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**МОДЕЛИРОВАНИЕ И УПРАВЛЕНИЕ КВАДРОКОПТЕРОМ БПЛА С 6  
СТЕПЕНЯМИ СВОБОДЫ С ИСПОЛЬЗОВАНИЕМ ПИД-РЕГУЛЯТОРОВ ПО  
ЦИГЛЕРУ-НИКОЛСУ**

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**Аннотация.** В данной статье представлены результаты исследований по разработке математических моделей и проектированию 6 ПИД-регуляторов для 6-DOF БПЛА квадрокоптера. Во-первых, математическая модель квадрокоптера БПЛА с 6 степенями свободы учитывает влияние ветровых возмущений и построена на основе уравнения Ньютона-Эйлера и уравнения Эйлера-Лагранжа. Далее в статье представлен метод проектирования 6 ПИД-регуляторов по Циглеру-Николсу. Наконец, в статье представлены результаты моделирования и моделирования 6-DOF системы управления квадрокоптером БПЛА на Matlab/Simulink. Результаты моделирования показывают, что предложенные ПИД-регуляторы для качества управления 6-степенным БПЛА квадрокоптером соответствуют предъявляемым требованиям.

**Ключевые слова:** UAV, PID, Quadcopter, Control, Model, Simulation, Matlab

**MODELING AND CONTROL OF 6-DOF UAV QUADCOPTER USING PID  
CONTROLLERS ACCORDING TO ZIEGLER-NICHOLS**

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**Abstract.** This article presents the results of research on developing mathematical models and designing 6 PID controllers for 6-DOF UAV quadcopter. First, the mathematical model of the 6-DOF UAV quadcopter considering the impact of wind disturbance is built based on the Newton-Euler equation and Euler-Lagrange equation. Then, the article presents the method of designing 6 PID controllers according to Ziegler- Nichols. Finally, the article presents the results of modeling and simulation of the 6-DOF UAV quadcopter control system on Matlab/Simulink. Simulation results show that the proposed PID controllers for 6-DOF UAV quadcopter control quality meet the requirements.

**Keywords:** UAV, PID, Quadcopter, Control, Model, Simulation, Matlab

**I. Introduction**

UAV quadcopter drones are the most common type of drones. They have four propellers attached to the body by the arm. The propellers are driven by electric motors, which allow the drone to fly up, down, forward, backwards, left, right and rotate. Quadcopter drones are versatile and can be used for various purposes, such as photography and videography, delivery, pesticide spraying, agricultural granulation, search and rescue, military service, security and defence, and other applications.

Studies of quadcopter UAVs often start from their basic dynamics model, incorporating more complex aerodynamic features [1,2]. Proposed and studied drone control methods include backstepping control [3,4], Fuzzy logic [5,6], Proportional-Integral-Derivative (PID) [7,8], Linear Quadratic Regulator (LQR)[9], Model Predictive Control (MPC)[10], Feedback linearization control[11], Sliding mode control (SMC)[12], .... These studies share the common goal of studying mathematical modelling of quadcopters and developing appropriate algorithms to stabilize and control the trajectory of movement of quadcopter UAVs.

The main content of this paper presents mathematical equations describing the dynamics of a 6-DOF UAV quadcopter based on the Newton-Euler equation and Euler-Lagrange equation. Then, it presents the PID controllers design method for the 6-DOF UAV quadcopter based on the Ziegler-Nichols method. The structure of this article consists of 3 main parts: Part 1 presents an overview of UAV quadcopter control. Part 2 introduces the mathematical model of a 6-DOF UAV quadcopter. Part 3 demonstrates the PID controller design method for the 6-DOF UAV quadcopter. Part 4 shows the 6-DOF UAV quadcopter control system modelling simulation results, conclusions, and references.

## II. Mathematical model of 6-DOF UAV quadcopter

The structural diagram of the UAV quadcopter is shown in Figure 1, including the angular velocity, torque, and the corresponding force generated by the four propellers, numbered from 1 to 4.

