Summary

The contest rules specify that you should include a one-page summary of your report. This page appears before the rest of the report, and will have a special header attached to it that takes up the top 2.5″ of the page.

By typing your summary inside a `summary` environment, TEX will handle the formatting of that page correctly, including leaving space at the top of the page and not numbering the page.

It will also reset the page numbers so that the first page of your report is labeled correctly.

What should you put here? Basically, you want a brief restatement of the problem followed by a largely *non-technical* description of what you've done. Try to avoid using mathematical notation.

You probably want to write a few paragraphs, around half to two-thirds of a page.

For 2009, the COMAP folks said the following about the summary:

> The summary is a very important part of your MCM paper. The judges place considerable weight on the summary, and winning papers are sometimes distinguished from other papers based on the quality of the summary. To write a good summary, imagine that a reader may choose whether to read the body of the paper based on your summary. Thus, a summary should clearly describe your approach to the problem and, most prominently, what your most important conclusions were. The summary should inspire a reader to learn the details of your work. Your concise presentation of the summary should inspire a reader to learn the details of your work. Summaries that are mere restatements of the contest problem, or are a cut-and-paste boilerplate from the Introduction are generally considered to be weak.
>
> To Summarize:
>
> **Restatement Clarification of the Problem** —
>
> **Assumptions with Rationale/Justification** —emphasize those assumptions that bear on the problem. List clearly all variables used in your model.

Model Design and justification for type model used/developed.

Model Testing and Sensitivity Analysis, including error analysis, etc.

Discuss strengths and weakness to your model or approach.

Provide algorithms in words, figures, or flow charts (as a step by step algorithmic approach) for all computer codes developed.

[1]

A Hexagonal Model of Repeater Coordination

ICM Contest Question B

Team # 15118

February 11, 2012

Contents

List of Figures

List of Tables

1 Introduction

In the Very High Frequency (VHF) communication system, the signal by itself cannot go much farther than the line-of-sight, because it travels as a straight line while the earth surface is curved, thereby limiting the range that two VHF users can directly communicate. The usage of repeaters can solve this problem by amplifying the weak signal it receives and, to avoid self-interference, transmits at a slightly different frequency but in a stronger transmitting power, hence extending the range of communication significanly. However, the two repeaters might interfere with each other and interrupt the comminucation for both of them. To solve this problem, we need to design a coordination system of repeaters to avoid the interference, either by placing them their locations or using sufficiently different frequencies. Further, we are given the technology of CCTCS or Private Line (PL) for a repeater to distinguish two signals with the same frequency but different PL tones, therefore reducing the number of repeaters needed at a place.

The goal to approach this problem is to design a repeater coordination system that minimizes the number of repeaters but is meanwhile capable to accommodate 1,000 people *simultaneously* (meaning that *after* we know where those 500 pairs of people live, we design a model to allow them to communicate to each other). Our approach to this problem is to design a mathematical model to simulate a uniform distribution of people and determine how many repeaters are required for each region.

——— TODO: the history and context of the problem, and your work and results. Your introduction should be more detailed and technical than your summary. You may also want to include an outline of your report, along the lines of

> In Section 1 we give our definitions and notation. Section 2 describes our numerical experiments.. . . .
> We prove our main result, Theorem 6, in Section 5. . . .

Of course you would replace the numbers in that example with appropriate `\ref` commands pointing to the correct `\labels` in your source.

2 Givens

1. We are given that Blah
2. We are given the usage of Private Line (PL).
3. The study area is flat and of the 40-mile radius.
4. The number of simultaneous users is 1,000.
5. The range of frequencies is 3 MHz (3,000 kHz), ranging from 145 to 148 MHz.
6. The offset is $\Delta(f) = 600$ kHz.
7. We have 54 PL tones in total.

3 Assumptions

1. A repeater can serve multiple usages for the same frequency, providing that the PL tones are different. If a repeater senses at least two signal with the same frequency and same PL tone, then the communication collapse and this mishap should not happen at at point of the system.

