

# Permanent Domain Memory and Faster KV-cache: Tauformer, the Topological Transformer

Lorenzo Moriondo

Independent Researcher - tuned.org.uk

ORCID: 0000-0002-8804-2963

1 January 2026

## Abstract

Starting from my research about vector search for topological spaces (spectral search with *taumode*), I introduce in this paper the definition, implementation and testing of a new class of Transformer architecture [1] focused on improving the attention mechanism. The Topological Transformer, or *Tauformer* applies all the features of current attention-based transformers (taking as a baseline *nanoGPT* and its architecture [3]) and redesigns the attention mechanism at its core to: (i) allow delivering domain-specific context at the level of the attention mechanism for more domain-relevant token generation, (ii) improve KV-cache memory by saving 50% compared to traditional KV-caching and (iii) draw a path to improve the Transformer performance on large context windows with high-dimensional heads, by substituting inner-product with *taumode*'s synthetic index-based distance, aiming to provide relevant linear gains in training and generation compared to current GPTs.

## 1 Introduction

Concepts and tools developed in my previous publication *ArrowSpace: introducing Spectral Indexing for vector search* [2] have been here reused to redesign the attention mechanism starting from the *nanoGPT* implementation of Transformer architecture; in particular *taumode*, a synthetic index based on the Rayleigh quotient [13] to compute the distribution of energy in the network defined by the embeddings space. Examples about how to compute the *taumode* distribution for any space of embeddings is available in the [pyarrowspace repository](#).

**Tauformer** makes possible downstream technical improvements in computing and memory usage for the Transformer (GPT) and they are detailed in the following sections. These improvements are a consequence of pursuing the concept of providing a memory layer to LLMs that brought forward the idea of leveraging **distilled knowledge graphs (dkb)** to deliver domain information to the attention mechanism at token generation level. For dkb I intend for example: (i) Text embeddings generated by feeding to an embedding model (in one of the examples TSDAE [10]) a representative corpus of the domain to map, this provides a vector space with manageable number of dimensions (384 in the example) that is a prerequisite to build the context windows at the attention level; (ii) Graph embeddings of triples generated using graph embeddings techniques like the SEPAL workflow demonstrated in [11], again the result is a vector space with a manageable dimensionality. Once a stable dkb is generated and its Graph Laplacian computed using *arrowspace* [2], it can be considered from the point of view of the LLM system as persistent memory that is founded on real-world-defined relations. Assuming a well-designed embedding pipeline, text embeddings from a curated text corpus or a curated Knowledge Base

turned into graph embeddings become the ground truths' numerical representations and a persistent memory of the topology of the context. Theoretically the Tauformer can also work, as any other transformer, on images and audio vectors or a mix of them but one of the critical points focuses on the process of producing the embeddings, how the vector space is designed and how its dimensionality fits in the attention mechanism. The choices for: embeddings, attention mechanism and seeding the latent space are taken as design constraints for the data pipeline from the corpus to the latent space of the model. What is meant with seeding the latent space? Once the attention mechanism is working with *taumode* synthetic scores the content of the dkb percolates in the context windows through the usual  $Q, K, V$  search iteration. Each distance metrics computed for  $Q, K, V$  is considered in the optics of the corpus/dataset from which the original embeddings were generated, delivering domain-specific metadata directly in the attention mechanism.

## 2 Statement of Need

This research is motivated by encoding at generation time an explicit (controllable and interpretable) domain structure beyond what the model can implicitly absorb into weights during training. Tauformer keeps the familiar  $Q, K, V$  and causal softmax pipeline but replaces the dot-product kernel with a distance metrics based on domain-specific manifold: each token/head is mapped to a topological signal and built into a scalar (*taumode* score) derived from the Graph Laplacian (manifold); this way attention is driven by distances in that manifold-aware scalar space rather than raw geometrical vector similarity. The hypothesis is that this process makes the model more scoped to the context in the training phase (forward) and in the generating phase (decoding), because weights and attention scores can be biased toward tokens that are similar under the learned domain manifold (i.e. a knowledge graph, citation graph, text embedding or any dataset originally passed to the embedding model). By hypothesis, this has the potential of improving contextual faithfulness relative to purely dot-product-based retrieval inside the context window. Not having permanent reference about the context is especially consequential in scientific and knowledge-intensive settings, where the desired notion of "relevance" is frequently defined less by raw geometrical proximity and more by the topology of domain relations and how information (in **arrowspace**, energy) propagates over them.