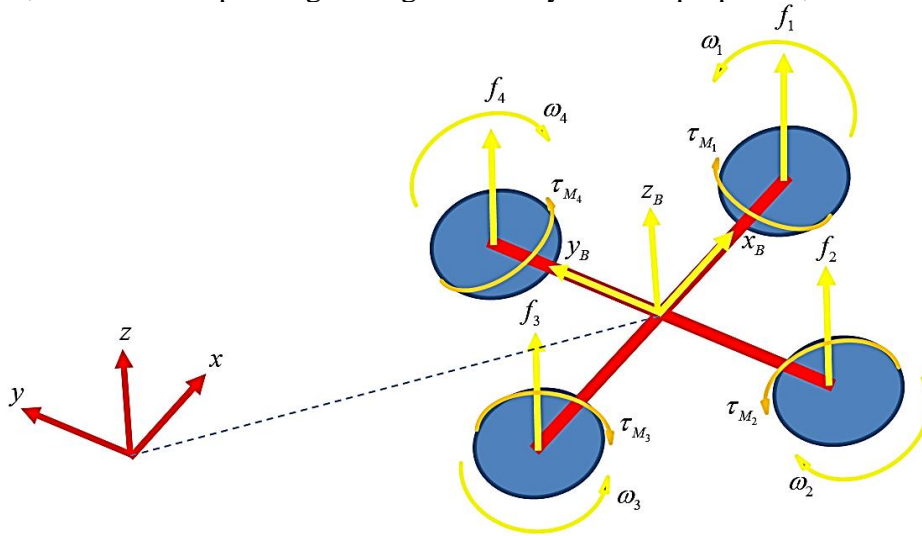


Figure 1 – Inertial reference frame and body reference frame of quadcopter UAVs

The centre of mass position of the UAV in the fixed coordinate system linked to the ground is denoted  $\xi$ ,  $\xi = [x \ y \ z]^T$ . The angle position is determined by the Euler angle  $\eta$ ,  $\eta = [\phi \ \theta \ \psi]^T$ . The roll angle ( $\phi$ ) determines rotation around the x-axis. The pitch angle ( $\theta$ ) determines the rotation of the quadcopter UAV around the y-axis and the yaw angle ( $\psi$ ) determines rotation around the z-axis.

Lagrangian function  $L$  is the sum of rotational energy  $E_{rot}$  and translational energy  $E_{trans}$  minus potential energy  $E_{pot}$  [13], [20]

$$L = E_{trans} + E_{rot} - E_{pot} = \frac{m}{2} \dot{\xi}^T \dot{\xi} + \frac{1}{2} \mathbf{v}^T \mathbf{I} \mathbf{v} - mgz \quad (1)$$

Lagrangian equation with external force and rotational moment [13], [20], [21].

$$\begin{bmatrix} \mathbf{f} \\ \boldsymbol{\tau} \end{bmatrix} = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\boldsymbol{\varepsilon}}} \right) - \frac{\partial L}{\partial \boldsymbol{\varepsilon}} \quad (2)$$

Where  $m$  is the mass of the four propellers,  $g$  is the gravitational acceleration,  $\boldsymbol{\varepsilon} = [\xi \ \eta]^T$  is the state vector;  $\mathbf{v}$  is angular velocity vector,  $\mathbf{v} = [p \ q \ r]^T$ ; and  $\mathbf{I}$  is inertia moment of UAV quadcopter, with assuming the quadcopter UAV has a symmetrical structure across the axes, the

diagonal matrix of inertial tensors is included only the main central moments of inertia;  $\tau$  are the moments of roll, pitch, yaw angles;  $\mathbf{f}$  is the translational force acting on the rotor.

