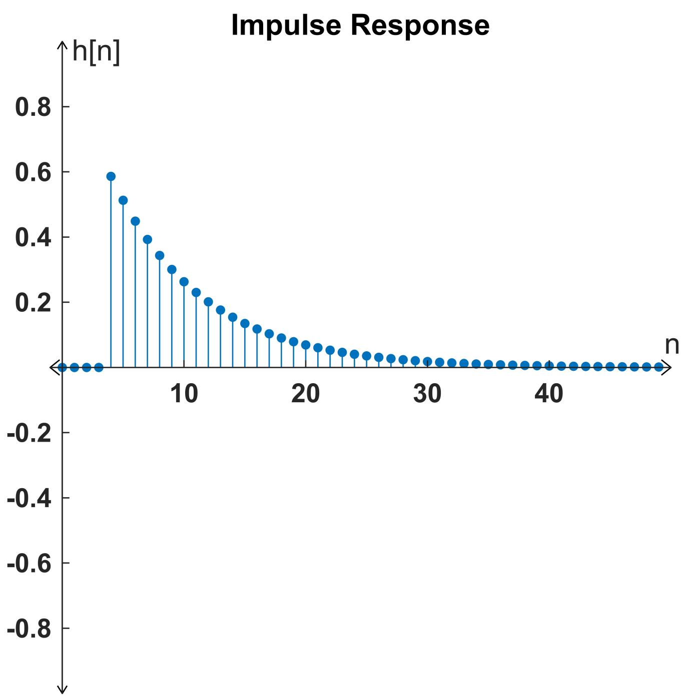
EEE 321 - Signals and Systems - Lab 2 - Tuna Şahin 22201730

Convolution Calculation and Plotting

Generate the h[n] impulse response function given as: $h[n] = \left(\frac{7}{8}\right)^n u[n-4]$

```
n = linspace(-100,100,201);
h = (7/8).^(n).*arrayfun(@u,n-4);
finestem(n,h,'Impulse Response','n','h[n]',[-1 50],[-1 1],'off',[400 400]);
```



We can see that $h[n] < 0, \forall\, n < 0$ so we can confirm that this system is causal. To check for stability we can calculate the following sum: $\sum_{-\infty}^{\infty} \left(\frac{7}{8}\right)^n u[n] = \sum_{0}^{\infty} \left(\frac{7}{8}\right)^n = \frac{1}{1-\frac{7}{8}} = 8$ which is finite therefore the system is stable.

Question 1:

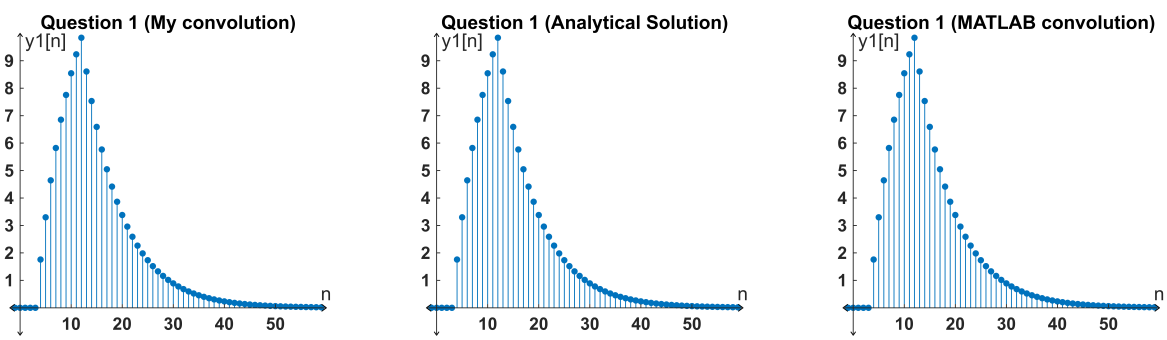
Analytical Calculation:

$$y[n] = \begin{cases} 0 & n \leq 4 \\ 3\sum_{i=4}^{n}(\frac{7}{8})^{n-4} & 4 \leq n \leq 12 \\ n[12].(\frac{7}{8})^{n-12} & 12 > n \end{cases}$$