Tauformer is also motivated by the practical difficulty of scaling traditional GPTs to very long contexts: full-sequence self-attention has  $O(T^2)$  compute in both the dot-product logits and the value aggregation, and decode-time still grows  $O(T)$  per token while KV-cache memory grows linearly with  $T$ . Tauformer's cache design reduces KV memory by storing values plus a compact key-side scalar  $(k_i, \lambda_i)$  instead of full  $K$  and  $V$  tensors, yielding roughly a halving of KV-cache memory per layer (up to a small overhead for the scalar stream). With a sparse Laplacian from a domain manifold, the extra cost of computing the tau scalars can shift from  $O(D^2)$  to a sparsity-dependent cost  $O(\text{nnz}(L))$ , making the incremental overhead largely independent of sequence length and therefore more attractive as context windows and embeddings dimensions grow.

Tauformer addresses this by redesigning attention around a synthetic, topology-aware index computed over the domain embedding space. Beyond accuracy and controllability, this substitution is also motivated by systems constraints: the approach reduces dependence on storing key/value histories for long contexts and enables more memory-efficient inference, opening space for higher-dimensional heads and richer internal representations under fixed hardware budgets. The Graph Laplacian is usually a DxD matrix where D is the number of dimensions that for the test dataset are defined by current SOTA embeddings models to 384. So basically a constant 384x384 sparse matrix ( $\text{nnz} \sim 10^4$  for a realistic sparsity of 1-8%) allows limiting the computing cost for the search of  $Q, K, V$  in the attention mechanism. The Graph Laplacian can be seen as a distillation of the domain knowledge and its generation takes a cheap pre-training step that for the test dataset is

$\sim 300$  seconds on a common laptop hardware for a dataset of  $3 * 10^5$  vectors on 384 dimensions.

The code for the GPT implementation is available at [4]. The code used to test and collect benchmark for the KV-cache is available at [5].

## 3 Memory Model and Algorithm

### 3.1 Generics

By leveraging the Graph Laplacian matrix ( $gl$ ) built on the vector space defined by the embeddings for a given domain, it is possible to define a synthetic score ( $\lambda\tau$ , lambda-tau or `taumode`) that can be used to spot approximate nearest neighbours of a given vector ( $x$ ) as:  $\lambda\tau = \text{taumode}(x, gl)$ . In the Tauformer architecture this distance metrics has been applied to the process of computing query, keys, value vectors ( $Q, K, V$ ) in the attention mechanism; allowing the substitution of  $Q^T K$  with  $-|\lambda_q - \lambda_k|$ . This also enable saving a relevant percentage of the memory space used by the KV-cache and opens to having vectors and attention heads with larger number of dimensions because part of the computing used by the inner-product operation can be reused to compute on higher dimensional vectors. These improvements sound already a nice step forward but, from knowledge engineering perspective, they are side-effects of the main concept brought forward by the implementation overall: *to develop a delivery system to bring distilled domain knowledge from the vector space (representing the knowledge graph, graph of citations, or any relations-representative structure) directly into the attention mechanism*. This to test the hypothesis that token generation can be made more context-relevant if domain-specific metadata is provided at the attention level. If this is made possible a new class of Transformers that are more or less narrowly scoped for context-specific generation in a given domain can be developed.

### 3.2 Tau Attention

In the process of causal attention using the *softmax* function, attention is driven by a transcendental normalisation over dot-product logits, where each new query must form inner-products against **all cached keys** (length  $T$ ) in head dimension  $D$ , i.e., the decode-time kernel cost scales as  $O(B H T D)$  for the  $QK^\top$  logits plus another  $O(B H T D)$  to aggregate values, with an additional  $O(B H T)$  for masking and the softmax itself. Where the letters are:

- **B**: Batch size (number of sequences processed in parallel).
- **H**: Number of attention heads.
- **T**: Sequence length / number of time steps (tokens) in the context window.
- **D**: Head dimension (the per-head vector size, typically  $D = C/H$  where  $C$  is the model embedding width).

In practice this also forces the KV-cache to store both  $K$  and  $V$  tensors, i.e.,  $2 \cdot B H_{kv} T D$  floats per layer, because the dot products cannot be reconstructed without the full key vectors. As context length grows, this "match against all previous keys and renormalise" pattern dominates both compute and memory, since every step repeats the same  $D$ -dimensional comparisons against an ever-growing set of cached vectors. Obviously this can become a problem for high  $D$ .

