1.

1. $x[n] = 2U[n] - 4U[n-1]$

$U[n]$ Noise $N(0,1)$

(a) To be w.s.s.

$E_x[n] = E[2U[n] - 4U[n-1]]$

$= 2E[U[n]] - 4E[U[n-1]]$

$= 0$

mean is constant

$R_x[n] = E[x[n]\, x[n+k]]$

$= E[(2U[n] - 4U[n-1])(2U[n+k] - 4U[n+k-1])]$

$= E[4U[n]U[n+k] + 16U[n-1]U[n+k-1]$
$- 8U[n-1]U[n+k] - 8U[n]U[n+k-1]]$

$=$

$k=0$ $= 4E[\underbrace{U[n]^2}_{Var}] + 16E[\underbrace{(U[n-1])^2}_{Var}] - 8[0] - 8[0]]$

$= 4 + 16 = 20.$

$k=1$ $= 0 + 0 - 0 - 8E[(U[n])^2]$

$= -8$

$k=-1$ $= 0 + 0 - 8[E(U[n-1])^2] - 0$

$= -8$

$$R_x[k] = \begin{cases} 20 & k=0 \\ -8 & k=\pm 1 \\ 0 & o.w \end{cases}$$

$\therefore$ The system is w.s.s.

(b)

$$P_x(f) = \sum_{k=-\infty}^{\infty} r_x[k]\, e^{-j2\pi f k}$$

$$= 20(1) + (-8)\, e^{-j2\pi f(1)} + (-8)\, e^{-j2\pi f(-1)}$$

$$= 20 - 8 \underbrace{\left(\frac{e^{-j2\pi f} + e^{j2\pi f}}{2} \right)}_{\cos(2\pi f)} \times 2$$

