1.

If $F(1) = 2$ and $F(n) = F(n-1) + \dfrac{1}{2}$ for all integers $n > 1$, then $F(101) =$

(A) 49
(B) 50
(C) 51
(D) 52
(E) 53

2.

Consider the following sequence of instructions.

1. Set $k = 999$, $i = 1$, and $p = 0$.
2. If $k > i$, then go to step 3; otherwise go to step 5.
3. Replace i with $2i$ and replace p with $p + 1$.
4. Go to step 2.
5. Print p.

If these instructions are followed, what number will be printed at step 5 ?

(A)   1
(B)   2
(C)   10
(D) 512
(E) 999

3.

For all real numbers x and y, the expression $\dfrac{x + y + |x - y|}{2}$ is equal to

(A) the maximum of x and y

(B) the minimum of x and y

(C) $|x + y|$

(D) the average of $|x|$ and $|y|$

(E) the average of $|x + y|$ and $x - y$

4.

Let B be a nonempty bounded set of real numbers and let b be the least upper bound of B. If b is <u>not</u> a member of B, which of the following is necessarily true?

(A) B is closed.
(B) B is not open.
(C) b is a limit point of B.
(D) No sequence in B converges to b.
(E) There is an open interval containing b but containing no point of B.

5.

A drawer contains 2 blue, 4 red, and 2 yellow socks. If 2 socks are to be randomly selected from the drawer, what is the probability that they will be the same color?

(A) $\frac{2}{7}$

(B) $\frac{2}{5}$

(C) $\frac{3}{7}$

(D) $\frac{1}{2}$

(E) $\frac{3}{5}$

6.

Which of the following are multiplication tables for groups with four elements?

I.	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

II.	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	a
d	d	c	a	b

III.	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	c	d
d	d	c	d	c

(A) None
(B) I only
(C) I and II only
(D) II and III only
(E) I, II, and III

7.

In the Euclidean plane, point A is on a circle centered at point O, and O is on a circle centered at A. The circles intersect at points B and C. What is the measure of angle BAC ?

(A)   60°
(B)   90°
(C)   120°
(D)   135°
(E)   150°

8.

When 20 children in a classroom line up for lunch, Pat insists on being somewhere ahead of Lynn. If Pat's demand is to be satisfied, in how many ways can the children line up?

(A) 20!

(B) 19!

(C) 18!

(D) $\dfrac{20!}{2}$

(E) $20 \cdot 19$

9.

How many integers from 1 to 1,000 are divisible by 30 but not by 16 ?

(A) 29
(B) 31
(C) 32
(D) 33
(E) 38

10.

For each real number x, let $\mu(x)$ be the mean of the numbers 4, 9, 7, 5, and x; and let $\eta(x)$ be the median of these five numbers. For how many values of x is $\mu(x) = \eta(x)$?

(A) None
(B) One
(C) Two
(D) Three
(E) Infinitely many

11.

A fair coin is to be tossed 8 times. What is the probability that more of the tosses will result in heads than will result in tails?

(A) $\dfrac{1}{4}$

(B) $\dfrac{1}{3}$

(C) $\dfrac{87}{256}$

(D) $\dfrac{23}{64}$

(E) $\dfrac{93}{256}$

12.

Consider the points $A = (-1, 2)$, $B = (6, 4)$, and $C = (1, -20)$ in the plane. For how many different points D in the plane are A, B, C, and D the vertices of a parallelogram?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

13.

Let x and y be uniformly distributed, independent random variables on $[0, 1]$. The probability that the distance between x and y is less than $\frac{1}{2}$ is

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

(E) $\frac{3}{4}$

14.

$$C = \frac{5}{9}(F - 32)$$

The equation above shows how temperature F, measured in degrees Fahrenheit, relates to a temperature C, measured in degrees Celsius. Based on the equation, which of the following must be true?

I. A temperature increase of 1 degree Fahrenheit is equivalent to a temperature increase of 59 degree Celsius.

II. A temperature increase of 1 degree Celsius is equivalent to a temperature increase of 1.8 degrees Fahrenheit.

III. A temperature increase of 59 degree Fahrenheit is equivalent to a temperature increase of 1 degree Celsius.

A) I only
B) II only
C) III only
D) I and II only

15.

$$\frac{8 - i}{3 - 2i}$$

If the expression above is rewritten in the form $a+bi$, where a and b are real numbers, what is the value of a? (Note: $i=-1$)

A) 1
B) 2
C) 3
D) 5
E) 7

16.

