The OLS method for estimating the parameters of the regression model

Econometrics for minor Finance, Lecture 3

Tunga Kantarcı, Fall 2025

OLS as an estimation method: Population parameter to estimate

Consider the population model:

$$y_i = \beta x_i + u_i$$

β is an unknown population parameter. It is a fixed number.

We want to estimate them given sample data. What method can we use for estimation?

OLS as an estimation method: The estimator

$$y_i = \beta x_i + u_i$$

Among several possible estimation methods, we focus on the most widely used: Ordinary Least Squares.

The idea is that the best estimate of β, is the one that makes

$$\beta x_i$$

as close as possible to

$$y_i$$

so that the error we make, u_i, is minimized.

Denote this estimate with $\hat{\beta}$.

OLS as an estimation method: The estimator

Consider the model with an intercept:

$$y_i = \beta_0 + \beta_1 x_i + u_i$$

The error term for observation i is

$$u_i = y_i - \beta_0 - \beta_1 x_i$$

For a sample of n observations, we want to minimize the sum of squared errors:

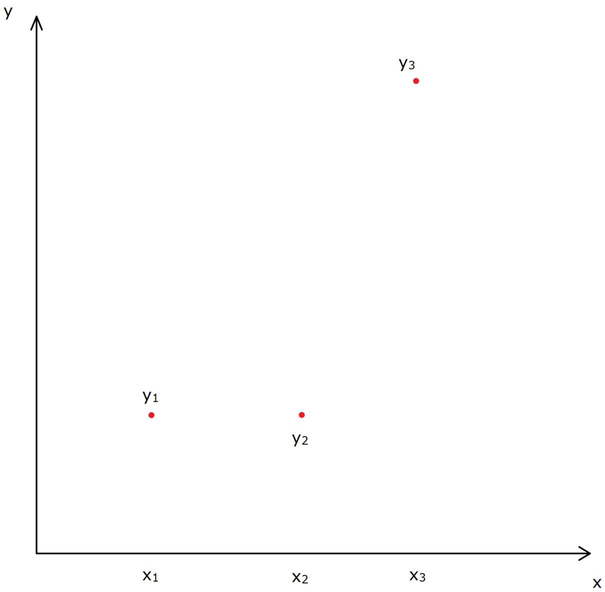
$$\sum_{i=1}^{n} \left(y_i - \beta_0 - \beta_1 x_i\right)^2$$

The values of β_0 and β_1 that minimize this criterion are denoted as

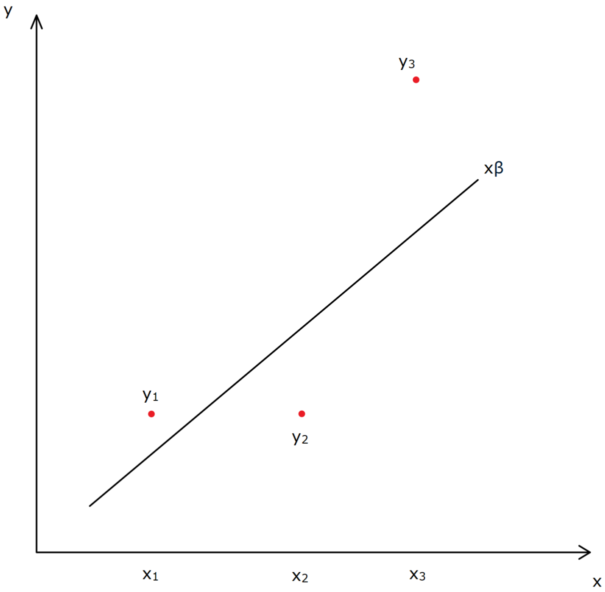
$$\hat{\beta}_0,\ \hat{\beta}_1$$

and called the OLS estimates.

