

# Game Theory

## Nash Equilibrium

(Static and Complete Information Games)

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# Nash Equilibrium

- ▶ Nash Equilibrium is simply the solution of a game where each player is not able to increase its utility by deviating from the strategy of the Nash Equilibrium.
- ▶ When the game is not solvable by eliminating the choices of players, the Nash Equilibrium can be used.

# Nash Equilibrium

- ▶ Consider this case, where we cannot eliminate any choice:

	A	B
A	(1,0)	(0,-2)
B	(-2,0)	(0,1)

- ▶ Since we cannot eliminate any choice, we can use Nash Equilibrium

# Nash Equilibrium

- ▶ Consider the first player choose  $A$ , then the second player would play  $A$  as well.
- ▶ In order to be this case to be a Nash Equilibrium, we have to examine this case from the point of view of first player  $A$  as well.
- ▶ If the first player expects second player to play  $A$ , will he/she will deviate from his/her choice of  $A$ ?
- ▶ The answer is no. Since the  $u_1(A, A) > u_1(B, A)$ , he/she will stick to his/her initial choice,  $A$ .

# Nash Equilibrium

- ▶ Consider that first player played  $B$ . Then, in order to maximize his/her payoff, second player will also play  $B$ .
- ▶ The same intuition that we should apply here. Assume that first player expects second player to play  $B$ . Then he/she will not deviate his/her initial choice of  $B$ , because in this case  $u_2(B, B) > u_2(B, A)$ .

# Nash Equilibrium

- ▶ So, the Nash Equilibriums of this game will be:

$$NE_1 = \{A, A\} , NE_2 = \{B, B\}$$

- ▶ This means that, under the assumption of rationality and every player knows each player is rational, both players will not change their decisions from these two solutions.

## Prisoners' Dilemma

- ▶ One of the most famous game, known as Prisoners' Dilemma. In this game, the Nash Equilibrium is unique. And main feature of this equilibrium is that even though there is more beneficial solution where both players will receive more payoff, this equilibrium occurs due to the rationality of players.
- ▶ The name actually comes from interrogation of two players, which are suspected on a crime. Two players do not know what other player will do and they didn't communicate beforehand. So it is a simultaneous game.
- ▶ Consider they have two choices: confess the crime or remain silent. The payoff matrix can be illustrated as:

	Remain Silent	Confess
Remain Silent	$(-5, -5)$	$(0, -10)$
Confess	$(-10, 0)$	$(-6, -6)$



# Prisoners' Dilemma

- ▶ The main idea behind this game is if both remain silent, then they will get 5 years of imprisonment, thus the payoff for each is  $-5$ . If one of them deviates, confessing the crime, then the player who remains silent will get 10 years of imprisonment and the player that confesses will not be punished. And if both of them confess, then they will be imprisoned, but get much longer imprisonment than the case of both of them remain silent, since they accepted the crime.
- ▶ So we can find the Nash Equilibrium as:

$$NE = \{\text{Confess}, \text{Confess}\}$$

## Pareto Optimality

- ▶ The classical view of utility indicates that, the rationality of individuals or players will lead to the best conclusion.
- ▶ However, we have seen that the rational choice for players sometimes leads to a less wanted conclusion when it comes to the payoff (as it is in the Prisoners' Dilemma example), when each player has opportunity to exploit the other player.
- ▶ Consider the parents (father and mother), which have two choices on the raising their child:

	C	NC
C	(0,0)	(-2,2)
NC	(2,-2)	(-1,-1)

# Pareto Optimality

- ▶ For instance, assume that first player (father) choose to care (C) he child. Then maximizing choice for the second player (mother) to not care (NC).
- ▶ However, if the father expects mother not care, then he will also choose not care. Thus we cannot say Nash Equilibrium.
- ▶ But for the choice of  $\{NC, NC\}$  is going to be unique Nash Equilibrium of this game.

# Pareto Optimality

- ▶ As you can understand, the payoffs of Nash Equilibrium for this game is not a Pareto Optimal solution.
- ▶ Since there is a choice where both players increase their payoffs, which is  $\{0, 0\}$
- ▶ I with mathematical notations, pareto optimality can be shown as:

$$\{\nexists \omega' \in \Omega : \forall i \ u_i(\omega') \geq u_i(\omega) \wedge i \in \{1, 2, 3 \dots n\}\}$$

$$n = 2$$

- This means, there is not an allocation ( $\omega'$ ) which is an element of all possible allocations ( $\Omega$ ) such that for all  $i$  its utility ( $u_i(\omega')$ ) is bigger than the utility that each player receive from the Nash Equilibrium allocation ( $u_i(\omega)$ ).

# Pareto Optimality

- ▶ As we have indicated, the main feature of Prisoners' Dilemma game is the equilibrium that game reaches is not a best solution for both of players.
- ▶ Hence the Prisoners' Dilemma will not result with a solution which is pareto optimal, which means there are, at least one, choice which increases payoffs of both players.