We can simply observe the result of this convolution. Firstly we will shift the rectangular area of x_1 starting from the left. The rectangle will act as an acumulator for the first 9 values. After that, all the values in the rectangular area will decay with a multiplier of $\frac{7}{8}$. We can then write the remaining area of $n > 12$ as we have.

```
x1 = [3 3 3 3 3 3 3 3 3]; % we use zero padding for the rest of the functions
y1 = convolution(x1,h);
y1a = arrayfun(@q1,n); % mapping the analytical solution to an array
n = linspace(-100,length(y1)-101,length(y1));

subplot(1,3,1)
finestem(n,y1,'Question 1 (My convolution)','n','y1[n]',[-2 60],[-1 10],'off',[1600
400])
subplot(1,3,2)
finestem(n,y1a,'Question 1 (Analytical Solution)','n','y1[n]',[-2 60],[-1 10],'off',
[1600 400])
subplot(1,3,3)
finestem(n,conv(x1,h),'Question 1 (MATLAB convolution)','n','y1[n]',[-2 60],[-1
10],'off',[1600 400])
```

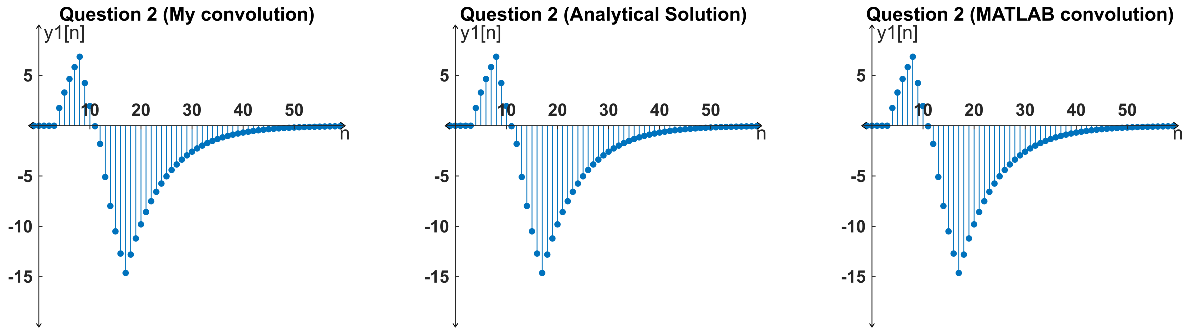


We can see all of the plots for this question are identical. Therefore, we can say that my analytical solution and my convolution functions were correct.

Question 2:

```
x2=[3 3 3 3 3 -3 -3 -3 -3 -6 -6 -6 -6 -6];
y2 = convolution(x2,h);
y2a = [y1a 0 0 0 0 0] - 2*[0 0 0 0 0 y1a]; %obtaining the analytical solution from
the first question.
n = linspace(-100,length(y2)-101,length(y2));
```

```
subplot(1,3,1)
finestem(n,y2,'Question 2 (My convolution)','n','y1[n]',[-2 60],[-20 10],'off',
[1600 400])
subplot(1,3,2)
finestem(n,y2a,'Question 2 (Analytical Solution)','n','y1[n]',[-2 60],[-20
10],'off',[1600 400])
subplot(1,3,3)
finestem(n,conv(x2,h),'Question 2 (MATLAB convolution)','n','y1[n]',[-2 60],[-20
10],'off',[1600 400])
```



We can see all of the plots for this question are identical. Therefore, we can say that my analytical solution and my convolution functions were correct.

Question 3:

Analytical solution:

$$x_3[n] = e^{\frac{jn}{3}}.\, u[n-2].\, u[20-n], \quad h[n] = \left(\frac{7}{8}\right)^n u[n-4]$$

$$y[n] = x_3[n] * h[n] \quad \Rightarrow \quad \sum_{k=-\infty}^{\infty} \left(\frac{7}{8}\right)^n u[n-4].\, e^{\frac{j(n-k)}{3}}.\, u[n-k-2].\, u[20-n+k]$$

$$\Rightarrow \quad e^{\frac{jn}{3}} \sum_{k=4}^{\infty} \left(\frac{7e^{\frac{-j}{3}}}{8}\right)^n u[n-k-2].\, u[20-n+k] \quad \Rightarrow \quad y[n] = \left\{ \begin{array}{ll} \sum_{k=4}^{n-2} \left(\frac{7e^{-\frac{j}{3}}}{8}\right)^k & n \leq 24 \\ \sum_{k=n-20}^{n-2} \left(\frac{7e^{-\frac{j}{3}}}{8}\right)^k & n > 24 \end{array} \right\}$$