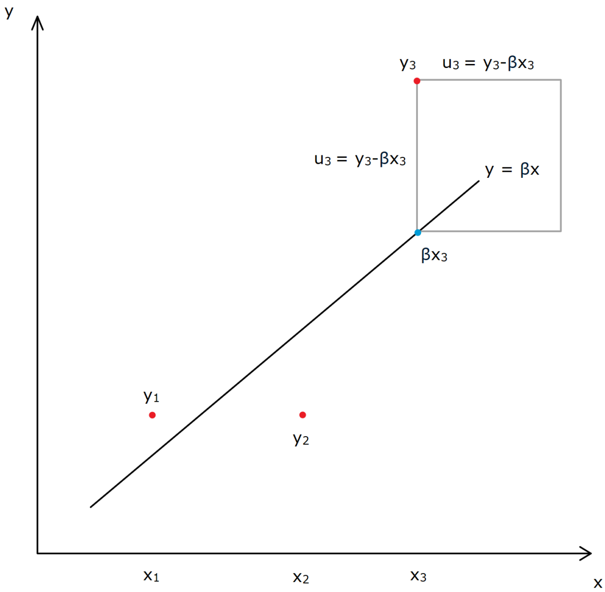
OLS as an estimation method: The estimator: Intuition



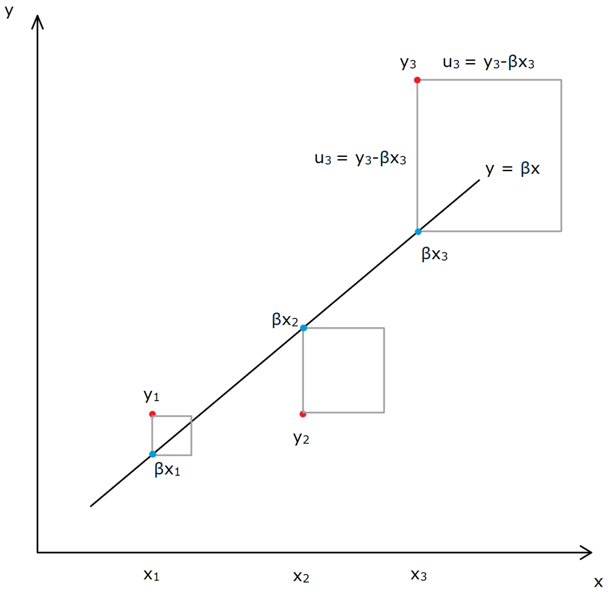
OLS as an estimation method: The estimator: Intuition



OLS as an estimation method: The estimator: Intuition



OLS as an estimation method: The estimator: Intuition



OLS as an estimation method: The estimator

By minimizing the sum of squared errors, OLS finds the

$$\hat{\beta}$$

that makes the regression line fit to the data points as good as possible.

OLS as an estimation method: The estimator

More formally, OLS chooses the parameters to solve

$$\hat{\beta}_0, \hat{\beta}_1 = \arg\min_{\beta_0,\beta_1} \sum_{i=1}^{n} \left(y_i - \beta_0 - \beta_1 x_i\right)^2$$

OLS as an estimation method: The estimator: Intuition

In the previous figure, imagine two possible lines, each corresponding to a different $\hat{\beta}$. Which line should we choose? The one that yields the smaller sum of squared deviations between the observations and their fitted values $x\hat{\beta}$. This is the essence of the OLS minimization problem: we search for the $\hat{\beta}$ that defines the line minimizing the total squared deviations.

OLS as an estimation method: The estimator

Using some algebra that we skip, the OLS estimator of β_0 is

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \, \bar{x}$$

and the OLS estimator of β_1 is

$$\hat{\beta}_1 = \frac{\displaystyle\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\displaystyle\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

OLS as an estimation method: Fitted value

The estimated population model becomes the fitted model:

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

It says: given the OLS estimates of population parameters β_0, β_1

$$\hat{\beta}_0, \hat{\beta}_1,$$

when

$$x = x_i,$$

our model predicts y_i as

$$\hat{y}_i$$

We call this the fitted value. It represents the part of y_i that our model is able to explain.

