

1.

$$1. \quad x[n] = 2U[n] - 4U[n-1]$$

$U[n]$ noise $N(0,1)$

(a) To be w.s.s.

$$E_x[n] = E[2U[n] - 4U[n-1]]$$

$$= 2E[U[n]] - 4E[U[n-1]]$$

$$= 0$$

mean is constant

$$R_x[n] = E[x[n]x[n+k]]$$

$$= E[(2U[n] - 4U[n-1])(2U[n+k] - 4U[n+k-1])]$$

$$= E[4U[n]U[n+k] + 16U[n-1]U[n+k-1] - 8U[n-1]U[n+k] - 8U[n]U[n+k-1]]$$

$$\begin{aligned} k=0 &= 4E[U[n]^2] + 16E[U[n-1]^2] - 8E[U[n-1]U[n]] - 8E[U[n]U[n-1]] \\ &= 4 + 16 = 20. \end{aligned}$$

$$\begin{aligned} k=1 &= 0 + 0 - 8E[U[n]^2] \\ &= -8 \end{aligned}$$

$$\begin{aligned} k=-1 &= 0 + 0 - 8E[U[n-1]^2] = 0 \\ &= -8 \end{aligned}$$

$$R_x[k] = \begin{cases} 20 & k=0 \\ -8 & k=\pm 1 \\ 0 & \text{o.w} \end{cases}$$

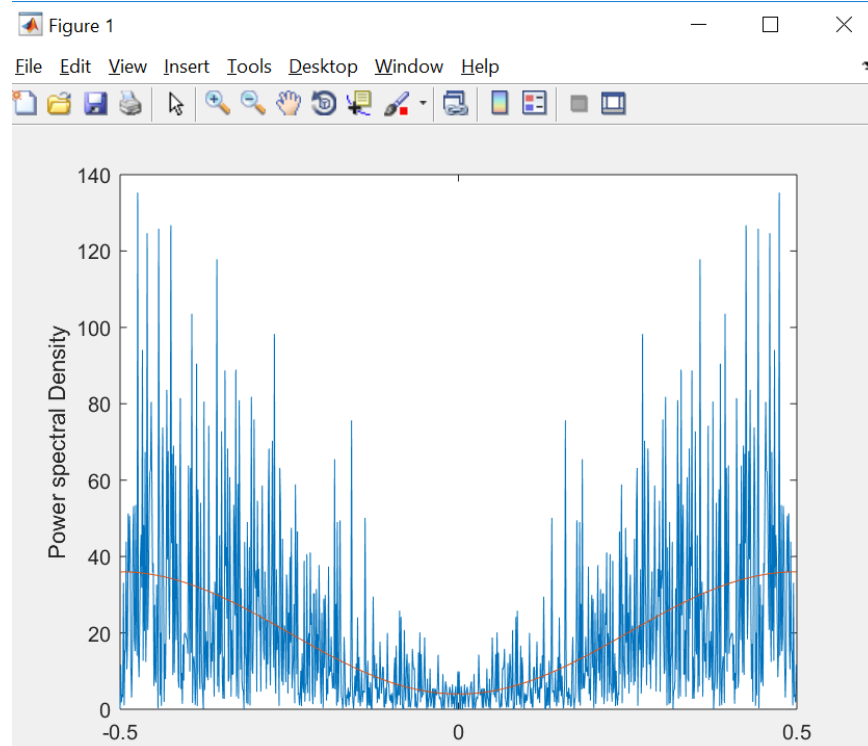
∴ The system is W.S.S.

$$\begin{aligned} \textcircled{b} \quad P_x(f) &= \sum_{k=-\infty}^{\infty} r_x[k] e^{j2\pi f k} \\ &= 20(1) + (-8) e^{-j2\pi f(1)} + (-8) e^{-j2\pi f(-1)} \\ &= 20 - 8 \left(\frac{e^{-j2\pi f} + e^{j2\pi f}}{2} \right) \times 2 \\ &= \underbrace{20 - 16 \cos(2\pi f)} \end{aligned}$$

