Game Theory

Mixed Strategies

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Mixed Strategies

- As you may remember that, in previous chapter we have evaluated the discrete static complete information games. However, we couldn't find an equilibrium for the Matching Pennies game.
- In order to find an equilibrium we should employ a different approach from the discrete understanding.

Mixed Strategies

- In this chapter, we will evaluate the continuous static games, in which there are infinite choices for each players.
- Consider a case where two firms decide to implement a price level. They have infinite amount of choices, with the constraint of not negative.
- Our starting point will be the infinite choices and then we will introduce the probability concept in terms of discrete games.

Mixed Strategies

- Consider a simple case where two players' utility functions are given as:

$$U_A(S_A, S_B) = -S_A^2 + 4S_A + 2S_AS_B + 3S_B - S_B^2$$

$$U_B(S_A, S_B) = -S_B^2 + 2S_B + S_AS_B + S_A - S_A^2$$

Mixed Strategies

- As you can observe, the utilities that each player will acquire depends on the other player's strategy.
- Since both players will try to maximize its utility, we should take the partial derivative of each utility with respect to their strategy and equalize both of them to 0:

$$\frac{\partial U_A}{\partial S_A} = -2S_A + 4 + 2S_B = 0$$

$$\frac{\partial U_B}{\partial S_B} = -2S_B + 2 + S_A = 0$$

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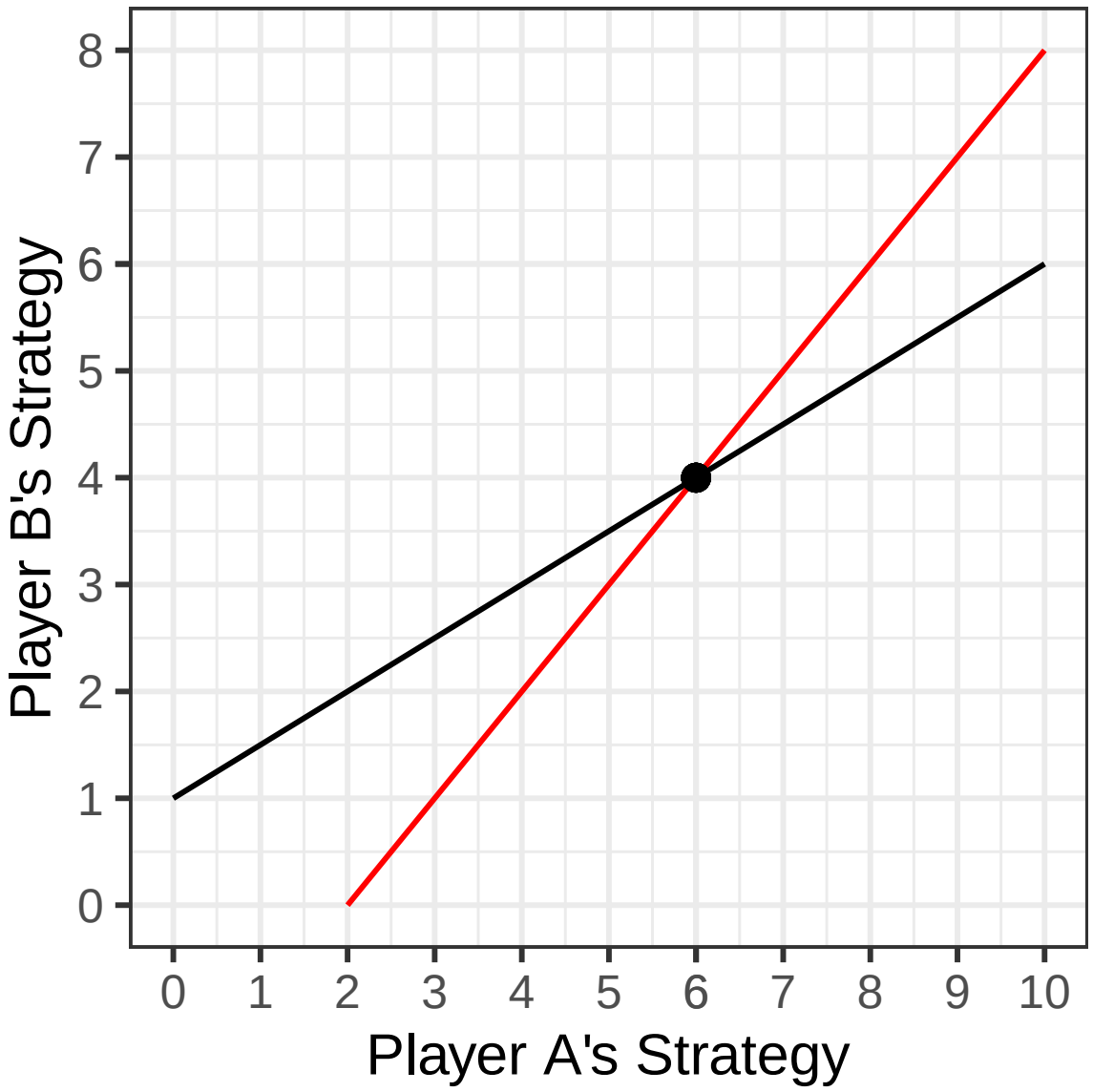
- If we rearrange the equations, we will get:

$$S_A = 2 + S_B$$

$$S_B = 1 + \frac{S_A}{2}$$

- These equations are called Best Response Functions for player A and player B.
- What we did was we have first taken the partial derivatives of each utility functions with respect to player's strategy and equalize to 0. This means that, we held constant other variables in the equation and tried to find optimal strategy which maximizes the utility.

Mixed Strategies



Mixed Strategies

- As it is illustrated through the graph, the Nash Equilibrium will be $\{6, 4\}$.
- Understand this conclusion as this: They will increase their strategies (in terms of numbers) until the Best Response Functions of both players intersect, which is $\{6, 4\}$.

Mixed Strategies

- In order to understand the mixed strategies, we will introduce the concept of Expected Payoff, in which players will try to maximize their expected payoff.
- Actually we have shown the implementation of Expected Payoff concept with continuous case in previous pages, in order to understand it there should also be an example where we implement it in discrete game.

Matching Pennies

- Remember from the previous chapter in which we were trying to solve Matching Pennies game and we indicated that we have to have a different tool to solve it.
- We will solve it through Expected Payoff, by attributing probabilities for each of strategies.
- Let's say probability of first player playing head is p, then playing tail will be $1 - p$. And for second player playing head q and tail will be $1 - q$.

		q	(1-q)
		H	T
p	H	(1,-1)	(-1,1)
(1-p)	T	(-1,1)	(1,-1)

Matchin Pennies

- According to this matrix, we would represent the Expected Payoff of first player as:
  $EP_1 = p \times q \times 1 + p \times (1-q) \times (-1) + (1-p) \times q \times (-1) + (1-p) \times (1-q) \times 1$

$$EP_1 = 4pq - 2p - 2q + 1$$

Matchin Pennies

- In order to find the equilibrium, we would take the partial derivative with respect to first player's probability and equalize to zero, to maximize the Expected Payoff:

$$\frac{\partial EP_1}{\partial p} = 4q - 2 = 0$$

$$q = 0.5$$

- Since the Expected Payoff functions will be symmetric, we would get the same answer for the Expected Payoff of second player ($p = 0.5$).

