Game Theory

Oligopolies

Contents

Monopolies

- A monopoly represents a case where only a firm produces a specific commodity or product.
- According to the Classical Marginal Theory, since there is only one firm in the industry, the price is exogenous. Meaning that the firm is the sole determinant of the market price rather than price-taker such as in perfect-competitive markets.

Monopolies

- A simple examination of monopoly is through the profit maximization problem. Consider this function of a profit for the monopoly:

$$\pi = P \times Q - MC \times Q$$

- And if we assume the demand function for this product as:

$$P = a - bQ$$

Monopolies

- As you can see, we can replace the price with the demand function.

$$\pi = (a - bQ) \times Q - MC \times Q$$

- Thus if we take the partial derivative of this function with respect to quantity (Q) and equalize it to zero, we would get the profit maximizing quantity:

$$\frac{\partial \pi}{\partial Q} = a - 2bQ - MC = 0$$

$$\frac{a - MC}{2b} = Q^*$$

Monopolies

- The reason why we have given a brief summary of monopolies is that We will examine the Cournot and Bertrand games in terms of profit maximization problems. So this tool will be beneficial on our examinatio of oligopolies.

Cournot

- We know that oligopolies consist of a few number of firms which dominate a specific industry.
- In Cournot Game, firms will compete with each other in terms of quantity produced. Where they are price takers, as in the perfect competitive markets.
- Consider two sellers of apples in a bazaar, who sell identical apples. They bring their apples at the same time, without knowing each others quantity.

Cournot

- Since the apples are identical, they will be sold at the same price level.
- And additionally and most importantly, the number of apples in the bazaar will affect the price level. We can show it as:

$$P = a - bQ \qquad Q = \{q_1 + q_2 : (q_1 > 0) \wedge (q_2 > 0)\}$$

Cournot

- Do not forget that Cournot and Bertrand games are static continuous games that players select their choices (quantity,price) simultaneously.
- Because in Cournot firms compete in terms of quantities, rather than prices (prices are determined by the total quantity), by giving the demand function for this product, we can find the equilibrium:

$$P = a - bQ$$

Cournot

- As we did in the monopoly, we can use the profit function for finding equilibrium:

$$\pi_1 = (a - bq_1 - bq_2) \times q_1 - MC_1 \times q_1$$

$$\pi_2 = (a - bq_1 - bq_2) \times q_2 - MC_2 \times q_2$$

$$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - MC_1 = 0$$

$$\frac{\partial \pi_2}{\partial q_1} = a - bq_1 - 2bq_2 - MC_2 = 0$$

$$q_1^* = \frac{a - bq_2 - MC_1}{2b}$$

$$q_2^* = \frac{a - bq_1 - MC_2}{2b}$$

Cournot

- Thus, if we assume these equalities for demand function and marginal costs of these two firms as:

$$P = 10 - 2Q \qquad MC_1 = 2 \qquad MC_2 = 3$$

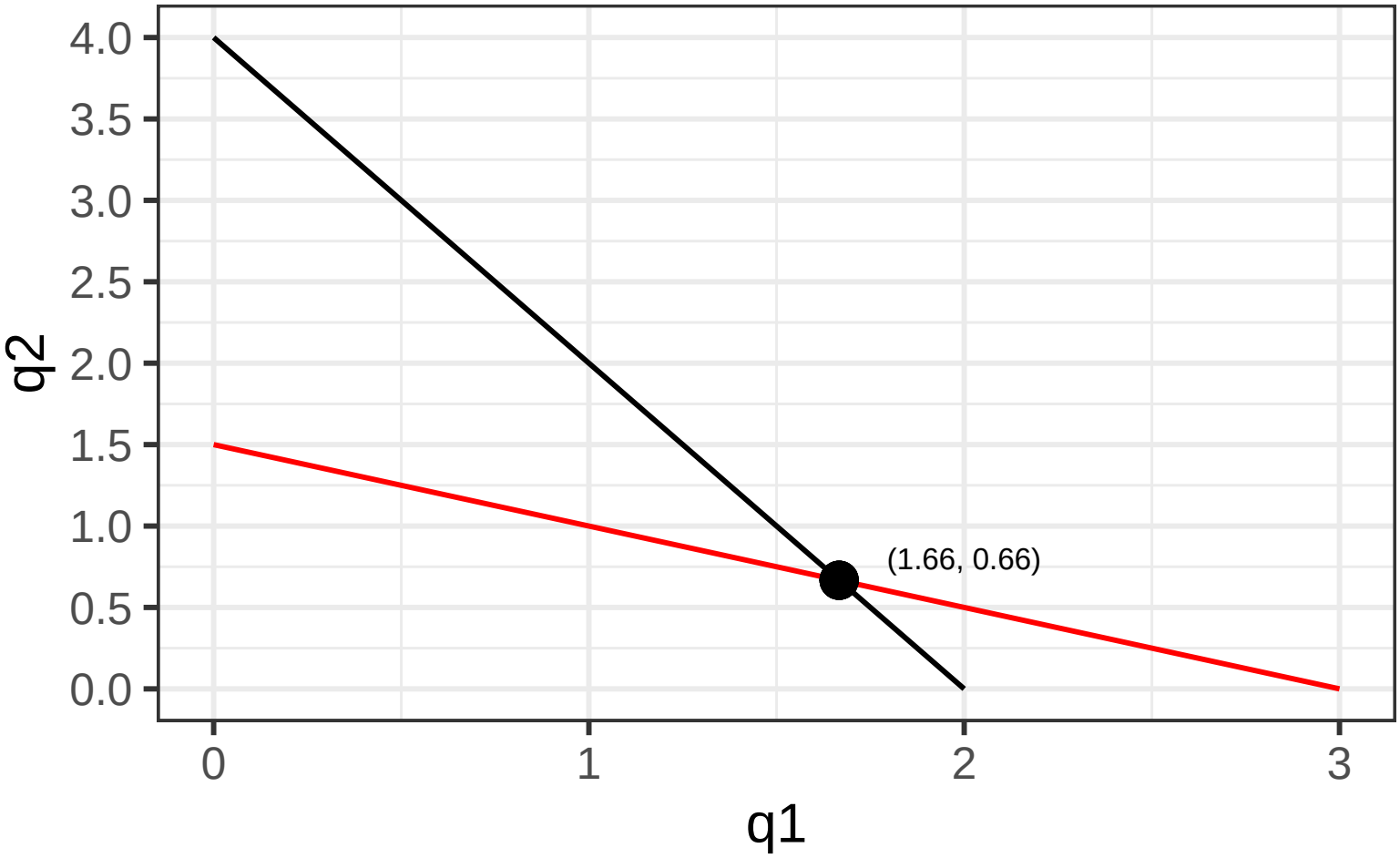
- Then the optimal solutions for firms will be:

