CSE 604: Artificial Intelligence
Quiz 2 Solutions
Instructor: Dr. Ahmedul Kabir
Institute of Information Technology
University of Dhaka

Full Marks: 50
Time: 50 minutes

Problem I. Quantifying Uncertainty [8 Points]

1. State whether the following statements are true or false. **Explain your answer**. [4 points]

   a. If P(a| b, c) = P(a), then P(b|c) = P(b)

      FALSE.
      If P(a| b, c) = P(a), that means a is independent of b and c. But that says nothing about the independence of b with regards to c, which is expressed by P(b|c) = P(b).

   b. If P(a|b) = P(a), then P(a| b, c) = P(a|c)

      TRUE.

      If a is independent of b, then a given b and c is the same as a given just c.

2. A doctor knows that the disease dengue causes the patient to have a severe headache 70% of the time. The doctor also knows some unconditional facts: the prior probability that a patient has dengue is 1/50, and the prior probability that any patient has a severe headache is 5%. Under these circumstances, what is the probability that a patient, who comes in with a severe headache, has dengue? [4 points]

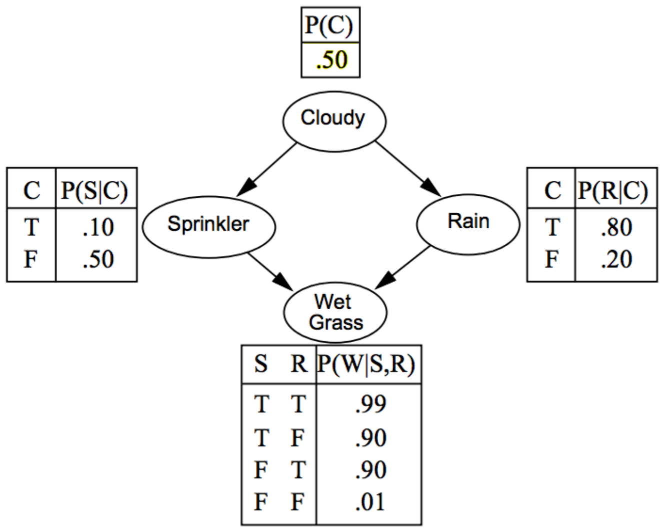
   P(dengue | headache) = P (headache | dengue) * P (dengue) / P(headache)

   = 0.7 * 0.02 / 0.05

   = 0.28

Problem II. Probabilistic Reasoning [14 Points]

A simple Bayesian Network, with corresponding conditional probability tables, is given below:



P(C)
.50

C	P(S|C)
T	.10
F	.50

C	P(R|C)
T	.80
F	.20

S	R	P(W|S,R)
T	T	.99
T	F	.90
F	T	.90
F	F	.01

From this Bayesian network, answer the following questions:

1. Which pairs of variables in this network are conditionally independent of each other given other variables? State your answers in the following format: "X and Y are cond. ind. given Z". Mention all such pairs [4 points]

   Sprinkler and Rain are cond. ind., given Cloudy

   Cloudy and Wet Grass are cond. ind., given Sprinkler

   Cloudy and Wet Grass are cond. ind., given Rain

2. What is the probability that it is cloudy, it is raining, the sprinkler is off, and the grass is wet? (Just write the corresponding values, you do not need a calculator) [4 points]

   P(c, r, s', w) = P(w | s', r) P(s' | c) P(r | c) P(c) = 0.90 * 0.90 * 0.80 * 0.50 = 0.324

   [Many students seem to have got this wrong, especially in evaluating P(s' | c). It's not 0.5 which is actually the value of P(s | c') in the table. P(s' | c) = 0.9 since P(s | c)= 0.1. This value can be inferred, it is not directly in the table]

3. What is the probability that it is raining, given the grass is wet? (Just write the corresponding values, you do not need a calculator) [6 points]

   P(r | w) = P(r, w) / P(w)

   Lets evaluate them separately:

   P(r, w) = P(r, w, s', c') + P(r, w, s', c) + P(r, w, s, c') + P(r, w, s, c)

   P(w) = P(w, c', s', r') + P(w, c', s', r) + ….. (all 8 combinations)

   Now evaluate each of the terms similar to Part 2 of this question

Problem III. Temporal Reasoning [20 Points]

1. For each of the following scenarios, mention whether or not the process can be modelled as a Markov chain / Markov process. **Explain your answer**. [6 points]