Matching Pennies

- This result that we have found indicates that, both players would meet at the point of equal probabilities for each strategy.
- If, assume that, second player attributed selecting first strategy as $q = 0.1$, then the Expected Payoff of first player will be:

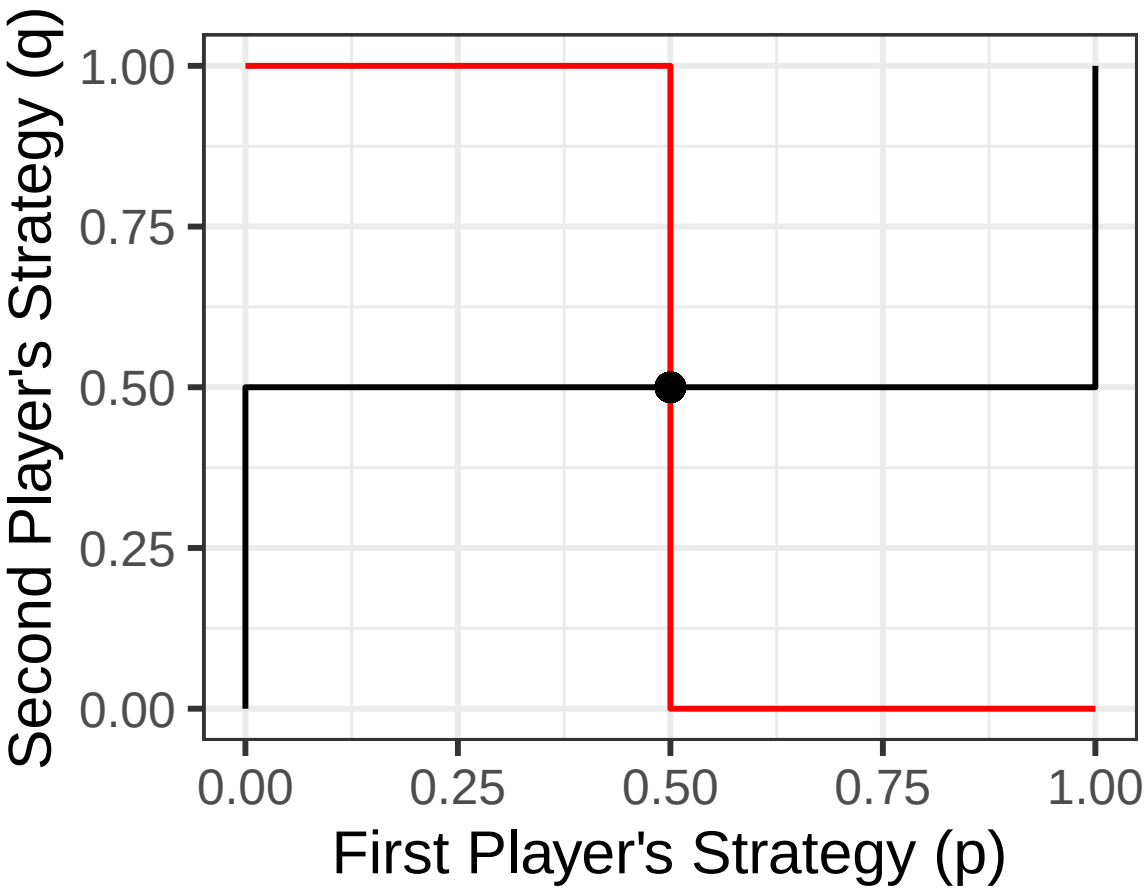
$$EP_1 = 0.4p - 2p - 0.2 + 1$$

$$EP_1 = -1.6p + 0.8$$

- In order to maximize this one, first player will decide as $p = 0$, which means that he/she will certainly choose tail.

Matching Pennies

- However, knowing that first player choose tail, the second player will choose head.
- Thus, as it can be seen that, the Best Response Functions of both players will be in a discrete shape, rather than continuous as our first example.



Matching Pennies

- To conclude the chapter, we can derive the formula for the Expected Payoff as:

$$EP = \sum_{i=1}^{n} p(X = x)\ x$$

- This tells us that Expected payoff is equal sum of all payoffs that will be attributed by the player for each case times the probability that the player attributes. So, the probability of selecting tail is 0.5 and its payoff is 1 and for head is 0.5 and its payoff is -1, then the expected payoff will be equal to : $0.5 \times 1 + 0.5 \times (-1) = 0$
- However, in the games, such as matching pennies, rather than marginal probabilities, we use the joint probabilities where we multiply the probabilities that is attributed by the players for a specific strategy.