ENERGY-AWARE MULTIPLE MOBILE CHARGERS COORDINATION FOR WIRELESS RECHARGEABLE SENSOR NETWORKS

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1. Wireless Rechargeable Sensor Networks (WRSNs)

- Use multiple Mobile Chargers (MCs)
- 3 main components: **stationary sensor nodes, MCs, 1 stationary base station (BS)**
- Charging cycle: MCs start from BS => Move to sensors & Charge => Return to BS
- Prior works:
- Single or Multiple objective (scheduling time, moving & charging time)
- Limited or Unlimited MC energy
- Distributed or Centralized mechanism
- Optimal or Non-optimal solution

2. Contributions

- Jointly optimize the **scheduling**, the **moving time & charging time** of MCs, guaranteeing the **perpetual** operation of WRSNs => Optimal multiple MCs (OMC)
- Derive a **lifetime-based charging sequence** for sensors, divide sensors into groups
- Determine whether a sensor requires charging in current cycle & How much energy should be replenished ?
- Decompose OMC into:
- Mater problem (MP): ILP => **MC scheduling**
- Slave problem (SP): LP => **MV moving & charging time**
- Solve MP, SP iteratively => **optimal solution**

3. System model and Problem formulation

3.1. System model:

- There are *n* sensor nodes (nodes); *m* MCs
- **Energy consumption rate:** $$r_i = \rho \sum_{k=1,k\neq i}^{n} f_{ki} + \sum_{j=1,j\neq i}^{n} C_{ij} f_{ij} + C_{ib} f_{ib}, \quad (1)$$

f_ij: data flow rate fron node i to node j
C_ij: energy consumption rate when transmittinga unit of data from node i to node j
p: energy consumption rate when receiving a unit of data

- **Received power:**

$$p_r = \frac{G_s G_r \kappa}{L_p}\left[\frac{\omega}{4\pi(d+\zeta)}\right]^2 p_0 = \frac{\varsigma}{(d+\zeta)^2} p_0, \quad (2)$$

$$\varsigma = \frac{G_s G_r \kappa \omega^2}{16 L_p \pi^2}$$

p0: source power of MC
Consider **point-2-point charging**

3.2. Problem formulation

- Definition:
  - **charging cycle:** n nodes charged once
  - **Charging round:** at most m nodes charged simultaneously when m MCs are scheduled
- n >> m => one charging cycle contains several charging round
- Scheme:
  - (1): find set of nodes to be charged in each cycle & each round
  - (2): define how much energy for each node
- **Sort nodes** according to their **lifetimes** in an increasing order $\boldsymbol{L}^k = \{L_1^k, L_2^k, \ldots, L_n^k\}$

$$L_i^k = \frac{e_i^k - e_{\min}}{r_i}$$

$e_{\min}$ minimum energy for a node to be operational

e_i^k initial energy of node s_i in cycle $\mathcal{C}^k$

- In cycle C_k, some nodes have enough energy until later cycle (C_k+1)
  => Non_serving set $\bar{\mathcal{S}}^k$

  Serving set $\mathcal{S}^k$,

3.2. Problem formulation

3.2.1. Determine charging sequence

Lemma 2.1: The cycle $\mathcal{C}^k$ contains at most σ^k rounds, where σ satisfies the inequalities:

$$\sum_{l=1}^{\sigma^k-1} m_{l,1}^k < n, \quad (4)$$

$$\sum_{l=1}^{\sigma^k} m_{l,1}^k \geq n. \quad (5)$$

$m_{l,1}^k$ number of schedulable MCs in the round $\mathcal{R}_l^k$ that estimated in round $\mathcal{R}_1^k$ $(l \geq 1)$

Lemma 2.2: If the lifetime of the node s_i $(1 \leq i \leq n)$ satisfies the inequality:

$$L_i^k < (\sigma_{i,1}^k + \sigma^{k+1} - 1)\varrho + \tau_d, \quad (7)$$

$s_i \in \mathcal{S}^k$, else, $s_i \in \bar{\mathcal{S}}^k$.

=> Charging sequence: $\tilde{L}^k = \{\underbrace{L_1^k, \ldots, L_{m_{1,1}^k}^k}_{\mathcal{R}_1^k}, \underbrace{L_{m_{1,1}^k+1}^k, \ldots, L_{m_{1,1}^k+m_{2,1}^k}^k}_{\mathcal{R}_2^k}, \ldots, L_{\tilde{n}^k}^k\}$

$\tilde{n}^k$ $(\tilde{n}^k \leq n)$ Number of nodes in the serving_set

3.2. Problem formulation

3.2.2. Optimize charging behaviour

$$E_{j,l}^{k} \geq \frac{e_{\max} - e_{\min}}{\eta} + 2d_{\max}\epsilon, \qquad (9)$$

the residual energy of the MC c_j at the beginning of the round *l* of cycle k

=> MC c_j has enough energy for this round
=> Total number of scheduable MCs for this round: $h_l^k \ (h_l^k \leq m)$

Lemma 2.3: If the replenished energy of node s_i satisfies the constraint:

$$\underline{E}_i^k \leq \sum_{j=1}^{h_l^k} p_r t_{ij,l}^k \leq \overline{E}_i^k, \qquad (10)$$

where

$$\underline{E}_i^k = [(\sigma_i^k + \sigma^{k+1} - 1)\varrho + \tau_d] r_i - (e_i^k - \tau_{l-1}^k r_i), \qquad (11)$$

$$\overline{E}_i^k = e_{\max} - (e_i^k - \tau_{l-1}^k r_i), \qquad (12)$$

no matter in which round of the next cycle this node is placed, it will never deplete its residual energy before being charged again.

