Rule-Compliant Trajectory Repairing using Satisfiability Modulo Theories

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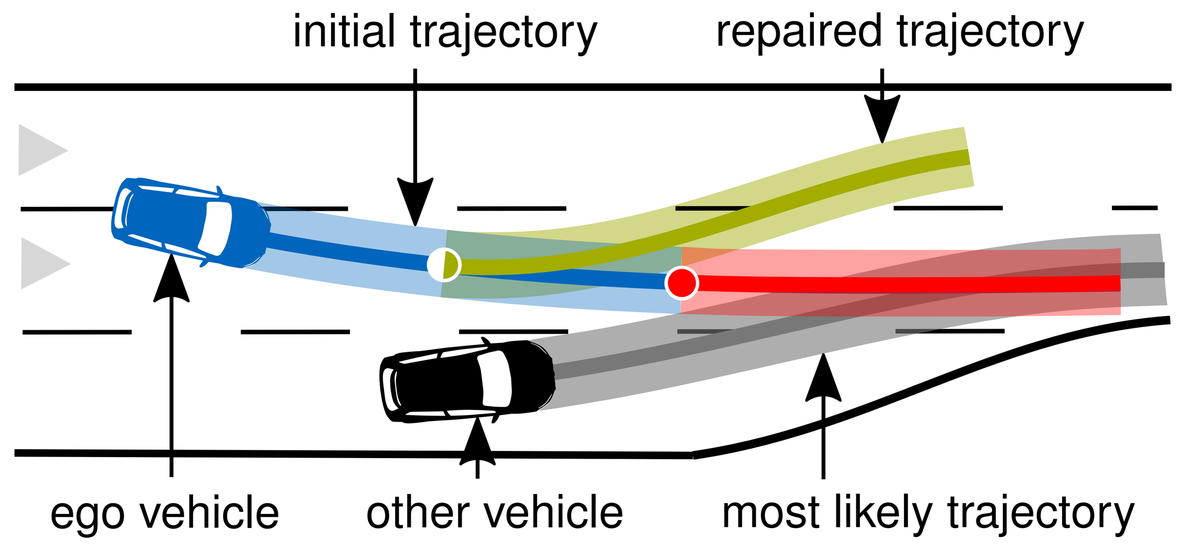
I. Trajectory Repairing

Scope: Autonomous vehicles must comply with traffic rules. Once traffic rule violations of an initially-planned trajectory are detected, replanning a complete trajectory is often unnecessary and time-consuming. To solve this problem, we propose to repair the initial trajectory.



- state of **time-to-violation**: the first time step at which the trajectory violates traffic rules
- state of **time-to-comply**: the last time step for which a rule-compliant trajectory exists

II. Traffic Rule Formalization

Definition (Metric Temporal Logic (MTL)) *Given a propositional variable σ, an associated interval I, the temporal previously, once, and globally operator $\mathbf{P}$, $\mathbf{O}_I$, and $\mathbf{G}_I$, an MTL formula φ is defined as:*