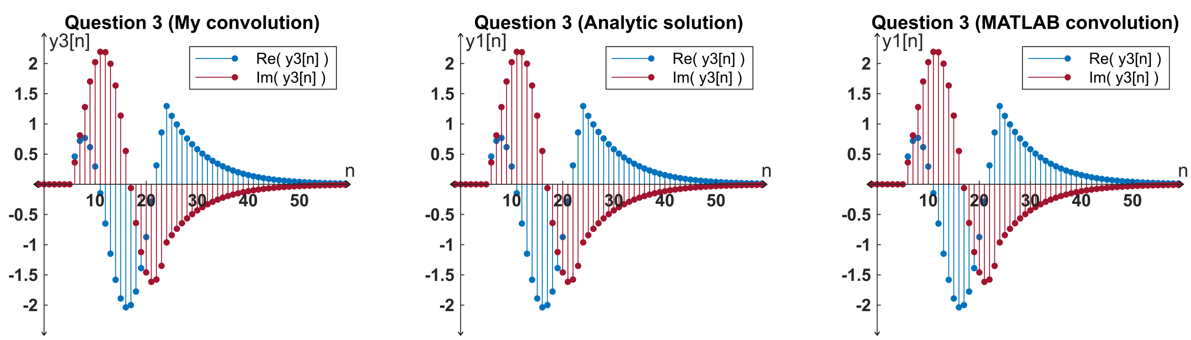
We can write the second part recursively to clean it up and substituting the formula for the sum of geometric series we get the following expression:

$$y[n] = \begin{cases} 0 & n < 6 \\ e^{\frac{jn}{3}} \sum_{i=4}^{n-2} \frac{\left(\frac{7e^{\frac{-j}{3}}}{8}\right)^{4} - \left(\frac{7e^{\frac{-j}{3}}}{8}\right)^{n-1}}{\left(1 - \frac{7e^{\frac{-j}{3}}}{8}\right)} & 6 \le n \le 24 \\ n[24].\left(\frac{7}{8}\right)^{n-24} & 24 > n \end{cases}$$

The decay with a multiplier of $\frac{7}{8}$ is again present.

```
x3 = [0 0 exp(1i*(1/3)*linspace(2,20,19))];
y3 = convolution(x3,h);
y3a = arrayfun(@q3,n); % mapping the analytical solution to an array
n = linspace(-100,length(y3)-101,length(y3));

subplot(1,3,1)
finestem(n,real(y3),'Question 3 (My convolution)','n','y3[n]',[-2 60],[-2.5
2.5],'on',[1600 400])
finestem(n,imag(y3),'Question 3 (My convolution)','n','y3[n]',[-2 60],[-2.5
2.5],'off',[1600 400])
legend('Re( y3[n] )','','','','','','Im( y3[n] )')
subplot(1,3,2)
finestem(n,real(y3a),'Question 3 (Analytic solution)','n','y1[n]',[-2 60],[-2.5
2.5],'on',[1600 400])
finestem(n,imag(y3a),'Question 3 (Analytic solution)','n','y1[n]',[-2 60],[-2.5
2.5],'off',[1600 400])
legend('Re( y3[n] )','','','','','','Im( y3[n] )')
subplot(1,3,3)
finestem(n,real(conv(x3,h)),'Question 3 (MATLAB convolution)','n','y1[n]',[-2 60],
[-2.5 2.5],'on',[1600 400])
finestem(n,imag(conv(x3,h)),'Question 3 (MATLAB convolution)','n','y1[n]',[-2 60],
[-2.5 2.5],'off',[1600 400])
legend('Re( y3[n] )','','','','','','Im( y3[n] )')
```



