NEURAL NETWORKS

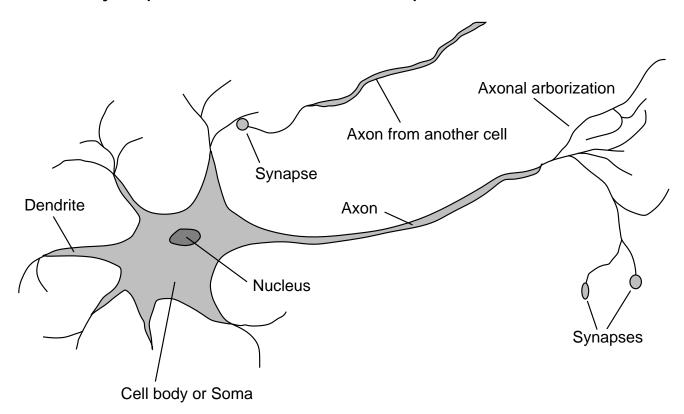
Chapter 20, Section 5

Outline

- \Diamond Brains
- ♦ Neural networks
- \Diamond Perceptrons
- ♦ Multilayer perceptrons
- \Diamond Applications of neural networks

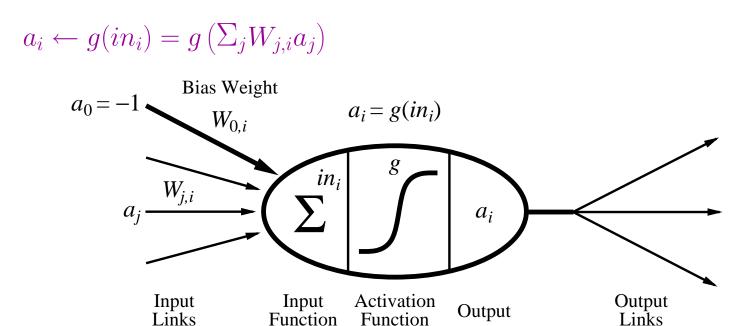
Brains

 10^{11} neurons of >20 types, 10^{14} synapses, 1ms–10ms cycle time Signals are noisy "spike trains" of electrical potential



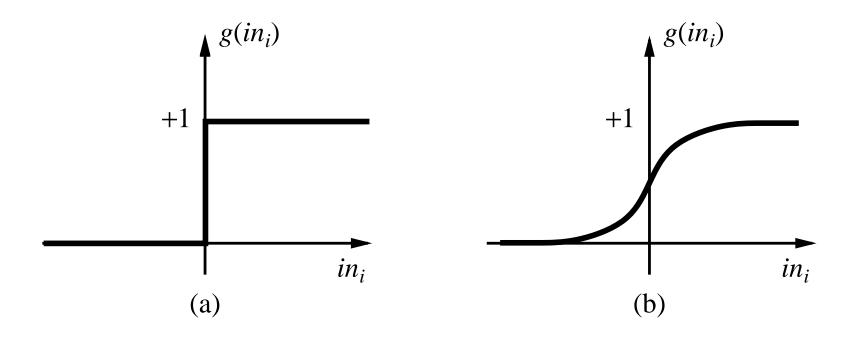
McCulloch-Pitts "unit"

Output is a "squashed" linear function of the inputs:



A gross oversimplification of real neurons, but its purpose is to develop understanding of what networks of simple units can do

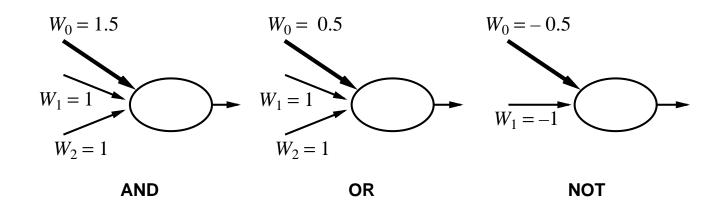
Activation functions



- (a) is a step function or threshold function
- (b) is a sigmoid function $1/(1+e^{-x})$

Changing the bias weight $W_{0,i}$ moves the threshold location

Implementing logical functions



McCulloch and Pitts: every Boolean function can be implemented

Network structures

Feed-forward networks:

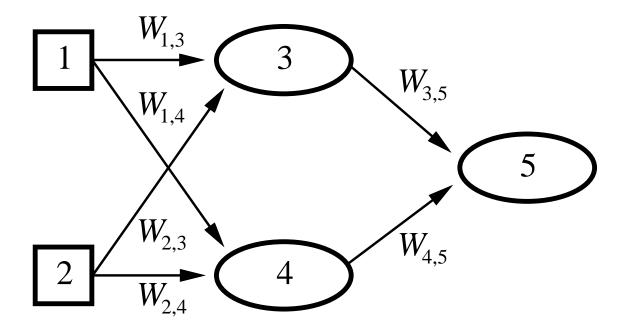
- single-layer perceptrons
- multi-layer perceptrons

Feed-forward networks implement functions, have no internal state

Recurrent networks:

- Hopfield networks have symmetric weights $(W_{i,j} = W_{j,i})$ g(x) = sign(x), $a_i = \pm 1$; holographic associative memory
- Boltzmann machines use stochastic activation functions, \approx MCMC in Bayes nets
- recurrent neural nets have directed cycles with delays
 - ⇒ have internal state (like flip-flops), can oscillate etc.