From [13]-[22], the translational and rotational components are independent, so they can be studied separately. The translational Euler–Lagrangian equations are as follows

$$\mathbf{f} = \mathbf{R}\mathbf{F}_B = m\ddot{\boldsymbol{\xi}} + mg \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (3)$$

Where,  $\mathbf{F}_B$  is the lift force of the UAV's four motors along the z-axis in the body reference frame;  $\omega_i$  The angular velocity of the  $i^{th}$  rotor,  $k$  is the lifting force coefficient of the rotors;  $\mathbf{R}$  is the transition matrix from the body reference frame to the inertial reference frame [13-16],  $S_\psi = \sin(\psi)$ ,  $C_\psi = \cos(\psi)$

$$\mathbf{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}; \quad \mathbf{R} = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\phi - S_\psi C_\theta & C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi C_\theta & S_\psi S_\theta S_\phi + C_\psi C_\theta & S_\psi S_\theta C_\phi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix}$$

$$\mathbf{F}_B = [0 \ 0 \ F]^T, F = \sum_{i=1}^4 f_i = k \sum_{i=1}^4 \omega_i^2,$$

To simplify the dynamics of quadcopter UAVs, the overall dynamics are divided into translational and rotational motion by considering the corresponding state vectors.

Translational motion equations can be represented by a fixed angles as follows [22].

$$\begin{aligned} m\ddot{x} + A_x \dot{x} &= F(\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) \\ m\ddot{y} + A_y \dot{y} &= F(\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi) \\ m\ddot{z} + mg + A_z \dot{z} &= F(\cos \phi \cos \theta) \end{aligned} \quad (4)$$

Where  $F$  is the thrust acting along the z-axis.  $A_x, A_y, A_z$  are the coefficients of air resistance in the corresponding directions of the axes of the absolute coordinate system.

Applying the Euler angles of the object to the equation (4), it can be seen that the acceleration ignoring air resistance would look like this [23]:

$$\begin{aligned} m\ddot{x} &= F(\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) \\ m\ddot{y} &= F(\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi) \\ m\ddot{z} &= F(\cos \phi \cos \theta) - mg \end{aligned} \quad (5)$$

Therefore, the translational motion equations of UAV quadcopter as below

$$\begin{aligned} \ddot{x} &= \frac{F}{m}(\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) \\ \ddot{y} &= \frac{F}{m}(\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi) \\ \ddot{z} &= \frac{F}{m}(\cos \phi \cos \theta) - g \end{aligned} \quad (6)$$

In addition, Jacobian matrix  $\mathbf{J}(\boldsymbol{\eta})$  from  $\mathbf{v}$  to  $\dot{\boldsymbol{\eta}}$  is defined as follows [13], [21], [22].

$$\mathbf{J}(\boldsymbol{\eta}) = \mathbf{J} = \mathbf{W}_\eta^T \mathbf{I} \mathbf{W}_\eta$$

$$\mathbf{J} = \begin{bmatrix} I_{xx} & 0 & -I_{xx} S_\theta \\ 0 & I_{yy} C_\phi^2 + I_{zz} S_\phi^2 & (I_{yy} - I_{zz}) C_\phi S_\phi C_\theta \\ -I_{xx} S_\theta & (I_{yy} - I_{zz}) C_\phi S_\phi C_\theta & I_{xx} S_\theta^2 + I_{yy} S_\phi^2 C_\theta^2 + I_{zz} C_\phi^2 C_\theta^2 \end{bmatrix} \quad (7)$$

The matrix that transforms the angular velocity from the inertial reference frame to the body reference frame is  $\mathbf{W}_\eta$  and from the body reference frame to the inertial reference frame is  $\mathbf{W}_\eta^{-1}$  [13,17].

$$\begin{aligned} \mathbf{v} &= \mathbf{W}_\eta \dot{\boldsymbol{\eta}} \\ \begin{bmatrix} p \\ q \\ r \end{bmatrix} &= \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\theta C_\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \\ \dot{\boldsymbol{\eta}} &= \mathbf{W}_\eta^{-1} \mathbf{v} \\ \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} &= \begin{bmatrix} 1 & S_\phi T_\theta & C_\phi T_\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi / C_\theta & S_\phi / C_\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \end{aligned} \quad (8)$$

Where,  $T_\phi = \tan(\phi)$ ;  $\mathbf{W}_\eta$  reversible in the case of  $\theta \neq (2k-1)\phi/2, k=1,2,\dots$