Tauformer's *taumode* mechanism replaces the dot-product kernel with a scalar spectral signature per head vector: for each query/key  $x \in \mathbb{R}^D$ , it computes a bounded Rayleigh quotient energy  $x^\top Lx / (x^\top x)$  (optionally blended with an item/edge dispersion statistic), producing a single  $\lambda$ -like score per token per head. Attention logits are then built by comparing the query's  $\lambda_q$  to **previously stored**  $\lambda_k$  values for cached keys:

$$a_{ij} = -|\lambda_{q,i} - \lambda_{k,j}|/\text{temp}$$

after which the causal mask and  $V$ -weighted sum are reused unchanged. This changes what must be cached: tauformer stores  $V$  and the **scalar**  $\lambda_k$  history rather than full  $K$ , reducing cache size from  $2B H_{kv} T D$  floats to  $B H_{kv} T D + B H_{kv} T$  floats (about 50% savings for typical  $D$ ). Computing shifts accordingly, instead of spending  $O(B H T D)$  on dot-products at each decode step, tauformer pays  $O(B H \text{nnz}(L))$  to compute the query's Rayleigh term with a sparse Laplacian and only  $O(B H T)$  to compare scalars against cached  $\lambda_k$ , making the scoring step dimension-light while keeping value aggregation  $O(B H T D)$  the same (equivalence of results compared to `nanoGPT` are available in the implementation's unit tests).

The key improvement is that the Graph Laplacian is designed to be a sparse matrix, so optimising the code for a sparse representation the  $O(D)$  terms become  $O(nnz)$  with  $nnz$  being the number of non-zero elements in the matrix.

Further computing advantages are discussed in 4.

### 3.3 Built-in Memory

Beside the additional potential computing advantages that are analysed below, the Tauformer and relative `tauGPT` concept allow the kind of persistent memory described in 3.1; **delivering a persistent (long-term, unchanging in this initial design) memory within the attention mechanism**. This follows current research for supplementing LLMs' memory with memory subsystems: for example as seen in [6] where a neural long-term memory module for 2M+ token contexts works in parallel and provides prefix sub-vectors to the context window; or in [7] that categorises memory into sensory, short-term and long-term within agentic workflows; or in [8] that uses a "Memory Bank" to store and retrieve long-form text for in-context learning. In addition, the papers that try to supplement LLMs with agentic workflows: for example [9] that evaluates "Agentic Memory" across multi-hop and temporal reasoning tasks. For the purpose of this paper I call the running attention mechanism the "momentary memory" and the Graph Laplacian against which the distance scores are computed "persistent memory". This aligns with the scope of my current research for building an "AI Memory Layer" based on topological vector search or, more briefly, "Memory Models" (MM) [12].

Tauformer/`tauGPT` is the only, at my current knowledge, Transformer architecture that delivers domain knowledge directly in the attention mechanism; leveraging the compression made possible by the *arrowspace* library and the *taumode* synthetic index. While technical and conceptual advantages are brought forward by this paper, further research is necessary to ascertain the improved accuracy in the training and token generation on very large context windows and the improved capability in avoiding deadlocks sooner than at reasoning time. As a follow-up version of this architecture, it is also possible to imagine defining an updating mechanism for the permanent memory that would make it into a long- or medium-term memory that can be adapted to the newly generated tokens.

### 3.4 Code

The first version of Tauformer/`tauGPT` [4] has been implemented using the Rust programming language [14] and the Burn Deep-Learning framework [15] as they provide fast, structured and type-safe development cycles with mature production ecosystem. Being a concept architecture this code is obviously not meant for production use at the publication of this paper.

**No inner-product** `tauGPT`'s most peculiar change is that it swaps dot-product attention for lambda-distance attention, where each token/head vector is compressed into a single scalar  $\lambda$  derived from a Laplacian energy, so that attention logits become  $-\lvert\Delta\lambda\rvert/T$ . The core of compression is in `lambdas_from_heads(...)`, which flattens  $[B, H, T, D]$  into  $[N, D]$ , computes  $xL$  via a matmul, then forms  $E_{\text{raw}} = (x^\top Lx)/(x^\top x + \epsilon)$  and bounds it as  $e_{\text{raw}}/(e_{\text{raw}} + \text{tau})$ . The scalar  $\lambda$  is

