Fundamentals of Learning and Information Processing

Session 12: Statistical Machine Learning (II)

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PAC Learning

The learner's task is to:

Input: training data $S = \{(x_1, y_1), \cdots, (x_m, y_m)\} \in (\mathscr{X} \times \mathscr{Y})^m$

Output: prediction rule $h : \mathscr{X} \to \mathscr{Y}$

Measure The error of a prediction rule $h : \mathscr{X} \to \mathscr{Y}$ can be defined as:

Generalization risk $L_{(\mathscr{D},f)}(h) \stackrel{def}{=} P_{x\sim\mathscr{D}}[h(x) \neq f(x)] \stackrel{def}{=} \mathscr{D}(\{x : h(x) \neq f(x)\})$

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The Statistical Learning Framework

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ERM

ERM comes up with a predictor h that minimizes $L_S(h)$

$$ERM_{\mathscr{Y}^{\mathscr{X}}}(S) \in \underset{h \in \mathscr{Y}^{\mathscr{X}}}{\operatorname{argmin}} L_S(h)$$

ERM with Inductive Bias comes up with any $h \in \mathscr{H}$ that minimizes $L_S(h)$

$$ERM_{\mathscr{H}}(S) \in \underset{h \in \mathscr{H}}{\operatorname{argmin}} L_S(h)$$

Can only be *Approximately* correct

For any training data S with m i.i.d. examples, we should not hope find an h s.t. $L_{(\mathscr{D},f)}(h) = 0$

Proof.

- For every $\epsilon \in (0,1)$ take $\mathscr{X} = \{x_1, x_2\}$ and $\mathscr{D}(\{x_1\}) = 1-\epsilon$, $\mathscr{D}(\{x_2\}) = \epsilon$
- The probability not to see x_2 at all among m i.i.d. examples in S is $(1-\epsilon)^m \approx e^{-\epsilon m}$
- So if $\epsilon \ll \frac{1}{m}$ we are likely not to see x_2 at all, but then we can not know its label.

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Relaxation.

- We would be happy with $L_{(\mathscr{D},f)}(h) < \epsilon$, where ϵ is the user-specified accuracy parameter.

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Can only be *Probably* correct

For any training data S with m i.i.d. examples, no algorithm can guarantee $L_{(\mathscr{D},f)}(h) \leq \epsilon$

Proof.

- Recall that the input to the learner is a set of randomly generated examples, there is always a (very small) chance to see the same example again and again.

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Relaxation.

- We would allow the algorithm to fail with probability δ, where $\delta \in (0,1)$ is the user-specified confidence parameter
- Here, the probability is over the random choice of examples

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Probably Approximately Correct (PAC) Learnability

A hypothesis class $\mathscr{H}$ is PAC learnable if there exists a function $m_{\mathscr{H}} : (0,1)^2 \to \mathscr{N}$ and a learning algorithm with the following property:

- For every $\epsilon, \delta \in (0,1)$, for every distribution $\mathscr{D}$ over $\mathscr{X}$, and for every labelling function $f : \mathscr{X} \to \{0,1\}$, if the realizable assumption holds with respect to $\mathscr{H}$, $\mathscr{D}$ and f, then when we run the algorithm on $m \geq m_{\mathscr{H}}(\epsilon, \delta)$ i.i.d. examples generated by $\mathscr{D}$ and labelled by f, the algorithm returns a hypothesis h such that, with probability of at least $(1-\delta)$, $L_{(\mathscr{D},f)}(h) \leq \epsilon$.

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Key Points.

- It is a distribution free model, i.e. no particular assumption about $\mathscr{D}$
- Training and test samples are drawn according to the same $\mathscr{D}$ (otherwise transfer learning)
- It deals with the question of learnability for $\mathscr{H}$, not a particular concept, namely the "*target labelling function*" f.

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- The learner should output a hypothesis h, s.t. with probability of at least $(1-\delta)$ it holds that $L_{(\mathcal{D},f)}(h) \leq \epsilon$.
  - the learner should be **P**robably (with probability at least $(1-\delta)$) **A**pproximately (up to accuracy ϵ) **C**orrect

Sample Complexity

The function $m_{\mathscr{H}} : (0,1)^2 \to \mathscr{N}$ determines the sample complexity of learning $\mathscr{H}$, namely, $m_{\mathscr{H}}(\epsilon, \delta)$ represents how many examples are required to guarantee a PAC solution:

- It is a function of the *accuracy* parameter ϵ and the *confidence* parameter δ
- It also depends on the properties of the hypothesis class $\mathscr{H}$.
  - If $\mathscr{H}$ is PAC learnable, there are many functions $m_{\mathscr{H}}$ that satisfy the requirements given in the PAC learnability definition.
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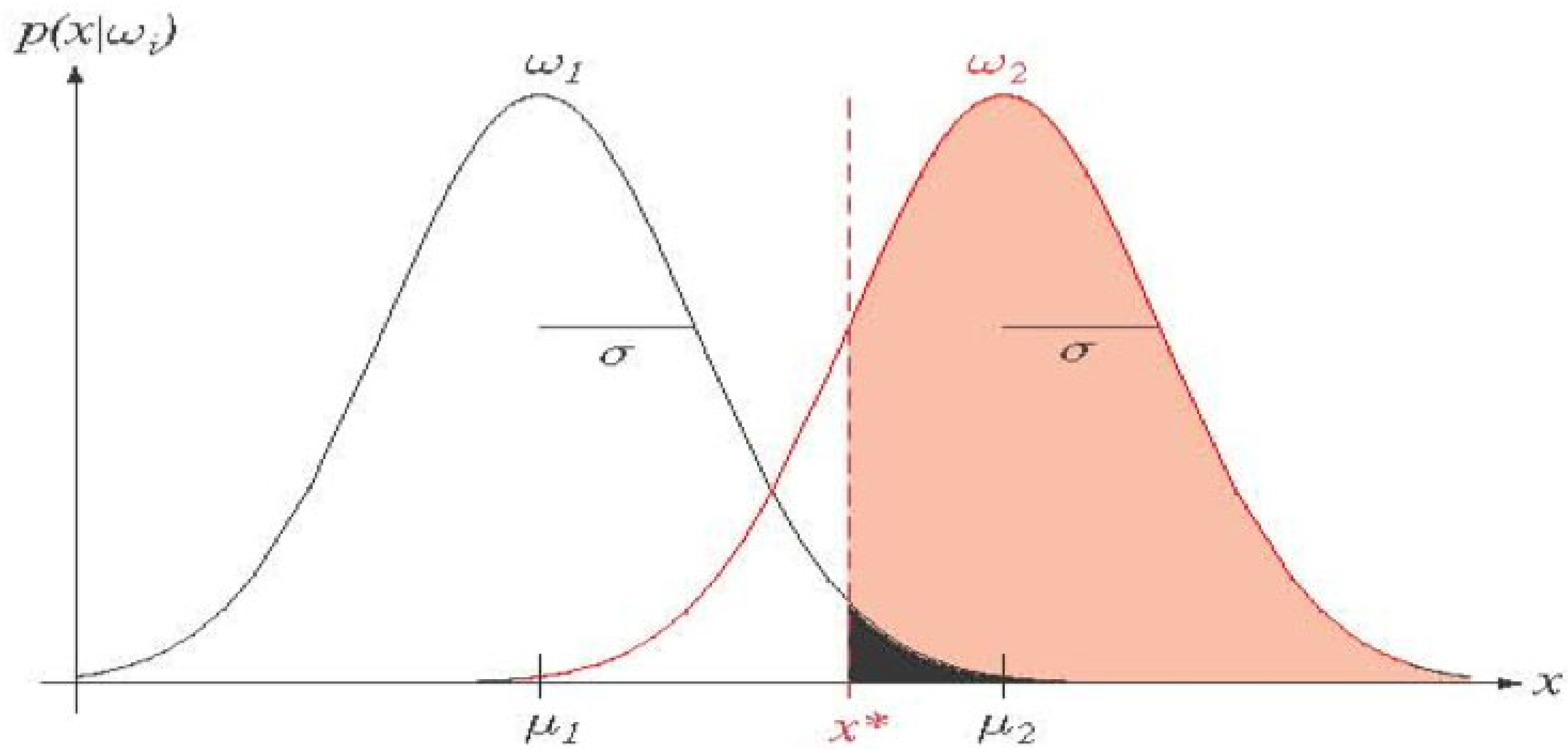
Every finite hypothesis class $\mathcal{H}$ is PAC learnable with the sample complexity:

$$m_{\mathcal{H}}(\epsilon, \delta) \leq \lceil \frac{\log(|\mathcal{H}|/\delta)}{\epsilon} \rceil = \lceil \frac{1}{\epsilon}[\log(|\mathcal{H}|) + \log(\frac{1}{\delta})] \rceil$$

Is there a learner?

In many scenarios, there is no perfect learner:



General PAC Learning Model

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Learning beyond Binary Classification

- Many learning tasks involve multiple class classification
- or even prediction of a real valued number.