If $\underline{E}_i^k$ < 0 => this node has enough energy to work until at the end of next cycle $s_i \in \bar{S}$.
=> Remove from charging sequence
=> Total number of required-charging nodes: $p_l^k \ (p_l^k \leq h_l^k)$

3.2. Problem formulation

3.2.2. Optimize charging behaviour

Find constraints

$$\sum_{i=1}^{p_l^k} q_{ij,l}^k \leq 1,\ 1 \leq j \leq h_l^k, \tag{13}$$

$$\sum_{j=1}^{h_l^k} q_{ij,l}^k = 1,\ 1 \leq i \leq p_l^k. \tag{14}$$

$$0 \leq t_{ij,l}^k \leq t_{j,l}^{\max} q_{ij,l}^k,\ 1 \leq i \leq p_l^k,\ 1 \leq j \leq h_l^k. \tag{15}$$

$$g_{ij,l}^k \geq q_{ij,l}^k \frac{d_{ij}}{v},\ 1 \leq i \leq p_l^k,\ 1 \leq j \leq h_l^k, \tag{16}$$

$$0 \leq t_{ij,l}^k + g_{ij,l}^k \leq \theta_l^k,\ 1 \leq i \leq p_l^k,\ 1 \leq j \leq h_l^k, \tag{17}$$

Variables	
$q_{ij,l}^k$	$=\begin{cases}1 & \text{if MC } c_j \text{ is scheduled to charge node } s_i \\ 0 & \text{otherwise}\end{cases}$
$t_{ij,l}^k$	time of MC c_j spends to charge node s_i
$g_{ij,l}^k$	moving time of MC from node s_j to node s_i

Lemma 2.4: To ensure that the interval between two adjacent rounds is smaller than ϱ, we can set

$$\theta_l^k = \min\{\varrho, \varpi_{l+1}^k, \ldots, \varpi_{l+\sigma_l^k}^k\}, \tag{18}$$

where $\varpi_j^k = L_{\iota_j^k} - \tau_{l-1}^k - (j-l)\varrho - \frac{d_{\max}}{v}$ and $\iota_j^k = n_{l-1}^k + \bar{n}_{l-1}^k + \sum_{i=l}^{j-1} m_{i,l}^k + 1$ $(l+1 \leq j \leq l+\sigma_l^k)$.

3.2. Problem formulation

3.2.2. Optimize charging behaviour

- Perpetual operation guarantee

Theorem 2.1: The sufficient condition to achieve perpetual operation is that the constraints (10), (16) – (18) are satisfied.

Proof: The constraint (10) makes sure that all nodes will not run out of their residual energies before being charged again, under the condition that the interval between two adjacent rounds is smaller than a predefined time threshold, which is guaranteed by the constraints (16) – (18). ■

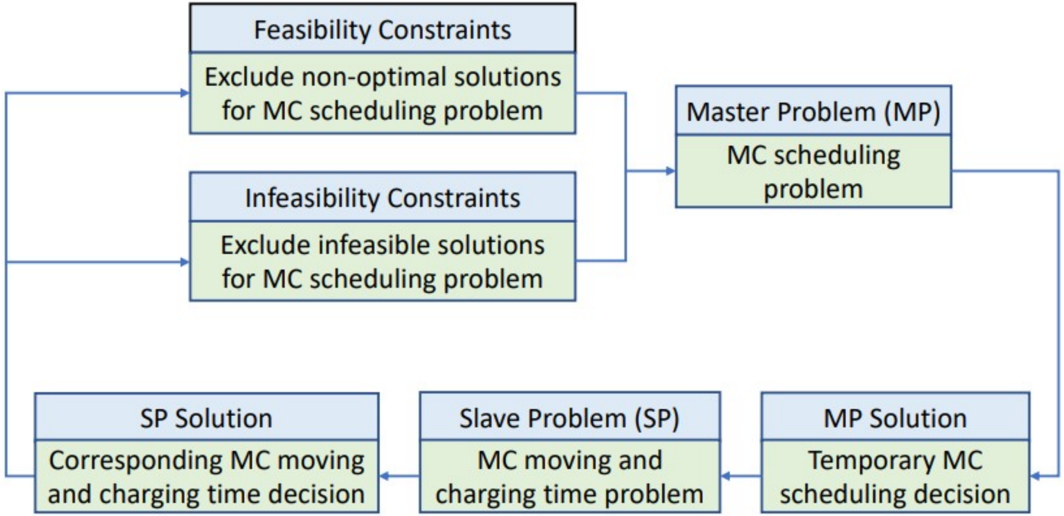
- Total energy consumption of the MCs minimization