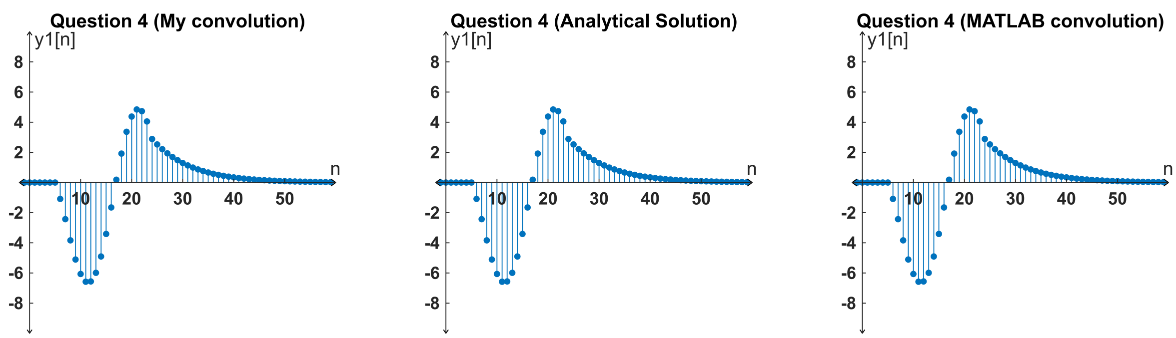
We can see all of the plots for this question are identical. Therefore, we can say that my analytical solution and my convolution functions were correct.

We can see all of the plots for this question are identical. Therefore, we can say that my analytical solution and my convolution functions were correct.

Question 4:

We know that $-3\sin\left(\frac{1}{3}n\right) = -3\mathrm{Re}\left(e^{\frac{j}{3}n}\right)$ therefore we can just use the values for the analytical solution of Q3 and multiply their real part with -3 as:

```
y4 = -3*imag(y3);
y4a = -3*imag(y3a);
subplot(1,3,1)
finestem(n,y4,'Question 4 (My convolution)','n','y1[n]',[-2 60],[-10 10],'off',
[1600 400])
subplot(1,3,2)
finestem(n,y4a,'Question 4 (Analytical Solution)','n','y1[n]',[-2 60],[-10
10],'off',[1600 400])
subplot(1,3,3)
finestem(n,-3*imag(conv(x3,h)),'Question 4 (MATLAB convolution)','n','y1[n]',[-2
60],[-10 10],'off',[1600 400])
```



We can see all of the plots for this question are identical. Therefore, we can say that my analytical solution and my convolution functions were correct.

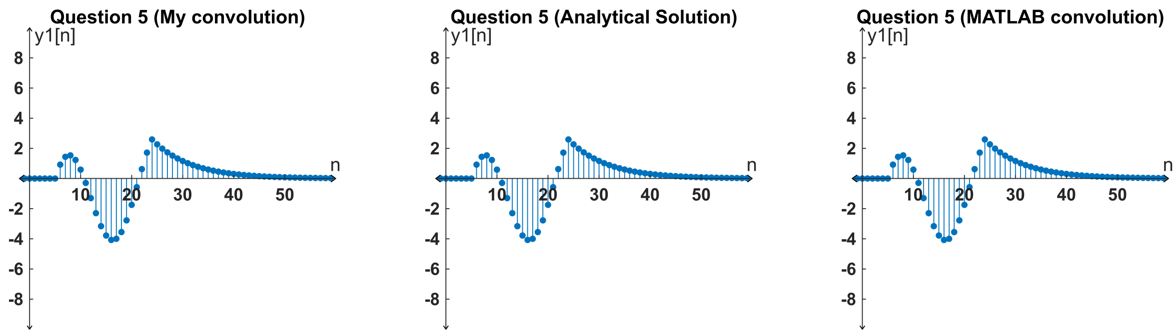
Question 5:

We know that $2\cos\left(\frac{1}{3}n\right) = 2\mathrm{Im}\left(e^{\frac{j}{3}n}\right)$ therefore we can just use the values for the analytical solution of Q3 and multiply their imaginary part with 2 as:

```
y5 = 2*real(y3);
y5a = 2*real(y3a);

subplot(1,3,1)
```

```
finestem(n,y5,'Question 5 (My convolution)','n','y1[n]',[-2 60],[-10 10],'off',
[1600 400])
subplot(1,3,2)
finestem(n,y5a,'Question 5 (Analytical Solution)','n','y1[n]',[-2 60],[-10
10],'off',[1600 400])
subplot(1,3,3)
finestem(n,2*real(conv(x3,h)),'Question 5 (MATLAB convolution)','n','y1[n]',[-2 60],
[-10 10],'off',[1600 400])
```