$$P_x(f) = 20 - 16 \cos(2\pi f)$$

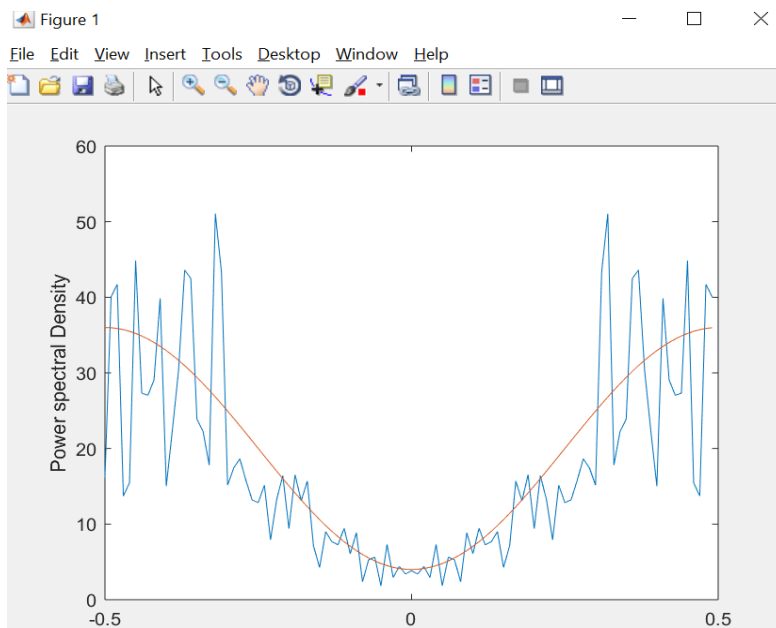
1.b. Matlab code

```
HW8matlab.m x +
1 -   clc;
2 -   clear;
3 -   N=1000;
4 -   u=randn(N,1);
5 -   for n = 1:N
6 -       if n==1 x(n)=2*u(n);
7 -       else x(n)=2*u(n)-4*u(n-1);
8 -       end
9 -   end
10
11 -   f=(0:(N-1))./N;
12 -   P=1/N*abs(fft(x)).^2;
13
14 -   figure(1)
15 -   plot(f-0.5,fftshift(P));
16 -   hold on;
17
18 -   plot((f-0.5),20-16*cos(2*pi*(f-0.5)));
19 -   xlabel("X[n]")
20 -   ylabel("Power spectral Density")
21
```



1.c Matlab Code

```
Editor - C:\Users\gowth\Documents\MATLAB\Untitled2.m*
HW8matlab.m  Untitled2.m*  +
1 -   clc;clear;
2 -   N=1000;
3 -   K=10;
4 -   Ns=N/K;
5 -   P=0;
6 -   u=randn(N,1);
7 -   for n=1:N
8 -       if n==1 x(n)=2*u(n);
9 -
10 -      else x(n)=2*u(n)-4*u(n-1);
11 -      end
12 -   end
13
14 -   for k=1:K-1
15 -       xi=x(1+k*Ns:k*Ns+Ns);
16 -       Pi=1/Ns*abs(fft(xi)).^2;
17 -       P=P+Pi;
18 -   end
19
20 -   P=1/K*P;
21 -   f=[0:Ns-1]./Ns;
22 -   figure(1)
23 -   plot(f-0.5,fftshift(P));hold on;
24 -   plot((f-0.5),20-16*cos(2*pi*(f-0.5)))
25 -   xlabel("X[n]")
26 -   ylabel("Power spectral Density")
27
```



2.a.

Handwritten mathematical derivation for the power spectral density of an AR(1) process:

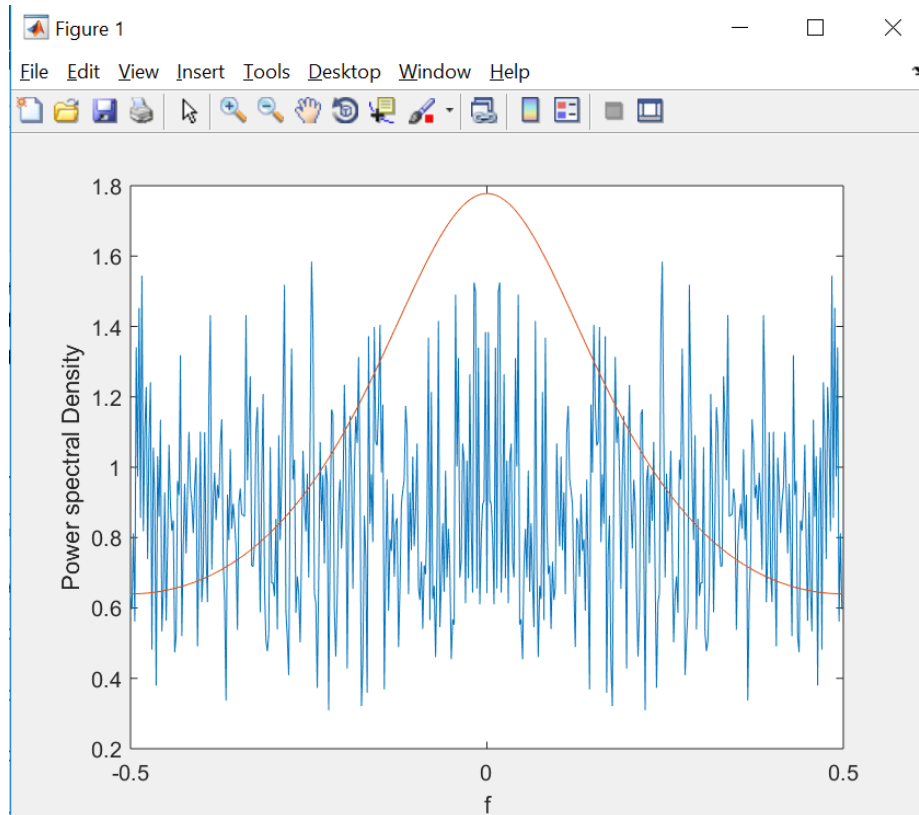
$$\begin{aligned} 2. \quad (a) \quad \sigma_x^2 &= 1 \quad a = 0.85 \\ y_n &= a y_{n-1} + x_n \\ S_y(f) &= \frac{\sigma_x^2}{1 + a^2 - 2a \cos(2\pi f)} \\ &= \frac{1}{1 + a^2 - 2a \cos(2\pi f)} \end{aligned}$$