2. The interference is discrete. If two repeaters are closer than R, the range of each repeater, then they interfere each other. On the other hand, if the distance is more than R, then the two repeaters are independent. If, however, the distance is approxiamtely R, we assume that they can either connected (not intefere) with each other. A reason for this is that the real usage of a chain of repeaters called "reverse splits" use this (TODO: explain).

3. Assume that the range of repeaters is fixed in this problem. According to (TODO: find some website that is more reliable than Wiki), the range depends on the antenna's height as follows:

$$R(\text{mile}) = \sqrt{1.5 \times A_f},$$

   where A_f is the antenna's height in foot. This equation is deviant when the are is mountainous, which we will explore in a later section. The typical heights of the antenna is about 15 to 50 feet, corresponding to the range R of 5 to 10 miles.

4. From the design of using 600 kHz as an offset frequency, we assume that 600 kHz is a threshold. The difference of frequencies lower than this threshould should interfere. (TODO: verify this!)

5. The distribution of people is a variable. However, we assume that once the people live, they are static. That is, we can think of those people as stations communicating to one another but not moving.

6. To avoid the ambiguity when one person, say A, wants to communicate to B but the voice also travels to C and D. We assume that A can overcome this ambiguity by indicating a code when starting the conversation, such as "Call B. This is A. Line number 4," so that C and D can stop listening and that B knows which line to contact back to A.

4 Model Design

4.1 Setup

1. The main part is about location. We need to configure a good location, and the PL tones and 3 different sets of frequencies can help reducing the number of repeaters later. (explain why. TODO)

2. We introduce the hexagonal-coordinate system. TODO: explain why we can use hexagons to approximate circles, and how the radius of 40 miles compared to R of 10 miles, plays a role in this model.

   Figure 1 is an example of our definition of the hexagonal coordinate. Notice that it needs not fit into the circle. The $y-$axis is 60° to the $x-$axis. Note that the point $(-1,-1)$ is in the circle where as the point $(1,1)$ is not, even though the origin of the circle is at $(0,0)$.

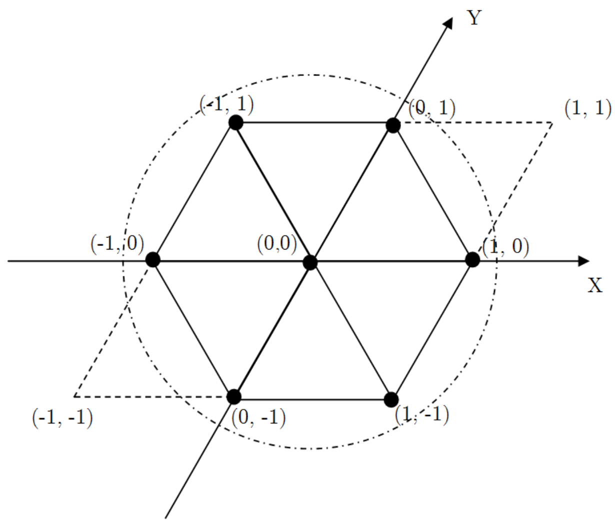

Figure 1: Hexagonal Coordinate System.

3. We generate a distribution of of people in two types: uniform and cluster (a city model).

4. For any pair of people, find the shortest distance s between the two and keep track the multiple possible paths. Add one to the nodes it travels through, meaning that that point needs an additional repeater for this path. Ultimately, these two people will choose only one way to communicate (over than this is a waste), so we need to divide the number of repeaters added from the previous step by the total number of possible paths. This method will keep the distribution of possible paths accurately while having the total number of repeaters added repeaters equal to s (since for each path we add s for the account of newly added repeaters.) (TODO: simplify this!)

