In [11]:
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import numpy as np
import matplotlib.pyplot as plt
from scipy.interpolate import RegularGridInterpolator
import pandas as pd
```

In [12]:
```
class SGD:

    def __init__(self, lr=0.01, max_iter=1000, batch_size=10,
                 tol=1e-3,theta=None):
        """
        Initializes the SGD optimizer.

        Parameters:
        - lr: Learning rate.
        - max_iter: Maximum number of iterations.
        - batch_size: Size of the mini-batch used in each iteration.
        - tol: Tolerance for convergence.
        - theta: Initial values for optimization.

        """
        self.learning_rate = lr
        self.max_iteration = max_iter
        self.batch_size = batch_size
        self.tolerance_convergence = tol
        self.theta = np.array(theta)

    def update_points(self, x1,y1,xval, yval):

        """
        Update the points using Stochastic Gradient Descent.

        Parameters:
        - x1, y1: Initial coordinates for optimization.
        - xval, yval: Data points for optimization.

        Returns:
        - Optimized coordinates.

        """
        for _ in range(self.max_iteration):
            indices = np.random.permutation(len(xval))
            x_batch = xval[indices]
            y_batch = yval[indices]

            for i in range(0, len(xval)-1, self.batch_size):
                xbatc = x_batch[i:i + self.batch_size]
                ybatc = y_batch[i:i + self.batch_size]
                x1, y1 = self.gradient(x1, y1, xbatc, ybatc)

            grad_x = np.mean(rgi_x((x1, yval)))
            grad_y = np.mean(rgi_y((xval, y1)))

            if np.linalg.norm([grad_x, grad_y]) < self.tolerance_convergence:
                break


        self.theta = np.array([x1,y1])
        return self.theta

    def gradient(self, x1, y1,xbatch,ybatch):
        """
        Calculates the gradient and updates the coordinates.

        Parameters:
        - x1, y1: Current coordinates.
        - xbatch, ybatch: Mini-batch coordinates.

        Returns:
        - Updated coordinates.

        """
        x_grad = np.mean(rgi_x((x1,ybatch)))
        y_grad = np.mean(rgi_y((xbatch,y1)))

        x1 = x1 - self.learning_rate*x_grad
        y1 = y1 - self.learning_rate*y_grad

        return x1,y1
```

In [13]:
```
# Definition of the cost function
def cost_function(x, y):
    term1 = 1.8 - np.exp(-0.1 * (2.5 * (x + 3)**2 + (y + 3)**2))
    term2 = -1.5 * np.exp(-0.05 * (2.5 * (x - 3)**2 + (y - 3)**2))
    return term1 + term2

# Create a 2D grid
x_values = np.arange(-8, 8.1, 0.1)
y_values = np.arange(-8, 8.1, 0.1)
X_grid, Y_grid = np.meshgrid(x_values, y_values)
grid = np.array(np.meshgrid(x_values, y_values)).T.reshape(-1,2)
values = cost_function(grid[:,0],grid[:,1])
interpolator = RegularGridInterpolator((x_values,y_values),values.reshape(161,161))

# Calculate the gradient on the grid
Z = cost_function(X_grid,Y_grid)
grad=np.gradient(Z)
rgi_x = RegularGridInterpolator((x_values, y_values), grad[0])
rgi_y = RegularGridInterpolator((x_values, y_values), grad[1])

# Calculate the cost values on the grid
Z_grid = interpolator((X_grid,Y_grid))

# Initial points for optimization
initial_points = [np.random.uniform(low=0,high=8,size=2),np.random.uniform(low=0,high=8,size=2),
                  np.random.uniform(low=0,high=8,size=2)]
```

In [14]:
```
# Plotting setup
fig = plt.figure(figsize=(12,11))
df = pd.DataFrame(columns=['x', 'y', 'z'])
i=-1

# Loop over initial points
for initial_point in initial_points:
    i+=1

    # Create and update the SGD model
    model = SGD(lr=0.1, max_iter=100, batch_size=10, tol=1e-4,theta=initial_point)
    model.theta = initial_point

    # Initialize arrays to store trajectory
    trajectory_x = np.zeros(model.max_iteration)
    trajectory_x[0] = model.theta[0]
    trajectory_y = np.zeros(model.max_iteration)
    trajectory_y[0] = model.theta[1]
    trajectory_z = np.zeros(model.max_iteration)
    trajectory_z[0] = interpolator((model.theta[0], model.theta[1]))

    # Store initial values in DataFrame
    init_point = (trajectory_x[0],trajectory_y[0])
    df.loc[f'Initial Values for iteration ({i + 1}) = '] = [trajectory_x[0], trajectory_y[0],
                                                             f'{interpolator(init_point):.6f}']

    # Perform SGD optimization
    for k in range(model.max_iteration-1):

        model.theta = model.update_points(model.theta[0], model.theta[1],x_values,y_values)

        trajectory_x[k+1] = model.theta[0]

        trajectory_y[k+1] = model.theta[1]

        trajectory_z[k+1] = interpolator((model.theta[0], model.theta[1]))