$$\mathbf{PP}: \min_{\boldsymbol{Q}_l^k, \boldsymbol{T}_l^k, \boldsymbol{G}_l^k} \Phi = \sum\nolimits_{i=1}^{p_l^k} \sum\nolimits_{j=1}^{h_l^k} (q_{ij,l}^k d_{ij} \epsilon + p_0 t_{ij,l}^k) \quad (19)$$
$$\text{s.t.} \quad (10) - (18).$$

where $\boldsymbol{q}_l^k = [q_{ij,l}^k]$, $\boldsymbol{t}_l^k = [t_{ij,l}^k]$, and $\boldsymbol{g}_l^k = [g_{ij,l}^k]$. Since the binary and the continuous variables are coupled with each other linearly, the PP is an MILP problem.

4. Multiple MCs Coordination Scheme

To solve PP, the most important step is to find a proper MC scheduling decision . If the value of is determined, PP is reduced to a LP problem, which has a simpler structure and it is easier to solve.



4. Multiple MCs Coordination Scheme

a. Formulations of MP and SP
b. Iterations between MP & SP
c. Convergence Analysis

4a. Formulations of MP and SP

For convenience, the matrices and the vectors are used to denote the constraints and the variables. Therefore, the PP is reformulated as

$$\mathbf{PP1}: \min_{\boldsymbol{x},\boldsymbol{y}} \ \Phi(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{g}^T\boldsymbol{x} + \boldsymbol{f}^T\boldsymbol{y}$$

$$\text{s.t.} \ \begin{cases} \boldsymbol{A}\boldsymbol{x} \preceq \boldsymbol{b}_1, \\ \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{y} \preceq \boldsymbol{b}_2, \end{cases}$$

+ x: vector of binary variables
+ y: vector of continuous variables
+ g, f: the vectors of the objective function coefficients x: vector of binary variables
+ A, C, D: the matrices of the coefficients in the constraints.
+ b1: u-dimensional vector
+ b2: v-dimensional vector.

4a. Formulations of MP and SP

To facilitate the iteration between the MP and the SP, we introduce an auxiliary (continuous) variable ˆ into the MP as the objective function, where ˆ and have the same physical meaning. Based on the structure of the PP1, the corresponding MP is

$$\mathbf{MP}: \Phi_L = \min_{\boldsymbol{x}, \hat{\Phi}} \ \hat{\Phi} \tag{21}$$

$$\text{s.t.} \begin{cases} \boldsymbol{A}\boldsymbol{x} \preceq \boldsymbol{b}_1, \\ C_1 : \hat{\Phi} \geq \boldsymbol{g}^T \boldsymbol{x} + \boldsymbol{\lambda}(i)^T (\boldsymbol{C}\boldsymbol{x} - \boldsymbol{b}_2), \ \forall i \in \mathcal{A}, \\ C_2 : 0 \geq \boldsymbol{\varphi}(j)^T (\boldsymbol{C}\boldsymbol{x} - \boldsymbol{b}_2), \ \forall j \in \mathcal{B}, \end{cases}$$

+ C1, C2: the sets of feasibility constraints (FCs) and infeasibility constraints (ICs) They are generated from the solution to the dual slave problem (DSP) (23).
+ A, B: the sets of iterations where the DSP has bounded and unbounded solutions, respectively.
λ(i): the solution to the DSP at the iteration
ϕ(j): the solution to the dual feasibility check problem (DFCP) (26) at the iteration

4a. Formulations of MP and SP

Let $(\boldsymbol{x}(l), \hat{\Phi}(l))$ denote the solution to the MP at the l^{th} iteration. Therefore, the corresponding MP is

$$\mathbf{SP}: \Phi_U = \min_{\boldsymbol{y} \succeq 0} \; \Phi(\boldsymbol{x}(l), \boldsymbol{y}) = \boldsymbol{g}^T \boldsymbol{x}(l) + \boldsymbol{f}^T \boldsymbol{y} \qquad (22)$$
$$\text{s.t.} \;\; \boldsymbol{C}\boldsymbol{x}(l) + \boldsymbol{D}\boldsymbol{y} \preceq \boldsymbol{b}_2,$$

Comparing the SP with the PP1, we observe that their formulations are the same, except that the binary variables x in the SP are fixed.

$$\mathbf{PP1}: \min_{\boldsymbol{x},\boldsymbol{y}} \; \Phi(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{g}^T \boldsymbol{x} + \boldsymbol{f}^T \boldsymbol{y}$$
$$\text{s.t.} \;\; \begin{cases} \boldsymbol{A}\boldsymbol{x} \preceq \boldsymbol{b}_1, \\ \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{y} \preceq \boldsymbol{b}_2, \end{cases}$$

4a. Formulations of MP and SP

Solving the MP we have , solving the SP we have . Let denote the optimal solution to the PP1.

Therefore, we have To reduce the gap between and , a new constraint FC (or IC) is added into C1 (or C2) at each iteration (due to Lemma 3.2). When the gap is smaller than a predefined threshold ε, the optimal solution ($x*$, $y*$) is found.