The rotational energy can be represented in the absolute coordinate system given by [13], [21], [22]

$$E_{rot} = \frac{1}{2} \mathbf{v}^T \mathbf{I} \mathbf{v} = \frac{1}{2} \dot{\boldsymbol{\eta}}^T \mathbf{J} \dot{\boldsymbol{\eta}} \quad (9)$$

The torque of the motor generates the angular force. Lagrangian angle equations, as follows

$$\boldsymbol{\tau} = \boldsymbol{\tau}_B = \mathbf{J} \ddot{\boldsymbol{\eta}} + \frac{d}{dt} (\mathbf{J}) \dot{\boldsymbol{\eta}} - \frac{1}{2} \frac{\partial}{\partial \boldsymbol{\eta}} (\dot{\boldsymbol{\eta}}^T \mathbf{J} \dot{\boldsymbol{\eta}}) = \mathbf{J} \ddot{\boldsymbol{\eta}} + \mathbf{C}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) \dot{\boldsymbol{\eta}} \quad (10)$$

Where, the matrix  $\mathbf{C}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

The elements  $C_{ij}$  of the  $\mathbf{C}$  matrix are as follows [13], [22].

$$C_{11} = 0$$

$$C_{12} = (I_{yy} - I_{zz}) (\dot{\theta} C_\phi S_\phi + \dot{\psi} S_\phi^2 C_\theta) + (I_{zz} - I_{yy}) \dot{\psi} C_\phi^2 C_\theta - I_{xx} \dot{\psi} C_\theta$$

$$C_{13} = (I_{zz} - I_{yy}) \dot{\psi} C_\phi S_\phi C_\theta^2$$

$$C_{21} = (I_{zz} - I_{yy}) (\dot{\theta} C_\phi S_\phi + \dot{\psi} S_\phi C_\theta) + (I_{yy} - I_{zz}) \dot{\psi} C_\phi^2 C_\theta + I_{xx} \dot{\psi} C_\theta$$

$$C_{22} = (I_{zz} - I_{yy}) \dot{\phi} C_\phi S_\phi$$

$$C_{23} = -I_{xx} \dot{\psi} S_\theta C_\theta + I_{yy} \dot{\psi} S_\phi^2 S_\theta C_\theta + I_{zz} \dot{\psi} C_\phi^2 S_\theta C_\theta$$

$$C_{31} = (I_{yy} - I_{zz}) \dot{\psi} C_\theta^2 S_\phi C_\phi - I_{xx} \dot{\theta} C_\theta$$

$$C_{32} = (I_{zz} - I_{yy}) (\dot{\theta} C_\phi S_\phi S_\theta + \dot{\phi} S_\phi^2 C_\theta) + (I_{yy} - I_{zz}) \dot{\phi} C_\phi^2 C_\theta + I_{xx} \dot{\psi} S_\theta C_\theta - I_{yy} \dot{\psi} C_\phi^2 S_\theta C_\theta - I_{zz} \dot{\psi} C_\phi^2 S_\theta C_\theta$$

$$C_{33} = (I_{yy} - I_{zz}) \dot{\phi} C_\phi S_\phi C_\theta^2 - I_{yy} \dot{\theta} S_\phi^2 C_\theta S_\theta - I_{zz} \dot{\theta} C_\phi^2 C_\theta S_\theta + I_{xx} \dot{\theta} C_\theta S_\theta$$

Finally, the rotaional motion equations of UAV quadcopter, from the Lagrange–Euler equation according to equation (10), as below

$$\ddot{\boldsymbol{\eta}} = \mathbf{J}^{-1} (\boldsymbol{\tau}_B - \mathbf{C}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) \dot{\boldsymbol{\eta}}) \quad (11)$$

### III. Design of PID controller for 6-DOF UAV quadcopter

#### 3.1. Design of the angles and attitude controllers for 6-DOF UAV quadcopter

The quadcopter UAV has four control inputs (four-rotor angular velocity,  $\omega_i$  but there are six states (position  $\boldsymbol{\xi}$  and angle  $\boldsymbol{\eta}$ ). Equations (10), (11), and (12) determine the dynamics of a quadcopter UAV, denoting the interaction between states and total thrust  $F$  and torque  $\boldsymbol{\tau}$  generated by the rotor.