then broadcast into a full attention matrix using `taumode_distance_logits(...)`, which literally builds logits as  $-(lq - lk).abs() / \text{temp}$  after reshaping into  $[B, H, Tq, 1]$  and  $[B, H, 1, Tk]$ .

```

1  // taumode.rs
2  pub fn lambdas_from_heads<B: Backend>(
3      x: Tensor<B, 4>,
4      lap: Param<Tensor<B, 2>>,
5      cfg: &TauModeConfig,
6  ) -> Tensor<B, 3> {
7      let [b, h, t, d] = x.dims();
8
9      // Flatten [B, H, T, D] -> [N, D]
10     let n = b * h * t;
11     let x_nd = x.reshape([n, d]);
12
13    // y = x L -> [N, D]
14    let y_nd = x_nd.clone().matmul(lap.val());
15
16    // numerator = sum_i x_i * (xL)_i
17    let numerator = (x_nd.clone() * y_nd).sum_dim(1); // [N]
18
19    // denominator = sum_i x_i^2 + eps
20    let denom = x_nd.powf_scalar(2.0).sum_dim(1) + cfg.eps; // [N]
21
22    let e_raw = numerator / denom; // [N]
23    let e_bounded = e_raw.clone() / (e_raw + cfg.tau); // [N]
24
25    e_bounded.reshape([b, h, t])
26 }
27
28 pub fn taumode_distance_logits<B: Backend>(
29     lambda_q: Tensor<B, 3>,
30     lambda_k: Tensor<B, 3>,
31     cfg: &TauModeConfig,
32 ) -> Tensor<B, 4> {
33     let lq = lambda_q.unsqueeze_dim::<4>(3); // [B, H, Tq, 1]
34     let lk = lambda_k.unsqueeze_dim::<4>(2); // [B, H, 1, Tk]
35     let temp = cfg.temperature.max(cfg.eps);
36     -((lq - lk).abs() / temp)
37 }
```

**Sparse Laplacian** The other distinctive design choice is that `TauModeAttention` is built to operate with either a dense "toy" Laplacian or, as intended by design, a sparse Laplacian loaded from a manifold (a `.parquet` file with the computed Laplacian). The code explicitly switches between those representations at runtime. In `TauModeAttention`, you can see the dual storage for the sparse matrix and the dense tensor. All tests are run using the sparse matrix from a pre-trained `arrowspace` (manifold file and embeddings for the test queries available in [5]).

```

1  // tauattention.rs (struct fields + sparse/dense selection)
2  pub struct TauModeAttention<B: Backend> {
3      // ...
4      laplacian_tensor: Option<Param<Tensor<B, 2>>>, // using dense
5      laplacian_matrix: Ignored<Option<CsMat<f64>>>, // using sparse
6      pub(crate) tau_mode: Ignored<Option<TauMode>>,
7      // ...
8  }
9
10 fn lambdas_from_heads_any(&self, heads: Tensor<B, 4>) -> Tensor<B, 3> {
11     let tau_cfg = self.get_tau_config();
12     if let Some(lap) = self.laplacian_matrix.0.as_ref() {
```

```

13     let mode =
14         self.tau_mode.0.unwrap_or(crate::pretraining::parquet::TauMode::Median);
15     crate::taumode::lambdas_from_heads_sparse::<B>(heads, lap, mode,
16         tau_cfg.eps)
17 } else {
18     let lap = self.get_laplacian_tensor().clone();
19     crate::taumode::lambdas_from_heads::<B>(heads, lap, &tau_cfg)
}
}

```

**KV-caching layout** KV-cache in tauGPT is also different compared to standard GPT: instead of caching  $K$  and  $V$ , each layer caches  $(V, \lambda_k)$  only, which matches the fact that scoring uses lambda scalars rather than key vectors. The cache type is defined as *pub type TauCacheLayer< $B$ > = Option<(Tensor< $B$ , 4>, Tensor< $B$ , 3>)>;* and tauGPT wraps a vector of these per layer in *TauKVCache store: Vec<TauCacheLayer< $B$ >, position: usize*.

```

1 // tauattention.rs (KV-cache payload + append logic)
2 pub type TauCacheLayer<B> = Option<(Tensor<B, 4>, Tensor<B, 3>)>;
3
4 let lambda_k_new = self.lambdas_from_heads_any(k_new); // [B, Hkv, 1]
5
6 // Cache management
7 let (v_full, lambda_k_full) = match cache_layer.take() {
8     Some((v_all, lk_all)) => (
9         Tensor::cat(vec![v_all, v_new.clone()], 2), // time axis
10        Tensor::cat(vec![lk_all, lambda_k_new.clone()], 2), // time axis
11    ),
12    None => (v_new.clone(), lambda_k_new.clone()),
13};
14
15 *cache_layer = Some((v_full.clone(), lambda_k_full.clone()));
16 let tk = v_full.dims()[2];
17 let y = self.scaled_tau_attention_decode(q, lambda_k_full, v_full, 1, tk);