To reduce the gap between , a new constraint FC (or IC) is added into C1 (or C2) at each iteration. When the gap is smaller than a predefined threshold ε, the optimal solution is found.

4b. Iterations between MP & SP

1) Initialization
2) Solving SP
3) Solving MP

4b. Iterations between MP & SP

1) Initialization:

Initialize the iteration counter , the MP solution x(0), the lower bound , and the upper bound . The sets C1 and C2 are set to null. The initial solution x(0) can be given arbitrarily, as long as it satisfies the constraint

4b. Iterations between MP & SP

2) Solving SP:

In this paper, rather than solving the SP directly, we solve its dual problem. This is because the SP and the DSP are equivalent due to the strong duality, and the new constraints can be constructed according to the solution of the DSP.

The SP:

$$\mathbf{SP}: \Phi_U = \min_{\boldsymbol{y} \succeq 0} \ \Phi(\boldsymbol{x}(l), \boldsymbol{y}) = \boldsymbol{g}^T \boldsymbol{x}(l) + \boldsymbol{f}^T \boldsymbol{y}$$
$$\text{s.t.} \ \ \boldsymbol{C}\boldsymbol{x}(l) + \boldsymbol{D}\boldsymbol{y} \preceq \boldsymbol{b}_2,$$

To construct the dual of the SP, we introduce Lagrange multipliers $\boldsymbol{\lambda} \triangleq [\lambda_i]$ $(1 \leq i \leq v)$ to the SP. Therefore, the DSP is

$$\mathbf{DSP}: \max_{\boldsymbol{\lambda} \succeq 0} \ \boldsymbol{g}^T \boldsymbol{x}(l) + \boldsymbol{\lambda}^T (\boldsymbol{C}\boldsymbol{x}(l) - \boldsymbol{b}_2) \qquad (23)$$
$$\text{s.t.} \ \ \boldsymbol{f} + \boldsymbol{D}^T \boldsymbol{\lambda} \succeq 0.$$

Since the DSP is an LP, it can be solved very fast using standard algorithms, such as simplex method or interior point method.

4b. Iterations between MP & SP

3) Solving SP:

Based on the solution to the DSP, we have:

- If the DSP is infeasible, the SP has an unbounded solution. Therefore, the PP1 is infeasible.
- If the DSP has a bounded solution

$$\Phi_U(l) = \min\{\Phi_U(l-1), \boldsymbol{g}^T \boldsymbol{x}(l) + \boldsymbol{\lambda}(l)^T (\boldsymbol{C}\boldsymbol{x}(l) - \boldsymbol{b}_2)\}$$

To avoid selecting again the non-optimal solution x(l), a new FC is generated and added into C1 (the sets of feasibility constraints (FCs))

$$\hat{\Phi} \geq \boldsymbol{g}^T \boldsymbol{x} + \boldsymbol{\lambda}(l)^T (\boldsymbol{C}\boldsymbol{x} - \boldsymbol{b}_2)$$

4b. Iterations between MP & SP

3) Solving SP:

- If the DSP has an unbounded solution. ( )

➔ the SP has no feasible solution under the given

➔ the positive (continuous) variables $\boldsymbol{\xi} \triangleq [\xi_i] \; (1 \leq i \leq v)$ introduced to relax the constraints s.t. $\boldsymbol{C}\boldsymbol{x}(l) + \boldsymbol{D}\boldsymbol{y} \preceq \boldsymbol{b}_2$

$$\begin{aligned} \textbf{FCP}: \min_{\boldsymbol{\xi}, \boldsymbol{y} \succeq 0} \; & \mathbf{1}^T \boldsymbol{\xi} \\ \text{s.t.} \; & \boldsymbol{C}\boldsymbol{x}(l) + \boldsymbol{D}\boldsymbol{y} \preceq \boldsymbol{b}_2 + \boldsymbol{\xi}. \end{aligned}$$

4b. Iterations between MP & SP

3) Solving SP:

- Instead of solving FCP, we solve its dual problem. To construct the dual of the FCP, we introducing Lagrange multipliers to the FCP. Therefore, the dual of the FCP (DFCP) is

$$\mathbf{FCP}: \min_{\boldsymbol{\xi}, \boldsymbol{y} \succeq 0} \ \mathbf{1}^T \boldsymbol{\xi} \quad \text{s.t.} \ \boldsymbol{C}\boldsymbol{x}(l) + \boldsymbol{D}\boldsymbol{y} \preceq \boldsymbol{b}_2 + \boldsymbol{\xi}. \longrightarrow \mathbf{DFCP}: \max_{\boldsymbol{\varphi} \succeq 0} \ \boldsymbol{\varphi}^T (\boldsymbol{C}\boldsymbol{x}(l) - \boldsymbol{b}_2) \quad \text{s.t.} \ \begin{cases} \mathbf{1} - \boldsymbol{\varphi} \succeq 0, \\ \boldsymbol{D}^T \boldsymbol{\varphi} \succeq 0. \end{cases}$$

- If SP exists infeasible constraints, the corresp e the others are zero. Due to the strong duality, we have

$$\mathbf{1}^T \boldsymbol{\xi}(l) = \boldsymbol{\varphi}(l)^T (\boldsymbol{C}\boldsymbol{x}(l) - \boldsymbol{b}_2) > 0.$$

- To avoid selecting again is generated and added into C2 (the sets of infeasibility constraints (FCs)):

$$0 \geq \boldsymbol{\varphi}(l)^T (\boldsymbol{C}\boldsymbol{x} - \boldsymbol{b}_2)$$

4b. Iterations between MP & SP

For the new generated FC (24) or IC (27), all the parameters are constant except ˆ and x. They are the variables to the MP. When the MP is solved, the iteration counter l increases, and Step 2 to Step 3 is repeated. The iteration stops when ε is satisfied.

