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Homework 2
CSE 2302-002

a. $2^{n+1} = O(2^n)$

We have $f(n) = 2^{n+1}$

$g(n) = O(2^n)$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{2^n}{2^{n+1}} = \lim_{n\to\infty} \frac{1}{2^{n+1-n}} = \lim_{n\to\infty} \frac{1}{2}$$

$= \frac{1}{2} \rightarrow$ is a constant

$\rightarrow 2^{n+1} = O(2^n)$ (limit theorem)

$\rightarrow$ Yes, it's valid

b. $2^{2n} = O(2^n)$

$f(n) = 2^{2n}$

$g(n) = O(2^n)$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{2^{2n}}{2^n} = \lim_{n\to\infty} 2^{2n-n}$$

$$= \lim_{n\to\infty} 2^n$$

$$= \lim_{n\to\infty} e^{n\ln(2)}$$

$\lim_{n\to\infty} (n\ln(2)) = \infty \Rightarrow \lim_{n\to\infty} e^{n\ln(2)} = \infty$

$\Rightarrow 2^{2n} \neq O(2^n)$ (2^{2n} grows faster than 2^n as n approaches infinity)

$\rightarrow$ Not valid