5. MORE ASSUMPTION: we assume that the repeaters locate at integer coordinates only!

6. Also we need to mention that we use python, I think.

4.2 Theorems

Theorem 1 *For a given point C, of all the paths from point A to point B, the number of repeaters needed at point C is equivalently increased by*

$$\frac{nPaths(A,C) \cdot nPaths(C,B)}{nPaths(A,B)},$$

where $nPaths(X,Y)$ is a function that returns the number of possible shortest *paths from X to Y, which is given in Theorem 2.*

Proof. It follows from the principle of counting:

$$\begin{aligned}\text{The weight for additional repeaters at C} &= \frac{\text{The number of paths from A to B via C}}{\text{The number of paths from A to B}} \\ &= \frac{\text{nPaths}(A,C) \cdot \text{nPaths}(C,B)}{\text{nPaths}(A,B)}.\end{aligned}$$

□

Theorem 2 *Given two points A and B in the hexagonal-coordinate system. Let Δx be subtraction of the x-component of B by that of A and Δy for y-component. The number of* shortest *paths from point A to B is given as follows:*

$$nPaths(A,B) = \begin{cases} \binom{|\Delta x + \Delta y|}{|\Delta x|}, & if\ \Delta x \Delta y \geq 0; \\ \binom{\max(|\Delta x|, |\Delta y|)}{\min(|\Delta x|, |\Delta y|)}, & otherwise, \end{cases}$$

and by a shortest path we mean the sum of lengths of its subpaths, not the Pythagorean distance.

Proof. (The Principle of Extreme.) We will use the principle of extreme to prove this. In each step, from the hexagonal coordinate, the point can move to an adjacent point only in these following three forms (recall Figure 1):

(X) Move in the x direction, either towards the more potitive or negative end.

(Y) Move in the y direction, either towards the more potitive or negative end.

(Z) Move in the x direction in one way (more positve or more negative), and, at the same time move in y direction in the opposite way (more negetive or more positive, respectively). This move is a diagonal movement.

We will show that at most *two* forms of those three forms are made when we seek the shortest possible paths (can be more than one path) from one point in the hexagonal coordinate to another point. Consider the extreme case: the case where path is shortest. This case exists by the Well Ordering Principle. Suppose to contrary that we can those three forms of movement each at least once. Note that the movement in either the x or the y direction, or both, should draw the starting point towards the destination. For otherwise we could move the point back one step and that indicated that we have wasted two steps, a contradiction to the assumption that the path we are considering is the shortest one. So, each move will draw the initial point to the destination, either in the x (by (X)) or the y (by (Y)) direction, or both (by (Z)). Next, because we use three forms at least once, we can combine one move by (X) and another one by (Y) to become just one move. Notice that no matter what order of steps, we yield the same result. Thus, this combining of (X) and (Y) is possible and so we could have saved one more step, a contradiction to the assumption that this path is the shortest one. Now we can conclude that we can use at most two forms.

Then, the rest of the proof follows from the fomula of the number of ways going two direction from the initial point towards the destination. If, for example, we need to use (X) form a times and use (Y) form b times. The way to count this is $\binom{a+b}{a}$ ways, by selecting which steps out of $a+b$ that we will choose (X). The rest of the proof is similar, but for the case for (X) and (Z), and (Y) and (Z), we need to check which axis is achieved first when we apply (Z) repeatedly. (TODO: simplify this whole page!) □

Theorem 3 *For the level of hexagons of n, the number of areas is*

$$6n^2.$$

Proof. There are six triangle subsectors of a hexagon. For each sector, the number of triangle is $1, 3, 5, \ldots, (2n-1)$. Each sector represents one area. Thus, the total number of areas is

$$6(1+3+5+\cdots+(2n-1)) = 6n^2.$$

□

5 Model Testing

Take a look at an example below, where we put the city into our hexagonal coordinate (Both X and Y range from -3 to 3):

X and Y	-3	-2	-1	0	1	2	3
3	15.53	34.87	36.03	32.00	0.00	0.00	0.00
2	36.73	76.12	104.57	81.22	43.00	0.00	0.00
1	37.13	94.27	137.75	138.05	91.57	54.00	0.00
0	31.0	82.1	134.47	**158.42**	140.13	83.07	35.00
-1	0.00	51.0	113.25	137.35	145.05	111.92	39.12
-2	0.00	0.00	49.00	89.37	99.00	92.42	46.47
-3	0.00	0.00	0.00	33.00	32.15	36.18	27.72

Table 1: The table of the number of conversations (reflecting the number of repeaters) at each point. The number of people is 1000. The level of hexagons is 3.