4c. Convergence Analysis

From the MP (21), we observe that the real constraints are and C2. C1 can be treated as the objective function. Therefore, the MP can be solved by only considering the binary variables x. Let λ(k) denote the bounded solution to the DSP at the iteration (k ∈ A). Comparing the following ILP problem.

$$\hat{\Phi}_r(k) = \min_{\boldsymbol{x}} \ \boldsymbol{g}^T\boldsymbol{x} + \boldsymbol{\lambda}^T(k)(\boldsymbol{C}\boldsymbol{x} - \boldsymbol{b}_2)$$

$$\text{s.t.} \ \begin{cases} \boldsymbol{A}\boldsymbol{x} \preceq \boldsymbol{b}_1, \\ C_2 : 0 \geq \boldsymbol{\varphi}(j)^T(\boldsymbol{C}\boldsymbol{x} - \boldsymbol{b}_2), \ \forall j \in \mathcal{B}, \end{cases}$$

4c. Convergence Analysis

- Lemma 3.1: The lower bound and the upper bound on the optimal objective function value * are derived from the solution to the MP and the SP, respectively.

$$
\begin{aligned}
\hat{\Phi}(l) = \hat{\Phi}_r(k) &= \min_{\boldsymbol{x}} \ \boldsymbol{g}^T \boldsymbol{x} + \boldsymbol{\lambda}^T(k)(\boldsymbol{C}\boldsymbol{x} - \boldsymbol{b}_2) \\
&\leq \boldsymbol{g}^T \boldsymbol{x}^* + \boldsymbol{\lambda}^T(k)(\boldsymbol{C}\boldsymbol{x}^* - \boldsymbol{b}_2) \\
&\leq \max_{\boldsymbol{\lambda} \succeq 0} \ \boldsymbol{g}^T \boldsymbol{x}^* + \boldsymbol{\lambda}^T(\boldsymbol{C}\boldsymbol{x}^* - \boldsymbol{b}_2) \\
&= \Phi^*,
\end{aligned}
$$

4c. Convergence Analysis

- Lemma 3.1: The lower bound and the upper bound on the optimal objective function value * are derived from the solution to the MP and the SP, respectively.

$$\begin{aligned}\Phi_U(l) &= \min\{\Phi_U(l-1), \boldsymbol{g}^T\boldsymbol{x}(l) + \boldsymbol{\lambda}(l)^T(\boldsymbol{C}\boldsymbol{x}(l) - \boldsymbol{b}_2)\} \\ &= \min_{1 \le i \le l} \{\boldsymbol{g}^T\boldsymbol{x}(i) + \boldsymbol{\lambda}(i)^T(\boldsymbol{C}\boldsymbol{x}(i) - \boldsymbol{b}_2)\}, \qquad (30)\end{aligned}$$

and

$$\begin{aligned}\boldsymbol{g}^T\boldsymbol{x}(i) + \boldsymbol{\lambda}(i)^T(\boldsymbol{C}\boldsymbol{x}(i) - \boldsymbol{b}_2) &= \min_{\boldsymbol{y} \succeq 0} \ \Phi(\boldsymbol{x}(i), \boldsymbol{y}) \\ &\ge \min_{\boldsymbol{y} \succeq 0} \ \Phi(\boldsymbol{x}^*, \boldsymbol{y}) = \Phi^*, \qquad (31)\end{aligned}$$

4c. Convergence Analysis

- Lemma 3.2: The lower bound sequence {(0), . . . (l)} is increasing, while the upper bound sequence {(0), . . . (l)} is decreasing.

+ Since the MP is a minimization problem, the non-optimal values of are excluded by the constraints C1 and C2, and, thus, (l+1) = ^(l+1) is larger than the previous lower bounds {(0), . . . , (l)}.

+ $$\Phi_U(l+1) = \min\{\Phi_U(l), \boldsymbol{\lambda}(l+1)^T(\boldsymbol{C}\boldsymbol{x}(l+1) - \boldsymbol{b}_2)\}.$$

4c. Convergence Analysis

- Lemma 3.1, 3.2 Theorem 3.1: With the FC (24) and the IC (27) added into the MP, the algorithm converges

- Theorem 3.2: The FC and the IC generated by solving the DSP with do not exclude the optimal solution (x ∗ , y ∗), where is an arbitrary feasible solution to the MP at the iteration.

$$\hat{\Phi} \geq \boldsymbol{g}^T \boldsymbol{x} + \bar{\boldsymbol{\lambda}}(l)^T (\boldsymbol{C}\boldsymbol{x} - \boldsymbol{b}_2). \tag{32}$$

$$0 \geq \bar{\varphi}(l)^T (\boldsymbol{C}\boldsymbol{x} - \boldsymbol{b}_2), \tag{33}$$

- Theorem 3.3: With the FC (32) and the IC (33) added into the MP, the algorithm converges.