Gender	Handedness	
	Left	Right
Female		
Male		
Total	18	122

The incomplete table above summarizes the number of left-handed students and right-handed students by gender for the eighth grade students at Keisel Middle School. There are 5 times as many right-handed female students as there are left-handed female students, and there are 9 times as many right-handed male students as there are left-handed male students. if there is a total of 18 left-handed students and 122 right-handed students in the school, which of the following is closest to the probability that a right-handed student selected at random is female? (Note: Assume that none of the eighth-grade students are both right-handed and left-handed.)
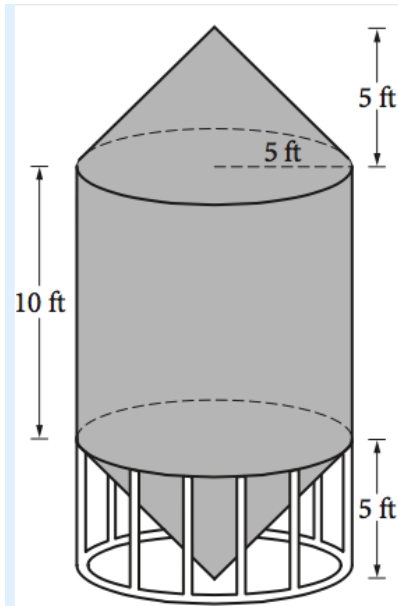
A) 0.410
B) 0.357
C) 0.333
D) 0.250

17.

In the xy-plane, the point (p,r) lies on the line with equation $y=x+b$, where b is a constant. The point with coordinates $(2p,5r)$ lies on the line with equation $y=2x+b$. If $p\neq0$, what is the value of $\frac{r}{p}$?

A) 2/5
B) 3/4
C) 4/3
D) 5/2

18.

A grain silo is built from two right circular cones and a right circular cylinder with internal measurements represented by the figure above. Of the following, which is closest to the volume of the grain silo, in cubic feet?
A) 261.8
B) 785.4
C) 916.3
D) 1047.2

19.

The price of an article was increased $p\%$. Later the new price was decreased $p\%$. If the last price was one dollar, the original price was:

A. $\dfrac{1-p^2}{200}$

B. $\dfrac{\sqrt{1-p^2}}{100}$

C. one dollar

D. $1 - \dfrac{p^2}{10000 - p^2}$

E. $\dfrac{10000}{10000 - p^2}$

20.
If x is the average (arithmetic mean) of m and 9, y is the average of $2m$ and 15, and z is the average of $3m$ and 18, what is the average of x, y, and z in terms of m?
A) $m+6$
B) $m+7$
C) $2m+14$
D) $3m+21$

21.
A company's manager estimated that the cost C, in dollars, of producing n items is $C = 7n + 350$. The company sells each item for $12. The company makes a profit when the total income from selling a quantity of items is greater than the total cost of producing that quantity of items. Which of the following inequalities gives all possible values of n for which the manager estimates that the company will make a profit?

A. $n < 70$
B. $n < 84$
C. $n > 70$
D. $n > 84$
E. $N < 55$

22. A paint store mixes 3/4 pint of red paint and 2/3 pint of blue paint to make a new paint color called Perfectly Purple. How many pints of red paint would be needed to make 34 pints of Perfectly Purple paint?

A. 1.42
B. 18.0
C. 36.13
D. 47.5
E. 48.12

23.

If $7 < x^2 - 4x + 12$, which of the following MUST be true?

i) $3^{x-2} > 0$

ii) $|x^3 + 1| > 0$

iii) $\sqrt{(x+2)^2} > 0$

A) i only
B) i and ii only
C) i and iii only
D) ii and iii only
E) i, ii and iii

24. A trip of 900 miles would have taken 1 hour less if the average speed for the trip had been greater by 10 miles per hour. What was the average speed for the trip?

(A) 40 miles per hour
(B) 45 miles per hour
(C) 60 miles per hour
(D) 75 miles per hour
(E) 90 miles per hour

25. f(x) has the properties, f(x+y)=x+f(y) and f(0)=5. What is the value of f(100)?

A.95
B.100
C.105
D.120
E.135

26. The average (arithmetic mean) age of the people in group G is 41 years, and the average age of the people in Group H is 36 years. The average age of the people in the two groups combined is 38. If no person is in both groups, what fraction of the people in the two groups combined are in group H?

A. 2/5
B. 1/2
C. 3/5
D. 2/3
E. ¾

27. If $r \le s \le t \le u \le v \le 110$ and the average, (arithmetic mean) of r, s, t, u, and v is 100, what is the least possible value of r?

A. 0
B. 20
C. 40
D. 60
E. 80

28. The first term of a sequence is 2005. Each succeeding term is the sum of the cubes of the digits of the previous term. What is the 2005th term of the sequence?
   A.29
   B.55
   C.85
   D.133
   E.250

29. The length of rectangle R is 50% greater than the length of rectangle S, and the width of rectangle R is twice the width of rectangle S. If the area of rectangle R is x, then in terms of x, the area of rectangle S is how much less than the area of rectangle R?
   A. x/3
   B. x2
   C.2x/3
   D. x
   E.2

30. A "palindrome" is any integer that reads the same backward and forward. A six-digit positive palindrome must be divisible by which of the following?

A. 3
B. 7
C. 9
D. 11
E. 13