```

Inside `TauModeAttention::forward_decode(...)`, the cache logic appends along the time axis and then runs attention against the full cached history via `scaled_tau_attention_decode(q, lambda_k_full, v_full, 1, tk)`.

```

1 // taugpt.rs (model-level decode uses cache.position for RoPE step
2 // slicing)
3 pub fn forward_decode(
4     &self,
5     last_ids: Tensor<B, 2, Int>, // [B, 1]
6     cache: &mut TauKVCache<B>,
7     use_softcap: bool,
8 ) -> Tensor<B, 3> {
9     let tpos = cache.position;
10    let d2 = self.cos.dims()[3];
11
12    // Slice RoPE for the current absolute position: [1, 1, 1, D/2]
13    let cos_step = self.cos.clone().slice([0..1, tpos..tpos + 1, 0..1,
14        0..d2]);
15    let sin_step = self.sin.clone().slice([0..1, tpos..tpos + 1, 0..1,
16        0..d2]);
17
18    for (i, block) in self.blocks.iter().enumerate() {
19        let layer_cache = &mut cache.store[i];
20        x = block.forward_decode(x, (&cos_step, &sin_step), layer_cache);
21    }
22}

```

```

20     logits
21 }
```

## 4 Results

Multiple tests have been run and results have been collected using [5].

Here some data and diagrams from the tests results. All this data was collected running tests on CPU hardware because the objective of this concept architecture's test results are preliminary and to be taken as a reference and a baseline, to demonstrate that Tauformer/tauGPT can work as any other Transformer but with delivering of domain-specific (topological) metadata directly in the attention mechanism and with some performance gains for larger windows and higher dimensional embeddings.

Considering generated tokens ( $0 \leq \text{token\_index} \leq \text{gen\_tokens}$ ), the results contains 500000 decoded-token latency rows spanning both engines ("nano", "tau"), both modes (`kv_cache`, `no_cache`), and `gen_tokens` (context window)  $\in 128, 256, 512, 1024$ . Per-token comparisons were computed by matching "nano" and "tau" tokens on `prompt_id`, `mode`, `gen_tokens`, `token_index` and then taking the ratio,

The full normalised token-latency results confirms that tauGPT is consistently faster in "kv-cache" mode (a proxy for inference decoding), and consistently slower in "no cache" (training/uncached proxy) for small context windows (up to 1024 tokens); with the gap widening in tauGPT's favour as generation length grows in "kv-cache" while the gap in "no cache" mode gets smaller (see 2). The difference in "no cache" between the two gets smaller as the lengths grow so there is room to make the hypothesis that at some threshold where context window and embeddings lengths are large enough tauGPT will be at least as fast as nanoGPT in training while being increasingly faster at inference time. Also to consider again that nanoGPT is a relatively much more mature implementation. This could confirm that Tauformer could be viable also for training on multimodal data where multiple modalities are concatenated in the same embedding space while keeps being faster and more efficient (-50% memory for KV-cache) at inference. The assumption that gains are provided as the context window and the length of the embeddings grows is for now supported only bu regressions run on 0.5M decoded-token rows across 7 runs ("variants" in the test jargon) of tests with different lengths.

See table 1 for summary statistics (repository [5] may contain more up-to-date data in the report/ directory).

### 4.1 Inference proxy (`kv_cache`) results

On a matched per-token basis, nano/tau speedup medians (p50) increase with `gen_tokens`:  $1.09 \times (128)$ ,  $1.12 \times (256)$ ,  $1.17 \times (512)$ , and  $1.22 \times (1024)$ , meaning tau's per-token latency advantage strengthens as sequences get longer. Tail behaviour also improves in tau's favour as sequences grow: the p95 of nano/tau per-token speedup rises from  $1.35 \times (128)$  to  $1.52 \times (1024)$ , suggesting tau reduces worse-case token stalls relative to nano at longer generations in `kv_cache` mode. Looking specifically at the last 25% of tokens (75–100% bins), the median-latency ratio nano/tau reaches  $1.33 \times$  at `gen_tokens=1024`, indicating the end of long generations is where tau helps most under `kv_cache`.