4c. Convergence Analysis

- Since MP is an ILP, this problem is still hard to solve directly compared with the LP-based SP. Moreover, the size of MP will increase with the number of iterations, since at each iteration a new FC or IC is added into the MP.

Therefore, the computational complexity of OMC is dominated by the cost of solving the MP at each iteration. Based on Theorem 3.3, we can circumvent the above difficulties by replacing the optimal solution to the MP x(k) with the feasible solution during the iteration between the MP and the SP.

4c. Convergence Analysis

Algorithm:

Algorithm 1: Optimal Multiple MCs Coordination (OMC) Algorithm

Input: System parameters $\boldsymbol{A}, \boldsymbol{C}, \boldsymbol{D}, \boldsymbol{f}, \boldsymbol{b}_1, \boldsymbol{b}_2$;

Output: Optimal scheduling decision $\boldsymbol{x}^*$, optimal charging and moving time decision $\boldsymbol{y}^*$;

4c. Convergence Analysis

Algorithm:

Set initial values: $l = 0$, $\Phi_L(0) = -\infty$, $\Phi_L(0) = +\infty$, $\{\boldsymbol{x}(0) | \boldsymbol{A}\boldsymbol{x}(0) \preceq \boldsymbol{b}_1\}$, ε;
C_1 and C_2 are set to null;
while $\Phi_U(l) - \Phi_L(l) > \varepsilon$ **do**
    Solve MP (21) to obtain temporary scheduling decision $\bar{\boldsymbol{x}}(k)$;
    Update lower bound $\Phi_L(l) = \max_{\forall k \in \mathcal{A}}\{\hat{\Phi}_r(k)\}$;
    Solve DSP (23) with $\bar{\boldsymbol{x}}(k)$;
    if *DSP has bounded or unbounded solution* **then**
        if *Solution is bounded* **then**
            $\mathcal{A} \leftarrow \{l\} \cup \mathcal{A}$;
            Add FC: $\hat{\Phi} \geq \boldsymbol{g}^T\boldsymbol{x} + \bar{\boldsymbol{\lambda}}(l)^T(\boldsymbol{C}\boldsymbol{x} - \boldsymbol{b}_2)$ to C_1;
            Update upper bound $\Phi_U(l) = \min\{\Phi_u(l-1), \bar{\boldsymbol{\lambda}}(l)^T(\boldsymbol{C}\bar{\boldsymbol{x}}^T(l) - \boldsymbol{b}_2)\}$;
        else
            $\mathcal{B} \leftarrow \{l\} \cup \mathcal{B}$;
            Add IC: $0 \geq \bar{\boldsymbol{\varphi}}(l)^T(\boldsymbol{C}\boldsymbol{x} - \boldsymbol{b}_2)$ to C_2;
        end
    else
        PP1 (20) is infeasible;
    end
    $l \leftarrow l + 1$;
end

5. Simulation

- Nodes and MCs randomly deployed in a 100m × 100m area
- BS at (50, 50)m.
- E_min = 18900 J
- E_max = 21600 J

<table>
<tr><td colspan="3">Sensor node s_i characteristics</td></tr>
<tr><td colspan="3">$r_i = \rho \sum_{k=1,k\neq i}^{n} f_{ki} + \sum_{j=1,j\neq i}^{n} C_{ij} f_{ij} + C_{ib} f_{ib}$</td></tr>
<tr><td>$\rho = 50$ nJ/b</td><td colspan="2">$f_{ij} \in [1, 10]$ kb/s</td></tr>
<tr><td>$C_{ij} = \lambda_1 + \lambda_2 (d_{ij})^\theta$</td><td colspan="2">$\lambda_1 = 50$ nJ/b</td></tr>
<tr><td>$\lambda_2 = 0.0013$ pJ/(b.m^4)</td><td colspan="2">$\theta = 4$</td></tr>
<tr><td>$e_{\min} = 18900$ J</td><td colspan="2">$e_{\max} = 21580$ J</td></tr>
<tr><td>$e_i^k \in [19000, 20000]$ J</td><td colspan="2">$\Delta_s = 1$ s</td></tr>
<tr><td colspan="3">Mobile charger c_j characteristics</td></tr>
<tr><td>$\eta = 6$ %</td><td colspan="2">$p_0 = 5$ W</td></tr>
<tr><td>$v = 1$ m/s</td><td colspan="2">$\epsilon = 1$ J</td></tr>
<tr><td>$E_j^k \in [1000, 5000]$ J</td><td colspan="2">$\tau_s = 10000$ s, $\tau_d = 50000$ s</td></tr>
<tr><td colspan="3">Tuned parameters</td></tr>
<tr><td rowspan="2">Min/Max/Step</td><td>n</td><td>m</td></tr>
<tr><td>25/50/5</td><td>5/15/5</td></tr>
</table>