matlab code

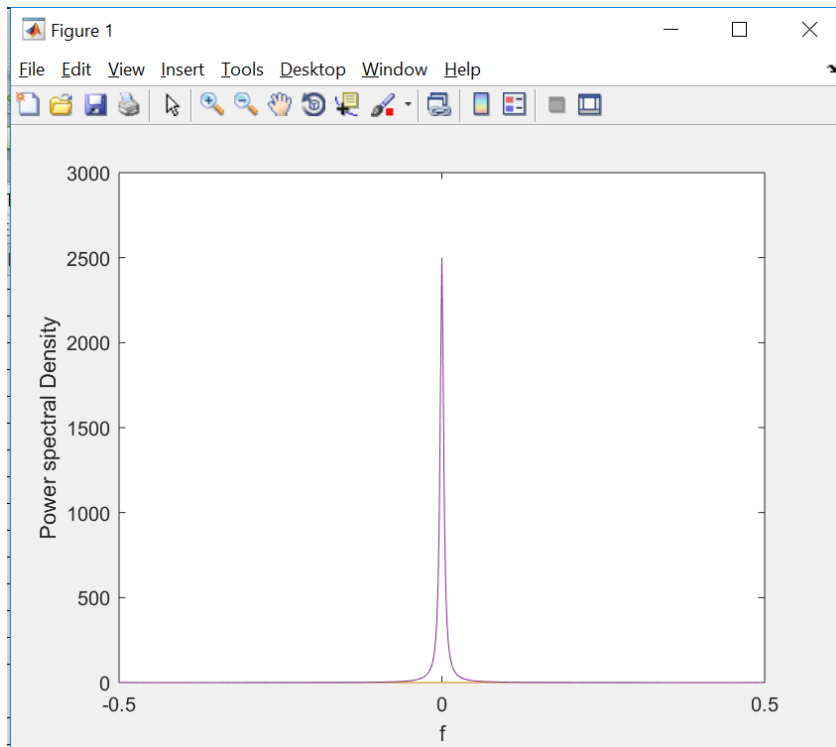
Editor - C:\Users\gowth\Documents\MATLAB\HW8matlab2prob.m*

```
1 - clc;clear;
2 - N=5000;K=10;
3 - a=0.98;Ns=N/K;
4 - P=0;
5 - x=randn(N,1);
6 - for n = 1:N
7 -     if n==1
8 -         y(n) = x(n);
9 -     else
10 -        y(n) = a*y(n-1) + x(n);
11 -    end
12 - end
13
14 - for k=1:K-1
15 -     xi=x(1+k*Ns:k*Ns+Ns);
16 -     Pi=1/Ns*abs(fft(xi)).^2;
17 -     P=P+Pi;
18 - end
19
20 - P=1/K*P;
21 - f=(0:(Ns-1))./Ns;
22
23 - figure(1)
24 - plot(f-0.5,fftshift(P));hold on;
25 - plot((f-0.5),1./(1+a^2-2*a*cos(2*pi*(f-0.5))));
26 - xlabel("f")
27 - ylabel("Power spectral Density")
```

