \frac{u'(R_{h1}(z)/P_2)}{P_2}$$

Since $R_{l1}(z) = R_{h1}(z)$ in the case where both decide to fully deplete at $t = 1$, the full depletion condition is satisfied for the low-cash agent as well. As a result, for an arbitrary $z \in [\underline{z}, \overline{z}]$, there are three possibilities: 1) both kinds of agent fully deplete; 2) both partially deplete; or 3) only the high-cash type partially depletes.

Revenues are not lower for low-cash agents than for high-cash ones:

I will show here that $\theta_{l1}(z) \geq \theta_{h1}(z)$. We analyze each of the possible cases in turn in turn:

1) Everyone fully depletes for $z \in [\underline{z}, \overline{z}]$: Naturally, in this case, $\theta_{l1}(z) = \theta_{h1}(z)$.

2) Everyone partially depletes for $z \in [\underline{z}, \overline{z}]$: Now, we have, for $i \in \{h, l\}$:

$$\theta_{i1}(z) = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{z^{1+\zeta} u'(C_{i1}(z))}{\gamma Y_t^{\zeta}} \right]^{\frac{\epsilon-1}{1+\zeta\epsilon}}$$

Since, naturally, $C_{i1}(z)$ strictly increases on $m_{i1}(z)$ given P_1, $\theta_{l1}(z) > \theta_{h1}(z)$.

3) Only the high-cash agent partially depletes for $z \in [\underline{z}, \overline{z}]$: Notice that, in this case, we have:

$$\theta_{i1}(z) = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{z}{\gamma} \beta \frac{P_1}{P_2} \frac{u'(C_{i2}(z))}{h_{i1}^{\zeta}}) \right]^{\epsilon - 1}, \tag{80}$$

Finally, assume by contradiction that $\theta_{l1}(z) \leq \theta_{h1}(z)$. This requires $p_{l1}(z) \geq p_{h1}(z)$ and, therefore, by (13):

$$\frac{u'(C_{h2}(z))}{h_{h1}^{\zeta}} \geq \frac{u'(C_{l2}(z))}{h_{l1}^{\zeta}},$$

As shown before, for $i \in \{h, l\}$:

$$\frac{u'(C_{i2}(z))}{h_{i1}(z)^{\zeta}} = \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta z} \frac{P_2}{P_1} \right]^{\frac{\zeta\epsilon}{1+\zeta\epsilon}} Y_1^{\frac{-\zeta}{1+\zeta\epsilon}} u'(C_{i2}(z))^{\frac{1}{1+\zeta\epsilon}},$$

which, in turn, implies that $C_{l2}(z) \geq C_{h2}(z)$. Since the low-cash agent fully depletes, we must have $R_{l1}(Z) \geq P_2 C_{l2}(z)$. Moreover, $\theta_{l1}(z) \leq \theta_{h1}(z)$ implies that $R_{h1}(z) \geq$

$R_{l1}(z)$. However, since $s_{h1}(z) > 0$ and $s_{l1}(z) = 0$, $C_{l2}(z) \geq C_{h2}(z)$ cannot happen, since consumption at $t = 2$ is strictly increasing on $m_{i2}(z)$, given the price P_2. As a result, $\theta_{l1}(z) > \theta_{h1}(z)$.

I have proved above that $\theta_{l1}(z) \geq \theta_{h1}(z)$. This implies that $p_{l1}(z) \leq p_{h1}(z)$, $y_{l1}(z) \geq y_{h1}(z)$ and $R_{l1}(z) \geq R_{h1}(z)$ for all $z \in [\underline{z}, \overline{z}]$, where the inequalities hold strictly whenever the high-cash agent partially depletes. Now, I will prove, by an induction argument, that these results hold for an arbitrary $t = \{1, 2, ...\}$ in which the economy is not in a stationary equilibrium. Therefore, it suffices to show that no reversion occurs, that is, if the inequalities above hold for an arbitrary t, then $m_{h,t+1}(z) \geq m_{l,t+1}(z)$, which will, in turn, imply that $\theta_{l,t+1}(z) \geq \theta_{h,t+1}(z)$.

Again, I will show it by cases. To begin, the case where both high- and low-cash agents fully deplete their cash at t for some $z \in [\underline{z}, \overline{z}]$ is trivial, as it directly implies that $m_{l,t+1}(z) = m_{h,t+1}(z)$. Now, consider the other two cases - that is, either both types partially deplete at t or only the high-cash agent does. Assume, by contradiction, that $t+1$ is the moment of reversion in roles, meaning that $m_{lt}(z) < m_{ht}(z)$ and $m_{l,t+1}(z) > m_{h,t+1}(z)$. This means that $\theta_{lt}(z) > \theta_{ht}(z)$. This requires $C_{l,t+1}(z) < C_{h,t+1}(z)$, which contradicts our assumption of reversion. I, therefore, conclude that $\theta_{lt}(z) \geq \theta_{ht}(z)$, $p_{lt}(z) \leq p_{ht}(z)$, $y_{lt}(z) \geq y_{ht}(z)$ and $R_{lt}(z) \geq R_{ht}(z)$ for all $z \in [\underline{z}, \overline{z}]$ and for all t such that the economy is not in a stationary equilibrium.

Characterization of inflation and relative revenues:

Integrating the individual price choices and imposing the result that the full depletion first-order condition holds for a non-negligible mass of agents implies that:

$$P_t = P_{t+1} \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} \left[\sum_{i \in \{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} \frac{z^{\epsilon-1} u'(C_{i,t+1}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}} dF_i(z) \right]^{\frac{1}{1-\epsilon}} \tag{81}$$

Now, notice that we can re-write:

$$\theta_{it}(z) = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{\beta}{\gamma} \frac{P_t}{P_{t+1}} \frac{z u'(C_{i,t+1}(z))}{h_{it}(z)^{\zeta}} \right]^{\epsilon-1}$$

Therefore:

$$\frac{\theta_{it}(z)}{\theta_{i0}(z)} = \left(\frac{1}{1 + \pi_{t+1}} \right)^{\epsilon-1} \frac{z^{\epsilon-1} u'(C_{i,t+1}(z))^{\epsilon-1} / h_{it}(z)^{\zeta(\epsilon-1)}}{z^{\epsilon-1} u'(C_{io}(z))^{\epsilon-1} / h_{i0}(z)^{\zeta(\epsilon-1)}} = \frac{U^{GAP}_{i,t+1}(z)}{U^{GAP}_{t+1}} \tag{82}$$

Characterization of relative monetary holdings per income bracket:

Notice that, for any $z \in [\underline{z}, \overline{z}]$, the result above implies that $P_t C_{ht}(z) > P_t C_{lt}(z)$ and

$R_{ht}(z) < R_{lt}(z)$ for as long as the high-cash agent does not fully deplete, which means, by (3), that:

$$\Delta m_{h,t+1}(z) = R_{ht}(z) - P_t C_{ht}(z) < R_{lt}(z) - P_t C_{lt}(z) = \Delta m_{l,t+1}(z), \tag{83}$$

where $\Delta m_{i,t+1}(z) = m_{i,t+1}(z) - m_{it}(z)$ for $i \in \{h, l\}$.