TABLE III. SYSTEM PARAMETERS

5. Simulation

1. System performance: **energy status of the nodes**
2. Trade-off between convergence iteration & computation time
3. Compare:
   Optimal approaches: Decentralized Benders decomposition (DBD), Branch&Bound (BB), Branch&Cut (BC)
   Evolutionary approach: GA
4. System performance: **energy consumption of the MCs** between OMC, m-TSP, region partition

5.1. Simulation

$\mathcal{R}_l^1$	$l = 1$	$l = 2$	$l = 3$	$l = 4$	$l = 5$
p_l^1	5	5	4	4	3
h_l^1	5	5	4	4	4

TABLE IV. THE REAL NUMBER OF CHARGED NODES IN CYCLE $\mathcal{C}^1$.

n = 25 and m = 5
Serving set S1 contains 22 nodes

5.1. Simulation

Let $g_l(k) = \sqrt{\sum_{s_i \in \mathcal{R}_l^1} (e_i^* - e_i(k))^2 / p_l^1}$ denote the charging error of the l th round at step k

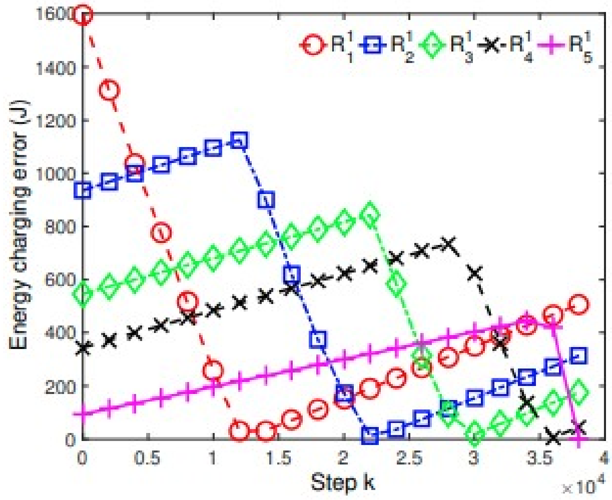


Fig. 3. The charging error of the nodes in the first cycle $\mathcal{C}^1$.

5.1. Simulation

$$g_e(k) = \sqrt{\sum_{i=1}^{n}(e_i(k) - e_{\min})^2/n}$$

MSE between the minimum optional energy and the residual energy levels

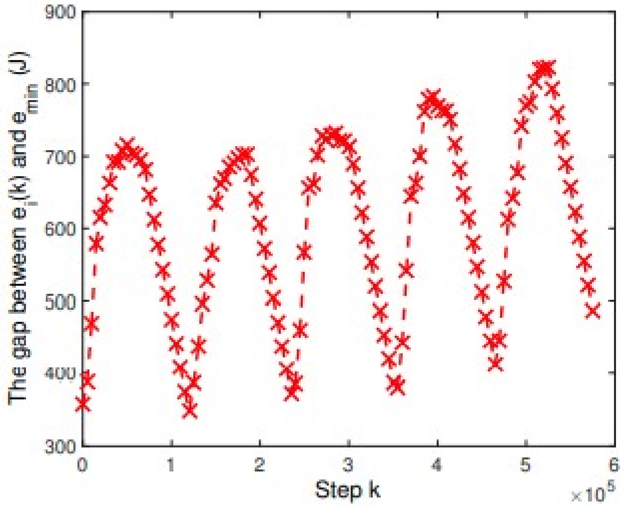


Fig. 4. The energy status of the nodes in five cycles.

5.2. Simulation

Convergence iteration is the **number of iterations** required by the OMC to **converge**

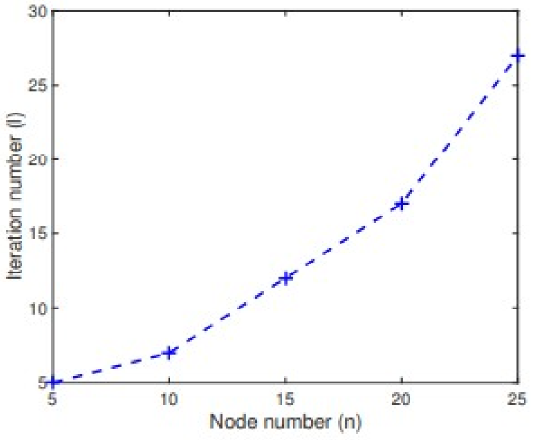


Fig. 5. Convergence iteration of OMC with n varying ($n = m$).

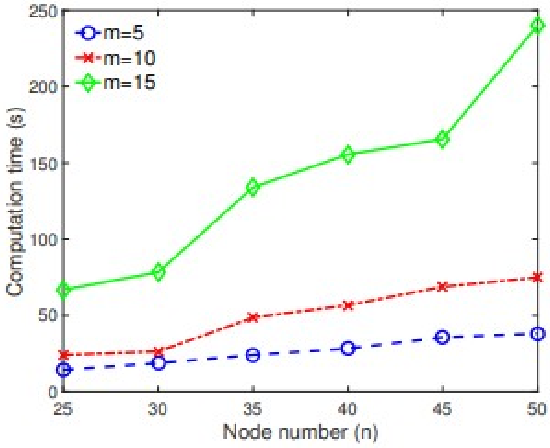


Fig. 6. Computation time of OMC and with m and n varying.

