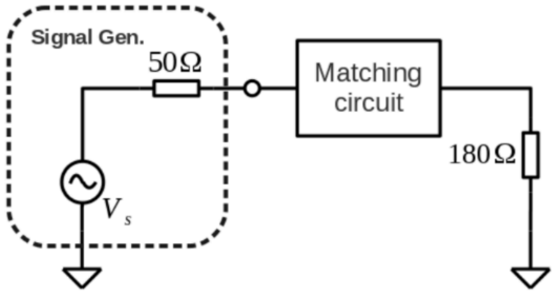
EEE 202 Lab 4 - Impedance Matching

Design at least two passive linear circuits to transfer maximum power to 180Ω load from a voltage source with output impedance 50Ω at a frequency between 5 and 10Mhz.



Introduction:

Impedance matching is used to maximise the power output on a desired component. This is done to save energy for the same work done on the resistor.

Theory:

Since we want to maximise the power on the resistor, we need to minimise other power consuming elements.

$$E_{source} = E_{heat} + \Delta E_{potential}$$

Minimising $\Delta E_{potential}$ would maximise the power output on the resistors. And in order to have $\Delta E_{potential} = 0$, we need to have a real impedance.

This can also be expressed from the formula for power. We need to find the real part for the power, since the imaginary part will give us the reactive power which does not dissipate heat.

$$Re(\frac{V^2}{Z}) = Re(V.i) = Re(i^2 Z)\ ;\ i, V, Z \in C$$

let $V = a$, $Z = b + jc$, $i = \frac{a}{b+jc}$, we can assume V is real in this case, because assuming V is real is choosing an arbitrary phase which the same as V's phase.

Using the values above, we can rewrite the expression as:

$$Re(\frac{V^2}{Z}) = Re(\frac{a^2}{b+jc}) = Re(\frac{a^2}{b} - j\frac{a^2}{c}) = \frac{a^2}{b}.$$

Notice that "a" is the voltage of the source and "b" in this case is the sum of input resistance and the real part of the circuit's impedance. We can call these R_1 and R_2 respectively. Rewriting the equation with these variables gives us:

$$\frac{V^2}{R_1+R_2}$$

This is the value supplied by the source. However, we need to maximise the power on the output so we can change the expression accordingly by the replacing the values for their respective equivalents for the resistor.

$$\frac{{V_{R2}}^2}{R_2} = V^2\frac{R_2}{Z^2} = \frac{V^2R_2}{(R_2+R_1)^2 + (X_1+X_2)^2}$$

X_1, X_2 are the reactances of the given circuit and the matching circuit respectively. X_1 was 0 in our case since the input impedance was only 50Ω. We can now maximise this expression to find the maximum power on our resistor. R_2 and X_2 are free variables so we need take its partial derivatives with respect to both.

$$\frac{\partial}{\partial X_2}\frac{V^2R_2}{(R_2+R_1)^2 + (X_1+X_2)^2} = \frac{-2V^2R_2}{(X_1+X_2)^3} \Rightarrow X_1 =- X_2$$

$$\frac{\partial}{\partial R_2}\frac{V^2R_2}{(R_2+R_1)^2 + (X_1+X_2)^2} = \frac{V^2((R_1+R_2)^2-2R_2(R_1+R_2) + (X_1+X_2)^2)}{((X_1+X_2)^2+(R_1+R_2)^2)^2} \Rightarrow R_1 = R_2$$

From these solutions we can conclude that the input and the matching circuit impedances should be their complex conjugates. The input impedance is 50Ω so the output impedance should also be 50Ω.

Design:

To implement our findings from the theory section, two impedance inverter circuits were designed. One of them being a pi-section and the other being a t-section. Both designs can be seen in Figure 1. Call the impedance that we want to invert Z_{out}. Then the impedance of the network can be calculated as follows:

$$Z_{eq} = \frac{Z^2}{Z_{out}}$$

Since we want this impedance to be equal to 50Ω, we can choose Z accordingly for a given frequency.

$$50 = \frac{Z^2}{180}$$

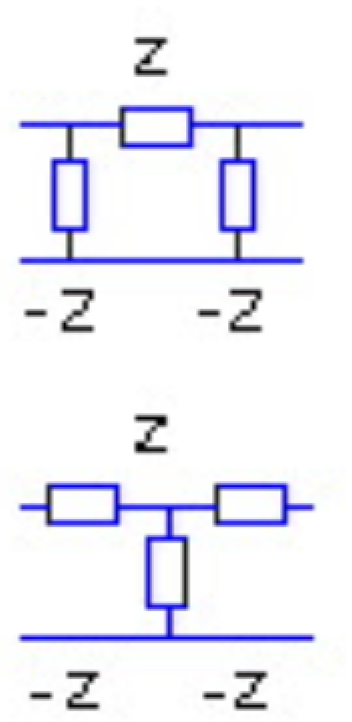


Figure 1: T and pi section impedance inverters

Software:

After choosing the frequency to be 7.5 MHz arbitrarily, we can determine the necessary L and C values.