<u>The economy converges to the new stationary equilibrium in finite time:</u>

Assume, by contradiction, that the mass of high-cash partially depleting agents remains forever bounded away from 0. Then, for these agents:

$$\lim_{t\to\infty} \frac{u'(C_{ht}(z))}{P_t} = \lim_{t\to\infty} \frac{u'(C_{h1}(z))}{\beta^{t-1} P_1} = \infty,$$

which implies that either $P_t \to 0$ or $C_{ht}(z) \to 0$. However, notice that, for partially depleting high-cash agents (who exist in positive mass by the contradiction assumption), $p_{ht}(z) > p_{lt}(z) \geq 0$, which implies that $P_t > 0$ for every $t = \{1, 2, ...\}$ by our contradiction assumption. This implies that $C_{ht} \to 0$, which cannot be the case either, since $m_{ht}(z) > m_{lt}(z) \geq 0$, which means that positive consumption is always available for these high-cash agents.

This proves that the mass of partially depleting agents goes to 0 in finite time, meaning that, at some time $T < \infty$, the economy reaches the new fundamental stationary equilibrium, in which the aggregate price is equal to $P_T = P^H(M_T) > 0$. This ensures that no agent, even with negligible mass, can partially deplete forever. Naturally, for any high-cash agent that partially depletes until $T - 1$, it is the case that $m_{hT}(z) > m_{lT}(z)$, meaning that $C_{hT}(z) = m_{hT}(z)/P^H((1 + \tau^A)M_0) > C_{h0}(z)$. As a result, consumption only returns to the stationary equilibrium level for all agents at $T + 1$. Finally, $Y_T = M_T / P^H(M_T) = Y_0$. □

A.3 Proposition 3

<u>There is a non-negligible mass of fully depleting agents:</u>

Now, I go on to prove that there is a non-negligible mass of fully depleting agents. Assume otherwise by means of contradiction. I begin by showing that, if there is no positive mass of fully depleting agents at t, then this must also be the case for $t + 1$. Again, by contradiction, assume that there is some low-cash agent that fully depletes at $t + 1$. This suffices as we have already shown that low-cash agents are more likely to fully deplete. As will be shown in more details below, due to the homothety of preferences, $R_{lt}(z) = \overline{R_{lt}} z^{\frac{\epsilon-1}{\epsilon}} / \mathcal{Z}_l^{\frac{\epsilon-1}{\epsilon}}$ and $C_{lt}(z) = \overline{C_{lt}} z^{\frac{\epsilon-1}{\epsilon}} / \mathcal{Z}_l^{\frac{\epsilon-1}{\epsilon}}$, where $\overline{R_{lt}}$ and $\overline{C_{lt}}$ are respectively the average revenue and consumption among low-cash agents[10]. As a result:

$$(\overline{R_{lt}} - P_t\overline{C_{lt}})\frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_l^{\frac{\epsilon-1}{\epsilon}}} = R_{lt}(z) - P_tC_{lt}(z) = \Delta m_{l,t+1}(z)$$

Now, notice that, since $\Delta m_{l,t+1}(z)$ must aggregate to 0 and that $\Delta m_{l,t+1}(z) > \Delta m_{h,t+1}(z)$ for every $z \in [\underline{z}, \overline{z}]$ according to Proposition 2, then it must be the case that $\Delta m_{l,t+1}(z) > 0$ for all $z \in [\underline{z}, \overline{z}]$. Now, notice that, for the low-cash agent that partially depletes her money at t, but fully depletes at $t+1$, it must be the case, by the first-order condition, that:

$$P_{t+1}C_{l,t+1}(z) = \beta P_tC_{lt}(z) < m_{lt}(z) < m_{l,t+1}(z),$$

a contradiction. Therefore, all low-cash agents that partially deplete at t must also partially deplete at $t+1$. As a consequence, if there is no positive mass of partially depleting agents at t, then there must be no positive mass of partially depleting agents at $t+1$ as well. This, however, means that the economy will never converge to the fundamental stationary equilibrium, which contradicts Proposition 2. Therefore, there must be, at any given t, a positive mass of fully depleting agents.

Relative revenues obtained by low- and high-cash agents:

To begin, notice that, since 1) some agents need to fully deplete in equilibrium, and 2) low-cash agents are more likely to do so, then a non-negligible mass of low-cash agents must choose full depletion. However, due to the homothety of preferences, this means that all of the low cash agents choose to do so too. I will prove so, by showing that all low-cash agents must choose to fully deplete whenever any of them finds it optimal to do so. I start by showing that, for any fully depleting low-cash agent, $p_{lt}(z)$ must satisfy (73). Therefore, using (11), (14) and the fact that $C_{l,t+1}(z) = \theta_{lt}\frac{M_t^C}{P_{t+1}}$, we obtain, for the low-cash agents:

$$\begin{aligned}\theta_{lt}(z) &= \left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}\frac{z}{\left(\theta_{lt}(z)^{\frac{\epsilon}{\epsilon-1}}Y_t/z\right)^{\zeta}}\frac{P_t}{P_t+1}\frac{P_{t+1}}{\theta_{lt}(z)M_t^C}\right]^{\epsilon-1} \\ &= \left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}\frac{1}{Y_t^{1+\zeta}}\right]^{\frac{\epsilon-1}{\epsilon(1+\zeta)}} z^{\frac{\epsilon-1}{\epsilon}} \qquad (84)\end{aligned}$$

[10] I will show later in this proof that this occurs both for fully and partially depleting agents, and it means that all agents of the same type, that is, either low- or high-cash, must make the same choice between full and partial depletion.