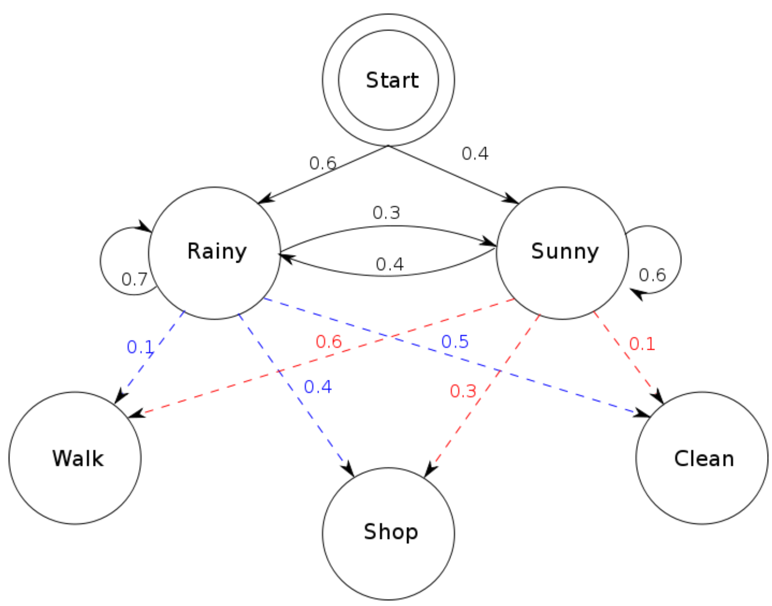
    a. Rolling a six-sided dice (the ones used in Ludo games) *n* times.

    No. These n events are independent of each other

    b. The movement of a Ludo marker ("guti") on the Ludo board.

    Yes. The position after the next move depends solely on the current position

2. Consider two friends, Alice and Bob, who live far apart from each other and who talk together daily over the telephone about what they did that day. Bob is only interested in three activities: walking in the park, shopping, and cleaning his apartment. The choice of what to do is determined exclusively by the weather on a given day. Alice has no definite information about the weather, that is, they are hidden from her. Since Bob tells Alice about his activities, those are the observations. A Hidden Markov Model (HMM) of this system, with corresponding transition and emission probability tables, is given below:



Answer the following questions from the given HMM: [14 points]

a) What are the hidden variables in this HMM? [1 point]

Rainy, Sunny

b) What are the observed variables? [1 point]

Walk, shop, clean

c) What is the probability that Bob will shop on a rainy day? [2 points]

0.4

d) What is the probability that Bob will NOT walk on a sunny day? [2 points]

1 – 0.6 = 0.4

e) What is the prior probability of rain, given no information? [1 points]

0.6

f) What is the probability of rain on the first day, given Bob walks on the first day? [3 points]

$$P(R1) = \langle 0.7, 0.3\rangle * 0.6 + \langle 0.4, 0.6\rangle * 0.4 = \langle 0.42, 0.18\rangle + \langle 0.16, 0.24\rangle$$
$$= \langle 0.58, 0.42\rangle$$

Now update with evidence:

$$P(R1 \mid w1) = \alpha P(w1 \mid R1)P(R1) = \alpha \langle 0.1, 0.6\rangle \langle 0.58, 0.42\rangle = \alpha\langle 0.058, 0.252\rangle$$
$$\approx \langle 0.19, 0.81\rangle$$

So there is ≈ 19% probability of rain on the first day

g) What is the probability of rain on the second day, given Bob walked on the first day, and cleaned on the second day? [4 points]

$$P(R2) = \langle 0.7, 0.3\rangle * 0.19 + \langle 0.4, 0.6\rangle * 0.81 = \langle 0.133, 0.057\rangle + \langle 0.324, 0.486\rangle$$
$$= \langle 0.457, 0.543\rangle$$

Now update with evidence:

$$P(R2 \mid c2) = \alpha P(c2 \mid R2)P(R2) = \alpha \langle 0.5, 0.1\rangle \langle 0.457, 0.543\rangle = \alpha\langle 0.2285, 0.0543\rangle$$
$$\approx \langle 0.81, 0.19\rangle$$

So there is ≈ 81% probability of rain on the second day

Problem IV. Decision Making [8 points]

Suppose you are participating in a game show. There are three doors. Behind each door is a prize, which you may win with a certain probability. If you choose door A, you will definitely win 1000 Tk. If you choose door B, there is a 60% chance of winning 5000 Tk, but there is also a 40% chance you will have to pay 5000 Tk. If you choose door C, there is a 30% chance of winning 10,000 Tk, but also a 10% chance you will have to pay 15,000 Tk. 60% times, you will win nothing.

1. Formulate this scenario as a Lottery. [3 points]

   L_A = [1.0, 1000]

   L_B = [0.6, 5000; 0.4, -5000]

   L_C = [0.3, 10000; 0.1, -15000, 0.6, 0]

2. Based on the theory of maximum expected utility, which should be the rational choice? Explain your answer through calculation. [5 points]

   $EU(L_A) = 1000$

   $EU(L_B) = 0.6*5000 + 0.4*(-5000) = 1000$

   $EU(L_C) = 0.3*10000 + 0.1*(-15000) + 0.6*0 = 1500$

   So, one should choose Door C