From Table 1, we have the total number of repeaters of 2,780. Hence, the actual number of repeaters needed (with help of PL tones) is 51.48. As there are $6 \times (3^2) = 54$ regions as well, the number of repeaters needed per 1 PL tone per 1 location is 0.953. This can be done easily since we have 3 frequency sets to serve. One concern is the service at the peak: the middle (point $(0, 0)$). The original number of repeaters needed is 158.42. By using PL tones, we have the avearge of 2.93 users using one PL tone. Again, this use can be accommadated by 3 different sets of frequencies. Thus, in this case, the total number of repeaters required is 51.48, approximately $\boxed{\text{52 repeaters}}$, each holding differnt 54 PL tones.

Figure 2 is an example of repeater coordination.

6 Sensitivity Analysis And Error Analysis

We do several cases and find the standard deviation. Also, we can try to plot a function of the minimum number of repeaters depending on the number of people. If we were to plot this graph, there would be a limit point in which no repeater coordination can serve that number of simultaneous users.

Figure 3 represents the data test for sensitivity. The reduced Chi-square

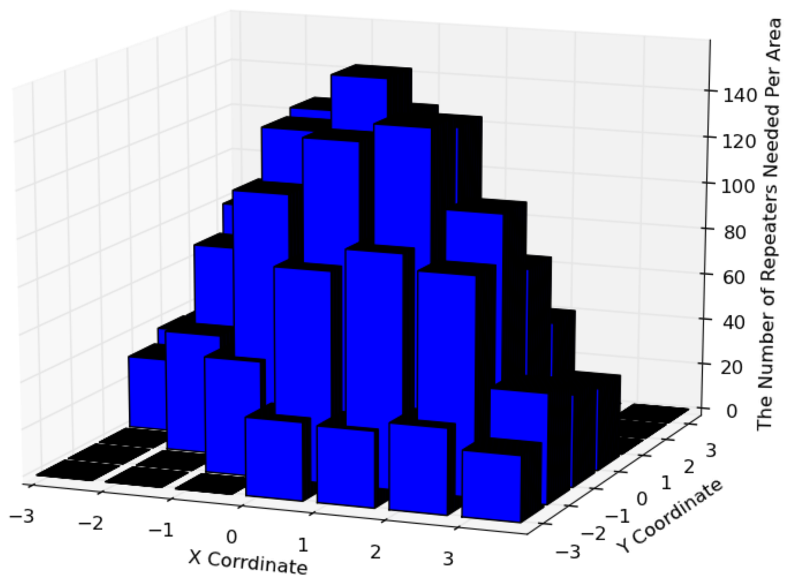

Figure 2: A Distribution of Repeaters (Maximum at the Middle).

value for this data set is 0.496. This is a good value, indicating that the number of repeaters is proportional to the number people, although the negative deviation this reduced Chi-square from 1.000 suggests that the line fits the data too well. The cause of this happening might be because we generate an almost completely randomized distribution of people. In the reaility, though, the distribution will be be as uniform as our model. For this particular data, the number of repeaters needed for 1,000 people is $\boxed{51.7 \pm 0.7 \text{ units.}}$

7 Comments

Strength: accuracy of counting! Weakness: need not be at the node.