Notice that it is decreasing on aggregate output. Now, notice that, at $t = 1$, full depletion requires:

$$\frac{1}{\beta}\theta_{l1}(z)M_1^C = \frac{1}{\beta}R_{l1}(z) \geq m_{l1}(z) = (1 + \mathbb{1}_{\tau<0}\tau)\theta_{l0}(z)M_0$$
$$\frac{1}{\beta}Y_t^{\frac{1-\epsilon}{\epsilon}}M_t^C \geq \frac{1}{\beta}Y_0^{\frac{1-\epsilon}{\epsilon}}M_0,$$

where $\mathbb{1}_{\tau<0}$ takes the value of 1 in the case of a contractionary shock, and 0 otherwise. Notice that this condition does not depend on z, meaning that it holds for all low-cash agents. A simple proof by induction generalizes this result for an arbitrary t along the transition toward the new stationary equilibrium. Now, I look into high-cash agents. Notice that, by imposing the partial depletion first-order condition to (13) and computing the resulting relative revenue, we obtain:

$$\theta_{ht}(z) = \left[\left(\frac{\epsilon - 1}{\epsilon}\right)\frac{z^{1+\zeta}}{\gamma C_{ht}(z)}\frac{1}{Y_t^{\zeta}}\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}} \tag{85}$$

Thus, it decreases their current consumption. As before, homothety implies that all high-cash agents choose to save a positive amount whenever one of them does so. A similar argument to the one made above suffices to prove it, as the condition for partial depletion by high cash agents does not depend on z. I will now arrive at an expression for the consumption, at $t = 1$, of any high-cash agent as a function of the contemporaneous average high-cash consumption.

Average individual variables

Notice that the intertemporal budget constraint of any high-cash agent along the transition path can be written as:

$$P_1 C_{h1}(z)\sum_{t=1}^{T}\beta^{t-1} = m_{h1}(z) + \sum_{t=1}^{T-1}\theta_{ht}(z)P_t Y_t,$$

where I have imposed the partial depletion first-order condition. Moreover, we can also impose it onto (85), which yields:

$$C_{h1}(z) = \left(\frac{1-\beta}{1-\beta^T}\right)\frac{1}{P_1}$$
$$\left\{(1 + \mathbb{1}_{\tau>0}\tau)m_0(z) + \left(\frac{z^{1+\zeta}}{C_{h1}(z)}\right)^{\frac{\epsilon-1}{1+\zeta\epsilon}}\left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{1}{\gamma P_1}\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}}\sum_{t=1}^{T-1}\beta^{\frac{(t-1)(1-\epsilon)}{1+\zeta\epsilon}}P_t^{\epsilon}Y_t^{\frac{1+\zeta}{1+\zeta\epsilon}}\right\},$$

which, using (22), can be simplified to:

$$C_{h1}(z) = Az^{\frac{\epsilon-1}{\epsilon}} + Bz^{\frac{(1+\zeta)(\epsilon-1)}{1+\zeta\epsilon}}C_{h1}(z)^{\frac{1-\epsilon}{1+\zeta\epsilon}}, \tag{86}$$

where A and B correspond to expressions that include only parameters and aggregate variables, which are common across all high-cash agents. Now, one can assume that $C_{h1}(z) = \overline{C_{h1}} z^D$, where $\overline{C_{h1}}$ is the average consumption by high-cash agents, and D is a constant. By plugging this in the equation above, we obtain:

$$C_{h1}(z) = \overline{C_{h1}} \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_h^{\frac{\epsilon-1}{\epsilon}}},$$

where $\overline{C_{h1}} = C_{h1}(\mathcal{Z}_h)$. By the first-order condition of partially depleting agents, this means that the whole consumption path is the same across high-cash agents as well as the moment where they decide to fully deplete, T. This, in turn, implies that $\theta_{ht}(z) = \overline{\theta_{ht}} \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_h^{\frac{\epsilon-1}{\epsilon}}}$, $m_{ht}(z) = \overline{m_{ht}} \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_h^{\frac{\epsilon-1}{\epsilon}}}$, $\overline{\theta_{ht}} = \theta_{ht}(\mathcal{Z}_h)$, and $\overline{m_{ht}} = m_{ht}(\mathcal{Z}_h)$. Moreover, (84) implies that $X_{l1}(z) = \overline{X_{l1}} \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_i^{\frac{\epsilon-1}{\epsilon}}}$ and $\overline{X_{l1}} = X_{l1}(\mathcal{Z}_h)$ for $X \in \{C, \theta, m\}$ as well.

Output and consumption paths:

The results above imply that $\theta_{lt}(z) > \theta_0(z) > \theta_{ht}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $t = \{1, ..., T-1\}$. In turn, equations (84) and (85) imply, respectively, that $Y_t < Y_0$ and $C_{ht}(z) > C_0(z) > C_{lt}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and t along the transition to the new stationary equilibrium, where the latter inequality relies on the former. Moreover, notice that, by (13):

$$p_{it}(z) = \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{z^{1+\zeta}\beta} P_t^{\zeta\epsilon} Y_t^{\zeta} P_{t+1} C_{i,t+1}(z) \right]^{\frac{1}{1+\zeta\epsilon}}$$

for $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$. Aggregating it, and computing the corresponding $\theta_{it}(z)$ gives:

$$\theta_{it}(z) = \frac{\left(\frac{z^{1+\zeta}}{P_{t+1} C_{i,t+1}(z)} \right)^{\frac{\epsilon-1}{1+\zeta\epsilon}}}{\sum_{j\in\{h,l\}} \eta_j \int_{\underline{z}}^{\overline{z}} \left(\frac{\hat{z}^{1+\zeta}}{P_{t+1} C_{j,t+1}(\hat{z})} \right)^{\frac{\epsilon-1}{1+\zeta\epsilon}} dF_j(\hat{z})}, \tag{87}$$

which means that, for any $z \in [\underline{z}, \overline{z}]$:

$$\frac{\theta_{lt}(z)}{\theta_{ht}(z)} = \left(\frac{P_{t+1} C_{h,t+1}(z)}{P_{t+1} C_{l,t+1}(z)} \right)^{\frac{\epsilon-1}{1+\zeta\epsilon}}$$

Since $R_{lt}(z) > R_{ht}(z)$ and $P_t C_{lt}(z) < P_t C_{ht}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $t = \{1, ..., T-1\}$, then we must have $R_{lt}(z) - P_t C_{lt}(z) > 0 > R_{ht}(z) - P_t C_{ht}(z)$ given the homothety of the utility function. As a result, $m_{l,t+1}(z) > m_{lt}(z)$ and $m_{h,t+1}(z) < m_{ht}(z)$ for every $z \in [\underline{z}, \overline{z}]$. This means that, for low-cash agents, $P_t C_{lt}(z) = m_{lt}(z)$ grows over time. For high-cash agents, on the other hand, $P_t C_{lt}(z) = \beta P_{t-1} C_{h,t_1}(z)$ for $t \in \{2, 3, ..., T-1\}$, meaning that it decreases over time. As a result, $\theta_{l0}(z)/\theta_{h0}(z) \leq \theta_{l,t+1}(z)/\theta_{h,t+1}(z) < \theta_{lt}(z)/\theta_{ht}(z)$ for $t \in \{1, 2, ..., T-1\}$.