Feed-forward example



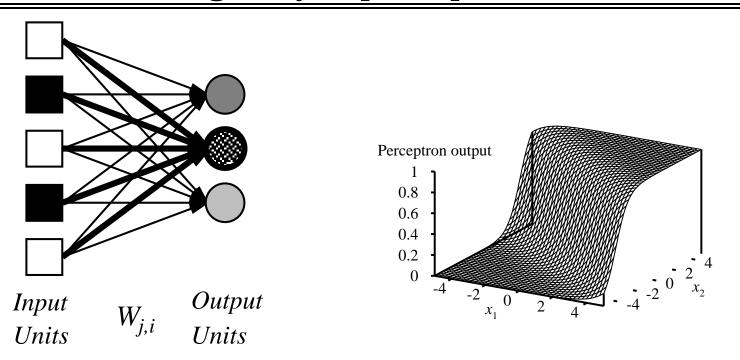
Feed-forward network = a parameterized family of nonlinear functions:

$$a_5 = g(W_{3,5} \cdot a_3 + W_{4,5} \cdot a_4)$$

= $g(W_{3,5} \cdot g(W_{1,3} \cdot a_1 + W_{2,3} \cdot a_2) + W_{4,5} \cdot g(W_{1,4} \cdot a_1 + W_{2,4} \cdot a_2))$

Adjusting weights changes the function: do learning this way!

Single-layer perceptrons



Output units all operate separately—no shared weights

Adjusting weights moves the location, orientation, and steepness of cliff

Expressiveness of perceptrons

Consider a perceptron with g = step function (Rosenblatt, 1957, 1960)

Can represent AND, OR, NOT, majority, etc., but not XOR

Represents a linear separator in input space:

Minsky & Papert (1969) pricked the neural network balloon

Perceptron learning

Learn by adjusting weights to reduce error on training set

The squared error for an example with input x and true output y is

$$E = \frac{1}{2}Err^2 \equiv \frac{1}{2}(y - h_{\mathbf{W}}(\mathbf{x}))^2 ,$$

Perform optimization search by gradient descent:

$$\frac{\partial E}{\partial W_j} = Err \times \frac{\partial Err}{\partial W_j} = Err \times \frac{\partial}{\partial W_j} \left(y - g(\sum_{j=0}^n W_j x_j) \right)$$
$$= -Err \times g'(in) \times x_j$$

Simple weight update rule:

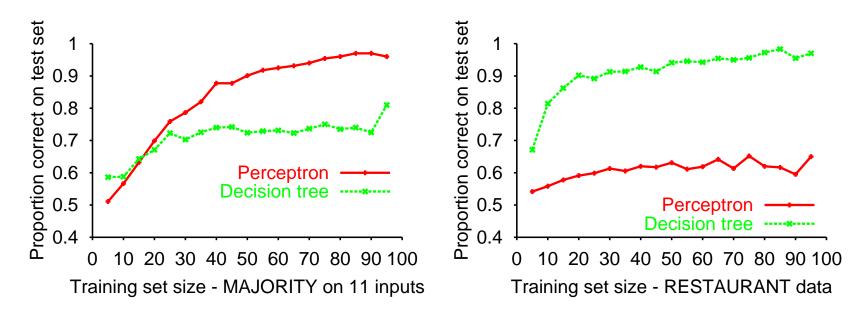
$$W_j \leftarrow W_j + \alpha \times Err \times g'(in) \times x_j$$

E.g., +ve error \Rightarrow increase network output

 \Rightarrow increase weights on +ve inputs, decrease on -ve inputs

Perceptron learning contd.

Perceptron learning rule converges to a consistent function for any linearly separable data set

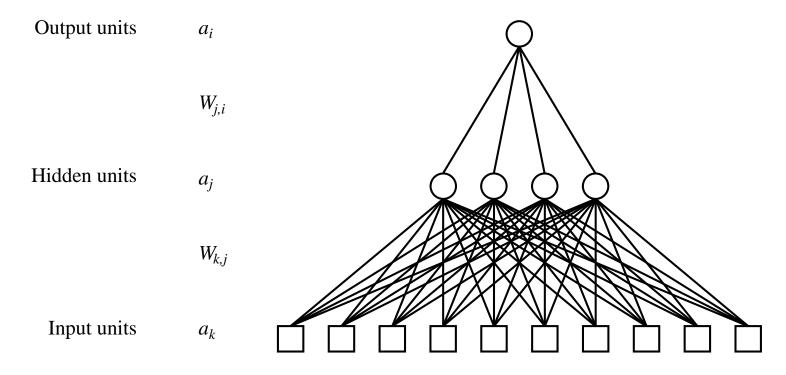


Perceptron learns majority function easily, DTL is hopeless

DTL learns restaurant function easily, perceptron cannot represent it

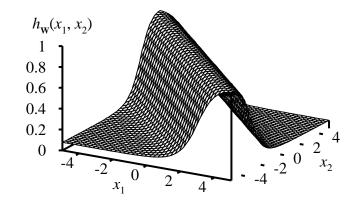
Multilayer perceptrons

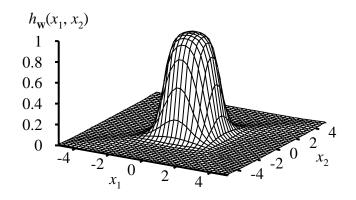
Layers are usually fully connected; numbers of hidden units typically chosen by hand