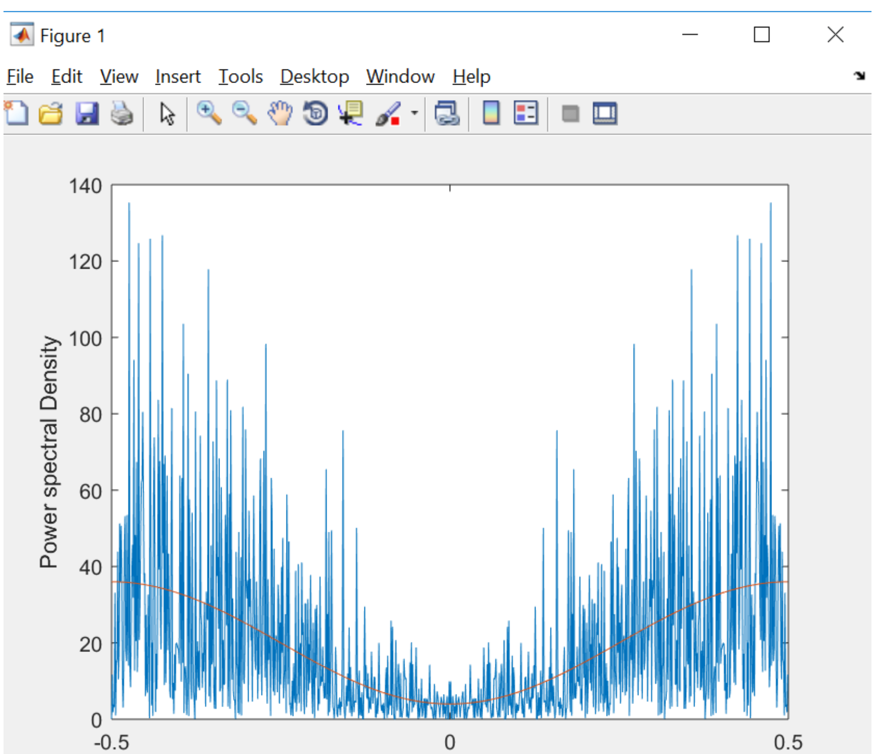
$$=$$

$$P_x(f) = 20 - 16\cos(2\pi f)$$

1.b. Matlab code

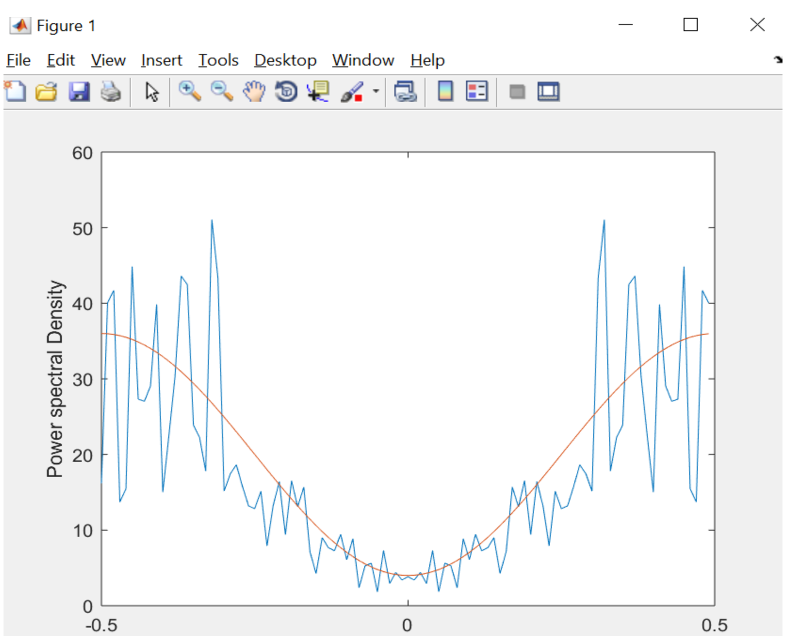
HW8matlab.m

```
clc;
clear;
N=1000;
u=randn(N,1);
for n = 1:N
    if n==1 x(n)=2*u(n);
    else x(n)=2*u(n)-4*u(n-1);
    end
end

f=(0:(N-1))./N;
P=1/N*abs(fft(x)).^2;

figure(1)
plot(f-0.5,fftshift(P));
hold on;

plot((f-0.5),20-16*cos(2*pi*(f-0.5)));
xlabel("X[n]")
ylabel("Power spectral Density")

```



1.c Matlab Code

```
clc;clear;
N=1000;
K=10;
Ns=N/K;
P=0;
u=randn(N,1);
for n=1:N
    if n==1 x(n)=2*u(n);

    else x(n)=2*u(n)-4*u(n-1);
    end
end

for k=1:K-1
    xi=x(1+k*Ns:k*Ns+Ns);
    Pi=1/Ns*abs(fft(xi)).^2;
    P=P+Pi;
end

P=1/K*P;
f=[0:Ns-1]./Ns;
figure(1)
plot(f-0.5,fftshift(P));hold on;
plot((f-0.5),20-16*cos(2*pi*(f-0.5)))
xlabel("X[n]")
ylabel("Power spectral Density")
```



2.a.

2. (a) $\sigma_x^2 = 1$ $\quad a = 0.85$

$$y_n = a\, y_{n-1} + X_n$$

$$S_y(f) = \frac{\sigma_x^2}{1 + a^2 - 2a\cos(2\pi f)}$$

$$= \frac{1}{1 + a^2 + 2a(f)}$$

matlab code

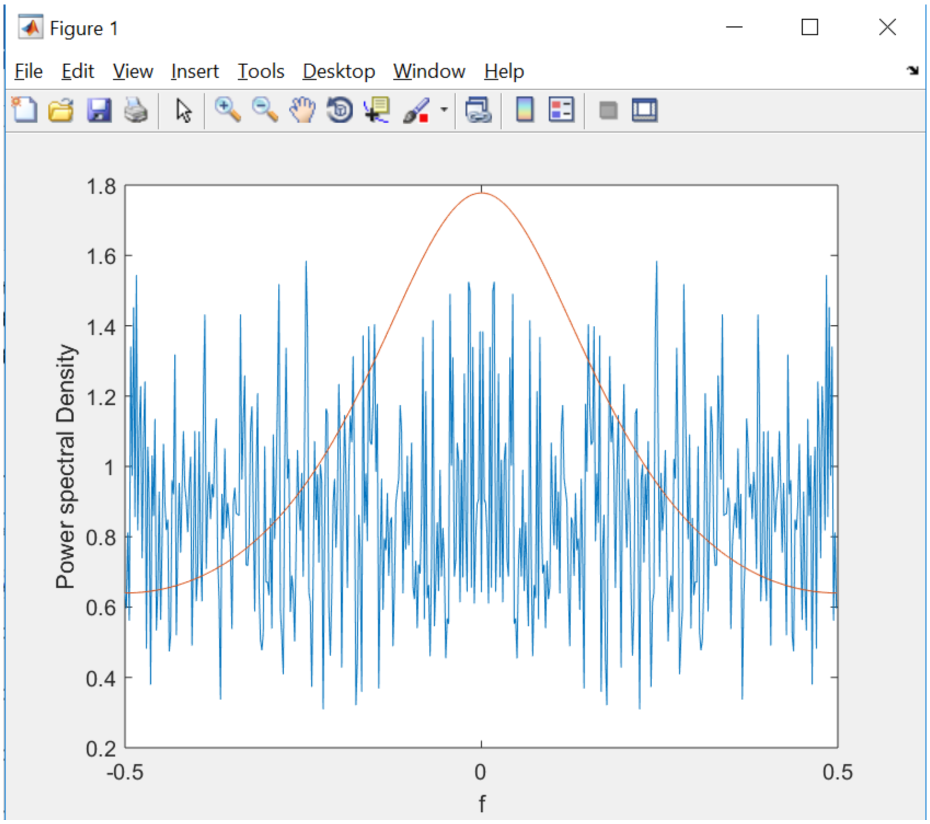
Editor - C:\Users\gowth\Documents\MATLAB\HW8matlab2prob.m*