Due to the homothety of preferences and the fact that $\theta_{it}(z)$ must integrate to 1, this means that $\theta_0(z) \leq \theta_{l,t+1}(z) < \theta_{lt}(z)$ and $\theta_0(z) \geq \theta_{h,t+1}(z) > \theta_{ht}(z)$. By (84), this implies that $Y_0 \geq Y_{t+1} > Y_t$ with strict inequality for $t = \{1, ..., T-2\}$ and equality for $t = T-1$. Finally, with this in mind, (85) implies that $C_0(z) \leq C_{h,t+1}(z) < C_{ht}(z)$.

Characterization of T:

Notice that $P_T C_{hT}(z) < \beta^{T-1} m_{h1}(z)$ for $z \in [\underline{z}, \overline{z}]$ by the first-order condition and budget constraint of high-cash agents. Given that, as proven in Proposition 2, $C_{hT}(z) > \theta_0(z)(M_0 + \eta\tau M_{c0})/P_T$, we obtain the condition:

$$\theta_0(z)(1+\tau^A)M_0 < \beta^{T-1} P_1 C_{h1}(z) < \beta^{T-1}\theta_0(z)(1+\mathbb{1}_{\tau>0}\tau)M_0$$

$$\beta^{T-1} > \frac{1+\tau^A}{1+\mathbb{1}_{\tau>0}\tau} \tag{88}$$

Let the maximum value of T that satisfies this condition be denoted by T^{MAX}. Then, $T \leq T^{MAX}$.

Characterization of prices:

Since low-cash agents always fully deplete their resources, we must have that, for $t = \{1, 2, ...T-1\}$:

$$P_{t+1} C_{l,t+1}(z) = R_{lt}(z) = \theta_{lt}(z) M_t^C > \theta_0(z) M_t^C$$

which means that:

$$p_{ht}(z) > p_{lt}(z) > p^H(z, M_t^C). \tag{89}$$

This means that $P(M_t^C) > P^H(M_t^C)$. Moreover, notice that (13) implies that:

$$p_{lt}(z) = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\frac{\left(\frac{y_{lt}(z)}{z}\right)^{\zeta}}{z} p_{lt}(z) y_{lt}(z) \quad \Longleftrightarrow \quad y_{lt}(z) = z\left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}\right]^{\frac{1}{1+\zeta}}, \tag{90}$$

meaning that $h_{l,t+1}(z) = h_{lt}(z) = h_{lt}(z')$ for all periods and any $z, z' \in [\underline{z}, \overline{z}]$. However, since low-cash agents fully deplete, this means that their prices are proportional to $p_{lt}(z) y_{lt}(z) = m_{l,t+1}(z)$. As a result, $p_{lt}(z) \leq p_{l,t+1}(z)$, with strict inequality for $t \in \{1, 2, ..., T-1\}$.

Lastly, by an argument already made in the proof of Proposition 1, one can show that:

$$\left[\sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} \frac{z^{\frac{\epsilon-1}{1+\zeta\epsilon}}}{C_{it}(z)^{\frac{\epsilon-1}{1+\zeta\epsilon}}} dF_i(z)\right]^{\frac{1+\zeta\epsilon}{\epsilon-1}} > \left(\frac{\epsilon}{\epsilon-1}\right)\gamma Y_t^{\zeta}$$

for every $t \in \{1, 2, ...\}$, where the strong inequality follows from the fact that, by the first-order condition of low-cash agents, $C_{l,t+1}(z) > \beta \frac{P_t}{P_{t+1}} C_{lt}(z)$. We can plug this into the aggregate price expression (63), which gives us:

$$\frac{P_{t+1}}{P_t} > \beta \left(\frac{Y_{t+1}}{Y_t} \right)^{\zeta} \geq \beta, \tag{91}$$

where the last inequality holds strictly for $t \in \{1, 2, ..., T-1\}$, since $Y_{t+1} > Y_t$ then. □

A.4 Proposition 4

I want to show that $T = \infty$ is not possible. I will do so through a simple proof by contradiction. Assume that $T = \infty$. As the borrowing constraint only binds for low-cash agents, we have:

$$\frac{u'(C_{ht}(z))}{P_t} = \frac{\beta}{q_t} \frac{u'(C_{h,t+1}(z))}{P_{t+1}} \quad \text{and} \quad \frac{u'(C_{lt}(z))}{P_t} > \frac{\beta}{q_t} \frac{u'(C_{l,t+1}(z))}{P_{t+1}} \tag{92}$$

which is valid for any $t = 1, 2, ...$ if $q_t \neq \beta$. This means that, (18) can be re-written as:

$$\frac{P_{t+1}}{P_t} > \frac{\beta}{q_{t+1}} \tag{93}$$

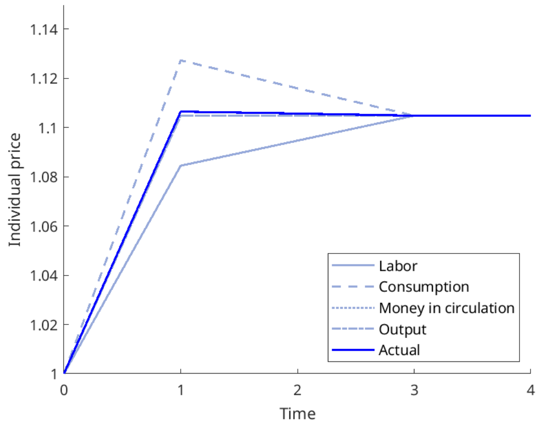
which follows from the fact that $\overline{U}^{GAP}_{t+2} < \overline{U}^{GAP}_{t+1} \frac{\beta}{q_{t+1}} \frac{P_{t+1}}{P_{t+2}}$ by the first-order conditions of above. Now, notice that that, by our simplifying assumption that $q_t = q_{t+1}$, which means that $P_{t+1} q_t / \beta > P_t$. This can be plugged into the first-order condition for the high-cash agent, as in (92), to obtain $u'(C_{ht}(z)) < u'(C_{h,t+1}(z))$, which means that $C_{h,t+1}(z) < C_{ht}(z)$ for all $t \in \{1, 2, ...\}$. I will now show that $\lim_{t\to\infty} C_{ht}(z) = 0$ for all high-cash agents. Assume that there is some high-cash agent with productivity $z \in [\underline{z}, \overline{z}]$ whose consumption does not converge to 0. Then, let $\lim_{t\to\infty} C_{ht}(z) =: C^{MIN}_{hT}(z) > 0$, meaning that $C^{MIN}_{hT}(z) = C^{MIN}_{h,T+1}(z)$[11] This means that, by (92), at the limit, $1 + \pi_{T+1} = \beta / q_T < 1$, meaning that the deflation rate remains bounded away from zero even at the limit. This, in turn, means that $P_t \to 0$ and, thus, by (59), we must have:

$$\lim_{t\to\infty} \sum_{i \in \{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} C_{it}(z) dF_i(z) = \infty,$$