The quadcopter UAV is kept aloft thanks to total F-thrust, which also affects acceleration along the z-axis. Acceleration of the angle  $\phi$  affected by torque  $\tau_\phi$ , acceleration of the angle  $\theta$  affected by torque  $\tau_\theta$ , and acceleration of the angle  $\psi$  Powered by torque  $\tau_\psi$  [13], [31-33].

The PID control system structure block diagram for the 6-DOF UAV quadcopter is presented below, Fig.2.

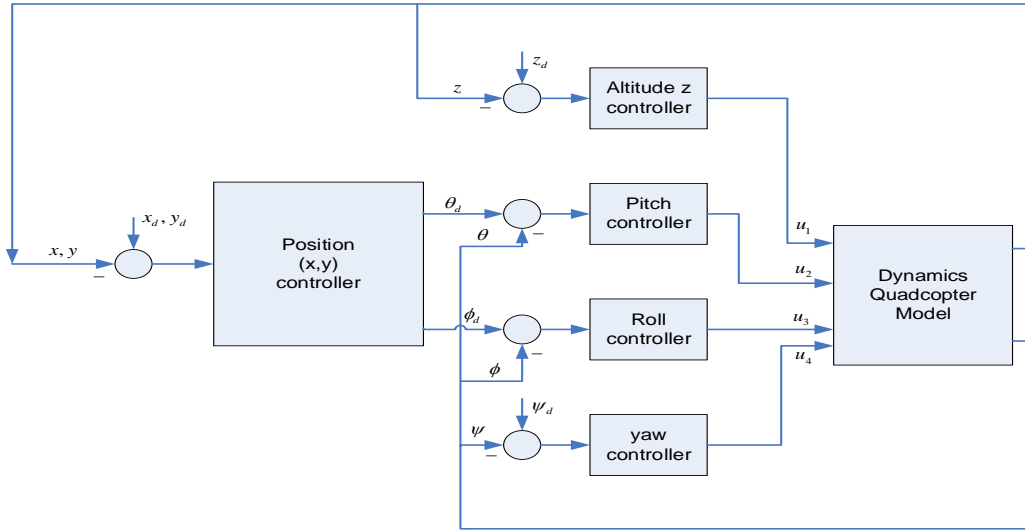


Figure 2 – Block diagram of PID controller structure for 6-DOF UAV quadcopter

The PID controller calculates the total thrust and rotor torque for the quadcopter UAV, which is determined as follows:

$$\begin{aligned}
 F &= (g + K_{Pz}(z_d - z) + K_{Iz} \int (z_d - z) + K_{Dz}(\dot{z}_d - \dot{z})) \frac{m}{C_\phi C_\theta} \\
 \tau_\phi &= (K_{P\phi}(\phi_d - \phi) + K_{I\phi} \int (\phi_d - \phi) + K_{D\phi}(\dot{\phi}_d - \dot{\phi})) I_{xx} \\
 \tau_\theta &= (K_{P\theta}(\theta_d - \theta) + K_{I\theta} \int (\theta_d - \theta) + K_{D\theta}(\dot{\theta}_d - \dot{\theta})) I_{yy} \\
 \tau_\psi &= (K_{P\psi}(\psi_d - \psi) + K_{I\psi} \int (\psi_d - \psi) + K_{D\psi}(\dot{\psi}_d - \dot{\psi})) I_{zz}
 \end{aligned} \tag{12}$$

Where,  $g$  is the gravity, mass  $m$  and moment of inertia  $I$  of the quadcopter UAV