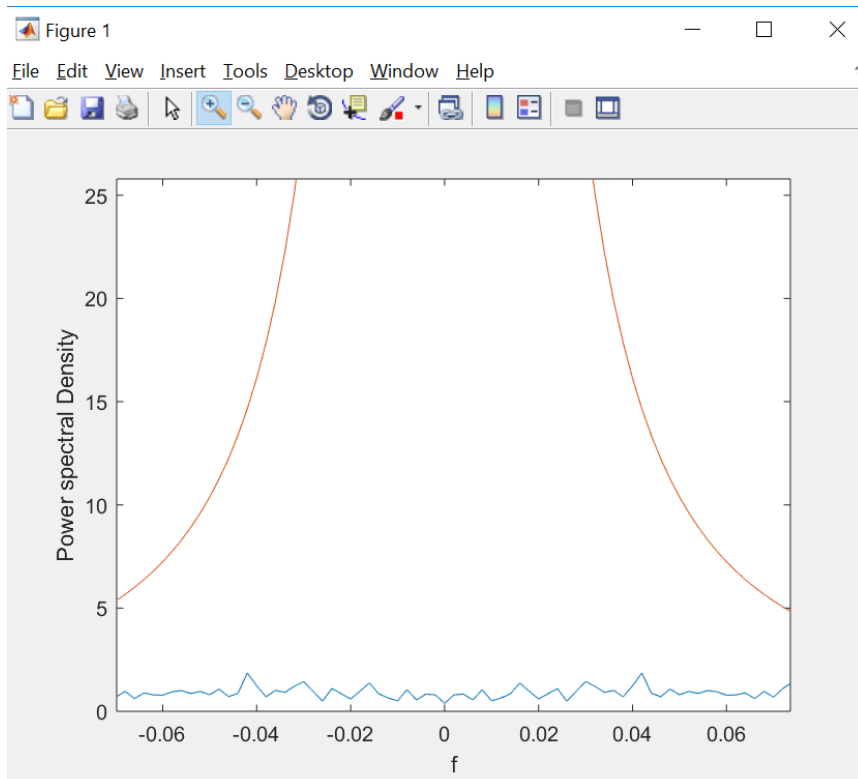
Output for
 $N=5000$;
 $K=10$;
 $a=0.25$;



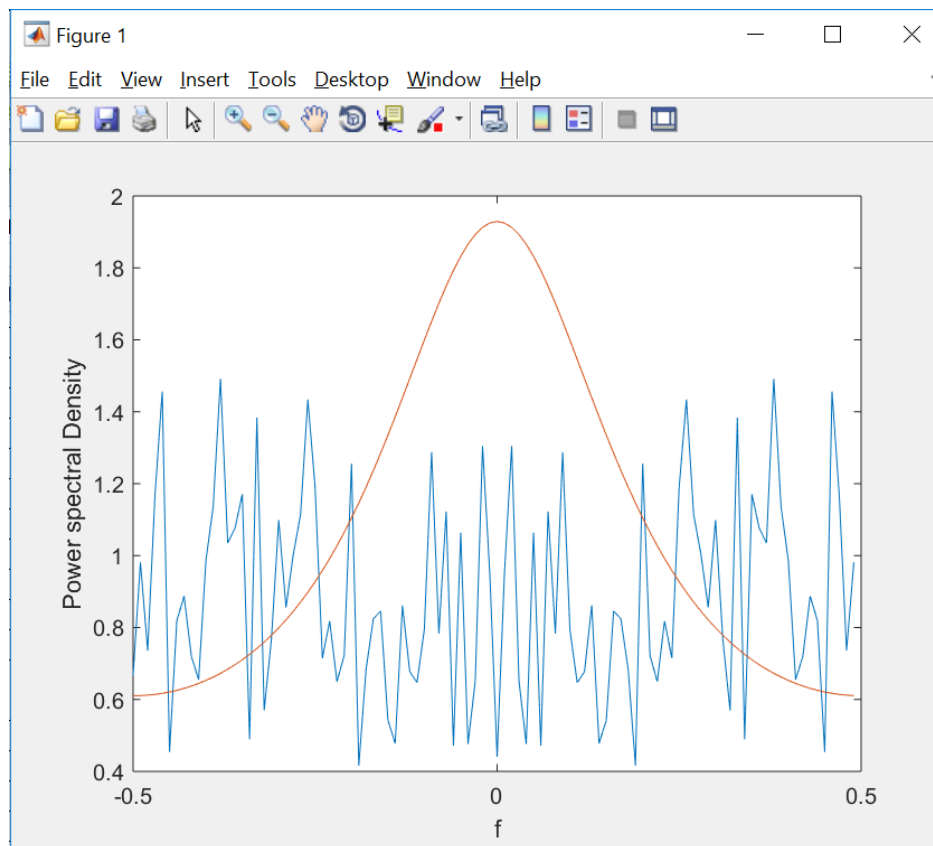
2.b for $N=5000$; $K=10$; $a=0.98$;



