

Objective →

Sparse Matrix- Vector Multiplication (SpMV)

$A(m \times n \text{ matrix}) \cdot X(n \times 1 \text{ vector}) = Y(m \times 1 \text{ vector})$

Algorithm:

- Number of non zeros is known (nz)

1-D Partitioning :

- Input matrix in CRS format => three arrays: I, J, K
- Number of processes => p
 - [Process => block level => thread level]
- Partitioning phase:
 - Task : Multiply a non-zero element K_e at (I_e, J_e) with X_{J_e} and add to Y_{I_e} .
 - (nz) is the number of tasks to be completed.
- Communication :
 - We assume that matrix A and vector X are available in the shared memory of the cluster. No other communication is required between the processes for this strategy.
- Agglomeration: Sparse Blocking
 - Group non-zeros into blocks of size $T \times T$. (we use, $T = 256$)
 - Rationale:
 - Operate on vectors X and Y with more locality => cache friendly.
 - Preprocessing => hashmap for T rows, each set of threads does lookup in it's hashmap (corresponding to the rows it processes)
 - Two levels:
 - Process **elements inside each block Hilbert Order** to maximise cache hits by preserving locality of input vector X.
 - Process **blocks in hilbert order**.
- Mapping phase:
 - Map blocks containing approximately **nz/p** elements (Tasks) to each process.
 - Process p_i determines its partition [number of consecutive rows] by binary search:
 - Lower limit :- index containing $i \cdot (nz/p)$ th element
 - Upper limit :- index containing $(i+1) \cdot (nz/p)$ th element
 - Threads work on consecutive elements in the predetermined hilbert order.

- Post-processing:
 - Each process communicates its computed portion of output vector to the master process, which concatenates these.

Approach 2: 2-dimensional Partitioning

1. Model the input matrix (A) as a hypergraph.
2. Recursive, adaptive, bipartitioning
 - **Aim** –
to obtain a p - way partition of the hypergraph (if there are p processors in the system)
 - At each step of partitioning, divide into 2 partitions, either row-wise or column wise depending on whichever partition leads to lesser increase in communication volume.
3. Metric for partitioning – load imbalance
4. Input, output vectors are also partitioned.
5. Broadcasted list – storing mapping of partition number against each cell of x and y, for fetching x and storing y.
6. Steps:
 1. Fetch the part of x vector not local.
 2. Compute the assigned part of output vector.
 3. Send the calculated y to appropriate processor,