OLS as an estimation method: Residual

Our population model for observation i is

$$y_i = \beta_0 + \beta_1 x_i + u_i$$

and hence the error term is

$$u_i = y_i - (\beta_0 + \beta_1 x_i)$$

The estimate of the second term is the fitted value

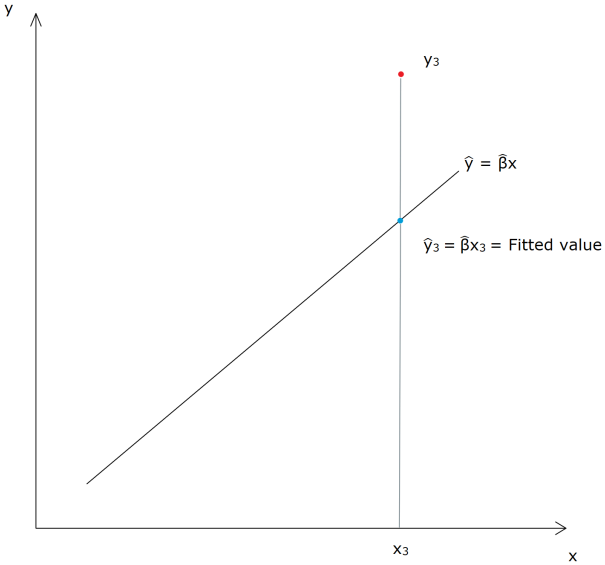
$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

Then, the difference between the observed and fitted value is

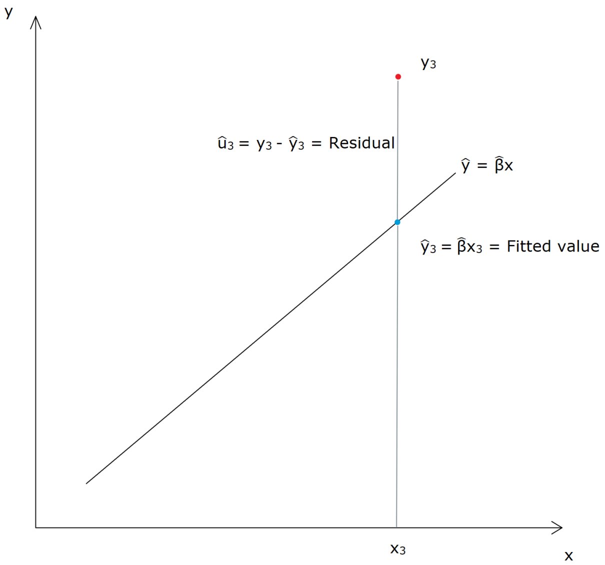
$$\hat{u}_i = y_i - \hat{y}_i$$

We call this the residual. It represents the part of y_i that the model does not explain.

OLS as an estimation method: Fitted value: Intuition



OLS as an estimation method: Residual: Intuition



OLS as an estimation method: Fitted value and residual: Intuition

So the solution to the least squares problem gives

$$y = \hat{y} + \hat{u}$$

It says that the observed outcome variable is fully explained by what our model can and cannot explain. It has to be. How else could it be?

OLS as an estimation method: The model and the method

OLS is an approximation, or estimation, method. It is not a model. There is no such thing as "OLS model", a term that appears even in good journals.

The model is the liner regression model.

We estimate the population, or slope, parameters of the regression model using the OLS method.

OLS as an estimation method: Algebraic properties of the OLS method

The solution of the OLS problem leads to three results. First, if the regression includes a constant, the residuals, or deviations from the regression line, sum to zero:

$$\sum_{i}^{n} \hat{u}_i = 0$$

Intuition: OLS chooses the line so that residuals cancel out in aggregate: there is no systematic tendency for them to be positive or negative.

OLS as an estimation method: Algebraic properties of the OLS method

Second,

$$y_i = \hat{\beta} x_i + \hat{u}_i$$

taking the average over i, we have

$$\bar{y} = \hat{\beta}\bar{x} + \bar{\hat{u}}$$

Since

$$\bar{\hat{u}} = 0$$

by the first result, we have

$$\bar{y} = \hat{\beta}\bar{x}$$

Intuition: OLS aligns the fitted line so that it goes exactly through the average values, ensuring the model captures the central tendency of the sample.