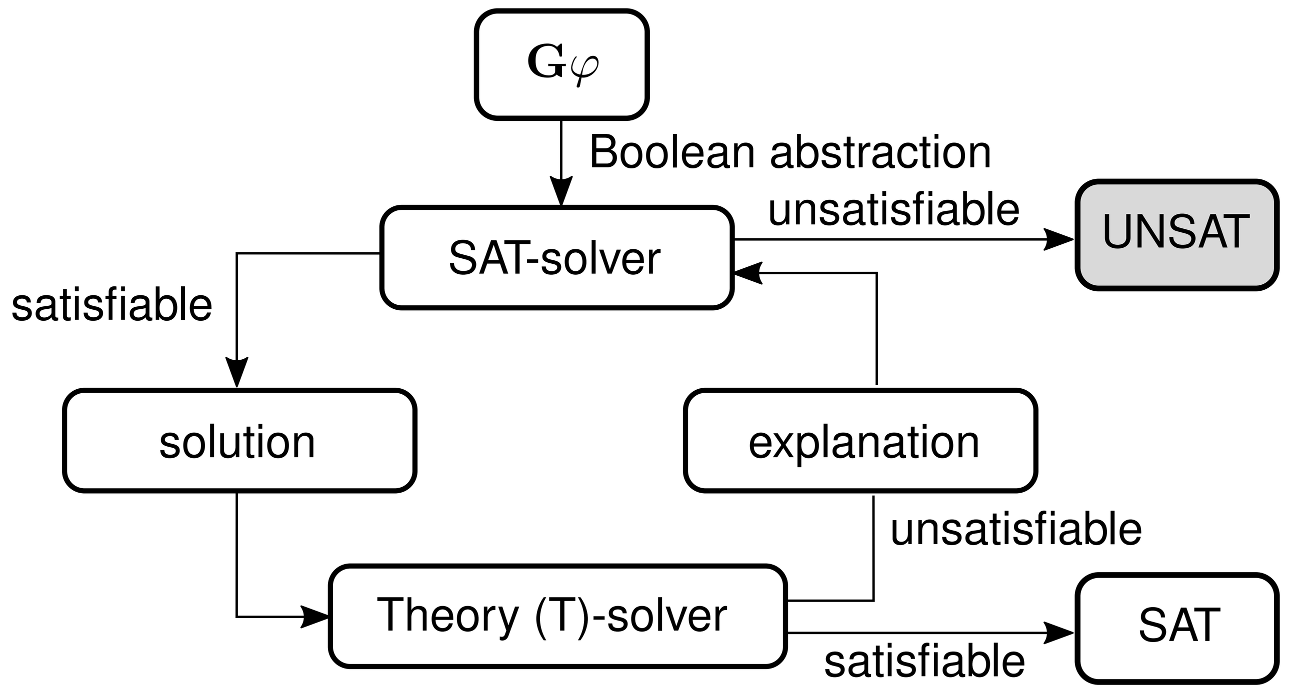
$$\varphi ::= \top \mid \sigma \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \varphi_1 \wedge \varphi_2 \mid \mathbf{P}\varphi \mid \mathbf{O}_I\varphi \mid \mathbf{G}_I\varphi.$$

- **Rule R_G1** - *Safe distance to preceding vehicle:*
  $$\mathbf{G}\big(\mathrm{in\_same\_lane}(x_{\mathrm{ego}}, x_b) \wedge \mathrm{in\_front\_of}(x_{\mathrm{ego}}, x_b) \wedge \neg\mathbf{O}_{[0,t_c]}\big(\mathrm{cut\_in}(x_b, x_{\mathrm{ego}}) \wedge \mathbf{P}(\neg\mathrm{cut\_in}(x_b, x_{\mathrm{ego}}))\big) \Rightarrow \mathrm{keeps\_safe\_distance\_prec}(x_{\mathrm{ego}}, x_b)\big)$$
- **Rule R_G2** - *Unnecessary braking:* $\mathbf{G}\big(\mathrm{brakes\_abruptly}(x_{\mathrm{ego}}) \Rightarrow \mathrm{braking\_justification}(x_{\mathrm{ego}}, \Omega)\big)$
- **Rule R_G3** - *Maximum speed limit:*
  $$\mathbf{G}\big(\mathrm{keeps\_lane\_speed\_limit}(x_{\mathrm{ego}}) \wedge \mathrm{keeps\_type\_speed\_limit}(x_{\mathrm{ego}}) \wedge \mathrm{keeps\_fov\_speed\_limit}(x_{\mathrm{ego}}) \wedge \mathrm{keeps\_braking\_speed\_limit}(x_{\mathrm{ego}})\big)$$

(x_{ego}: state of ego vehicle, x_b: state of the rule relevant vehicle, Ω: environment model)

III. Satisfiability-Modulo-Theories-based Solution

Definition (Satisfiability Modulo Theories (SMT)) *SMT is about checking the satisfiability of formulas with respect to decidable background theories by combining SAT solving and theory-specific decision procedures (T-solver).*

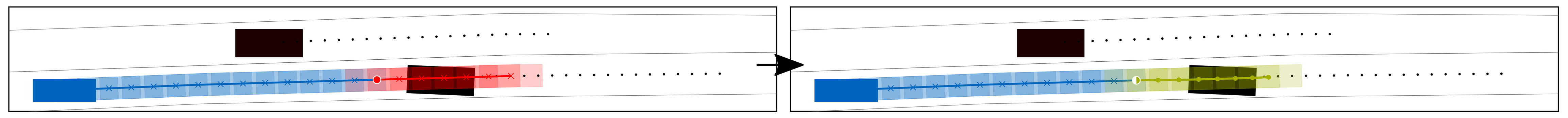


Artictecture of the lazy SMT solver.