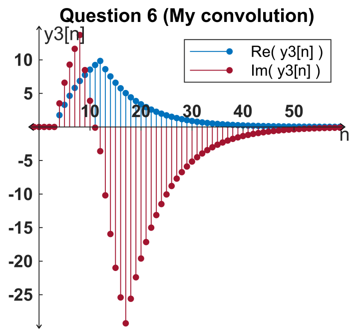


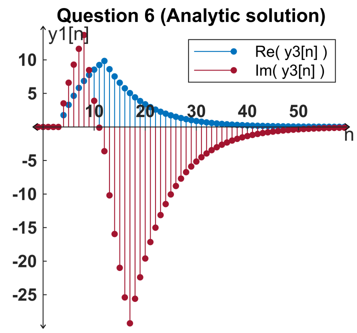
We can see all of the plots for this question are identical. Therefore, we can say that my analytical solution and my convolution functions were correct.

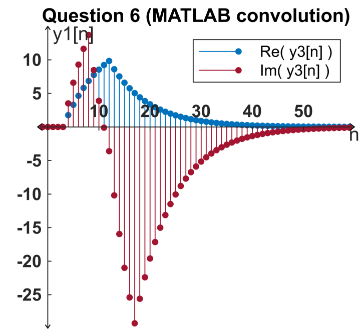
Question 6:

```
y6 = [y1 0 0 0 0 0] + 2j*y2; %we use zero padding for y1 to match sizes with y2
y6a = [y1a 0 0 0 0 0] + 2j*y2a;
n = linspace(-100,length(y6)-101,length(y6));

subplot(1,3,1)
finestem(n,real(y6),'Question 6 (My convolution)','n','y3[n]',[-2 60],[-30 15],'on',
[1600 400])
finestem(n,imag(y6),'Question 6 (My convolution)','n','y3[n]',[-2 60],[-30
15],'off',[1600 400])
legend('Re( y3[n] )','','','','','','Im( y3[n] )')
subplot(1,3,2)
finestem(n,real(y6a),'Question 6 (Analytic solution)','n','y1[n]',[-2 60],[-30
15],'on',[1600 400])
finestem(n,imag(y6a),'Question 6 (Analytic solution)','n','y1[n]',[-2 60],[-30
15],'off',[1600 400])
legend('Re( y3[n] )','','','','','','Im( y3[n] )')
subplot(1,3,3)
finestem(n,real(conv(([x1 0 0 0 0 0]+2j*x2),h)),'Question 6 (MATLAB
convolution)','n','y1[n]',[-2 60],[-30 15],'on',[1600 400])
finestem(n,imag(conv([x1 0 0 0 0 0]+2j*x2,h)),'Question 6 (MATLAB
convolution)','n','y1[n]',[-2 60],[-30 15],'off',[1600 400])
legend('Re( y3[n] )','','','','','','Im( y3[n] )')
```







We can see all of the plots for this question are identical. Therefore, we can say that my analytical solution and my convolution functions were correct.

Functions Used:

Question 1 Analytical Solution:

```
function y = q1(n)
    if n < 4
        y = 0;
    elseif n > 12
        y = q1(12)*((7/8)^(n-12)); %using recursion for n = 12
    else
        y = 0;
        for i = linspace(4,n,n-3)
            y = y + (7/8)^(i);
        end
        y = y*3;
    end
end
```

Question 3 Analytical Solution:

```
function y = q3(n)
    if n < 6
        y = 0;
    elseif n > 24
        y = q3(24)*((7/8)^(n-24));
    else
        base = (7*exp(-1j/3))/8;
        sum1 = (1 - (base^(n-1)))/(1-base);
        sum2 = (1-(base^4))/(1-base);
        y = (sum1-sum2) * exp(n*1j/3);
    end
end
```

Unit Step | Function:

```
function un = u(n)
    if n < 0
        un = 0;
    else
        un = 1;
    end
end
```

convolution | Function:

```
function y = convolution(x,h);
    y = zeros([0 length(h)]); %initial size is the same as impulse function
    y = [y 0]; %extend size of result array to be compatible
    for i = x;
        y = y + i*h; % add the weighted impulse responses to the result
        h = [0 h]; %shift impulse function for the next operation
        y = [y 0]; %extend size of result array to be compatible
    end
end
```

finestem | Function:

The stem plotting function with other configurations to get the same style as the book:

- titlename,xaxisname,yaxisname all take string values that determine the corresponding text.
- xlimits,ylimits take 1x2 arrays that hold the limits of the plot in arbitrary order
- holdstate should take holdstate = 'off' as an input if you don't want to plot to be held any other input will not hold the plot

```
function finestem(n,x,titlename,xaxisname,yaxisname,xlimits,ylimits,holdstate,size)
    %even if the limits are reverse this section corrects the order
    if ylimits(1) > ylimits(2)
        ylimits = [ylimits(2) ylimits(1)];
    end
    if xlimits(1) > xlimits(2)
        xlimits = [xlimits(2) xlimits(1)];
    end

    %these measures are necessary for positioning arrows, labels etc.
    dx = (xlimits(2)-xlimits(1))/65;
    dy = (ylimits(2)-ylimits(1))/85;

    %this section is to not plot values that overlap with the arrows
    lowindex = n(1);
    highindex = n(1);
    for i = n
        if (i > xlimits(1) + dx) * (lowindex == n(1))
            lowindex = find(n==i);
        end
```

```
        if (i > xlimits(2) - dx) * (highindex == n(1))
            highindex = find(n==i)-1;
        end
    end
    if n(lowindex) > 0 %we allow points to be drawn at the origin
        lowindex = lowindex - 1;
    end
    if n(highindex) < 0 %we allow points to be drawn at the origin
        highindex = highindex + 1;
    end
    n = n(lowindex:highindex);
    x = x(lowindex:highindex);

    %this section is responsible for axis configuration
    stem(n,x,'filled','MarkerSize',3)
    xlim(xlimits)
    ylim(ylimits)
    set(get(gca,'XLabel'),'Visible','on')
    set(gca,'XAxisLocation','origin', 'box','off')
    set(gca,'YAxisLocation','origin')
    set(get(gca, 'XAxis'), 'FontWeight', 'bold')
    set(get(gca, 'YAxis'), 'FontWeight', 'bold');
    set(get(gca,'YLabel'),'Visible','on')
    set(gca,'Layer','top')
    set(gcf,'position',[(xlimits(2)-xlimits(1))/2 , (ylimits(2)-ylimits(1))/2 ,
size(1) , size(2)])

    %deletes the ticks that overlap with arrows
    xticks('auto');
    xt = xticks;
    xticks(xt(2:length(xt)-1));
    yticks('auto');
    yt = yticks;
    yticks(yt(2:length(yt)-1));

    % determining the ylevel to draw the arrows on the x axis
    if ((ylimits(1) * ylimits(2)) < 0)
        xal = 0;
    elseif ylimits(1) < 0
        xal = ylimits(2);
    else
        xal = ylimits(1);
    end

    %plotting the arrows
    hold on
    if xlimits(2) > 0
        plot([(xlimits(2)-dx) xlimits(2) (xlimits(2)-dx)],[xal+dy xal xal-dy],'k')
%xaxis right arrow
    end
```

```
    if xlimits(1) < 0
        plot([xlimits(1)+dx xlimits(1) xlimits(1)+dx],[xal+dy xal xal-dy],'k')
%xaxis left arrow
    end
    plot([0 0],[ylimits(2) ylimits(1)],'k')% y axis
    if ylimits(2) > 0
        plot([-dx/2 0 dx/2],[ylimits(2)-dy ylimits(2) ylimits(2)-dy],'k') %yaxis
top arrow
    end
    if ylimits(1) < 0
        plot([-dx/2 0 dx/2],[ylimits(1)+dy ylimits(1) ylimits(1)+dy],'k') %yaxis
bottom arrow
    end

    %repositioning title & label locations
    label_h1 = xlabel(xaxisname);
    label_h1.Position(1) = xlimits(2)+dx; % change horizontal position of xlabel.
    label_h1.Position(2) = dy; % change vertical position of xlabel.c5
    label_h2 = ylabel(yaxisname,rotation=0);
    label_h2.Position(1) = dx; % change horizontal position of ylabel.
    label_h2.Position(2) = ylimits(2)+dy; % change vertical position of ylabel.
    title(titlename);
    if ((xlimits(1) * xlimits(2)) < 0) && (xlimits(1)+xlimits(2) < ((xlimits(2)-
xlimits(1))/3))
        set(get(gca,'title'),'Position', [20*dx ylimits(2)-dy]) %prevents the title
from colliding with ylabel
    end

    %holds or doesnt hold
    if strcmp(holdstate,'off')
        hold off
    else
        hold on
    end
end
```