

1 Constant Turn Rate and Acceleration Model (CTRA)

Modelled after Schubert, R.; Richter, E.; Wanielik, G.: *Comparison and evaluation of advanced motion models for vehicle tracking* in 2008 11th International Conference on Information Fusion

1.1 System Model

$$\vec{x}_{t+T} = \begin{pmatrix} x_{t+T} \\ y_{t+T} \\ \theta_{t+T} \\ v_{t+T} \\ a \\ \omega \end{pmatrix} = \vec{x}_t + \begin{pmatrix} \Delta x(T) \\ \Delta y(T) \\ \omega T \\ aT \\ 0 \\ 0 \end{pmatrix}$$

with

$$\begin{aligned} \Delta x(T) &= \frac{1}{\omega^2} [(v_t\omega + a\omega T) \sin(\theta_t + \omega T) - v_t\omega \sin(\theta_t) + a \cos(\theta_t + \omega T) - a \cos(\theta_t)] \\ &= \frac{1}{\omega^2} [a\omega T \sin(\theta_t + \omega T) + v_t\omega (\sin(\theta_t + \omega T) - \sin(\theta_t)) + a(\cos(\theta_t + \omega T) - \cos(\theta_t))] \end{aligned}$$

and

$$\begin{aligned} \Delta y(T) &= \frac{1}{\omega^2} [-(v_t\omega + a\omega T) \cos(\theta_t + \omega T) + v_t\omega \cos(\theta_t) + a \sin(\theta_t + \omega T) - a \sin(\theta_t)] \\ &= \frac{1}{\omega^2} [-a\omega T \cos(\theta_t + \omega T) - v_t\omega (\cos(\theta_t + \omega T) - \cos(\theta_t)) + a(\sin(\theta_t + \omega T) - \sin(\theta_t))] \end{aligned}$$

1.1.1 Simplified System Model for $\omega \rightarrow 0$

$$\begin{aligned} \Delta x(T) &= \frac{1}{2}T(2v_t + aT) \cos(\theta_t) \\ \Delta y(T) &= \frac{1}{2}T(2v_t + aT) \sin(\theta_t) \end{aligned}$$

1.2 System Model Jacobian

$$F = \begin{pmatrix} 1 & 0 & \frac{\partial}{\partial \theta_t} x_{t+T} & \frac{\partial}{\partial v_t} x_{t+T} & \frac{\partial}{\partial a} x_{t+T} & \frac{\partial}{\partial \omega} x_{t+T} \\ 0 & 1 & \frac{\partial}{\partial \theta_t} y_{t+T} & \frac{\partial}{\partial v_t} y_{t+T} & \frac{\partial}{\partial a} y_{t+T} & \frac{\partial}{\partial \omega} y_{t+T} \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

1.2.1 x -Derivatives

Standard

$$\begin{aligned} \frac{\partial}{\partial \theta_t} x_{t+T} &= \frac{1}{\omega^2} [(v_t \omega + a \omega T) \cos(\theta_t + \omega T) - v_t \omega \cos(\theta_t) - a \sin(\theta_t + \omega T) + a \sin(\theta_t)] \\ &= \frac{1}{\omega^2} [a \omega T \cos(\theta_t + \omega T) + v_t \omega (\cos(\theta_t + \omega T) - \cos(\theta_t)) - a (\sin(\theta_t + \omega T) - \sin(\theta_t))] \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial v_t} x_{t+T} &= \frac{1}{\omega^2} [\omega \sin(\theta_t + \omega T) - \omega \sin(\theta_t)] \\ &= \frac{1}{\omega} [\sin(\theta_t + \omega T) - \sin(\theta_t)] \end{aligned}$$

$$\frac{\partial}{\partial a} x_{t+T} = \frac{1}{\omega^2} [\omega T \sin(\theta_t + \omega T) + \cos(\theta_t + \omega T) - \cos(\theta_t)]$$

