

## Exercise – Understanding the consistency of the OLS estimator using simulation

### 1. Aim of the exercise

To understand the consistency of the OLS estimator.

### 2. Theory

The consistency of the OLS estimator means that as the sample size increases, the estimator converges to the true parameter value. In other words, with a sufficiently large sample, the OLS estimator provides an estimate that is very close to the actual population parameter. This concept of consistency is underpinned by the notion of the probability limit. To better understand this, review the simulation exercise on the law of large numbers, which illustrates the concept of the probability limit.

### 3. Application

#### 3.1. Clear the memory

Set the number of simulations to be carried out.

```
8 % 3.1. Clear the memory
9 clear;
```

#### 3.2. Set the number of simulations

Set the number of simulations to be carried out.

```
11 % 3.2. Set the number of simulations
12 N_sim = 1000;
```

#### 3.3. Set the sample size

Assume that we have a linear regression model that contains a constant term and an independent variable. Assume also that we have `N_obs` observations for the variables of this model.

```
14 % 3.3. Set the sample size
15 N_obs = [1000 10000 100000];
16 N_obs_j = size(N_obs,2);
```

#### 3.4. Set true values for the coefficients of the intercept and the independent variable

Assume that we know the true values of the coefficients of the variables of the linear regression model we consider, and that these values are as indicated at the end of the section.

```
19 % 3.4. Set true values for the coefficients
20 B_true = [0.2 0.5]';
```

### 3.5. Define the number of coefficients to be simulated

Define the number of coefficients to be simulated

```
22 % 3.5. Define the number of coefficients to be simulated
23 N_par = 1;
```

### 3.6. Preallocate matrices for storing the simulated OLS coefficient estimates

The code presented at the end of the section creates an empty vector, and an empty matrix. The empty vector is  $N\_sim \times N\_par$  because the vector is to store  $N\_sim$  coefficient estimates of the only independent variable  $x_1$  from a given simulation using a certain number of observations (sample size). The empty matrix is to store in each of its columns the  $N\_sim$  simulated coefficient estimates from three different scenarios featuring different numbers of observations. Therefore the matrix is  $N\_sim \times N\_obs\_j$ , where  $N\_obs\_j$  is the number of different scenarios of numbers of observations.

```
25 % 3.6. Preallocate matrices for storing the simulated statistics
26 B_hat_1_sim = NaN(N_sim,N_par);
27 B_hat_1_sim_j = NaN(N_sim,N_obs_j);
```

### 3.7. Create sampling distributions for the OLS coefficient estimates using different sample sizes

The for loop presented at the end of the section creates three different sampling distributions for the OLS coefficient estimates using three different sample sizes. A note for avoiding a computational hurdle is the following. The presented for loop makes use of the user-written function `exercisefunctionlss`. The function calculates a set of OLS statistics. However, the for loop used here only needs the OLS coefficient estimates, and therefore other OLS statistics need not be calculated. To avoid an unnecessary waiting time for the for loop to finish its iterations, go to the function file, and mask the code except the part of it calculating the OLS coefficient estimates. You can mask the code by selecting the code, and then by pressing the 'Comment' button located on the toolbar of the Editor of your open script file.

```
29 % 3.7. Create sampling distributions for the OLS estimator
30 for j = 1:N_obs_j
31     for i = 1:N_sim
32         u = random('Normal',0,1,[N_obs(1,j) 1]);
33         x_0 = ones(N_obs(1,j),1);
34         x_1 = random('Uniform',-1,1,[N_obs(1,j) 1]);
35         X = [x_0 x_1];
36         y = X*B_true+u;
37         LSS = exercisefunctionlss(y,X);
38         B_hat_1_sim(i,1) = LSS.B_hat(2,1);
39         B_hat_1_sim_j(:,j) = B_hat_1_sim(:,1);
40     end
41 end
```

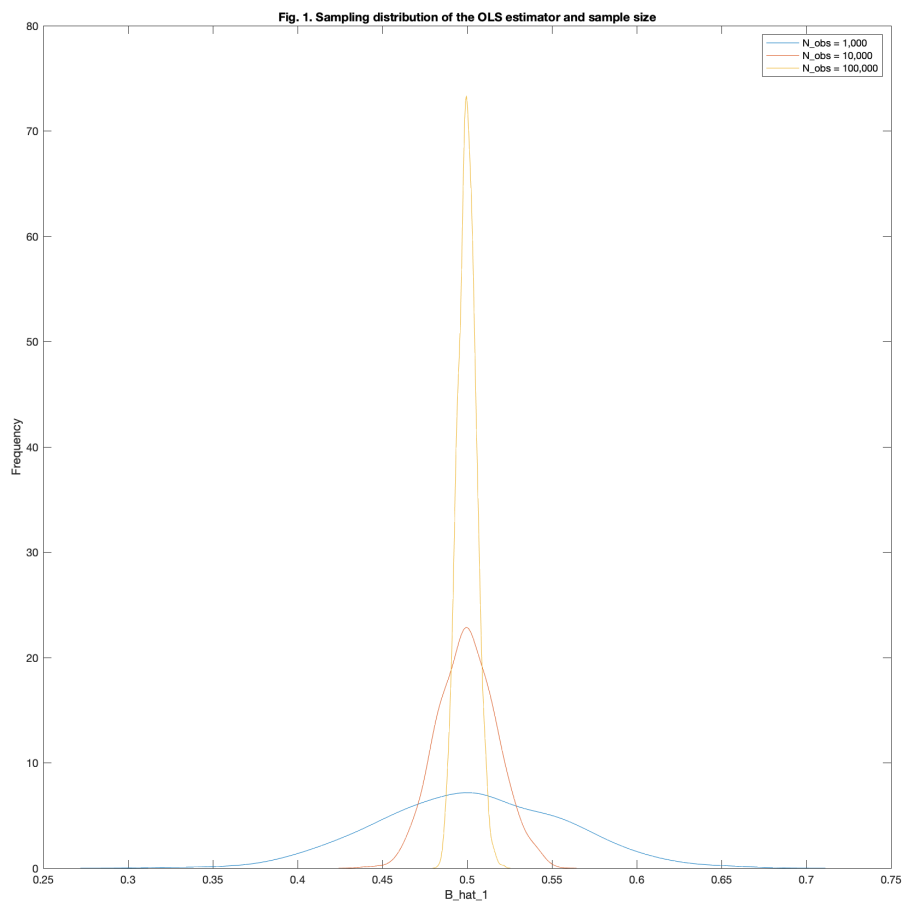
## 4. Plot the sampling distributions of the OLS coefficient estimates at different sample sizes

Here we plot three different sampling distributions featuring three different sample sizes. It demonstrates the consistency property of the OLS estimator. Note that this exercise shows consistency of the estimator without doing any theoretical derivation such as taking the probability limit of the estimator. Indeed, when it is not possible to theoretically prove the consistency, or some other property, of an estimator, one can rely on a simulation study, as done here, to investigate the properties of the estimator of interest.

```

43 %% 4. Plot the sampling distributions of the OLS estimator
44 ksdensity(B_hat_1_sim_j(:,1))
45 hold on
46 ksdensity(B_hat_1_sim_j(:,2))
47 hold on
48 ksdensity(B_hat_1_sim_j(:,3))
49 title(['Fig. 1. Sampling distribution of the OLS estimator ' ...
50       'and sample size'])
51 legend('N\_obs = 1,000', 'N\_obs = 10,000', 'N\_obs = 100,000')
52 ylabel('Frequency')
53 xlabel('B\_hat\_1')

```



## 5. Final notes

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