### 4.2 Training/uncached proxy (`no_cache`) results

For `no_cache`, matched per-token speedups are  $\leq 1$  across all lengths (tau slower): median nano-/tau speedup is  $0.75 \times (128)$ ,  $0.75 \times (256)$ ,  $0.76 \times (512)$ , and  $0.80 \times (1024)$ . The late-token-bin summary also stays  $\leq 1$  for `no_cache` (e.g., last-25% median ratio is  $\sim 0.86$  at 1024), but the ratio

Table 1: Benchmark Summary Statistics

<b>Throughput (tokens/sec) – Higher is Better</b>					<b>Speedup: Tau/Nano (&gt; 1.0 = Tau faster)</b>	
<b>Engine</b>	<b>Mode</b>	<b>Mean</b>	<b>Min</b>	<b>Max</b>	<b>Metric</b>	<b>Factor</b>
nano	kv_cache	37.47	13.91	59.96	Tokens/sec	1.068×
nano	no_cache	4.55	0.89	9.71	TTFT	0.997×
tau	kv_cache	49.39	15.04	71.21	p50 latency	1.066×
tau	no_cache	3.43	0.81	6.91	p95 latency	1.104×
					Decode time	1.067×
					Total time	1.028×

<b>Per-Token Latency (ms) – Lower is Better</b>					<b>Cache Gain (&gt; 1.0 = Cache helps)</b>	
<b>Engine</b>	<b>Mode</b>	<b>Mean</b>	<b>p50</b>	<b>p95</b>	<b>Metric</b>	<b>Factor</b>
nano	kv_cache	36.25	36.26	41.35	Tokens/sec	16.26×
nano	no_cache	327.58	295.88	550.58	TTFT	9.22×
tau	kv_cache	30.44	30.42	33.13	p50 latency	15.13×
tau	no_cache	390.04	359.90	636.52	p95 latency	22.67×
					Decode time	16.25×
					Total time	10.93×

moves upward with length, which suggests tauGPT’s relative penalty shrinks at longer sequences even without caching. In the `no_cache` distribution, even the p99 nano/tau speedup stays below 1 ( $\simeq 0.95 \sim 0.97$ ), meaning tauGPT is slower almost everywhere token-by-token in this mode rather than losing only in a small tail, the situation improves when the window and embeddings lengths grow.

### 4.3 Matched per-token speedup distribution (nano/tau)

This is the test matrix and the relative results:

<b>mode</b>	<b>gen_tokens</b>	<b>matched token pairs</b>	<b>speedup</b>	<b>p50</b>	<b>speedup p95</b>	<b>speedup p99</b>
kv_cache	128	96,768	1.093	1.346	1.437	
kv_cache	256	165,888	1.124	1.395	1.494	
kv_cache	512	62,208	1.166	1.443	1.557	
kv_cache	1024	359,424	1.223	1.523	1.658	
no_cache	128	96,768	0.747	0.889	0.965	
no_cache	256	165,888	0.749	0.898	0.968	
no_cache	512	62,208	0.762	0.901	0.954	
no_cache	1024	359,424	0.805	0.924	0.965	

Table 2: Matched per-token speedup distribution (nano/tau) across cache modes and sequence lengths.

This is a diagram that compares the gains as the context windows grow up to  $10^9$ , this is only indicative as for nanoGPT is running on CPU and tauGPT is unoptimised and a concept model but I think it can provides hints to frame the orders of magnitude at which Tauformer can become performant also in `no_cache` mode:

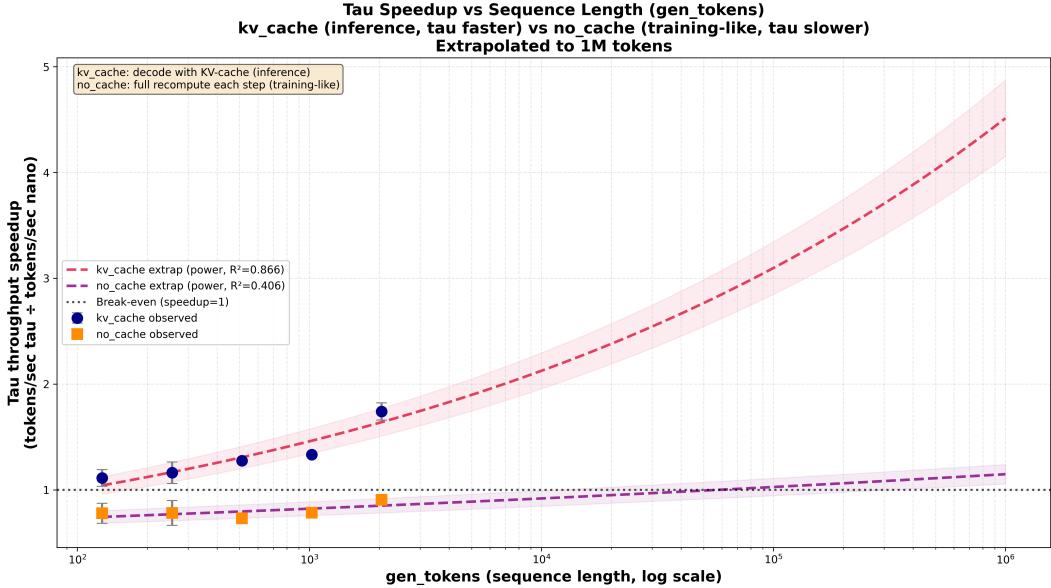


Figure 1: Regressions for tau/nano speedups as the context window grows

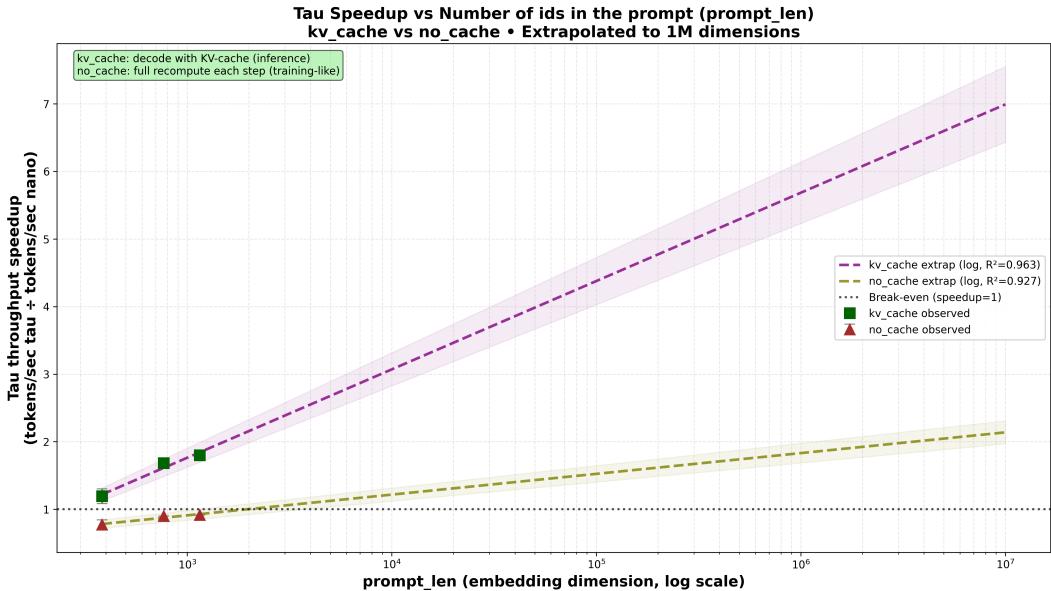


Figure 2: Regressions for tau/nano speedups as the prompt length grows

At current time I am running the same kind of tests but with the embeddings dimensions (`prompt_len`) growing to the same order of magnitude to simulate concatenated multimodal embeddings. The hypothesis is that similar advantages as above could be demonstrated.

## 5 Conclusion: A Beginning

Potential for larger improvements are tested in larger windows generation tests (256, 512, 1024, ...,  $10^6$  tokens). The driving hypothesis is that when caching becomes too expensive memory-wise (like for very long windows with high-dimensional embeddings), tauGPT can provide a better solution both in terms of contextual accuracy and performance. Current research is exploring at the order of magnitude of  $10^4$  window length (average for LLMs is 6000 to 14000) with values for embeddings dimensions in the same order of magnitude. The idea is to make it possible to have a

prompt length (embeddings length) that is of the same order of magnitude of number of non-zero values of the sparse representation in the pre-trained manifold, so that the model can adapt to the number of dimensions of the manifold processing an element in a single query ( $Q$ ).