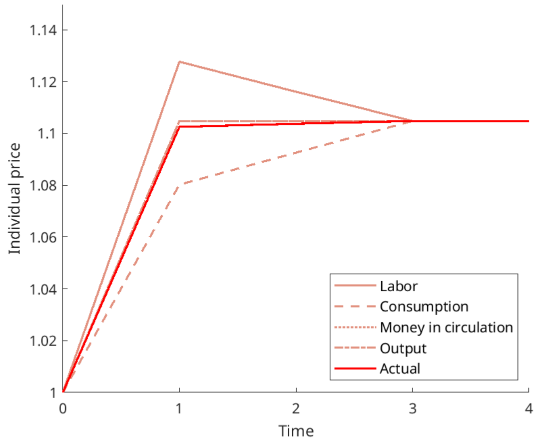
which cannot happen, as the consumption of all high cash agents is finite and below $C_{h1}(z)$. Hence (93) implies that $\lim_{t\to\infty} C_{ht}(z) = 0$ for every $z \in [\underline{z}, \overline{z}]$. This is a contradiction, however, since $C_{ht}(z) > C_{lt}(z) \geq 0$ for as long as high-cash agents do not fully deplete, which leads to the conclusion that $T < \infty$.

[11] I look at the limit for simplicity, but this intuitively means that, after some $t < \infty$, the differences between $C_{ht}(z)$ and $C_{h,t+1}(z)$ are nearly zero.

B Outstanding Graphs



(a) Connected agents' prices, $p_{ct}(\mathcal{Z})$



(b) Unconnected agents' prices, $p_{ut}(\mathcal{Z})$

Figure 6: Decomposition of individual prices in the baseline economy

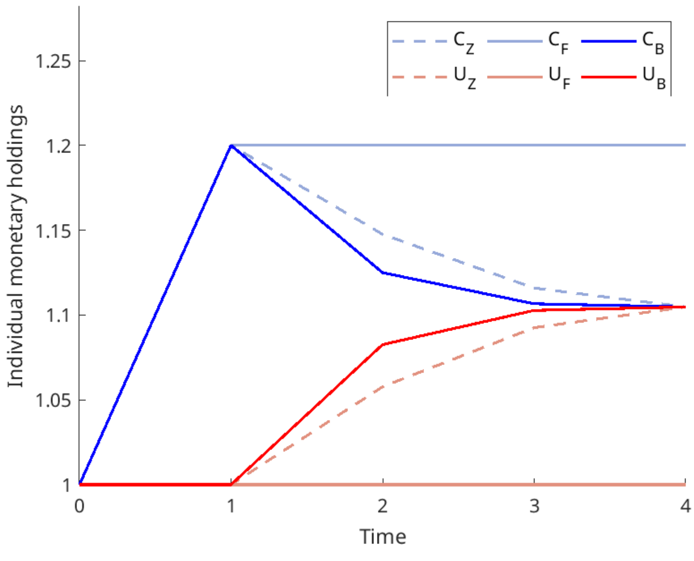
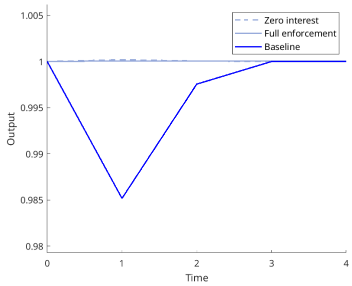
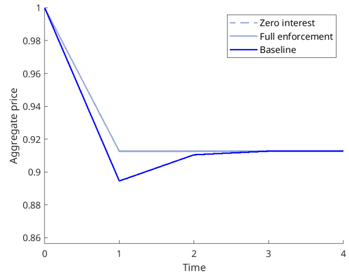


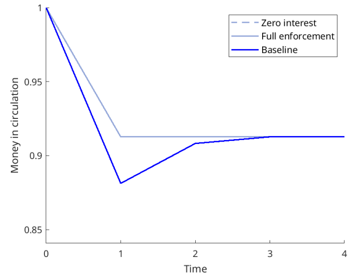
Figure 7: Comparison of monetary holdings, m_t, across agents