HW8matlab.m | Untitled2.m | HW8matlab2prob.m*

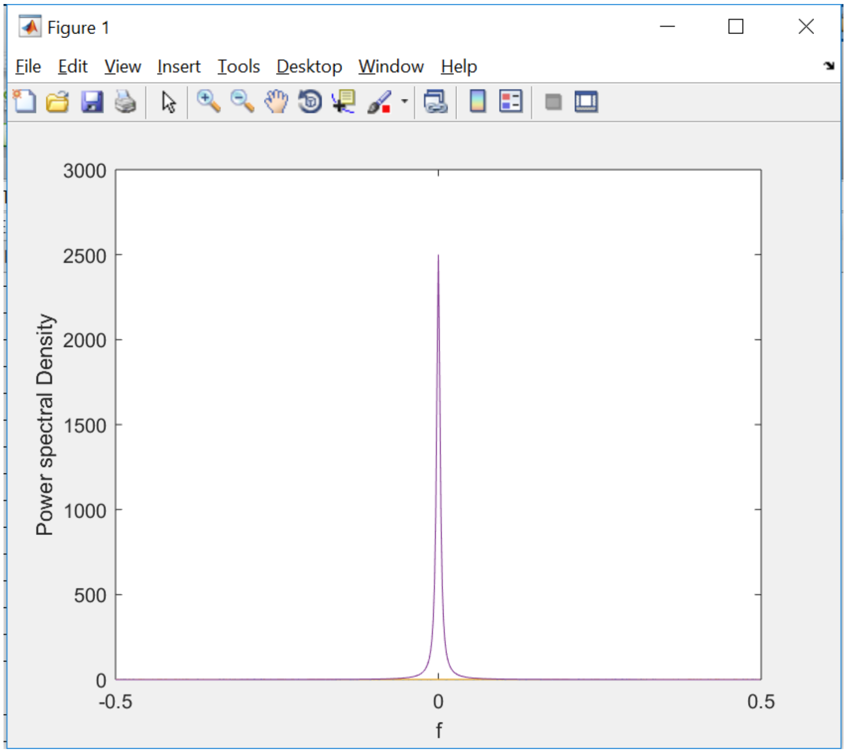
```
clc;clear;
N=5000;K=10;
a=0.98;Ns=N/K;
P=0;
x=randn(N,1);
for n = 1:N
    if n==1
        y(n) =  x(n);
    else
        y(n) = a*y(n-1) + x(n);
    end
end

for k=1:K-1
    xi=x(1+k*Ns:k*Ns+Ns);
    Pi=1/Ns*abs(fft(xi)).^2;
    P=P+Pi;
end

P=1/K*P;
f=(0:(Ns-1))./Ns;

figure(1)
plot(f-0.5,fftshift(P));hold on;
plot((f-0.5),1./(1+a^2-2*a*cos(2*pi*(f-0.5))));
xlabel("f")
ylabel("Power spectral Density")
```