OLS as an estimation method: Algebraic properties of the OLS method

Third, we showed that

$$y_i = \hat{y}_i + \hat{u}_i$$

Taking the average over i, we have

$$\bar{y} = \bar{\hat{y}} + \bar{\hat{u}}$$

Since $\bar{\hat{u}} = 0$ by the first result, we obtain

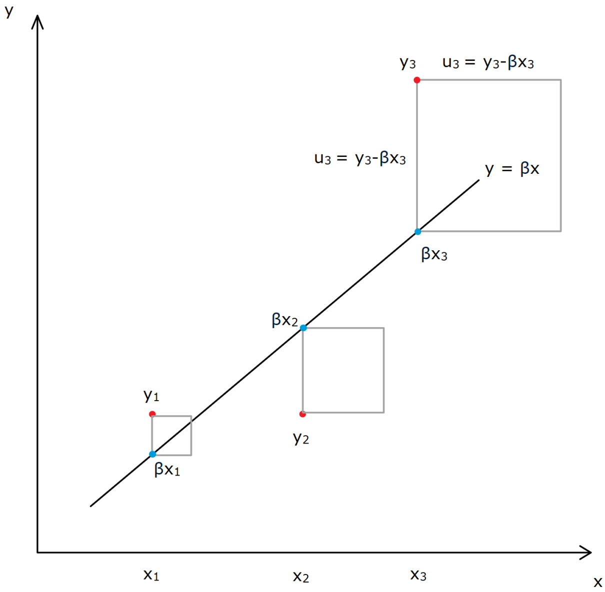
$$\bar{y} = \bar{\hat{y}}$$

Intuition: the average of the fitted values equals the average of the actual values, so OLS ensures the model reproduces the sample mean without bias.

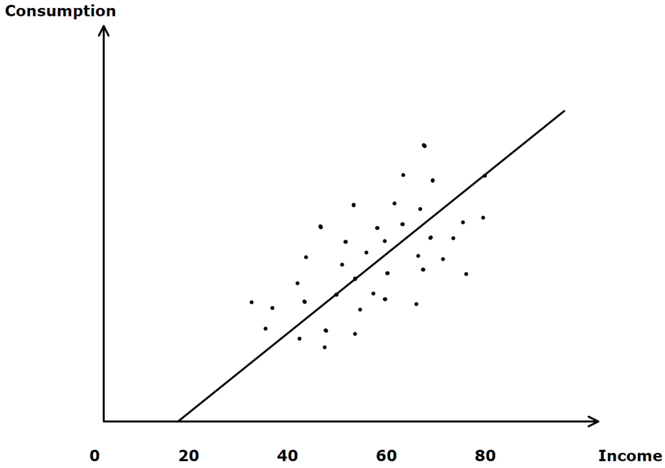
OLS as an estimation method: Insights: Outlier

The OLS estimator is not robust to outliers. y_3 contributes too much to the sum of squared errors, the minimization problem of OLS. It pulls the regression towards itself. A given observation is not supposed to dictate a contribution much larger than other observations do.

OLS as an estimation method: Insights: Outlier



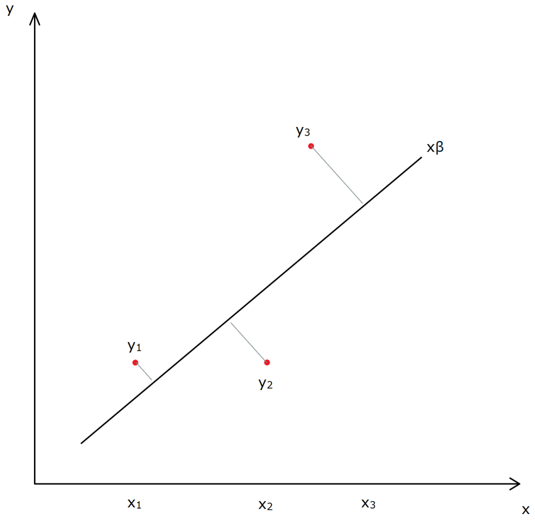
OLS as an estimation method: Insights: Models are approximations



OLS as an estimation method: Insights: Models are approximations

For incomes between 40 and 80, the consumption function can be approximated by the model. Does the model describe the consumption-income relationship for all incomes, or only for the those around the center? Only in the center. What is the predicted consumption when income is 10? A negative value. Models are approximations. Approximations do not work well if we move too far away from the point of approximation. OLS is a good approximation around the average value of x.