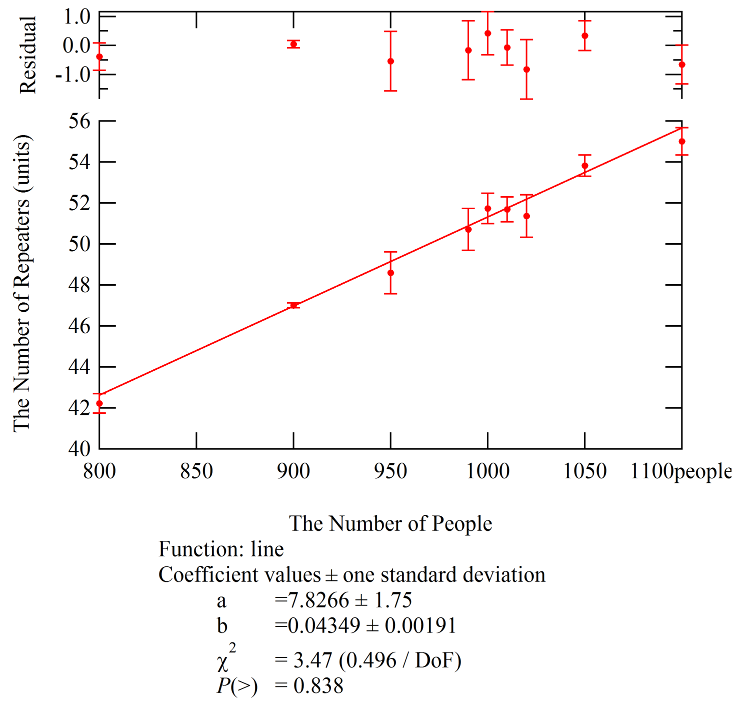

Figure 3: Sensitivity Analysis with a Change in the Number of People.

8 The Model Algorithm

Long!

9 Adapted Models

9.1 The population of 10,000 people

In this case, I think the center of the town will have a lot of problems because we don't have enough PL tones and sets of frequencies to distinquish to coming signals. Our program reports the number of repeaters needed at the center as large as 1377, corresponding to the actual required number of repeaters of 514 with help of PL tones, and so each location needs approximately 10 sets of frequencies. We only have 3. There are many ways to solve this, such as extending the range of frequency, extending the area, and increasing the number of PL tones.

9.2 Mountainous Area

For the area that is mountainous, the range R of each repeater is decreased. Therefore, we need more chains of repeaters in order to accomplish the same task. The level of haxagons will increase. If we set it to five, here is a result when we have 1,000 people:

Figure 4 is an example of repeater coordination for a mountainous area. Total is 4831.0. The repeaters per 1PL tone is is 89.46 (This value reflects

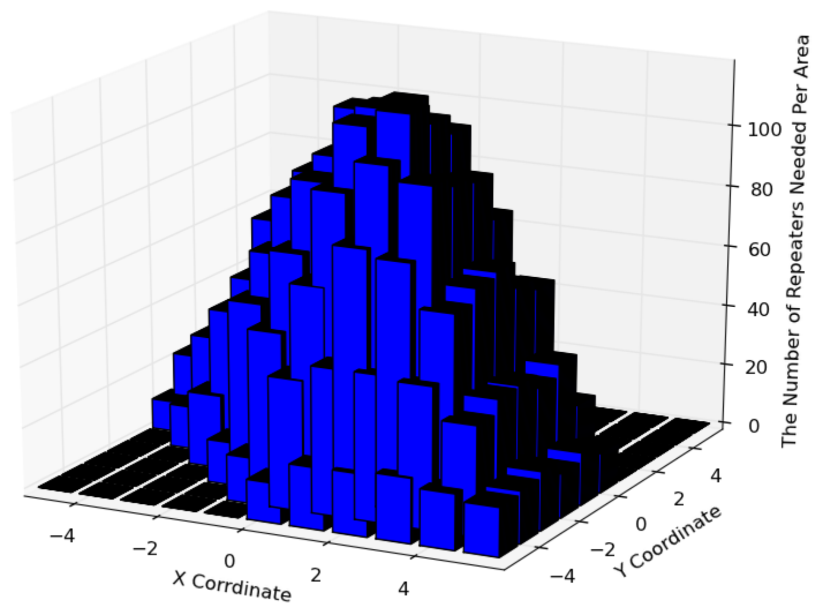

Figure 4: Distribution of Repeaters for a Mountainous Area (Maximum at the Middle).

the total number of repeaters needed). Per one location is 0.596. We have 3 sets of frequencies, so this case will be easy to deal with. The peak at the middle is about 158.42. The number of sets of frequencies needed is

$$\frac{158.42}{(\text{The number of areas})(\text{The number of PL tones})},$$

which is

$$\frac{158.42}{(6 \cdot 5^2)(54)} = \boxed{0.020.}$$

This value means that we can use only one frequency even at the middle of the town. Although this data suggests that we need to use more repeaters (as much as 90 repeaters, as opposed to 52 repeaters), we perform better in terms of avoiding frequency interference.