Relaxing the Realizability Assumption:

General PAC Learning — Relaxing the Realizability Assumption

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Intuition.

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- The notation of "*approximately correct*" is now defined as:

$$L_{\mathscr{D}}(h) \leq \min_{h^* \in \mathscr{H}} L_{\mathscr{D}}(h^*) + \epsilon$$

□

Scope of Learning Problems.

General PAC Learning — Beyond Binary Classification

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Muticlass categorization $\mathscr{Y}$ is a finite set representing $|\mathscr{Y}|$ different classes.

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Regression $\mathscr{Y} = \mathscr{R}$

- For example, one wishes to predict the marks of a student based on the resources access pattern.

□

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Cost-sensitive loss $l(h,(x,y)) = C_{h(x),y}$, where C is $|\mathscr{Y}| \times |\mathscr{Y}|$ matrix.

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- The learner does not know $\mathscr{D}$ but can sample $S \sim \mathscr{D}^m$
- Using S the learner outputs some hypothesis $h \in \mathscr{H}$, with probability of at least $(1-\delta)$ it holds that $L_{\mathscr{D}}(h) \leq \min_{h^* \in \mathscr{H}} L_{\mathscr{D}}(h^*) + \epsilon$.

PAC versus Agnostic PAC Learning

Table 1: Comparison of PAC and Agnostic PAC

	PAC	**Agnostic PAC**
Distribution	$\mathscr{D}$ over $\mathscr{X}$	$\mathscr{D}$ over $\mathscr{X} \times \mathscr{Y}$
Truth	$f \in \mathscr{H}$	not in class or does not exist
Risk	$L_{(\mathscr{D},f)}(h) = \mathscr{D}(\{x : h(x) \neq f(x)\})$	$L_{\mathscr{D}}(h) = \mathscr{D}(\{x : h(x) \neq y\})$
Training set	$(x_1, \cdots, x_m) \sim \mathscr{D}^m$, $\forall i,\ y_i = f(x_i)$	$((x_1, y_1), \cdots, (x_m, y_m)) \sim \mathscr{D}^m$
Goal	$L_{(\mathscr{D},f)}(h) \leq \epsilon$	$L_{\mathscr{D}}(h) \leq \min_{h^* \in \mathscr{H}} L_{\mathscr{D}}(h^*) + \epsilon$

$\mathscr{X}$: Domain $\mathscr{Y}$: Range $\mathscr{H}$: Hypothesis Class
L: Loss function ϵ: accuracy parameter m: sample size

Agnostic Learning Finite Hypothesis Classes

Representative Sample

A training set S is called ϵ-representative w.r.t. domain Z, hypothesis class $\mathcal{H}$, loss function l and distribution $\mathcal{D}$, if

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- This concept ensures that: uniformly over *all hypotheses* in the hypothesis class $\mathcal{H}$, the empirical risk will be *close to the true* risk.

□

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Assume that a training set S is $\frac{\epsilon}{2}$-representative w.r.t. domain Z, hypothesis class $\mathscr{H}$, loss function l and distribution $\mathscr{D}$, then, any output of $ERM_{\mathscr{H}}(S)$, namely any $h_S \in \operatorname{argmin}_{h \in \mathscr{H}} L_S(h)$

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- $L_S(h^*) \le L_{\mathcal{D}}(h^*) + \frac{\epsilon}{2}$
- Combine them together, we have

$$\begin{aligned} L_{\mathcal{D}}(h_S) &\le L_S(h_S) + \frac{\epsilon}{2} \\ &\le L_S(h^*) + \frac{\epsilon}{2} \\ &\le L_{\mathcal{D}}(h^*) + \frac{\epsilon}{2} + \frac{\epsilon}{2} \\ &= L_{\mathcal{D}}(h^*) + \epsilon \end{aligned}$$

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Uniform Convergence

A hypothesis class $\mathscr{H}$ has the uniform convergence property if there exists a function $m_{\mathscr{H}}^{UC} : (0,1)^2 \rightarrow \mathscr{N}$, such that for every $\epsilon, \delta \in (0,1)$, and every distribution $\mathscr{D}$, we have: if S is a sample with $m \geq m_{\mathscr{H}}^{UC}(\epsilon, \delta)$ examples drawn i.i.d. according to $\mathscr{D}$, then with probability of at least $(1-\delta)$, S is ϵ-representative.