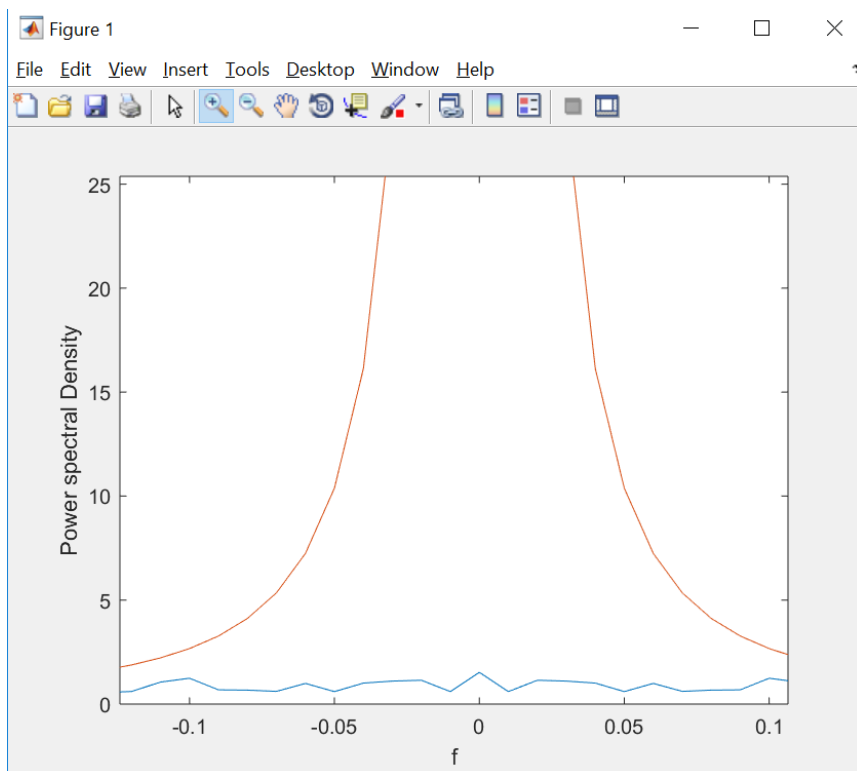
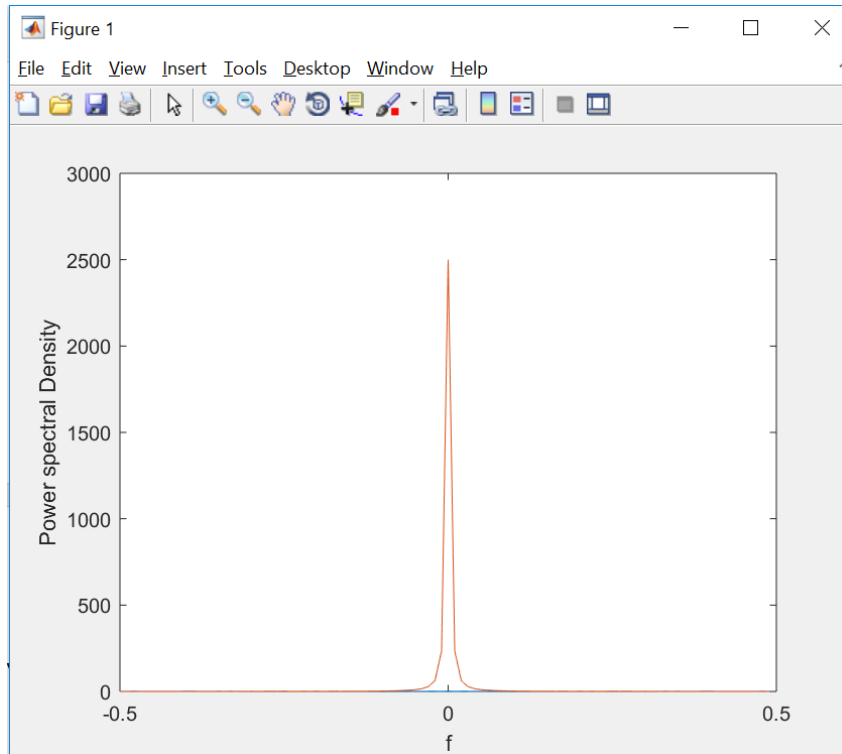
Zoom in Screenshot



For values $N=1000$; $K=10$; $a=0.25$;



For values $N=1000; K=10; a=0.98;$



By comparing all the values for a ,
Correlation increases with the increase in the value of a .

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