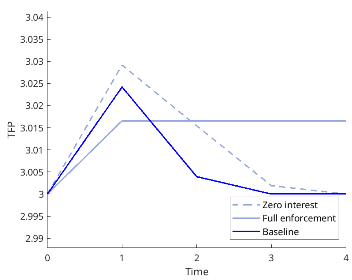
(a) Path of output, Y_t



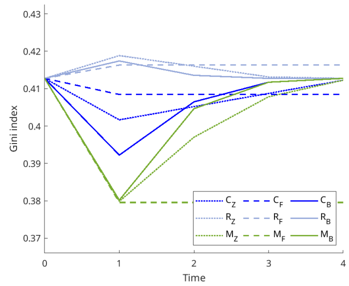
(b) Path of output, P_t



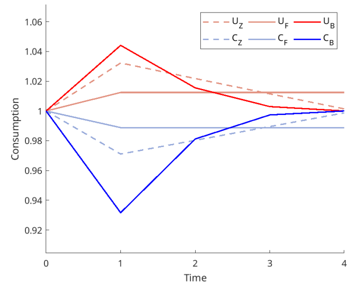
(c) Money in circul., M_t^C



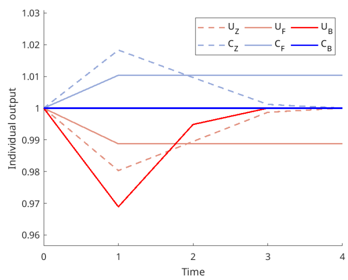
(d) Total Factor Productivity



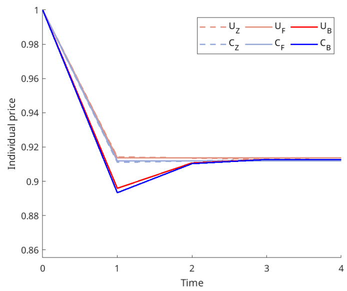
(e) Gini indexes



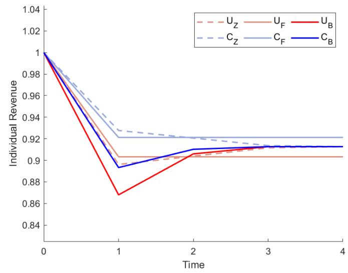
(f) Indiv. consumption, $C_{it}(z)$



(g) Individual output, $y_{it}(z)$

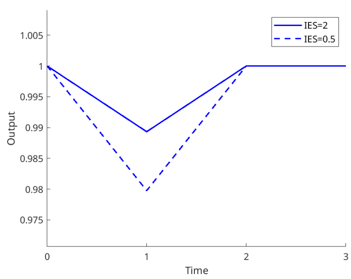


(h) Individual prices, $p_{it}(z)$

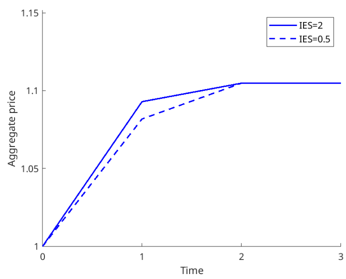


(i) Indiv. revenues, $R_{it}(z)$

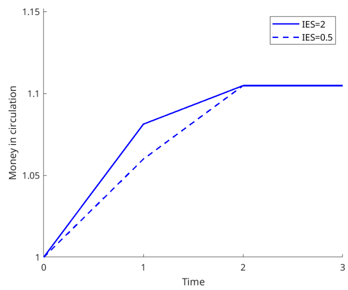
Figure 8: Paths for aggregate and individual variables under a negative shock



(a) Path of output, Y_t

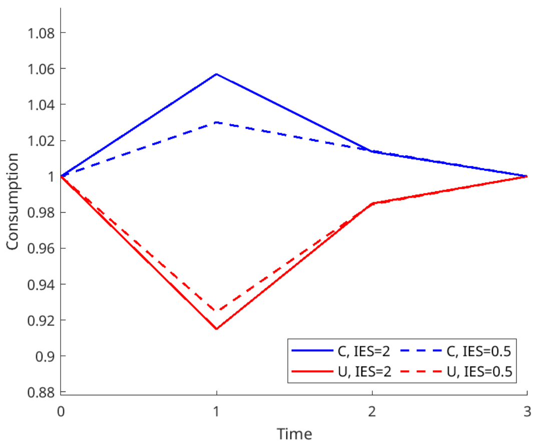


(b) Aggregate price, P_t

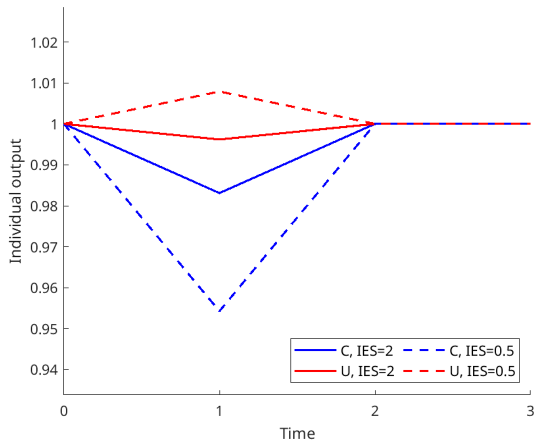


(c) Money in circul., M_t^C

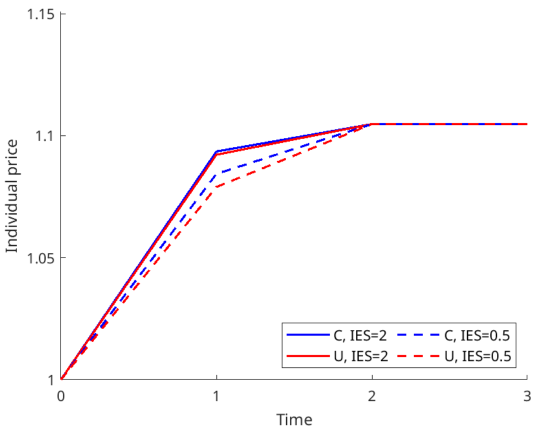
Figure 9: Paths for aggregate variables under CRRA utility



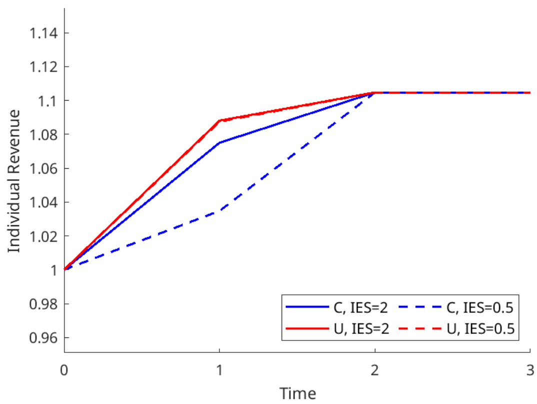
(a) Individual consumption, $C_{it}(\mathcal{Z})$



(b) Individual output, $y_{it}(\mathcal{Z})$

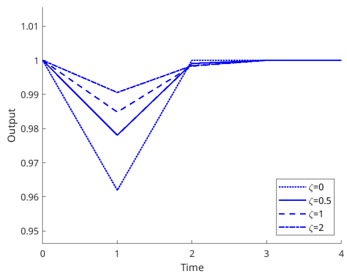


(c) Individual prices, $p_{it}(\mathcal{Z})$

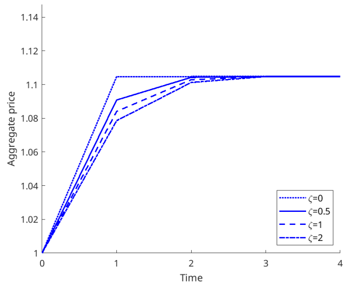


(d) Indiv. revenues, $R_{it}(\mathcal{Z})$

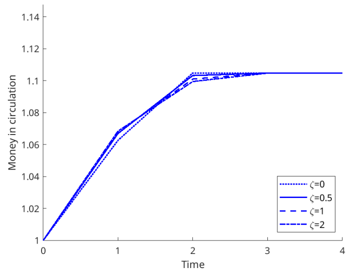
Figure 10: Paths for individual variables under CRRA utility