OLS as an estimation method: Insights: Why not minimize orthogonal deviations?



OLS as an estimation method: Insights: Why not minimize orthogonal deviations?

Why not minimize the sum of orthogonal deviations but squared deviations? Such a method leads to an estimator that does not have statistical properties as desirable.

Population regression function

Our population model for observation i is

$$y_i = \beta_0 + \beta_1 x_i + u_i$$

We do not want to work with the entire distribution of the random variable y_i, but a summary measure of it, the expected value. So take the expected value conditional on x_i, to obtain

$$\mathrm{E}(y_i \mid x_i) = \mathrm{E}(\beta_0 + \beta_1 x_i \mid x_i) + \mathrm{E}(u_i \mid x_i)$$

Under the exogeneity assumption, that is, $\mathrm{E}(u_i \mid x_i) = 0$, we obtain

$$\mathrm{E}(y_i \mid x_i) = \mathrm{E}(\beta_0 + \beta_1 x_i \mid x_i)$$

Using the con. exp. property 1, we obtain

$$\mathrm{E}(y_i \mid x_i) = \beta_0 + \beta_1 x_i$$

This is the population regression function.

Population regression function: Intuition

$$\mathrm{E}(y_i \mid x_i) = \beta_0 + \beta_1 x_i$$

On average, what outcome do we expect at a specific value of the explanatory variable?

Sample regression function

The population regression function

$$\mathsf{E}(y_i \mid x_i) = \beta_0 + \beta_1 x_i$$

is not known to us. Using the OLS estimates of the population parameters, we can estimate it as

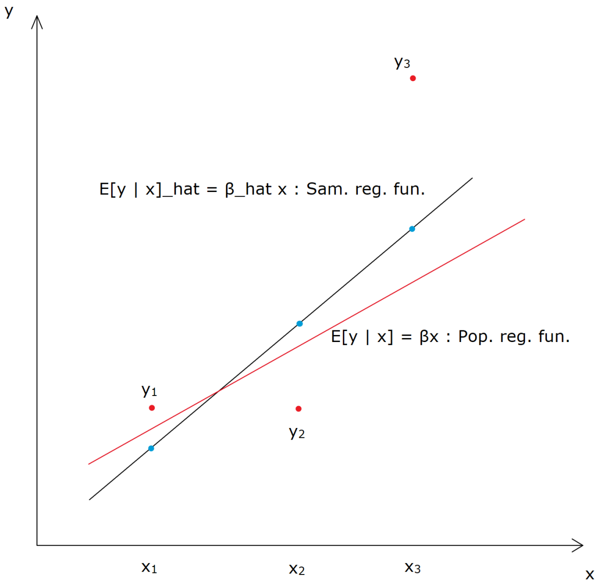
$$\begin{aligned} \widehat{\mathsf{E}(y_i \mid x_i)} &= \hat{\beta}_0 + \hat{\beta}_1 x_i \\ &= \hat{y}_i \end{aligned}$$

which is just the prediction we have learned.

$$\widehat{\mathsf{E}(y_i \mid x_i)} = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

is the sample regression function. It is the estimate of the population regression function.

