Landing an ellipse

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Problem Setup

Consider an ellipse with semi-major axis $a = 1.0$ and semi-minor axis $b = 0.5$, density $\rho_s = 1.2$, and moment of inertia $I = 1.0$. The surrounding fluid has density $\rho_f = 1.0$ and gravity $g = 9.81,\ \mathrm{m/s}^2$. Derived non-dimensional parameters are as follows:

$$\beta = \frac{b}{a}, \quad \rho_* = \frac{\rho_s}{\rho_f}$$

Forces and torque parameters are given by

- Constants: $A = 1.4$, $B = 1.0$
- Viscous terms: $\mu = 0.2$, $\nu = 0.2$
- Torque constants: $C_T = 1.2$, $C_R = \pi$

The system is simulated with a time step $\Delta t = 0.01$ and total time $T = 5$.

Dynamics Equations

Define the translational velocities u and v, the angular velocity w, and orientation θ. The forces F_u, F_v and moment M are computed as:

$$F = \frac{1}{\pi}\left(A - B\frac{u^2 - v^2}{u^2 + v^2 + \epsilon}\right)\sqrt{u^2 + v^2 + \epsilon}$$

$$M = 0.2(\mu + \nu|w|)w$$

$$\Gamma = \frac{2}{\pi}\left(C_R w - C_T\frac{uv}{\sqrt{u^2 + v^2 + \epsilon}}\right)$$

where ϵ is a small constant to avoid division by zero.

Equations of Motion

The equations of motion governing u, v, and w are given by:

$$\dot{u} = \frac{(I + \beta^2)vw - \Gamma v - \sin\theta - Fu}{I + \beta^2}$$

$$\dot{v} = \frac{-(I + 1)uw + \Gamma u - \cos\theta - Fv}{I + 1}$$

$$\dot{w} = \frac{-(1 - \beta^2)uv - M}{0.25\left(I(1 + \beta^2) + 0.5(1 - \beta^2)^2\right)}$$

The position (x, y) and orientation θ are updated as follows:

$$x \leftarrow x + (u\cos\theta - v\sin\theta)\Delta t$$

$$y \leftarrow y + (u\sin\theta + v\cos\theta)\Delta t$$

$$\theta \leftarrow \theta + w\Delta t$$

1 Reinforcement Learning

Our objective is the following

- $theta_G = \pi/4$
- $x_G = 100$
- $y_G = -50$

We want to model the torque action is constant control torque

$$\tau_t = \tanh(a_t) \in \{-1, 1\}.$$

Numerical Simulation

The system is numerically simulated over a fixed time period with a time step of $\Delta t = 0.01$. Initial conditions are:

$$x = 0, \quad y = 0, \quad \theta = \frac{\pi}{4}, \quad u = 0, \quad v = 0, \quad w = 0$$

Results

The trajectory of the ellipse can be visualized by plotting x vs y, showing the dynamics influenced by the forces and torques as described.