$$30\sqrt{10} = Z$$

$$\frac{1}{wC} = Z \Rightarrow C \simeq 0.223685\text{nF}$$

$$wL = Z \Rightarrow L \simeq 2.103168\mu\text{H}$$

The LTSpice schematic with these values can be seen in Figure 2.1

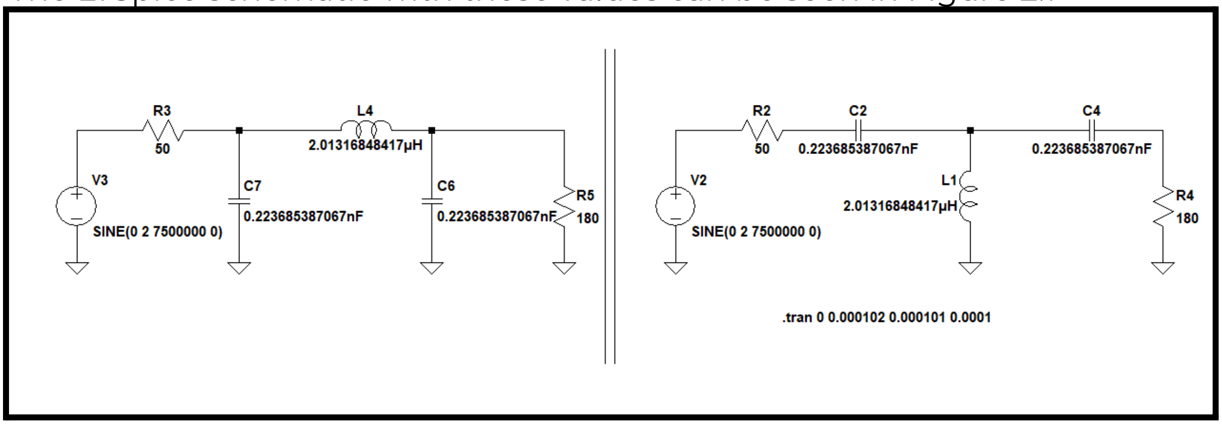


Figure 2.1: Schematic of pi and t section impedance matching circuits.

The power on both resistors are be expected to be equal. The simulation values for both resistors' power values can be seen in Figure 3.

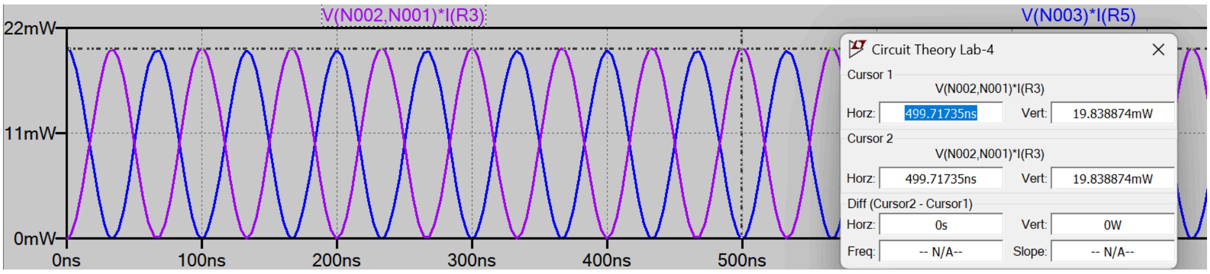


Figure 2.2: Simulation power values for both resistors.

Hardware:

The inductors were implemented using the T38-8 toroidal core in order to

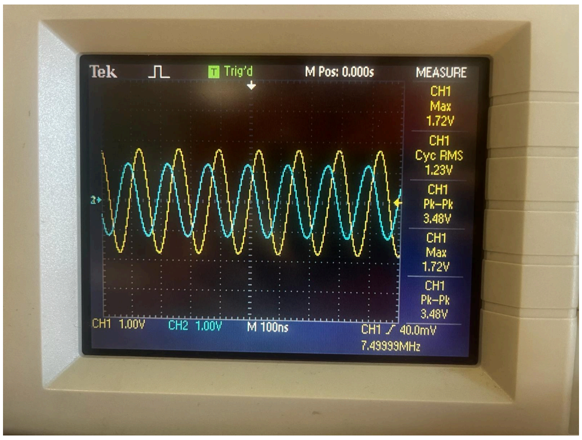


Figure 3.1: T-section output

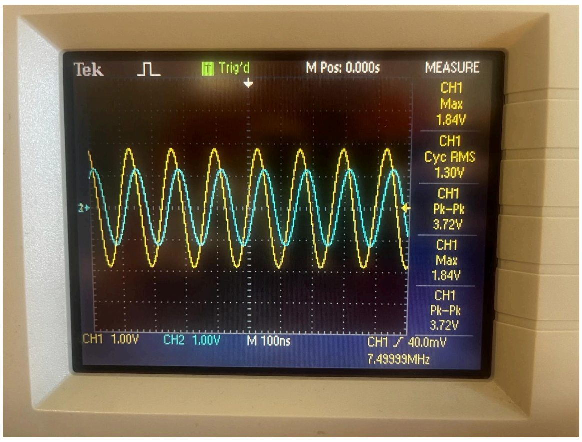


Figure 3.2: Pi-section output

Error Calculation:

The two voltage values measured were 1.88V and 1.72V. The power for these values are 19.6 and 16.6 mW. These provide an error of 1% and 15% respectively. The reason for these errors mainly come from the fact that the values for the inductors and capacitors were not exact.