The normalised token latencies indicate tau's current value proposition is "decode-heavy inference on long windows and high dimensions", with KV-cache enabled, because both median and tail per-token improvements increase with `gen_tokens`. By contrast, if training work is well proxied by `no_cache`, tauGPT currently underperforms across essentially the entire per-token distribution; so demonstrating training wins require architectural/implementation changes (or a true forward-backward benchmark rather than `no_cache`), or for the model to be tested on very long context windows and very high embeddings dimensions. The next experiments that would most strengthen Tauformer position are *(i)* sweeping `prompt_len` (prefill/context length) separately from `gen_tokens` up to lengths of 4000-10000 and *(ii)* pushing `prompt_len` in the same order of magnitude for longer windows. This is suggested by the per-token advantage trend in `kv_cache` that is still climbing at 1024 and the `no_cache` mode improving as `gen_tokens` grows.

These results demonstrate the promising characteristics of a Topological Transformer architecture, Tauformer; designed to deliver

- (i)* domain-specific metadata and
- (ii)* provide topological search (using `arrowspace`'s `taumode`)

at the level of the attention mechanism. To synthesise in few words, Tauformer leverages `taumode` as attention score, allowing for a limited need of inner-products in the attention computation, potentially making possible very large context windows and to run attention on higher dimensional (multimodal) embeddings.

## 6 Acknowledgments

The author is an independent researcher who self-funded this work. All works available at [his research page](#). Thanks to my supporting network of peers and to all the backers and sponsors to my research effort.

## References

- [1] Vaswani, Ashish and Shazeer, Noam and Parmar, Niki and Uszkoreit, Jakob and Jones, Llion and Gomez, Aidan N and Kaiser, Lukasz and Polosukhin, Illia *Attention is All You Need*, Advances in Neural Information Processing Systems (NeurIPS), volume 30, 2017
- [2] Lorenzo Moriondo, *ArrowSpace: introducing Spectral Indexing for vector search*, 2025. <https://github.com/tuned-org-uk/arrowspace-rs> DOI: 10.21105/joss.09002
- [3] Andrej Karpathy *nanoGPT: The simplest, fastest repository for training open-source GPT models*. <https://github.com/karpathy/nanoGPT>
- [4] Lorenzo Moriondo (@Mec-iS)      Tauformer . <https://github.com/tuned-org-uk/taugpt-kvcache-bench>
- [5] Lorenzo Moriondo (@Mec-iS)      KV-cache bench for Tauformer. <https://github.com/tuned-org-uk/taugpt-kvcache-bench>
- [6] Ali Behrouz, Peilin Zhong, and Vahab Mirrokni. Titans: Learning to Memorize at Test Time. *arXiv preprint arXiv:2501.00663*, 2025.
- [7] Lianlei Shan, Shixian Luo, Zezhou Zhu, Yu Yuan, Yong Wu Cognitive Memory in Large Language Models. *arXiv preprint arXiv:2504.02441*, 2025.
- [8] Weizhi Wang, Li Dong, Hao Cheng, Xiaodong Liu, Xifeng Yan, Jianfeng Gao, Furu Wei Augmenting Language Models with Long-Term Memory. *arXiv preprint arXiv:2306.07174*, 2024 (Updated).
- [9] Wujiang Xu, Zujie Liang, Kai Mei, Hang Gao, Juntao Tan, Yongfeng Zhang A-MEM: Agentic Memory for LLM Agents. *arXiv preprint arXiv:2502.12110*, 2025.
- [10] Kexin Wang, Nils Reimers, and Iryna Gurevych. 2021. TSDAE: Using Transformer-based Sequential Denoising Auto-Encoder for Unsupervised Sentence Embedding Learning. In *Findings of the Association for Computational Linguistics: EMNLP 2021*, pages 671–688, Punta Cana, Dominican Republic. Association for Computational Linguistics.
- [11] Félix Lefebvre and Gaël Varoquaux. 2025. SEPAL: Scalable Embedding Propagation ALgorithm for large knowledge graphs. In *Proceedings of the 13th International Conference on Learning Representations (ICLR 2025)*, Singapore.
- [12] Blog posts at tuned.org.uk. Tuned Blog. [www.tuned.org.uk/blog](http://www.tuned.org.uk/blog), 2025. Accessed: December 29, 2025.
- [13] J. W. Strutt (Lord Rayleigh), *The Theory of Sound*, Vol. 1, London: Macmillan and Co., 1877.
- [14] The Rust Project Developers, *The Rust Programming Language*, 2024. [Online]. Available: [www.rust-lang.org](http://www.rust-lang.org)
- [15] Tracel-AI, *Burn: A Deep Learning Framework Designed from Engineers' Perspectives*, 2025. [Online]. Available: <https://github.com/tracel-ai/burn>