Expressiveness of MLPs

All continuous functions w/2 layers, all functions w/3 layers





Combine two opposite-facing threshold functions to make a ridge

Combine two perpendicular ridges to make a bump

Add bumps of various sizes and locations to fit any surface

Proof requires exponentially many hidden units (cf DTL proof)

Back-propagation learning

Output layer: same as for single-layer perceptron,

$$W_{j,i} \leftarrow W_{j,i} + \alpha \times a_j \times \Delta_i$$

where
$$\Delta_i = Err_i \times g'(in_i)$$

Hidden layer: back-propagate the error from the output layer:

$$\Delta_j = g'(in_j) \sum_i W_{j,i} \Delta_i .$$

Update rule for weights in hidden layer:

$$W_{k,j} \leftarrow W_{k,j} + \alpha \times a_k \times \Delta_j$$
.

(Most neuroscientists deny that back-propagation occurs in the brain)

Back-propagation derivation

The squared error on a single example is defined as

$$E = \frac{1}{2} \sum_{i} (y_i - a_i)^2 ,$$

where the sum is over the nodes in the output layer.

$$\frac{\partial E}{\partial W_{j,i}} = -(y_i - a_i) \frac{\partial a_i}{\partial W_{j,i}} = -(y_i - a_i) \frac{\partial g(in_i)}{\partial W_{j,i}}
= -(y_i - a_i) g'(in_i) \frac{\partial in_i}{\partial W_{j,i}} = -(y_i - a_i) g'(in_i) \frac{\partial}{\partial W_{j,i}} \left(\sum_j W_{j,i} a_j\right)
= -(y_i - a_i) g'(in_i) a_j = -a_j \Delta_i$$

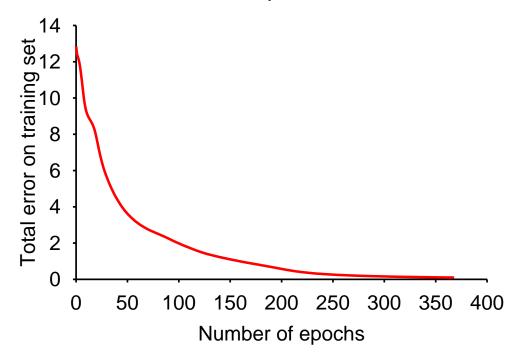
Back-propagation derivation contd.

$$\frac{\partial E}{\partial W_{k,j}} = -\sum_{i} (y_i - a_i) \frac{\partial a_i}{\partial W_{k,j}} = -\sum_{i} (y_i - a_i) \frac{\partial g(in_i)}{\partial W_{k,j}}
= -\sum_{i} (y_i - a_i) g'(in_i) \frac{\partial in_i}{\partial W_{k,j}} = -\sum_{i} \Delta_i \frac{\partial}{\partial W_{k,j}} \left(\sum_{j} W_{j,i} a_j \right)
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= -\sum_{i} \Delta_i W_{j,i} g'(in_j) a_k = -a_k \Delta_j$$

Back-propagation learning contd.

At each epoch, sum gradient updates for all examples and apply

Training curve for 100 restaurant examples: finds exact fit



Typical problems: slow convergence, local minima

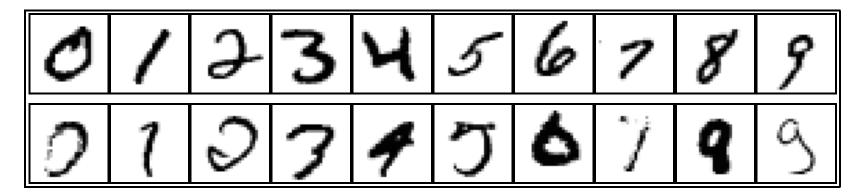
Back-propagation learning contd.

Learning curve for MLP with 4 hidden units:



MLPs are quite good for complex pattern recognition tasks, but resulting hypotheses cannot be understood easily

Handwritten digit recognition



3-nearest-neighbor = 2.4% error

400-300-10 unit MLP = 1.6% error

LeNet: 768-192-30-10 unit MLP = 0.9% error

Current best (kernel machines, vision algorithms) $\approx 0.6\%$ error

Summary

Most brains have lots of neurons; each neuron \approx linear–threshold unit (?)

Perceptrons (one-layer networks) insufficiently expressive

Multi-layer networks are sufficiently expressive; can be trained by gradient descent, i.e., error back-propagation

Many applications: speech, driving, handwriting, fraud detection, etc.

Engineering, cognitive modelling, and neural system modelling subfields have largely diverged