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If a class $\mathcal{H}$ has the *uniform convergence property* with the sample complexity $m_{\mathcal{H}}^{UC}$, then $\mathcal{H}$ is *agnostically PAC learnable* with the sample complexity

$$m_{\mathcal{H}}(\epsilon,\delta) \leq m_{\mathcal{H}}^{UC}(\frac{\epsilon}{2},\delta)$$

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- $m_{\mathcal{H}}^{UC}$ measures the minimal sample complexity of obtaining the uniform convergence.

Agnostic Learning Finite Hypothesis Classes

Assume $\mathscr{H}$ is finite and the range of the loss function is $[0,1]$, then $\mathscr{H}$ is agnostic PAC learnable using the $ERM_{\mathscr{H}}$ algorithm with sample complexity:

$$m_{\mathscr{H}}(\epsilon,\delta) \leq \lceil \frac{2\log(2|\mathscr{H}|/\delta)}{\epsilon^2} \rceil = \lceil \frac{2}{\epsilon^2}[\log(2|\mathscr{H}|) + \log(\frac{1}{\delta})] \rceil$$

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2. From the union bound, we have:

$$\begin{aligned} &\mathcal{D}^m(\{S : \exists h \in \mathcal{H}, |L_S(h) - L_{\mathcal{D}}(h)| > \epsilon\}) \\ &= \mathcal{D}^m(\bigcup_{h\in\mathcal{H}} \{S : |L_S(h) - L_{\mathcal{D}}(h)| > \epsilon\}) \\ &\le \sum_{h\in\mathcal{H}} \mathcal{D}^m(\{S : |L_S(h) - L_{\mathcal{D}}(h)| > \epsilon\}) \end{aligned}$$

$\square$

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7. So if $m \geq \frac{\log(2|\mathcal{H}|/\delta)}{2\epsilon^2}$, we have the right hand side is at most δ as required. $\square$

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- While not very elegant, it is a great tool for upper bounding *sample complexity*.

Quiz

Theoretical analysis:

1. If the range of the loss function is $[a, b]$, then the sample complexity satisfies:

$$m_{\mathcal{H}}(\epsilon, \delta) \leq m_{\mathcal{H}}^{UC}(\epsilon/2, \delta) \leq \lceil \frac{2 \log 2|\mathcal{H}/\delta|(b-a)^2}{\epsilon^2} \rceil .$$

Theoretical analysis:

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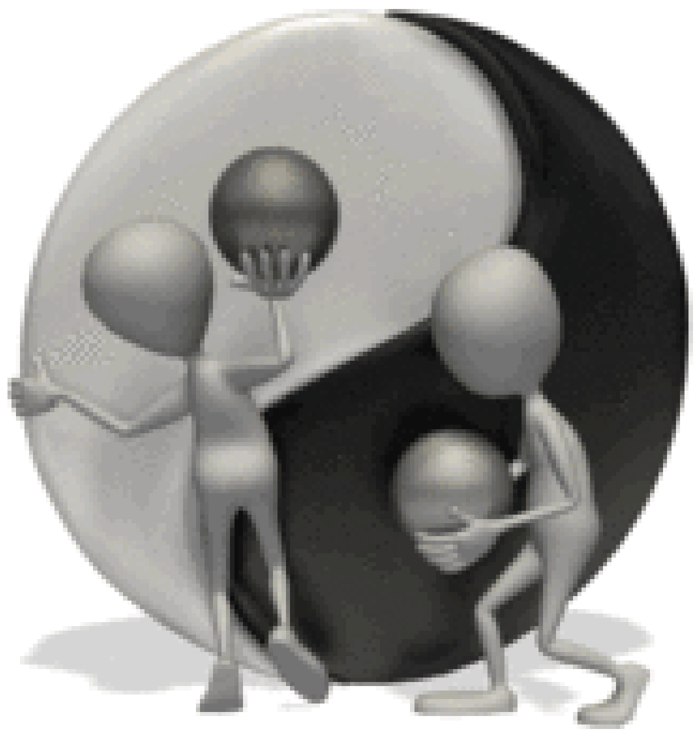
2. Given any probability distribution $\mathscr{D}$ over $\mathscr{X} \times \{0, 1\}$, the *Bayes Optimal Predictor* is defined as: $f_{\mathscr{D}}(x) = \begin{cases} 1 & \text{if } P[y=1|x] \ge \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$ Show that for every probability distribution $\mathscr{D}$, the *Bayes Optimal Predictor* $f_{\mathscr{D}}$ is optimal, in the sense that for every classifier g from $\mathscr{X}$ to $\{0, 1\}$, we have $L_{\mathscr{D}}(f_{\mathscr{D}}) \le L_{\mathscr{D}}(g)$.

Questions?



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