$$q_1^* = \frac{10 - 2q_2 - 2}{4}$$

$$q_2^* = \frac{10 - 2q_1 - 4}{4}$$

- Thus, if we plug the one of the equations to other, we would get the Cournot Nash Equilibrium where both firms will not change its strategies.

Cournot



- Since the marginal cost of second firm MC_2 is bigger than the first firm's MC_1, the optimal quantity is lower than first firm's.

Bertrand

- In Bertrand Competition, rather than based on quantities, firms compete in terms of prices, which is considered as more realistic, since firms in real life are not bound to any rules for prices (except for price controls by governments).
- You may think that by establishing the prices through the competition based on quantities with the Cournot Competition, it is unreasonable to construct a competitive environment based on price. But the results will be very different from Cournot.

Bertrand

- Let's look at the assumptions that are collected from ***Introduction to Game Theory and The Bertrand Trap*** by ***Benjamin E. Hermalin***.
  - Firms produce homogenous products
  - Customers know the prices that firms selected
  - Customers do not incur any costs from switching firms
  - Both firms have same marginal cost
  - No capacity constraints on production
  - No future considerations

Bertrand

- Two important assumptions, fourth and fifth, makes Bertrand an important type of competition.
- By having equal marginal costs for each firm, we prevent the cost advantages between firms. It will be seen that the cost advantageous firm will be the only player in the industry.
- By adding the fifth assumption, we do not constrain any firm on producing. Since there will be sharp transition of customers, this assumption is important.

Bertrand

- Let's look into the intuition behind the Bertrand. Do not forget, again, that the Bertrand Game is also a static type game as Cournot Game, where players choose simultaneously.
- Consider that the first firm choose \$10 for a product which has \$5 of marginal cost (for both firms).
- Then second firm will choose a number that is so close to \$10 but lower than that (let's say \$9.8). This is because, since consumers know the prices, the logical to thing them to do is purchasing at lower prices. So the second firm will spur all the market and the first will make 0 profit.
- Thus, on the other hand, since the first firm makes 0 profit, it will change its choice and will determine the price at a lower level of \$9.8 (let's say \$9.7)

Bertrand

- By understanding this process, one can predict where the equilibrium will be set.
- Since neither firm would not determine price level below the marginal cost, the prices will be set exactly at the marginal cost, where marginal cost is same for both firms.
- At the marginal cost price both firms will make 0 profits. This is argued as the one of the most important conclusions of perfect competitive markets, where firms had to compete in terms of prices.
- The marginal cost price is Nash Equilibrium. Consider a firm that decreases its price below marginal cost. Even though it satisfies the whole demand in market, because marginal cost is higher than price, it will make negative profits.
- Consider, on the other hand, one of them increased price. Since the other firm has lower prices, this firm which increased its price will not be able to sell any product.

Bertrand

- From the function, the information that we have given can be seen.
- In this case n represent the firm number, m represents the number of firms that determined their prices as minimum. D represents the total demand for product.
- Then, we can represent the choices and profit function of firms as this:

$$\pi_n = \begin{cases} 0 & \text{if } P_n > P_{min} \\ \frac{1}{m} \times D \times P_n - (MC \times q_n) & \text{if } P_n = P_{min} \\ D \times P_n - (MC \times q_n) & \text{if } P_n < P_{min} \end{cases}$$

- Then, we can conclude that in Bertrand Competition, firms will make zero profit, which is considered as undesirable from the point of view of firms.