Output for

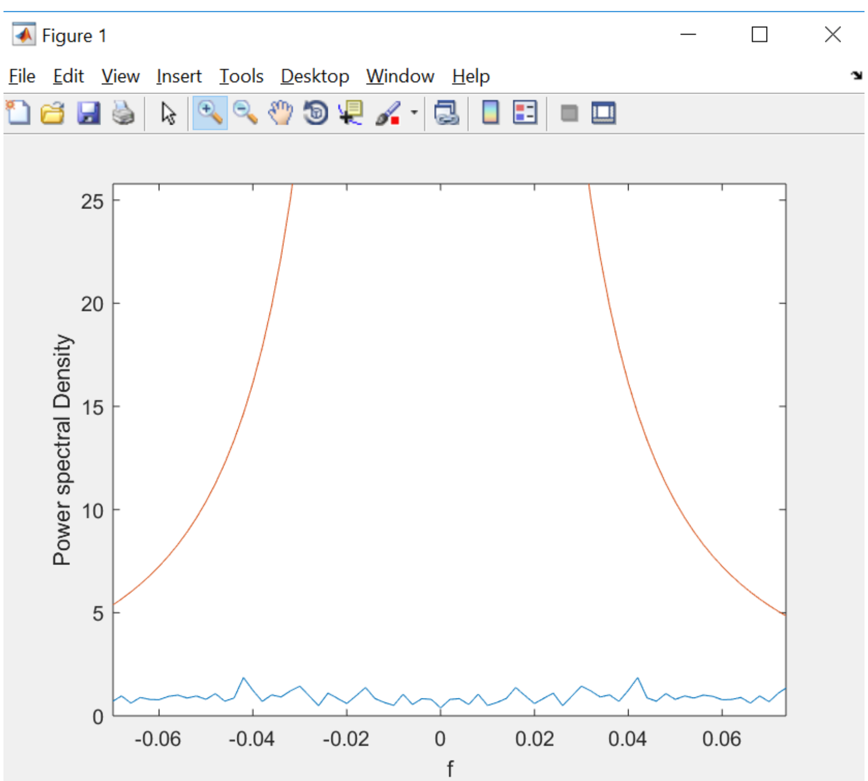
```
N=5000;
K=10;
a=0.25;
```



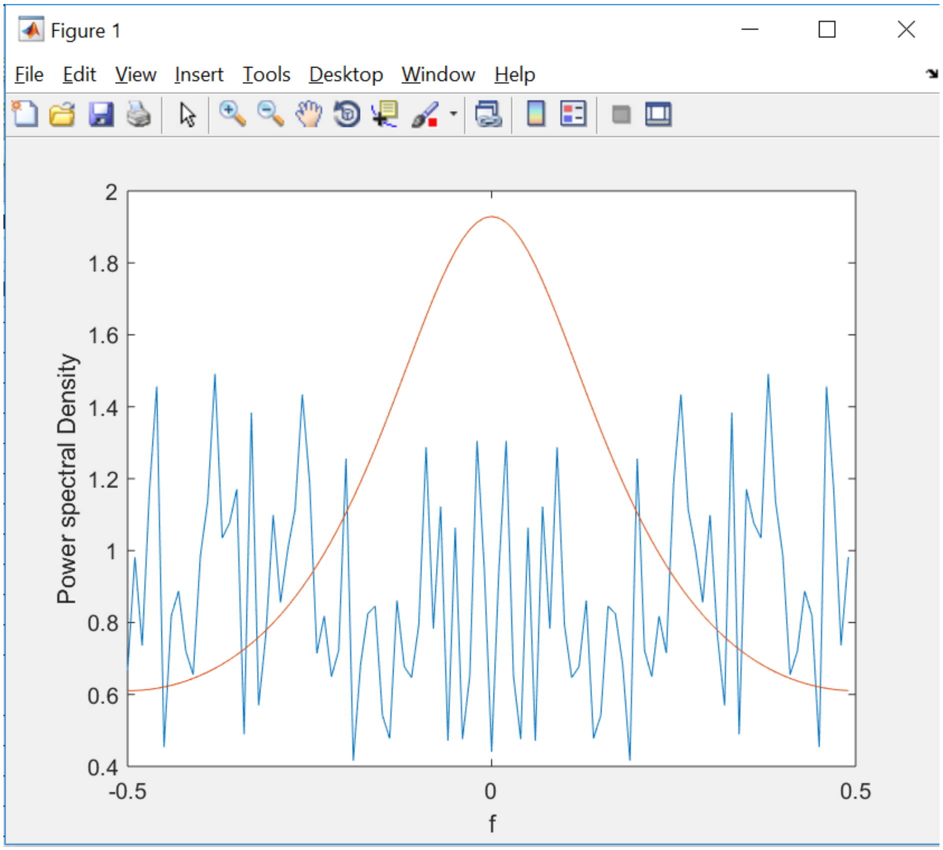
2.b for `N=5000;K=10;a=0.98;`



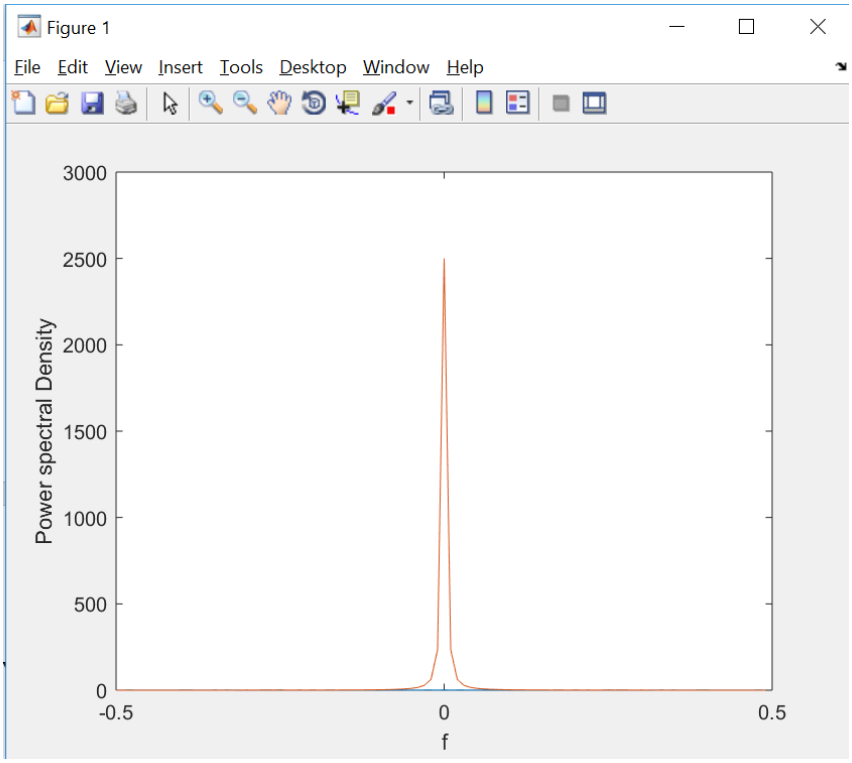
Zoom in Screenshot