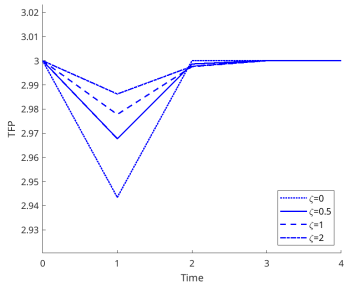
(a) Path of output, Y_t



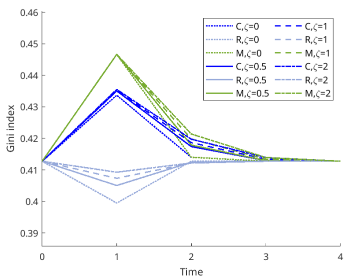
(b) Aggregate price, P_t



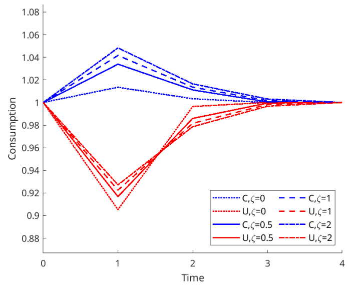
(c) Money in circulation, M_t^C



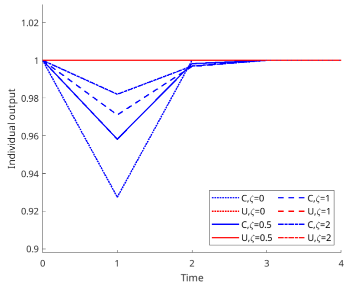
(d) Total Factor Productivity



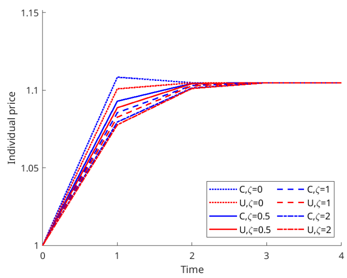
(e) Gini indexes



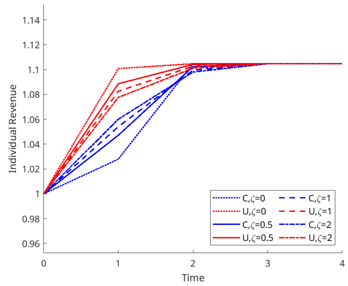
(f) Indiv. consumption, $C_{it}(\mathcal{Z})$



(g) Individual output, $y_{it}(\mathcal{Z})$

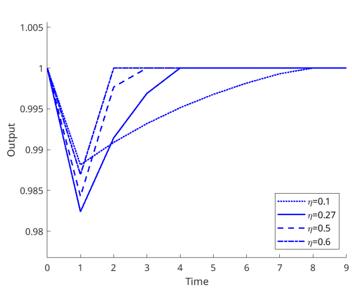


(h) Individual prices, $p_{it}(\mathcal{Z})$

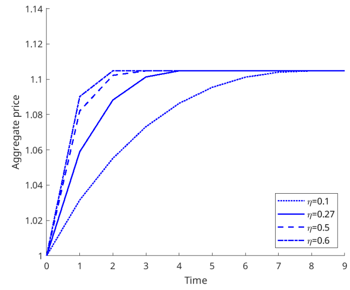


(i) Indiv. revenues, $R_{it}(\mathcal{Z})$

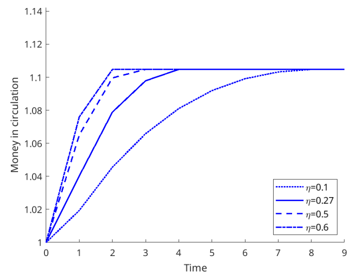
Figure 11: Paths for aggregate and individual variables under different Frisch elasticities



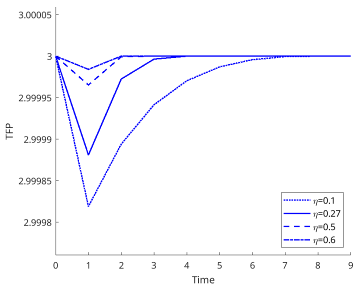
(a) Path of output, Y_t



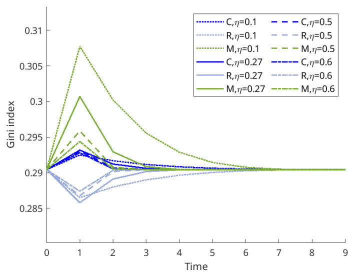
(b) Aggregate price, P_t



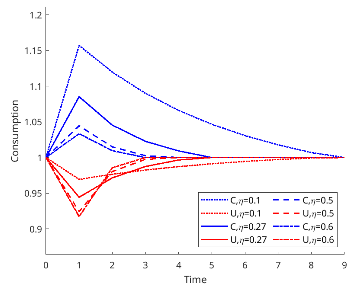
(c) Money in circulation, M_t^C



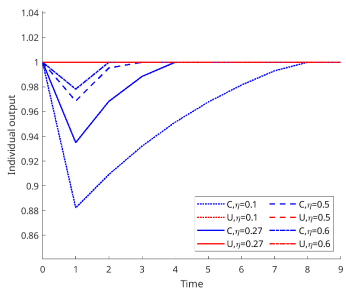
(d) Total Factor Productivity



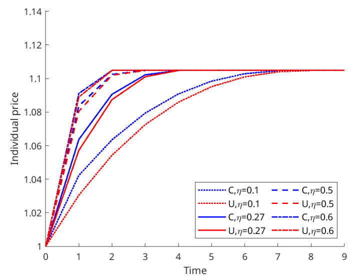
(e) Gini indexes



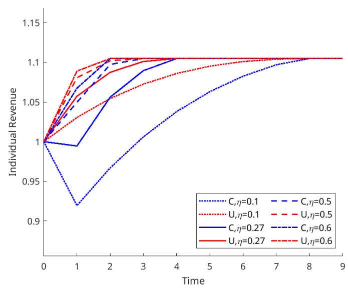
(f) Indiv. consumption, $C_{it}(\mathcal{Z})$