According to the Ziegler-Nichols method [30], the PID controller parameter control the UAV quadcopter according to the angles  $(\phi, \theta, \psi)$  control loops, and the altitude ( $z$ ) control loop of the UAV quadcopter are defined belows

$$\begin{aligned}
 k_{P\phi} &= 0.6k_{th\phi}; k_{I\phi} = \frac{2k_{P\phi}}{\tau_{th\phi}}; k_{D\phi} = \frac{k_{P\phi}\tau_{th\phi}}{8} \\
 k_{P\theta} &= 0.6k_{th\theta}; k_{I\theta} = \frac{2k_{P\theta}}{\tau_{th\theta}}; k_{D\theta} = \frac{k_{P\theta}\tau_{th\theta}}{8} \\
 k_{P\psi} &= 0.6k_{th\psi}; k_{I\psi} = \frac{2k_{P\psi}}{\tau_{th\psi}}; k_{D\psi} = \frac{k_{P\psi}\tau_{th\psi}}{8} \\
 k_{Pz} &= 0.6k_{thz}; k_{Iz} = \frac{2k_{Pz}}{\tau_{thz}}; k_{Dz} = \frac{k_{Pz}\tau_{thz}}{8}
 \end{aligned} \tag{13}$$

### 3.2. Design of the position controller for 6-DOF UAV quadcopter

According to the above mentioned control structure, the internal feedback loop controls 4-DOF are three angles and altitude. An external feedback loop is performed to adjust the  $x$  and  $y$  positions of the UAV quadcopter. The desired roll and pitch angles are the output of the external control loop, and they serve as the input of the inner control loops, respectively, the desired angle position,  $\phi_d, \theta_d$  (Fig.2)

When the UAV is stable in the air, the roll angle  $\phi$  and pitch angle have small values. Therefore, by using small angle assumptions ( $S_{\phi_d} \equiv \phi_d, S_{\theta_d} \equiv \theta_d, C_{\phi_d} = C_{\theta_d} = 1$ ), the kinematic equations for the x and y coordinates are simplified in equation as belows [34,35].

$$\begin{aligned}\ddot{x} &= \frac{U_1}{m}(\theta_d \cos \psi + \phi_d \sin \psi) \\ \ddot{y} &= \frac{U_1}{m}(\theta_d \sin \psi - \phi_d \cos \psi)\end{aligned}\quad (14)$$

With  $U_1$  is the UAV quadcopter altitude control input signal

The altitude control PID controller is selected as follows:

$$U_1 = K_{Pz}(z_d - z) + K_{Iz} \int (z_d - z) + K_{Dz}(\dot{z}_d - \dot{z}) \quad (15)$$

Equation (15) is written as a matrix as follows.

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \frac{U_1}{m} \begin{bmatrix} S_{\psi} & C_{\psi} \\ -C_{\psi} & S_{\psi} \end{bmatrix} \begin{bmatrix} \phi_d \\ \theta_d \end{bmatrix} \quad (16)$$

Therefore, the desired pitch and roll control angle is determined by the formula below [34,35].

$$\begin{aligned}\phi_d &= (U_x \sin \psi - U_y \cos \psi) \\ \theta_d &= (U_x \cos \psi + U_y \sin \psi)\end{aligned}\quad (17)$$

Where  $U_x$  and  $U_y$  are input control signals using the PID law.

$$\begin{aligned}U_x &= K_{Px}(x_d - x) + K_{Ix} \int (x_d - x) + K_{Dx}(\dot{x}_d - \dot{x}) \\ U_y &= K_{Py}(y_d - y) + K_{Iy} \int (y_d - y) + K_{Dy}(\dot{y}_d - \dot{y})\end{aligned}\quad (32)$$

According to the Ziegler-Nichols method [30], the PID controller parameters for the position (x, y) are defined below.

$$\begin{aligned}k_{Px} &= 0.6k_{thx}; k_{Ix} = \frac{2k_{Px}}{\tau_{thx}}; k_{Dx} = \frac{k_{Px}\tau_{thx}}{8} \\ k_{Py} &= 0.6k_{thy}; k_{Iy} = \frac{2k_{Py}}{\tau_{thy}}; k_{Dy} = \frac{k_{Py}\tau_{thy}}{8}\end{aligned}\quad (18)$$

#### IV. Results of modeling and simulation of control systems for 6-DOF UAV quadcopter

The study in the paper utilizes the parameters of the UAV quadcopter in Table 1.