9.3 A Non-Uniform Distribution

In this section, we will explore cases where the distribution of people is not uniform anymore. People live together, more closely in the capital. This adaptation in the model might reflect upon this reality better. In our model, we use the Normal Distribution that has the center at some point, say the point that is of scale 0.2 on both x- and y-axes considering the possible range. We use the standard deviation of 0.5, not too big to let the point fall outside the scope. Below is the result:

X and Y	-3	-2	-1	0	1	2	3
3	78.23	41.65	39.38	24.00	0.00	0.00	0.00
2	100.78	99.27	65.13	46.98	15.00	0.00	0.00
1	130.98	133.83	130.20	83.75	43.30	19.00	0.00
0	107.00	167.32	**172.68**	161.28	89.97	40.73	22.00
-1	0.00	97.00	153.37	158.77	134.52	68.23	33.60
-2	0.00	0.00	89.00	151.90	118.47	89.75	41.27
-3	0.00	0.00	0.00	100.00	113.43	84.47	62.75

Table 2: The table of the number of conversations (reflecting the number of repeaters) at each point. The number of people is 1000. The level of hexagons is 3.

Figure 5 represents the data test for sensitivity. From the 2, we have that the peak is at point $(0, -1)$. With help of PL tones, the number of repeaters needed for this case is $\frac{172.68}{54} = 3.19$. This means that the 3 sets of frequencies is not enough and we might have some inteference as we can use at most 3 sets of frequencies at this point.

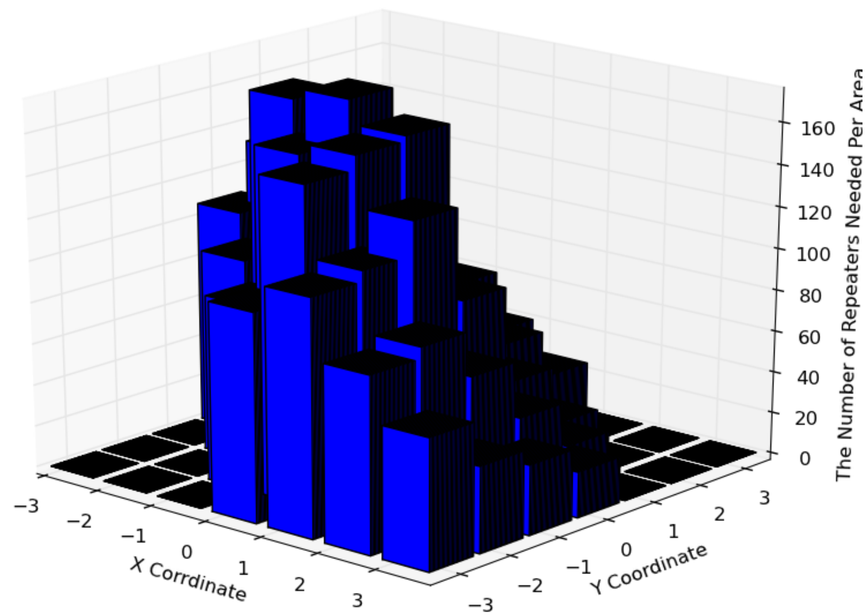

Figure 5: Non-Uniform Distribution of People And Its Effect on the Number of Repeaters Needed.

10 Conclusion

All's well that ends well.

References

[1] COMAP. Contest registration and instructions. Website, 2009. URL `http://www.comap.com/undergraduate/contests/mcm/instructions.php`. Viewed on 2009 February 5.