(g) Individual output, $y_{it}(\mathcal{Z})$

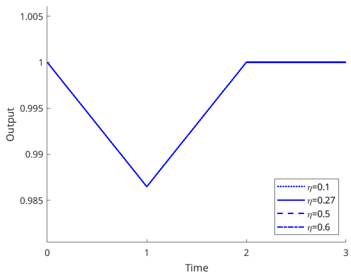


(h) Individual prices, $p_{it}(\mathcal{Z})$

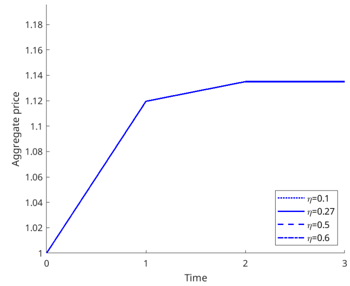


(i) Indiv. revenues, $R_{it}(\mathcal{Z})$

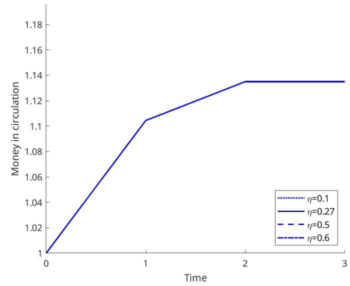
Figure 12: Paths for aggregate and individual variables for different values of η and τ



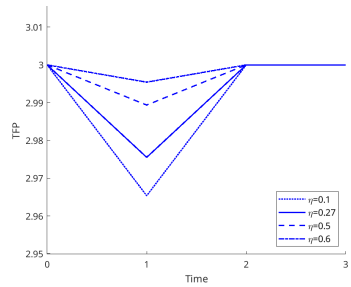
(a) Path of output, Y_t



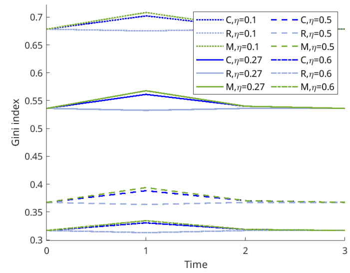
(b) Aggregate price, P_t



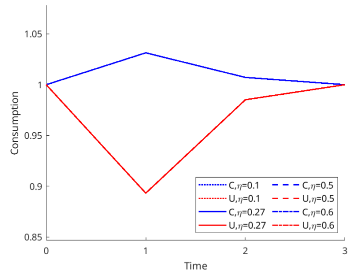
(c) Money in circulation, M_t^C



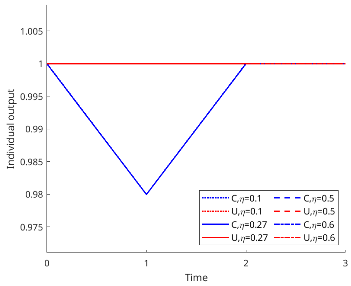
(d) Total Factor Productivity



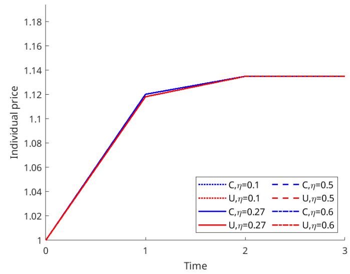
(e) Gini indexes



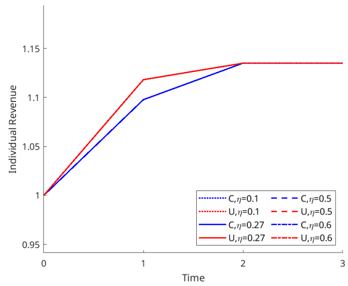
(f) Indiv. consumption, $C_{it}(\mathcal{Z})$



(g) Individual output, $y_{it}(\mathcal{Z})$



(h) Individual prices, $p_{it}(\mathcal{Z})$



(i) Indiv. revenues, $R_{it}(\mathcal{Z})$

Figure 13: Paths for aggregate and individual variables for different values of η and $\frac{M_{c0}}{M_0}$

C Outstanding Tables

Quarter	ΔM_t
1999-Q4	10.0036%
2008-Q4	83.1813%
2009-Q4	12.4917%
2011-Q1	18.7556%
2011-Q2	10.5707%
2020-Q1	13.3256%
2020-Q2	28.8095%
2021-Q1	12.1483%
2022-Q2	-10.2375%

Table 4: Shocks to the U.S. monetary base of more than 10%

Source: Board of Governors of the Federal Reserve System (US), retrieved from FRED, Federal Reserve Bank of St. Louis

Fraction of Connected	Model	Constant output	No inequality
$M_{c0}/M_0 = 1.94$	-4.9684%	-4.8068%	-0.1905%
$M_{c0}/M_0 = 1$	-0.3965%	-0.2067%	-0.1905%
$M_{c0}/M_0 = 0.6$	6.2557%	6.4881%	-0.1905%

The second column shows the welfare in the economy that does not receive a monetary shock. The third column contains the values for the welfare function under the benchmark specification of the baseline economy. The fourth column presents the counterfactual exercise of assuming that output is constant at the initial level, but keeping the degree of inequality across the connected and unconnected agents. The last column stands for the opposite exercise: it removes inequality between connected and unconnected agents with the same productivity but maintains the fall in output.

Table 5: Counterfactual welfare analysis of the baseline economy

Fraction of Connected	Model	Counterfactual
$M_{c0}/M_0 = 1.94$	-4.4928%	-0.091437%
$M_{c0}/M_0 = 1$	-0.0021272%	-4.23e-05%
$M_{c0}/M_0 = 0.6$	6.5237%	0.12584%

The second column shows the welfare in the economy that does not receive a monetary shock. The third column contains the values for the welfare function under the benchmark specification of the full enforcement economy. The counterfactual corresponds to the exercise of assuming that, after period T, as the baseline economy returns to equilibrium, the full enforcement economy returns as well.

Table 6: Counterfactual welfare analysis of the full enforcement economy

Fraction of Connected	Model	Baseline output	Baseline inequality	No inequality
$M_{c0}/M_0 = 1.94$	-4.5545%	-4.7108%	-5.3741%	0.0026%
$M_{c0}/M_0 = 1$	-0.1%	-0.2923%	-0.2167%	0.0026%
$M_{c0}/M_0 = 0.6$	6.3706%	6.1238%	7.342%	0.0026%

The second column shows the welfare in the economy that does not receive a monetary shock. The third column contains the values for the welfare function under the benchmark specification of the zero interest rate economy. The fourth column stands for the exercise of keeping the degree of consumption and labor inequality in the zero interest rate, but imposing that the aggregate output be equal to the one in the baseline economy. The penultimate column stands for the opposite exercise: keeping the output level, but modifying consumption inequality between connected and unconnected. The last column stands for the counterfactual removal of inequality across connected and unconnected agents with the same productivity.

Table 7: Counterfactual welfare analysis of the zero interest rate economy