Table 1. Parameters of the UAV quadcopter

| Parameter                   | Symbol           | Value                       |
|-----------------------------|------------------|-----------------------------|
| Quad. mass                  | $m$              | 0.468 kg                    |
| Arm length                  | $l$              | 0.225 m                     |
| Gravity                     | $g$              | 9.81 m/s <sup>2</sup>       |
| Inertia moment of the rotor | $I_m$            | 3.357e-5 kg.m <sup>2</sup>  |
| Thrust factor of rotor      | $k$              | 2.980e-6 N.s <sup>2</sup>   |
| Drag coeffi.                | $b$              | 1.140e-7 N.m.s <sup>2</sup> |
| Inertial constants          | $I_{xx}, I_{yy}$ | 4.856e-3 kg.m <sup>2</sup>  |

|  |          |                           |
|--|----------|---------------------------|
|  | $I_{zz}$ | $8.801e-3 \text{ kg.m}^2$ |
|--|----------|---------------------------|

Carry out experimental simulations to determine the parameters of 6PID controllers for the 6-DOF UAV quadcopter, including 6 control loops: yaw angle  $\psi$ , pitch angle  $\theta$ , roll angle  $\phi$ , attitude  $z$ , and position  $x$ ,  $y$ .

Finally, the parameters of the 6 PID controllers for the 6-DOF UAV quadcopter, as shown in Table 2.

Table 2. Parameters of 6 PID controllers for 6-DOF UAV quadcopter

| Parameter | Position<br>( $x,y$ ) | Altitude<br>( $z$ ) | Roll<br>( $\phi$ ) | Pitch<br>( $\theta$ ) | Yaw<br>( $\psi$ ) |
|-----------|-----------------------|---------------------|--------------------|-----------------------|-------------------|
| $k_p$     | 0.00414               | 75                  | 2.1                | 2.4                   | 0.804             |
| $k_i$     | 0.0000345             | 42.8571             | 0.84               | 0.48                  | 0.29236           |
| $k_D$     | 0.1242                | 32.8125             | 1.3125             | 3.09                  | 0.55275           |

Simulation of the 6-DOF UAV quadcopter control system with the above 6 PID controllers, with the setpoint  $\phi_d=0$ ,  $\theta_d=0$ ,  $\psi_d=1$ ,  $x_d=0.5$ ,  $y_d=0.5$ ,  $z_d=2$ , we obtained the system responses as shown in Figure 3-4.