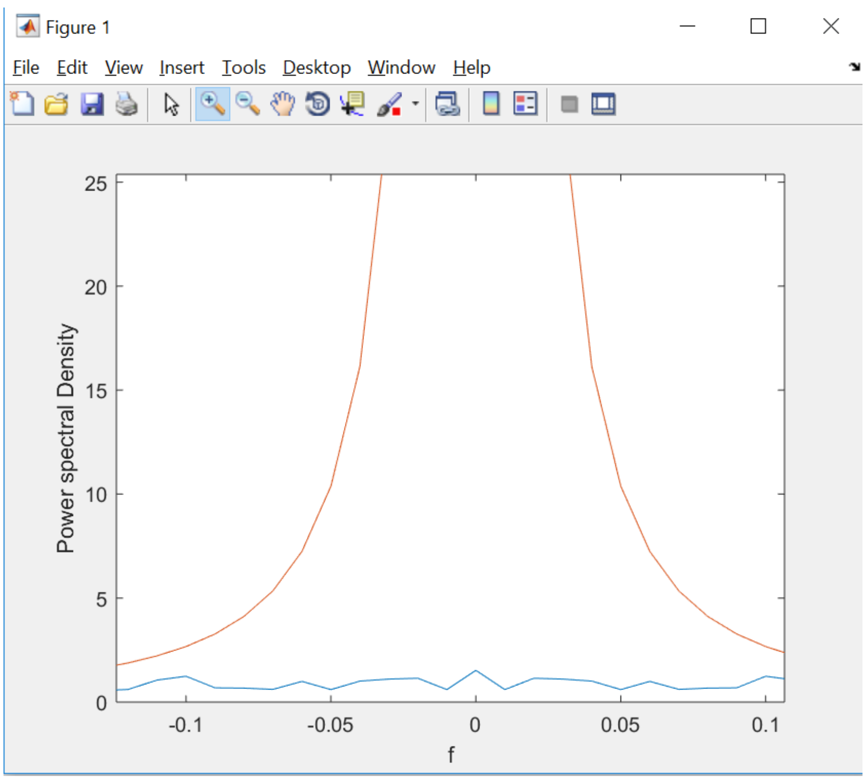


For values N=1000;K=10;a=0.25;



For values N=1000;K=10;a=0.98;





By comparing all the values for a,

Correlation increases with the increase in the value of a.

GOWTHAM TUMMALA

G01123244