

EXERCISES PROPOSITIONAL LOGIC

1. [R+N, exercise 7.8] Decide whether each of the following sentences is valid, unsatisfiable or neither. Use truth tables or equivalence rules
 - a. $\text{Smoke} \Rightarrow \text{Smoke}$
 - b. $\text{Smoke} \Rightarrow \text{Fire}$
 - c. $(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\neg \text{Smoke} \Rightarrow \neg \text{Fire})$
 - d. $\text{Smoke} \vee \text{Fire} \vee \neg \text{Fire}$
 - e. $((\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{Fire}) \Leftrightarrow ((\text{Smoke} \Rightarrow \text{Fire}) \vee (\text{Heat} \Rightarrow \text{Fire}))$
 - f. $(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow ((\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{Fire})$
 - g. $\text{Big} \vee \text{Dumb} \vee (\text{Big} \Rightarrow \text{Dumb})$
 - h. $(\text{Big} \wedge \text{Dumb}) \vee \neg \text{Dumb}$
2. Determine whether each of the wffs in propositional logic
 - a) $((P \rightarrow (Q \vee R)) \wedge \neg(Q \vee R)) \rightarrow \neg P$
 - b) $((P \vee Q) \rightarrow \neg R) \wedge (\neg R \vee (Q \vee P))$are i) a tautology.
ii) satisfiable but not a tautology. In this case give an interpretation that is a model and another interpretation that is not a model.
iii) unsatisfiable.
Justify your answer choosing one of the methods presented in the course.
3. Justify your answers using the tools of formal logic and giving the steps of your solution in detail. Name all the rules used in your derivations.
 - a. Is the following reasoning correct? Translate to wff's and apply inference to verify it.
"If the temperature and the pressure are constant then it does not rain. The temperature remained constant. Thus, if it rained, then the pressure did not remain constant."
 - b. Is the following reasoning correct? Translate to wff's and apply inference to verify it.
"Men eat when they are hungry. John always wears his best suit to eat. At this moment John is not hungry. Therefore, John is not wearing his best suit".
 - c. Is the following statement correct? Demonstrate it
 $\text{False} \models w$, for any wff w
4. Consider the text:
"There are only two formats for photos: round and square. Photos are either color or black and white. Let me tell you about the photo I found yesterday. If the photo is square, then it is a black and white picture. If it is round, it is a digital color picture. If the photo is digital or in black and white, then it is a portrait. If it is a portrait, then it is a picture of my friend"
 - a) Build a knowledge base to represent the statements about the photo the author of the previous text found yesterday.
Use the atoms A,B,C,D,E,F,G

Atom	Elementary statement about the world
A	The photo is in color
B	The photo is in black and white
C	The photo is square
D	The photo is round
E	The photo is digital
F	The photo is a portrait
G	The photo is a picture of my friend

Known world facts (True in our world)	wff
1. There are only two formats for Photos: round and square	
2. Photos are either color or black and white	
3. If the photo is square, then it is a black and white picture.	
4. If it is round, it is a digital color picture	
5. If the photo is digital or in black and white, then it is a portrait	
6. If it is a portrait, then it is a picture of my friend .	

- b) Using inference, can you answer the author's question: Is the photo I found yesterday the picture of my friend?

5. [R+N, exercise 7.9] Consider the text
 "If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned."

Can we prove that the unicorn is mythical? that the unicorn is magical? that the unicorn is horned?

6. [R+N, exercise 7.11] Let's play minesweeper on a rectangular grid of $[1..N_f] \times [1..N_c]$ squares with M invisible mines scattered among them.
- Let the atom X_{ij} have value *True* when the square $[i,j]$ contains mine. Write down the assertion that there are exactly two mines adjacent to the square $[1,1]$ as a wff involving some atoms of the form X_{ij} .
 - Generalize the expression obtained in (a) by explaining how to construct a CNF sentence asserting that k of n neighbors contain mines.
 - How can an agent use DPLL to prove that a square has (or has not) a mine, ignoring the global constraint that there are exactly M mines.
 - Suppose that the global constraint is constructed via your method from part (b). How does the number of clauses depend on M , N_f , N_c ? Suggest a way to modify DPLL to prove that the global constraint does not need to be represented explicitly.