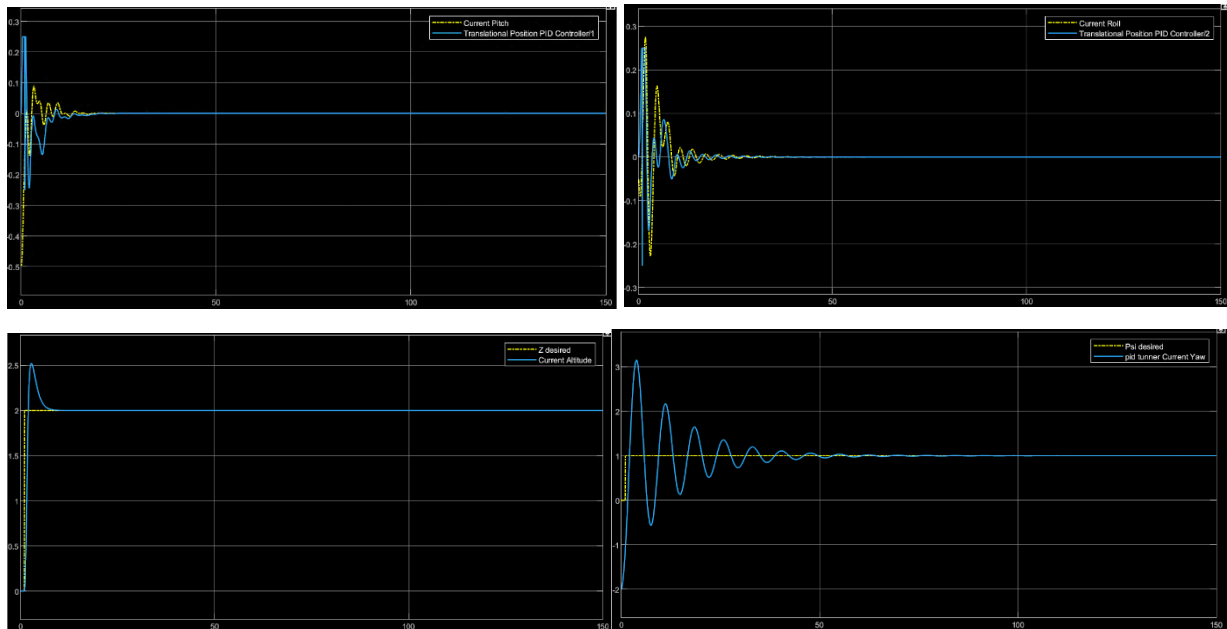


Figure 3 – Responses of the pitch, roll, yaw angles and altitude of the UAV quadcopter

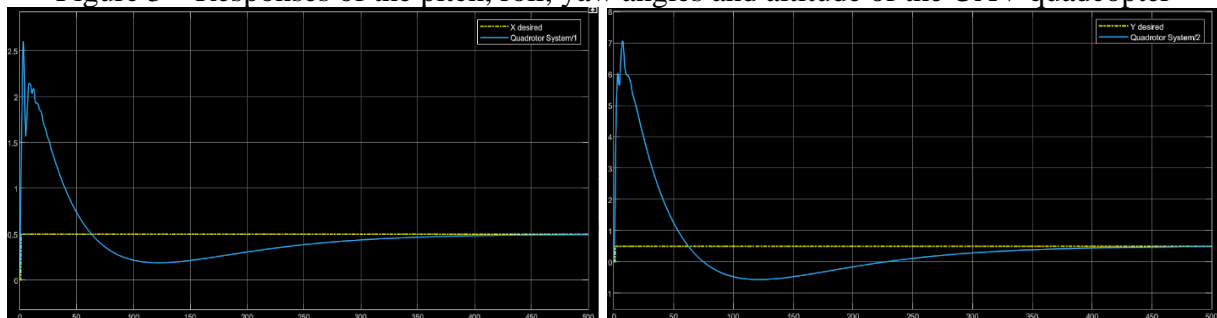




Figure 4 – Response of position (x, y) của UAV quadcopter

The UAV quadcopter control system simulation results show that the Euler angle, altitude, and position responses for the UAV quadcopter met the requirements. The control system quickly brought the UAV quadcopter to the set value of the altitude and the three angles pitch, roll, yaw, and then stabilized positions x, y. In Figure 3, the response of altitude (z) is very fast, ~7s, followed by responses of pitch angles, ~20s, roll angle, ~32s and yaw angle, ~70s. Position responses (x,y) have a long time, ~350s, as shown in Figure 4.

## V. Conclusion

Research on mathematical model development and design of 6 PID controllers for 6-DOF UAV quadcopter was given in this article. Using the Newton–Euler and the Euler–Lagrange equations, a system of differential equations describing the dynamics of the 6-DOF UAV quadcopter was introduced here, thereby allowing easy modelling of the 6-DOF UAV quadcopter on Matlab/Simulink. After that, the design process of 6 PID controllers for 6-DOF UAV quadcopter, based on the Ziegler-Nichols II method, was researched and successfully applied in this study. The simulation results show that the proposed PID controllers has stabilized the 6-DOF UAV quadcopter at the desired position and altitude, meeting the requirements.

Future research will continue to propose proposals to implement different methods of PID parameter adjustment, development of nonlinear, modern and intelligent control algorithms, application of fuzzy logic, neural networks, artificial intelligence ... in position control and balance stabilization, trajectory tracking control for the 6-DOF UAV quadcopter.

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