5.2. Simulation

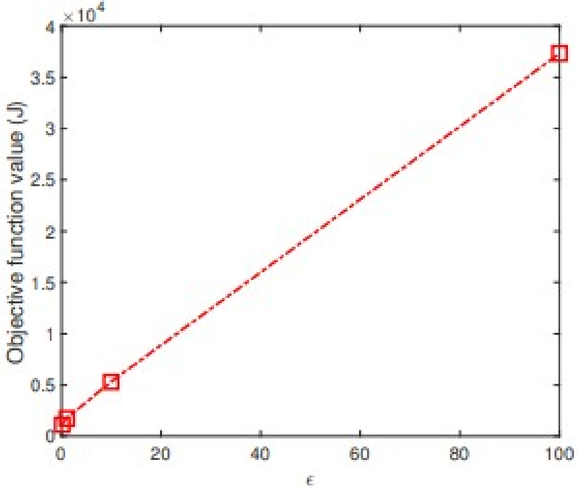


Fig. 7. Solution quality of OMC with ε varying.

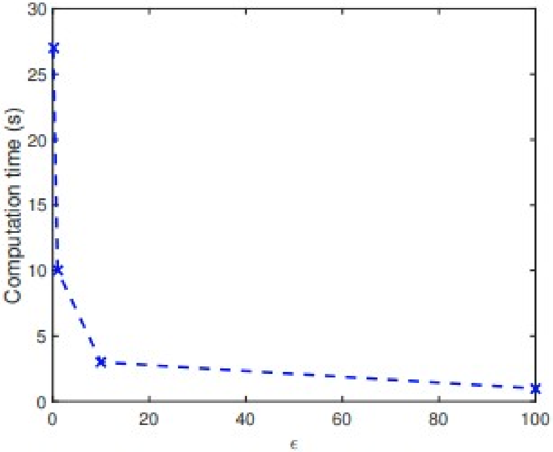


Fig. 8. Computation time of OMC with ε varying.

n = 50, m = 15, ε in [0.1, 1, 10, 100].

5.3. Simulation

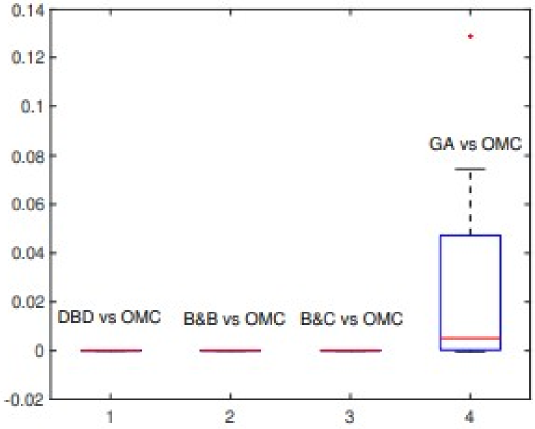


Fig. 9. Energy gain of OMC, DBD, B&B, B&C and GA.

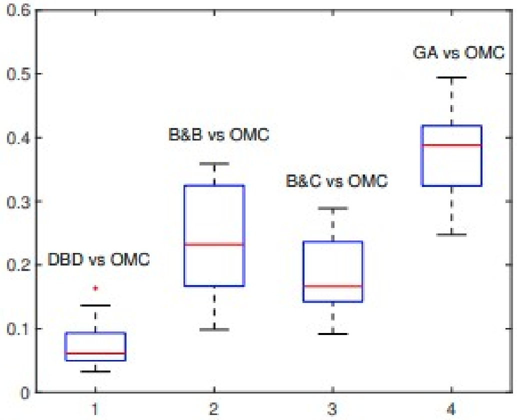


Fig. 10. Time gain of OMC, DBD, B&B, B&C and GA.

5.4. Simulation

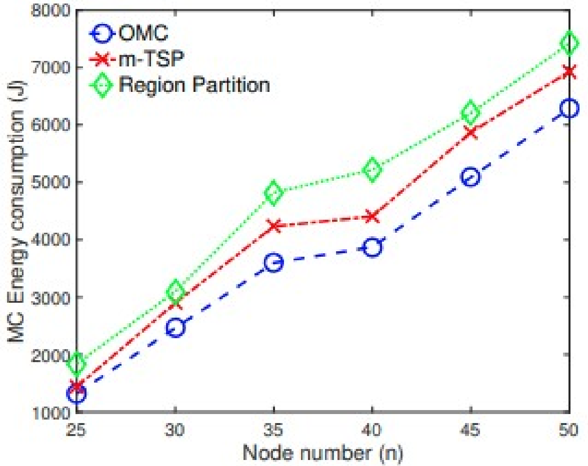


Fig. 11. MC energy consumption with OMC, m-TSP and region partition.