$$\begin{aligned} \frac{\partial}{\partial \omega} x_{t+T} &= \frac{1}{\omega^2} [a T \sin(\theta_t + \omega T) + a \omega T^2 \cos(\theta_t + \omega T) + v_t (\sin(\theta_t + \omega T) - \sin(\theta_t)) + v_t \omega T \cos(\theta_t + \omega T) - a T \sin(\theta_t + \omega T)] \\ &\quad - \frac{2}{\omega^3} [a \omega T \sin(\theta_t + \omega T) + v_t \omega (\sin(\theta_t + \omega T) - \sin(\theta_t)) + a (\cos(\theta_t + \omega T) - \cos(\theta_t))] \\ &= \frac{1}{\omega^2} [(a \omega T^2 + v_t \omega T) \cos(\theta_t + \omega T) + v_t (\sin(\theta_t + \omega T) - \sin(\theta_t))] - \frac{2}{\omega^3} [(a \omega T + v_t \omega) \sin(\theta_t + \omega T) - v_t \omega \sin(\theta_t) + a (\cos(\theta_t + \omega T) - \cos(\theta_t))] \\ &= \frac{1}{\omega^2} [(a T + v_t)(\omega T) \cos(\theta_t + \omega T) - (2a T + v_t) \sin(\theta_t + \omega T) + v_t \sin(\theta_t)] - \frac{2a}{\omega^3} (\cos(\theta_t + \omega T) - \cos(\theta_t)) \end{aligned}$$

Limit $\omega \rightarrow 0$

$$\frac{\partial}{\partial \theta_t} x_{t+T} = -\frac{1}{2} T (2v_t + aT) \sin(\theta_t)$$

$$\frac{\partial}{\partial v_t} x_{t+T} = T \cos(\theta_t)$$

$$\frac{\partial}{\partial a} x_{t+T} = \frac{1}{2} T^2 \cos(\theta_t)$$

$$\frac{\partial}{\partial \omega} x_{t+T} = -\frac{1}{6} T^2 (3v_t + 2aT) \sin(\theta_t)$$

1.2.2 y -Derivatives

Standard

$$\begin{aligned}\frac{\partial}{\partial \theta_t} y_{t+T} &= \frac{1}{\omega^2} [(v_t \omega + a \omega T) \sin(\theta_t + \omega T) - v_t \omega \sin(\theta_t) + a \cos(\theta_t + \omega T) - a \cos(\theta_t)] \\ &= \frac{1}{\omega^2} [a \omega T \sin(\theta_t + \omega T) + v_t \omega (\sin(\theta_t + \omega T) - \sin(\theta_t)) + a (\cos(\theta_t + \omega T) - \cos(\theta_t))]\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial v_t} y_{t+T} &= \frac{1}{\omega^2} [-\omega \cos(\theta_t + \omega T) + \omega \cos(\theta_t)] \\ &= \frac{1}{\omega} [-(\cos(\theta_t + \omega T) - \cos(\theta_t))]\end{aligned}$$

$$\frac{\partial}{\partial a} y_{t+T} = \frac{1}{\omega^2} [-\omega T \cos(\theta_t + \omega T) + \sin(\theta_t + \omega T) - \sin(\theta_t)]$$

$$\begin{aligned}\frac{\partial}{\partial \omega} y_{t+T} &= \frac{1}{\omega^2} [-a T \cos(\theta_t + \omega T) + a \omega T^2 \sin(\theta_t + \omega T) - v_t (\cos(\theta_t + \omega T) - \cos(\theta_t)) + v_t \omega T \sin(\theta_t + \omega T) + a T \cos(\theta_t + \omega T)] \\ &\quad - \frac{2}{\omega^3} [-a \omega T \cos(\theta_t + \omega T) - v_t \omega (\cos(\theta_t + \omega T) - \cos(\theta_t)) + a (\sin(\theta_t + \omega T) - \sin(\theta_t))] \\ &= \frac{1}{\omega^2} [(a \omega T^2 + v_t \omega T) \sin(\theta_t + \omega T) - v_t (\cos(\theta_t + \omega T) - \cos(\theta_t))] - \frac{2}{\omega^3} [-(a \omega T + v_t \omega) \cos(\theta_t + \omega T) + v_t \omega \cos(\theta_t) + a (\sin(\theta_t + \omega T) - \sin(\theta_t))] \\ &= \frac{1}{\omega^2} [(a T + v_t) (\omega T) \sin(\theta_t + \omega T) + (2a T + v_t) \cos(\theta_t + \omega T) - v_t \cos(\theta_t)] - \frac{2a}{\omega^3} (\sin(\theta_t + \omega T) - \sin(\theta_t))\end{aligned}$$

Limit $\omega \rightarrow 0$

$$\frac{\partial}{\partial \theta_t} y_{t+T} = \frac{1}{2} T (2v_t + aT) \cos(\theta_t)$$

$$\frac{\partial}{\partial v_t} y_{t+T} = T \sin(\theta_t)$$

$$\frac{\partial}{\partial a} y_{t+T} = \frac{1}{2} T^2 \sin(\theta_t)$$

$$\frac{\partial}{\partial \omega} y_{t+T} = \frac{1}{6} T^2 (3v_t + 2aT) \cos(\theta_t)$$