    # Plotting trajectories
    for j in range(2):

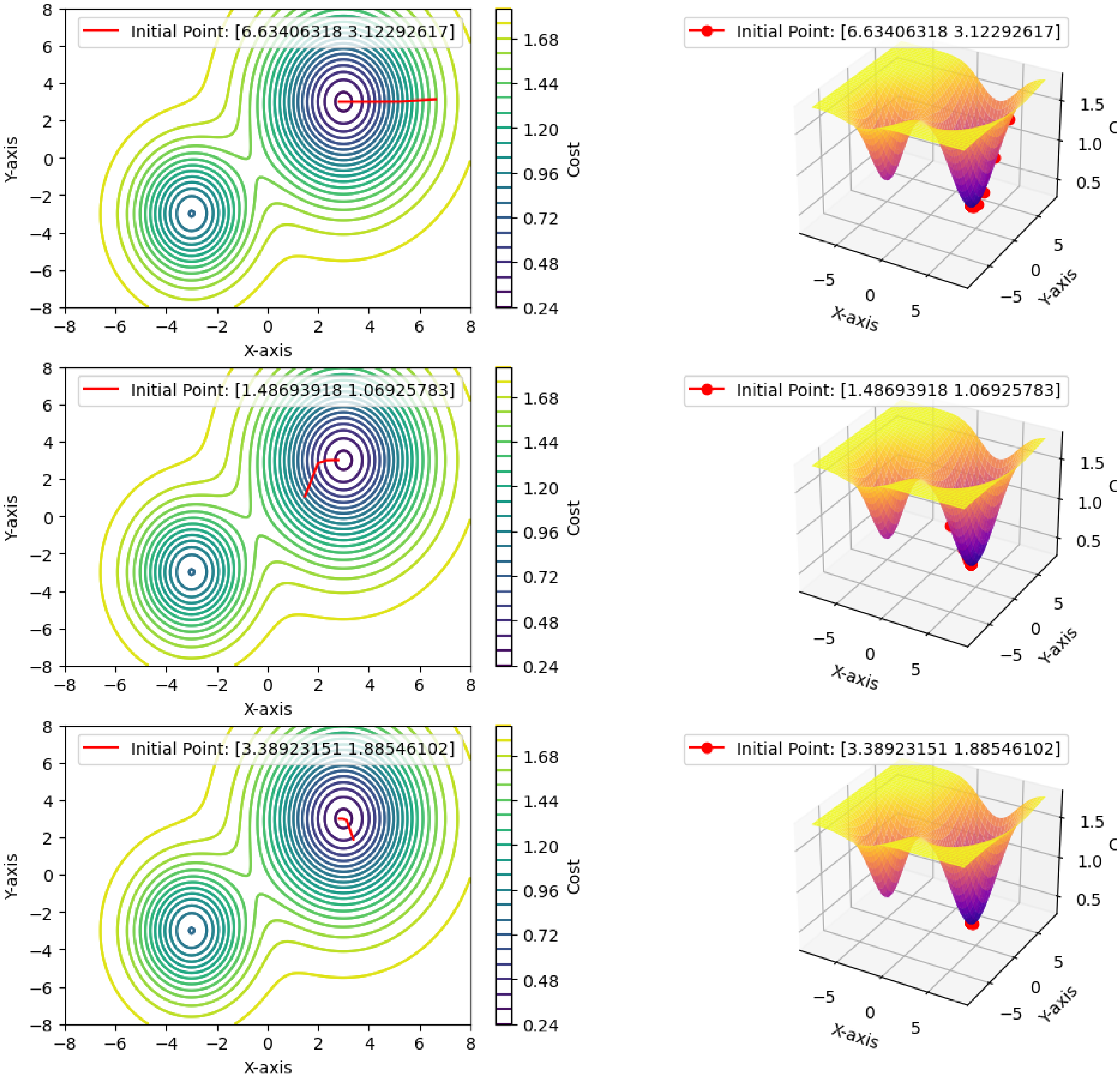
        if j == 1:
            ax = fig.add_subplot(3, 2, i * 2 + j + 1, projection='3d')
            ax.plot_surface(X_grid, Y_grid, Z_grid, cmap='plasma', alpha=0.9)
            ax.set_xlabel('X-axis')
            ax.set_ylabel('Y-axis')
            ax.set_zlabel('Cost')
            ax.plot(trajectory_x, trajectory_y, trajectory_z, marker="o",
                    label=f'Initial Point: {initial_point}',color="r")
            handles, labels = plt.gca().get_legend_handles_labels()
            by_label = dict(zip(labels, handles))
            plt.legend(by_label.values(), by_label.keys())

        else:
            ax = fig.add_subplot(3, 2, i * 2 + j + 1)
            contour = ax.contour(X_grid, Y_grid, Z_grid, levels=20, cmap='viridis')
            ax.set_xlabel('X-axis')
            ax.set_ylabel('Y-axis')
            plt.colorbar(contour,label="Cost")
            plt.contour(X_grid, Y_grid, Z_grid, levels=20, cmap='viridis')
            ax.plot(trajectory_x, trajectory_y, label=f'Initial Point: {initial_point}',color="r")
            handles, labels = plt.gca().get_legend_handles_labels()
            by_label = dict(zip(labels, handles))
            plt.legend(by_label.values(), by_label.keys())


    final_point = (trajectory_x[-1],trajectory_y[-1])
    df.loc[f'Final Values for iteration ({i + 1}) = '] = [final_point[0], final_point[1],
                                                           f'{interpolator(final_point):.6f}']

print(df)
plt.show()
```

```
                                       x         y         z
Initial Values for iteration (1) =   6.634063  3.122926  1.512213
Final Values for iteration (1) =     2.826342  2.999257  0.306004
Initial Values for iteration (2) =   1.486939  1.069258  0.863758
Final Values for iteration (2) =     2.795780  2.999257  0.307873
Initial Values for iteration (3) =   3.389232  1.885461  0.417007
Final Values for iteration (3) =     2.825970  2.999257  0.306025
```



When we experimented with different learning rates while keeping the tolerance constant, significant changes occurred in the program's execution speed.Stochastic gradient descent works much faster for large datasets, but in our experience, we couldn't determine its speed compared to the final version of the code.Also We observed that it was quite fast when we printed the visuals one by one instead of printing them at the same time.