Sample regression function



Sample regression function

Consider the sample regression function

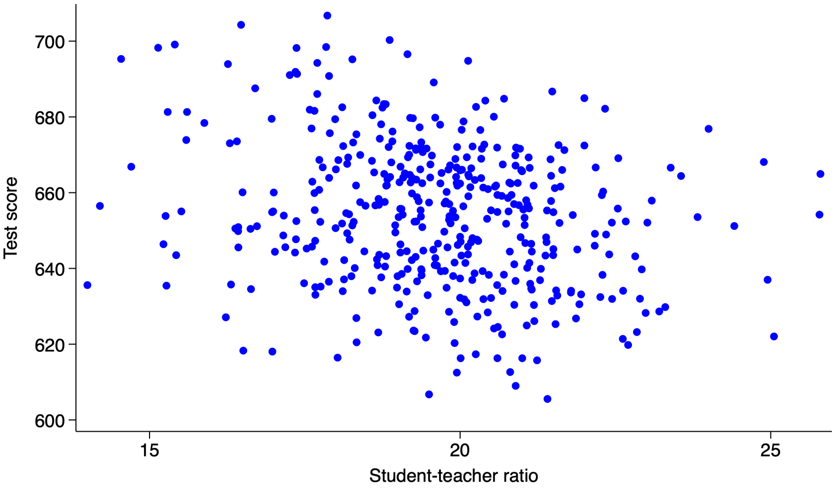
$$\widehat{\mathsf{E}[y \mid x]} = \hat{\beta}_0 + \hat{\beta}_1 x$$

The discrete change in the predicted dependent variable with respect to a discrete change in the independent variable is

$$\frac{\Delta \widehat{\mathsf{E}[y \mid x]}}{\Delta x} = \hat{\beta}_1$$

This shows that when the independent variable changes by some unit, the dependent variable changes on average by our OLS estimate.

Sample regression function: Example



Sample regression function: Example

Assume that the population regression model is

$$\mathrm{E}(testscr \mid str) = \beta_0 + \beta_1 str$$

We want to estimate this model. Given sample data, we can use the OLS method in Stata.

Sample regression function: Example

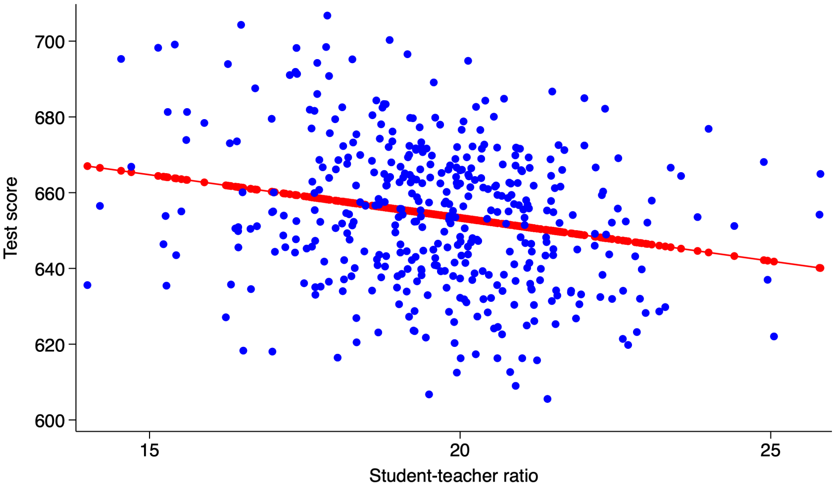
```
. regress testscr str

      Source |       SS           df       MS      Number of obs   =       420
-------------+----------------------------------   F(1, 418)       =     22.58
       Model |  7794.11012         1  7794.11012   Prob > F        =    0.0000
    Residual |  144315.484       418  345.252353   R-squared       =    0.0512
-------------+----------------------------------   Adj R-squared   =    0.0490
       Total |  152109.594       419  363.030056   Root MSE        =    18.581

------------------------------------------------------------------------------
     testscr | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         str |  -2.279808   .4798256    -4.75   0.000     -3.22298   -1.336637
       _cons |    698.933   9.467491    73.82   0.000     680.3231    717.5428
------------------------------------------------------------------------------
```

Sample regression function: Example

Our sample regression model, or the fitted model, becomes

$$\mathrm{E}(\widehat{testscr \mid str}) = 698.93 - 2.28 str$$

Sample regression function: Example



Sample regression function: Example

Interpretation of the estimated coefficient of the independent variable:

$$\frac{\Delta \widehat{\mathrm{E}(testscr \mid str)}}{\Delta str} = -2.28$$

A unit increase in student-teacher ratio is associated with a -2.28 points decrease in test scores, on average.