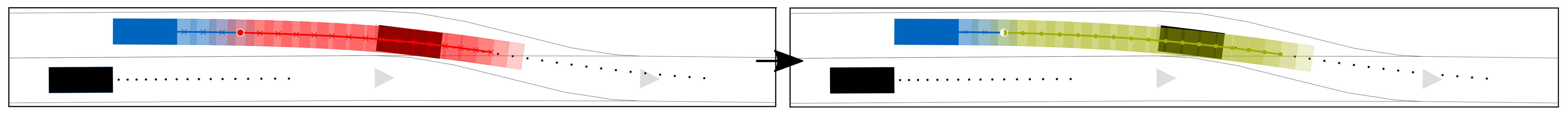
- **Boolean Abstraction**: abstracting temporal logic formulas to Boolean propositional formulas in conjunctive normal form
- **SAT-Solver**: determining whether there exists a solution that satisfies a Boolean formula
- **T-Solver**: checking the satisfiability of the obtained assignment from SAT-solver in a trajectory-repairing framework
  - *Time-To-Comply Search*: underapproximated by compliant maneuvers using a point-mass vehicle model
  - *Optimization-based Trajectory Repairing*: using convex linear-quadratic programs including the robustness degree of temporal logic specifications as an objective

IV. Case Studies

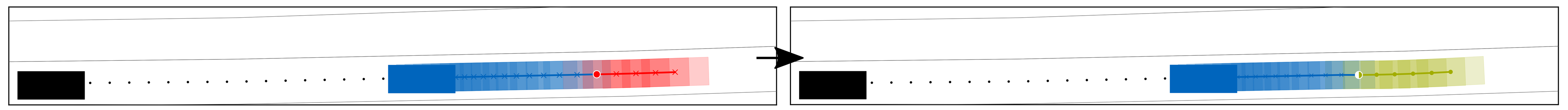
R_G1 (CommonRoad ID: DEU_Gar-1_1_T-1)

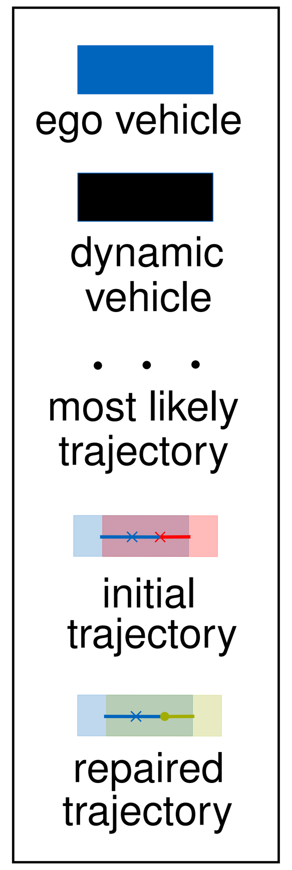


R_G2 (CommonRoad ID: ZAM_Zip-1_56_T-1)



R_G3 (CommonRoad ID: DEU_Muc-4_2_T-1)



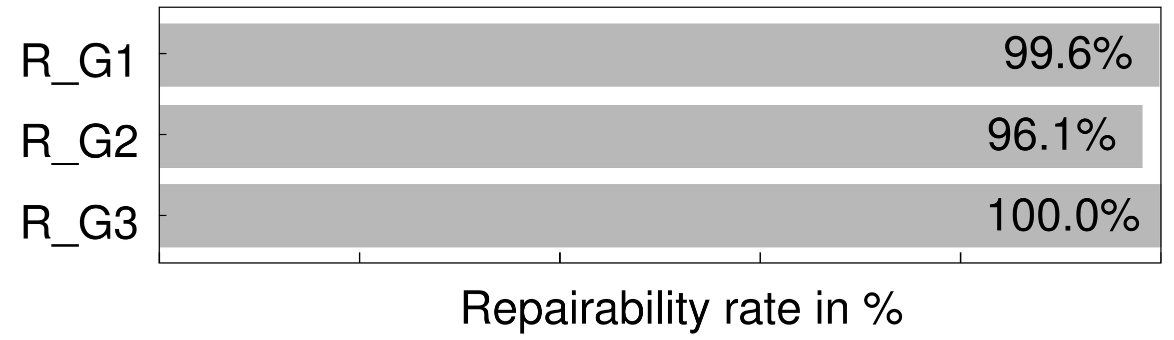


Performance Evaluation

Comparison to trajectory replanning:

Computation Time in ms	R_G1	R_G2	R_G3
Repairing	223	168	147
Replanning	887	399	398

Evaluation with the highD dataset on over 1,000 rule violating trajectories:



All our scenarios are available at `commonroad.in.tum.de`, which provides open-source benchmarks for trajectory planning/repairing.

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