- e. Are any conclusions derived by the method in part (c) invalidated when the global constraint is taken into account?
 - f. Give examples of configurations of probe values that induce long-term dependencies such that the contents of a given unprobed square would give info about the contents of a far-distant square (Hint: assume $N_f = N$, $N_c = 1$)
7. [From Nilsson 14.2] “Heads you win; tails I lose” Express these statements (plus other statements that you may need about how coins are, what is losing and winning) in the propositional calculus and then use resolution to answer the question “Who wins?”.
8. [From Nilsson 14.3] The following wffs are instances of axioms that are sometimes used in the propositional calculus.
- a. Implication introduction: $P \Rightarrow (Q \Rightarrow P)$
 - b. Implication distribution: $(P \Rightarrow (Q \Rightarrow R)) \Rightarrow ((P \Rightarrow Q) \Rightarrow (P \Rightarrow R))$
 - c. Contradiction realization: $(Q \Rightarrow \neg P) \Rightarrow ((Q \Rightarrow P) \Rightarrow \neg Q)$

Use resolution refutation to prove each of these formulas.

9. [From Nilsson 14.4] Consider the following set of clauses
 $\{P \vee Q, P \vee \neg Q, \neg P \vee Q, \neg P \vee \neg Q\}$
 Try to determine whether they are satisfiable by
- a. Set-of-support resolution (the initial set of support is the last clause)
 - b. Ancestry-filtered resolution
 - c. A strategy that violates both set-of-support and ancestry-filtered resolution
- Show that input resolution is not a valid resolution strategy for this problem.
10. Using the methods of formal logic, formalize the following problems and solve the riddles using inference

Let's hear Alceo, Safo and Catulo

Alceo says: “The only ones who speak the truth here are Catulo and I”

Safo states: “Catulo is a liar”

Catulo replies: “Safo speaks the truth. Or maybe it is Alceo who lies”

Assuming that the person who lies always lies and that the person who speaks the truth is always truthful, who is sincere? Who lies?

Let's hear Anaximandro, Parménides and Heráclito

Anaximandro says: “One should not trust Heráclito”

Parménides states: “Anaximandro and Heráclito never lie”

Heráclito replies “Parménides has spoken the truth”

Assuming that the person who lies always lies and that the person who speaks the truth is always truthful, who is sincere? Who lies?

11. Using propositional logic and inference (no reasoning in natural language is allowed), solve the following enigma

In the Roman Senate

- (I) Mark Anthony: "It was Cassius or Brutus (or both)."
- (II) Cassius: "I did not do it. Mark Anthony is lying"
- (III) Brutus: "I did not do it"

Assuming that liars always lie, that truthful Romans always speak the truth, and that only one of them is telling the truth, who did it? Who is telling the truth?

12. There are three chests (red, blue and green in color) with the inscriptions:

Red: The treasure is here.

Blue: The treasure is not here.

Green: The treasure is not in the red chest.

Knowing that only one chest contains a treasure and that, at most, one of the inscriptions is correct, can you decide which chest contains the treasure?

Using the atoms

Atom R means "The red chest contains the treasure".

Atom B means "The blue chest contains the treasure".

Atom G means "The green chest contains the treasure".

Atom RR means "The inscription in the red chest is correct".

Atom BB means "The inscription in the blue chest is correct".

Atom GG means "The inscription in the green chest is correct".

- a) Translate the sentences to wffs.
- b) Using only inference (no truth tables or semiformal reasoning):
 - i) Demonstrate that $\neg (RR \wedge \neg BB \wedge \neg GG)$ is entailed by the knowledge base.
 - ii) Demonstrate that $\neg (\neg RR \wedge BB \wedge \neg GG)$ is entailed by the knowledge base.
 - iii) Demonstrate that $\neg (\neg RR \wedge \neg BB \wedge \neg GG)$ is entailed by the knowledge base.
 - iv) Demonstrate that $(\neg RR \wedge \neg BB \wedge GG)$ is entailed by the knowledge base.
- c) Using these results, can you decide which chest contains the treasure?